Issues with linear optics in X-ray FEL oscillator (XFELO) cavity

Gunn Tae Park

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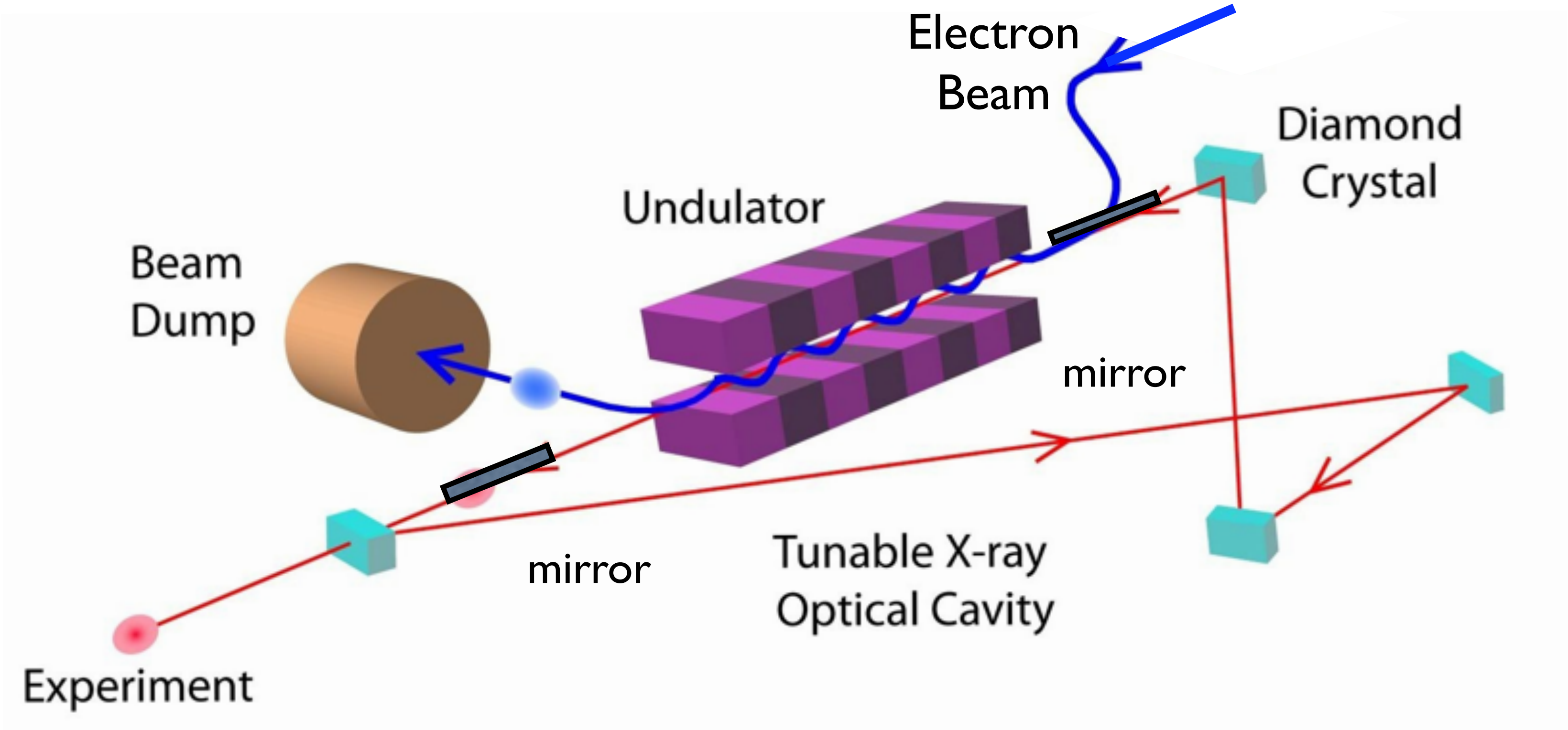
1. Issues for X-ray FEL oscillator Optical Cavity



XFELO (X-ray FEL Oscillator)

Optical cavity affects the performance of XFELO via net power gain P per pass

$$\mathcal{P} = g - \alpha$$

gain/pass   power loss/pass

- Conditions for Optical Cavity Design

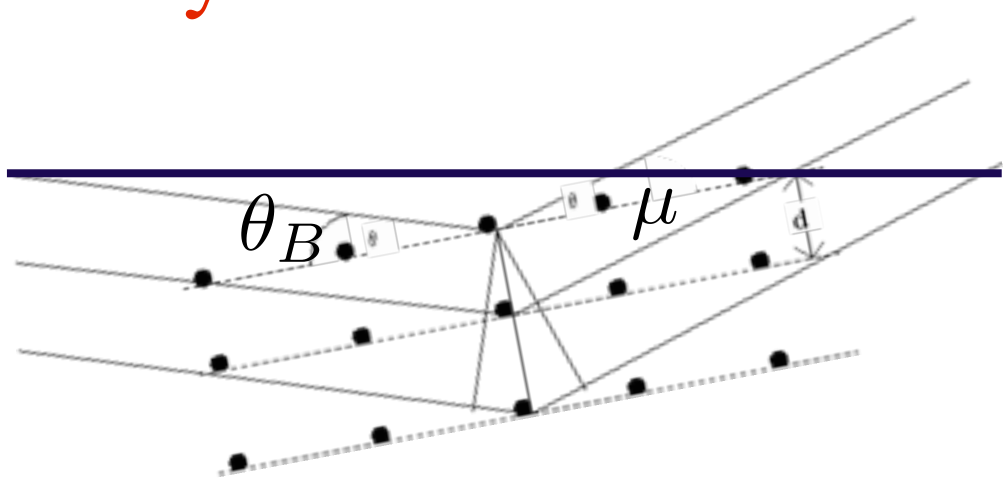
(1) For maximum gain, focusing of X-ray in the undulator to maximally overlap with electron beam

waist location — electron beam rms length

$$Z_R(z_0) = \beta_e(z_0), \quad \sigma_R = \sigma_e$$

Rayleigh length of radiation — beta function of electron beam — radiation pulse lentgh

(2) For minimum power loss, X-ray must be well collimated at each crystal for narrow angular acceptance of crystal. Also heat load on the crystal must be relieved.



(3) Beam profile must be periodic (thus stable) after every turn.

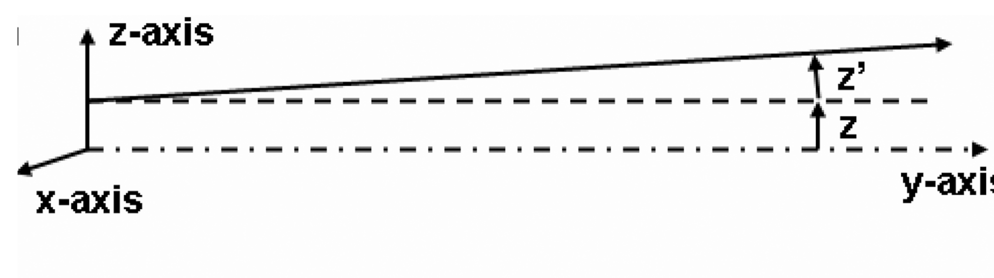
2. Matrix Formulation

Geometrical optics can be formulated in phase space P of rays. In P, a ray and its propagation through an optical element is represented by a column vector and matrix.

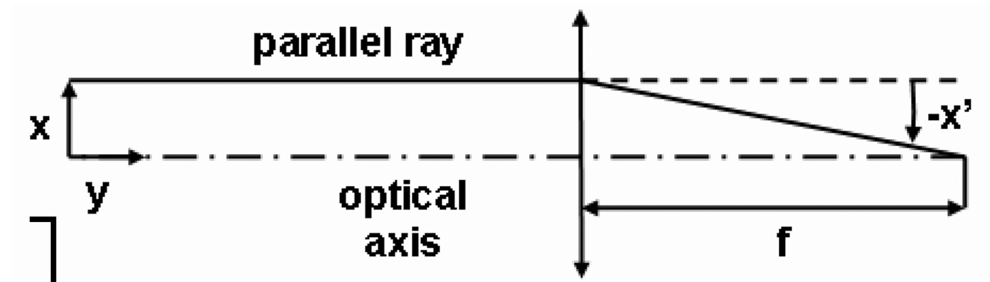
$$V' = MV, \quad V = \begin{bmatrix} x \\ x' \\ t \\ \xi \end{bmatrix}, \quad \xi = \delta\lambda/\lambda$$

Free Space Transform

$$\begin{bmatrix} x_i \\ x_i' \\ t_i \\ \xi_i \end{bmatrix} \rightarrow \begin{bmatrix} x_o \\ x_o' \\ t_o \\ \xi_o \end{bmatrix} = \begin{bmatrix} 1 & l & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i \\ x_i' \\ t_i \\ \xi_i \end{bmatrix}$$



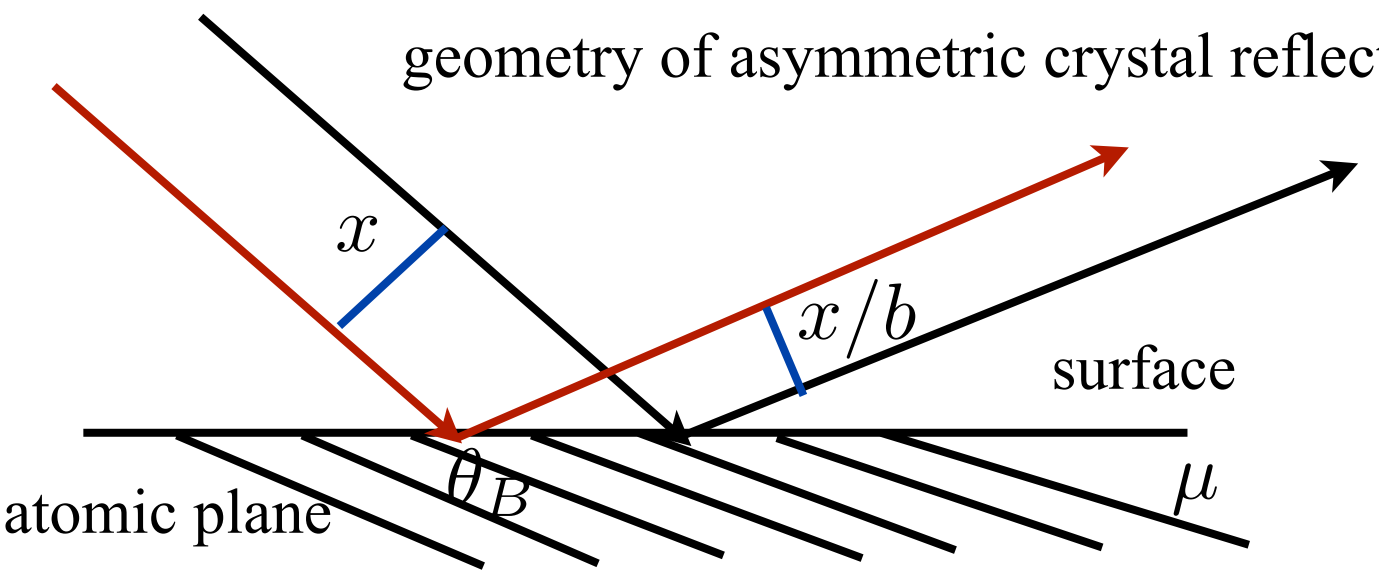
Lens (Curved Mirror) Transform



$$\begin{bmatrix} x_i \\ x'_i \\ t \\ \xi \end{bmatrix} \rightarrow \begin{bmatrix} x_o \\ x'_o \\ t_o \\ \xi_o \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1/f & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i \\ x'_i \\ t_i \\ \xi_i \end{bmatrix}$$

Asymmetric Crystal Transform

geometry of asymmetric crystal reflection



μ asymmetric angle

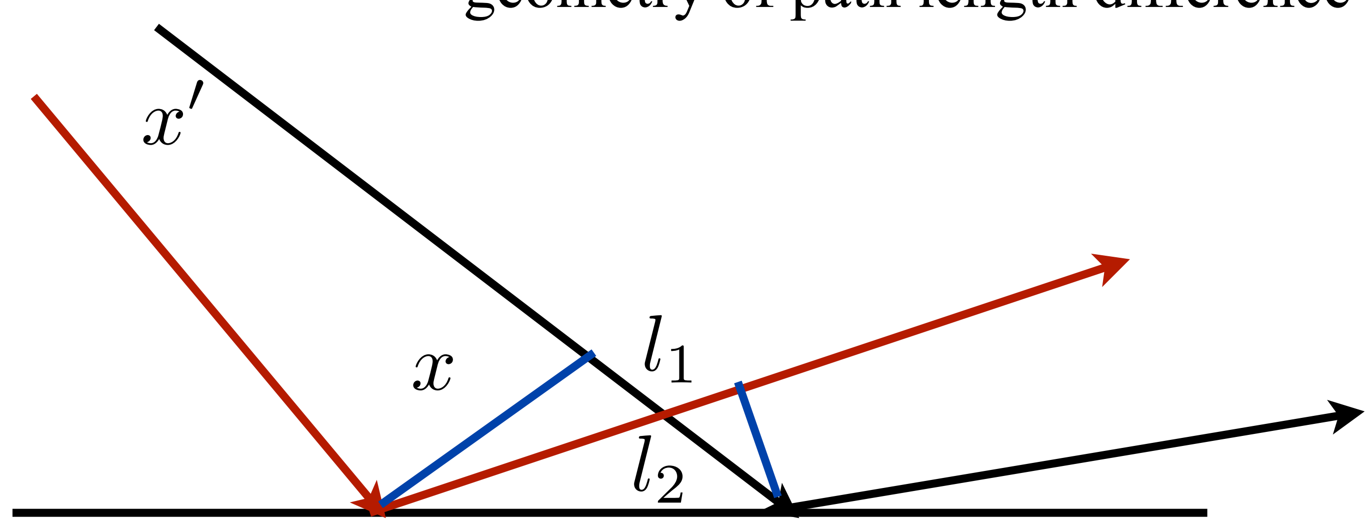
θ_B Bragg's angle,

$$b = \frac{\sin(\theta_B + \mu)}{\sin(\theta_B - \mu)}$$

Path length difference

$$\Delta = l_1 - l_2 = -\frac{2\sin\theta_B \sin\mu}{\sin(\theta_B + \mu)} x$$

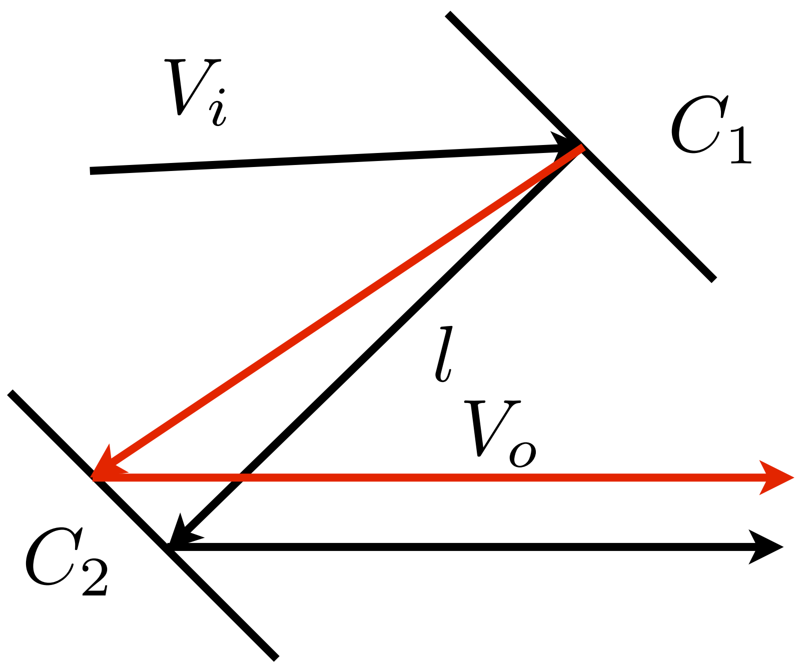
geometry of path length difference



$$\begin{bmatrix} x_i \\ x_i' \\ t_i \\ \xi \end{bmatrix} \rightarrow \begin{bmatrix} x_o \\ x_o' \\ t_o \\ \xi_o \end{bmatrix} = \begin{bmatrix} 1/b & 0 & 0 & 0 \\ 0 & b & 0 & (1-b)\tan\theta_B \\ a & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i \\ x_i' \\ t_i \\ \xi \end{bmatrix}$$

$$a = -\frac{2\sin\theta_B \sin\mu}{\sin(\theta_B + \mu)}$$

Pulse Length Dilation



$$V_i \rightarrow V_o = M_{C_1} M_V M_{C_2} V_i$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ \xi \end{bmatrix} \rightarrow \begin{bmatrix} 2\frac{\sin\mu \sin\theta_B \sin(\theta+\mu)}{\sin^2(\theta+\mu)} l\xi \\ 0 \\ 4\frac{\sin^2\theta_B \sin^2\mu}{\sin^2(\theta_B-\mu)} l\xi \\ \xi \end{bmatrix}$$

A cavity is a periodic system consistsing of a series of optical elements and described by one-turn matrix **M**.

$$\mathbf{M} = M_1 \cdots M_n$$

In a general configuration of cavity (with asymmetric crystal), one-turn matrix ***M*** is written as

$$M = \begin{bmatrix} C & S & 0 & D \\ C' & S' & 0 & D' \\ E & F & 1 & G \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$CS' - SC' = 1$$
$$E = C'D - CD'$$
$$F = DS' - SD'$$

Symplecticity constraint

X-ray profile and correlations of rays are described by beam matrix, whose elements are 2nd order moments.

$$\Sigma = \langle VV^T \rangle = \begin{bmatrix} \langle x^2 \rangle & \langle xx' \rangle & \langle xt \rangle & \langle x\xi \rangle \\ \langle x'x \rangle & \langle x'^2 \rangle & \langle x't \rangle & \langle x'\xi \rangle \\ \langle tx \rangle & \langle tx' \rangle & \langle t^2 \rangle & \langle t\xi \rangle \\ \langle \xi x \rangle & \langle \xi x' \rangle & \langle \xi t \rangle & \langle \xi^2 \rangle \end{bmatrix}$$

Beam matrix transform through an optical element

$$\Sigma \to \Sigma' = \langle V' V'^T \rangle = M \Sigma M^T$$

Optimal Gain Condition in matrix formulation

$$\Sigma = \begin{bmatrix} \varepsilon_x \beta_e & 0 & 0 & 0 \\ 0 & \varepsilon_x / \beta_e & 0 & 0 \\ 0 & 0 & \tau^2 & 0 \\ 0 & 0 & 0 & \xi^2 \end{bmatrix} \qquad \mathcal{M} \Sigma \mathcal{M}^T = \Sigma$$

pulse length remains constant if and only if

$$E = F = G = 0$$ isochronous

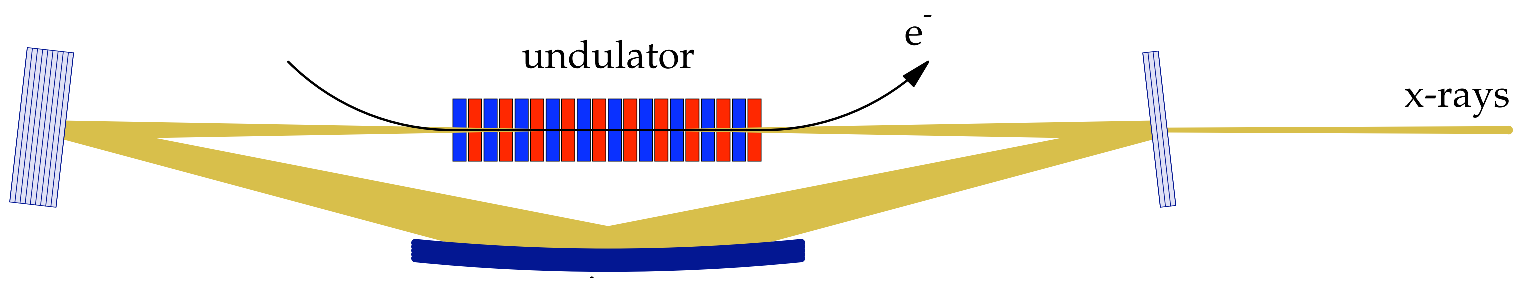
$$D = D' = 0$$ non-dispersive

stability & waist matching

$$-1 < \frac{C + S'}{2} < 1, \quad Z = \frac{2S}{\sqrt{4 - (C + S')^2}}$$

• Examples of Configuration

2-crystal 1-mirror



$$M_{2,1} = L_1 C_2 L_2 F L_2 C_1 L_1$$

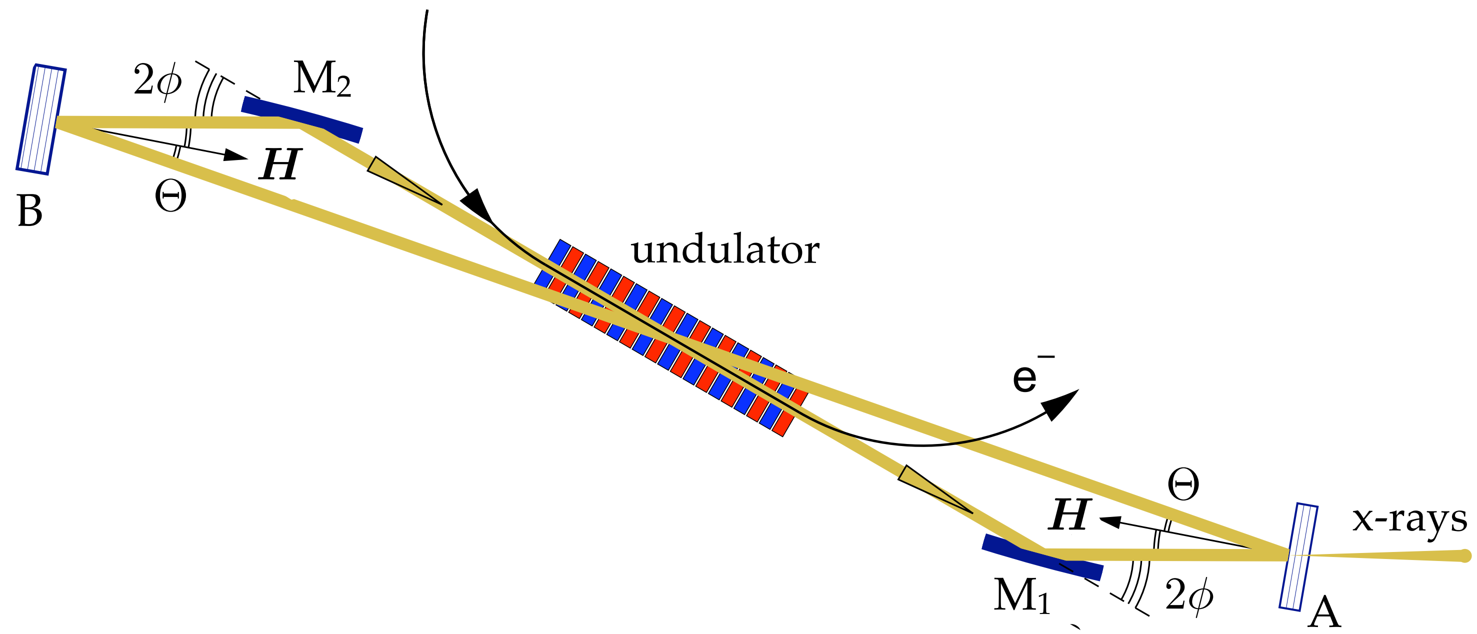
$$= \begin{bmatrix} X_L & b^2 L(2 - L/f) & 0 & \mathcal{A}(1-b) b \tan\theta_B L \\ -1/fb^2 & X_L & 0 & \mathcal{A}(1-b)\tan\theta_B / b \\ a\mathcal{A} & ab^2 \mathcal{A} L & 1 & l_2 \mathcal{A} ab(1-b)\tan\theta_B \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathcal{A} = 2 - l_2/f, L = l_1/b^2 + l_2, X_L = 1 - L/f$$

Constraints $\quad l_1 = l_2 \cos 2\theta_B \approx l_2, \quad l_2 = 2f, \quad -1 < X_L < 1$

This is not stable with $X_L = -1 - 2/b^2 < -1$

2-crystal 2-mirror (I)



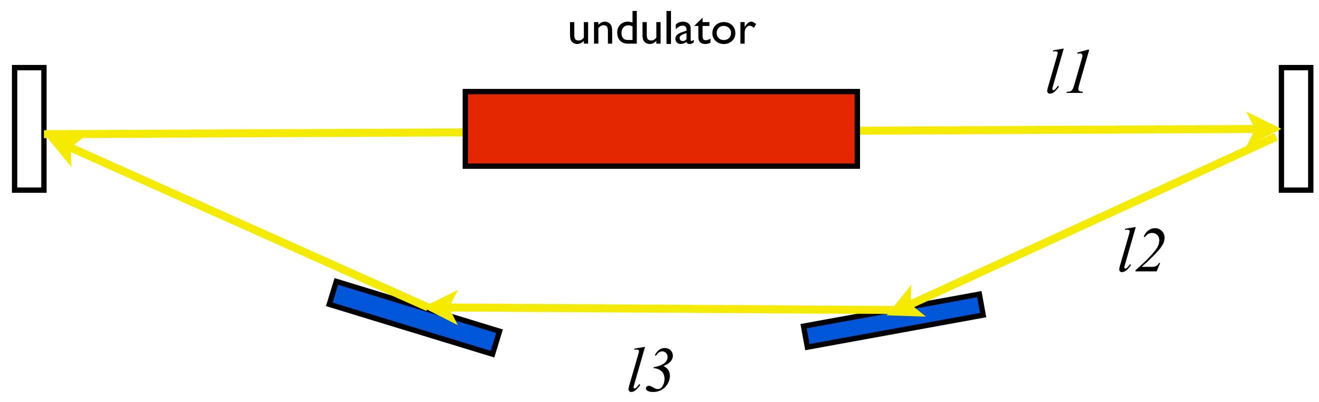
$$M_{2,2} = L_1 F L_2 C_2 L_3 C_1 L_2 F L_1$$

$$= \begin{bmatrix} 2X_1X_2 - 1 & 2fb^2X_1(1 - X_1X_2) & 0 & b(1-b)\mathcal{B}\tan\theta_B \\ -2X_2/b^2 f & 2X_1X_2 - 1 & 0 & 2(1/b - 1)\mathcal{B}X_2\tan\theta_B \\ 2aX_s & 2aL_s & 1 & abl_3(1-b)\tan\theta_B \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$X_s = L_s/f, L_s = l_2 + b^2 l_3/2$$

This is not isochronous or non-dispersive with $L_s \neq 0$

2-crystal 2-mirror(II)



$$M'_{2,2} = L_1 C_2 L_2 F L_3 F L_2 C_1 L_1$$

$$= \begin{bmatrix} 2X_1X_2 - 1 & 2fb^2X_1(1 - X_1X_2) & 0 & 2fb(1-b)\mathcal{B}(1 - X_1X_2)\tan\theta_B \\ -2X_2/b^2f & 2X_1X_2 - 1 & 0 & 2(1/b - 1)\mathcal{B}X_2\tan\theta_B \\ 2a\mathcal{B}X_2 & 2fb^2a\mathcal{B}(1 - X_1X_2) & 1 & 2fb(1-b)a\mathcal{B}\tan\theta_B(1 - \mathcal{B}X_2) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

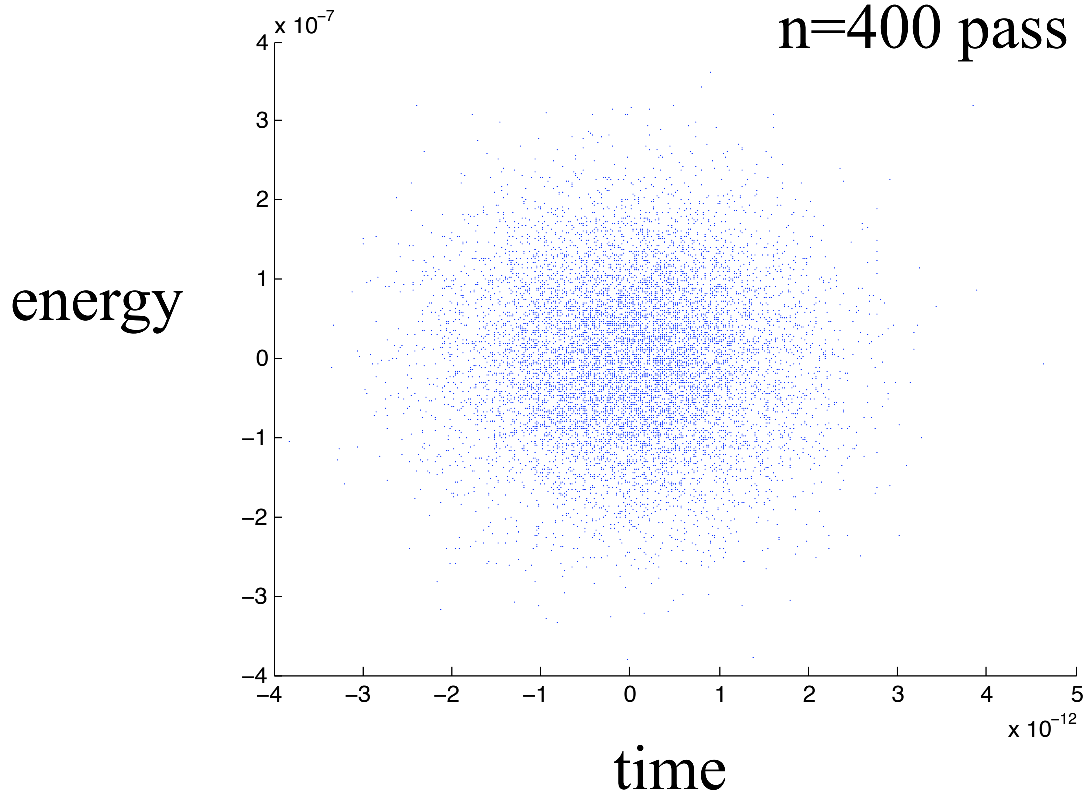
$$\mathcal{B} = 1 - l_2/f, X_1 = 1 - (l_2 + l_1/b^2)/f, X_2 = 1 - l_3/2f$$

constraints $\quad l_1 = l_2 + l_3/2, \quad l_2 = f, \quad 0 < X_1X_2 < 1$

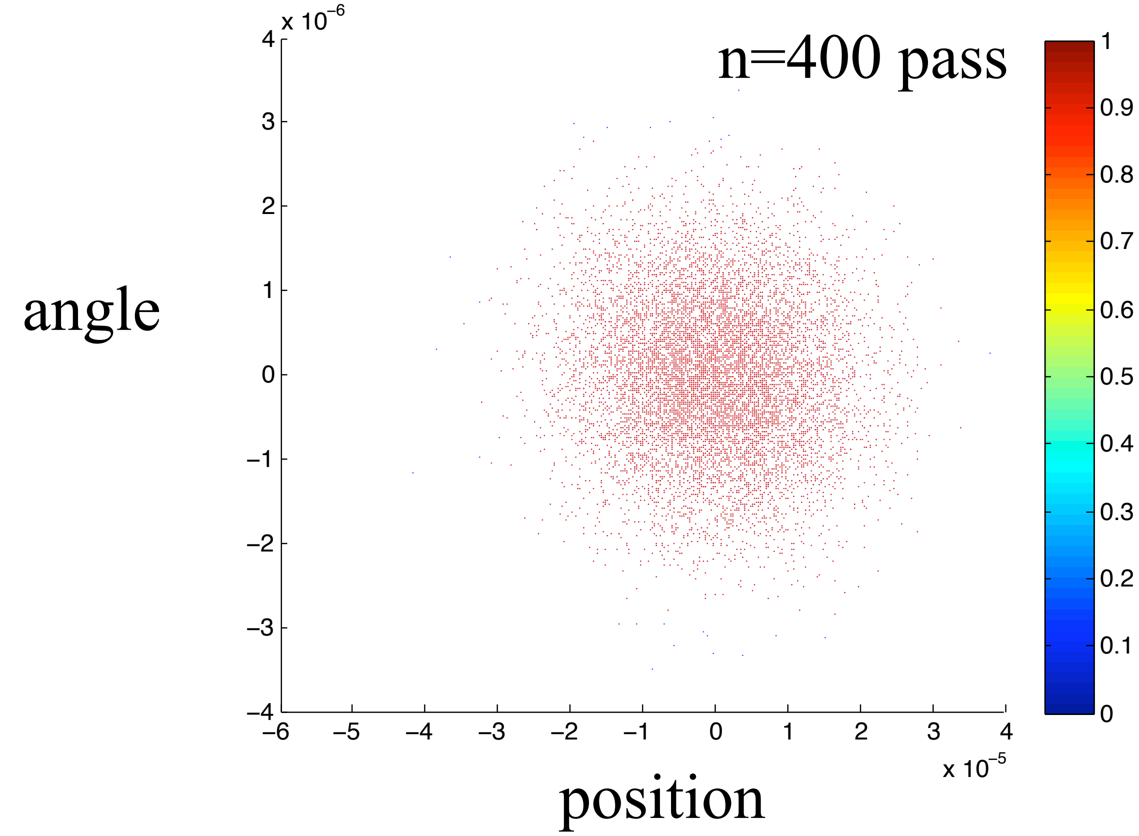
$$Z = fb^2\sqrt{\frac{X_1(1 - X_1X_2)}{X_2}} = \sqrt{l_1^2 + \frac{l_1 b^2 f^2}{l_1 - 2f}}$$

solution: $f = 20\text{m}, \quad l_1 = 47.929\text{m}, \quad l_2 = 20\text{m}, \quad l_3 = 55.858\text{m}$

X-ray profile at waist

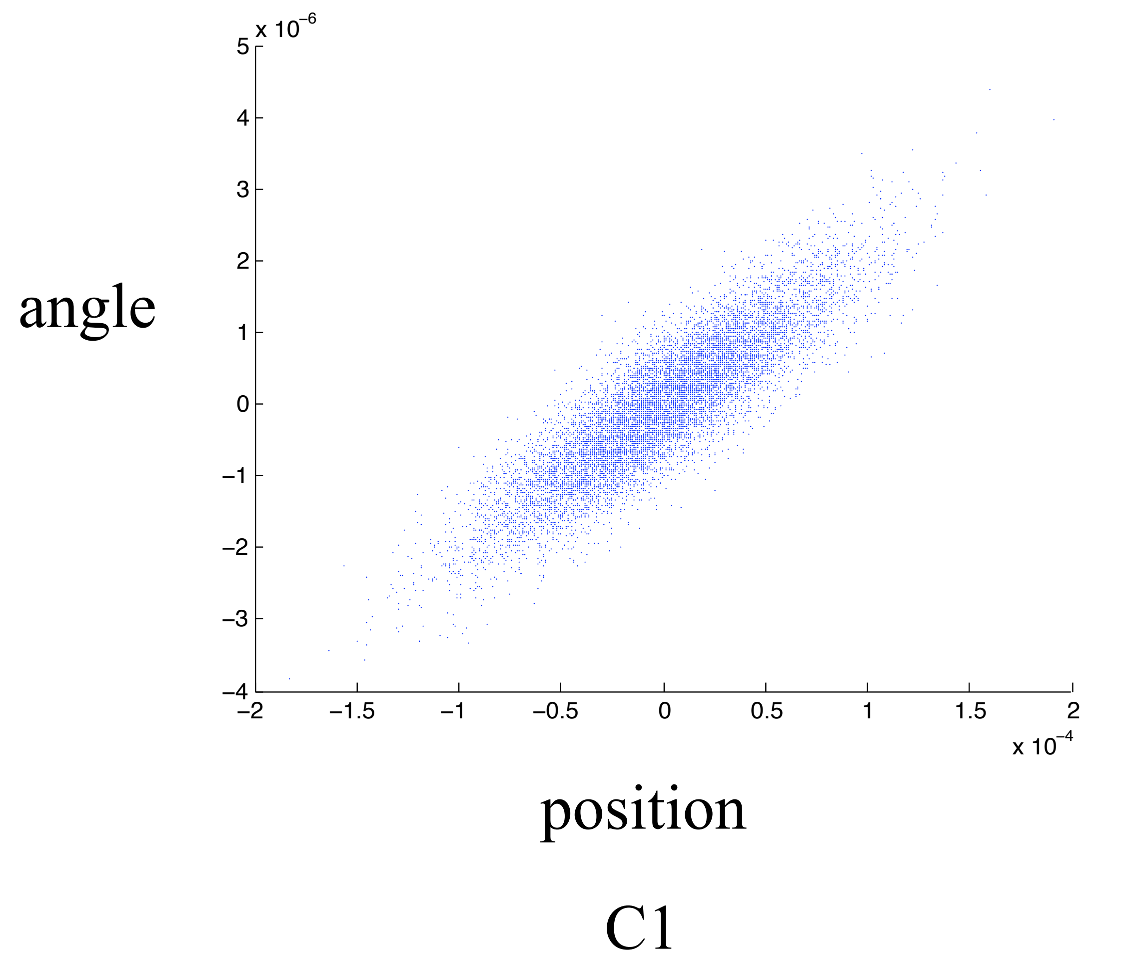


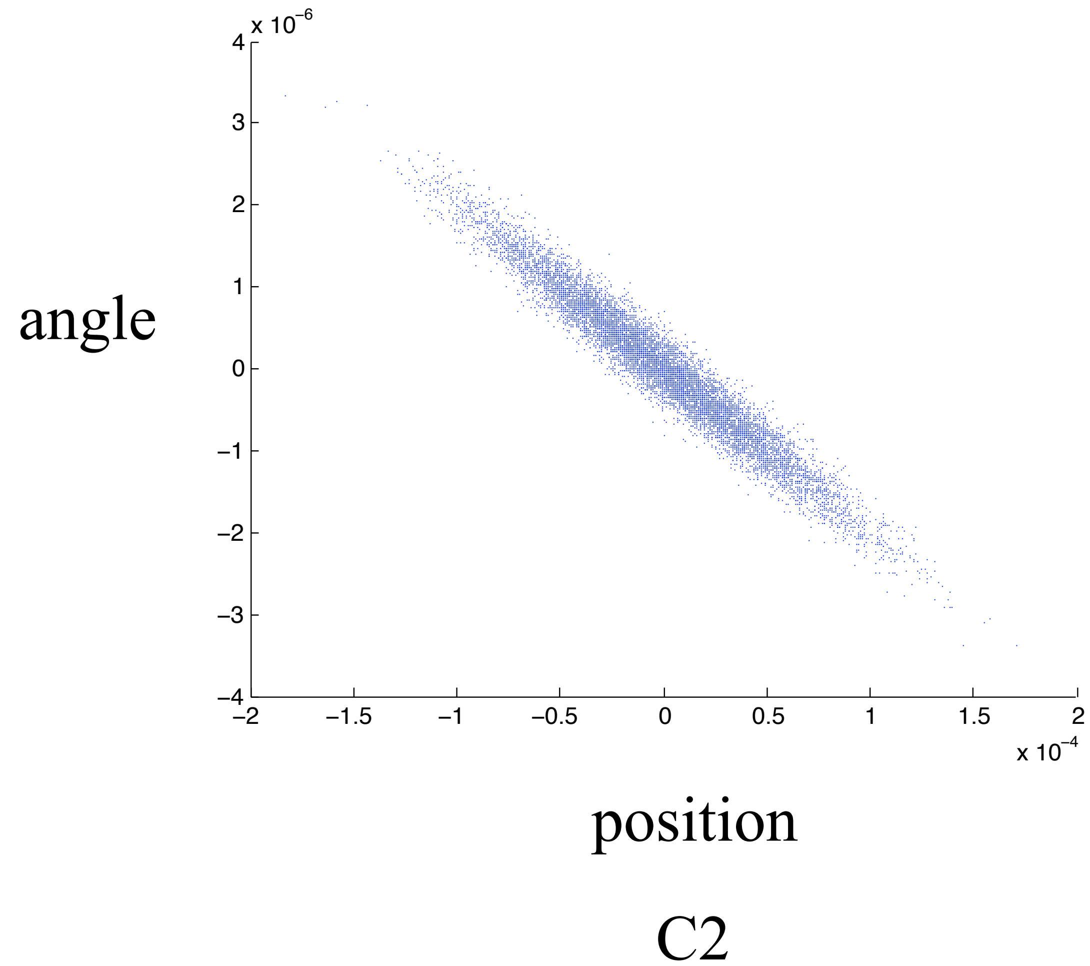
longitudinal profile



transverse profile

Angular Divergence at Crystals

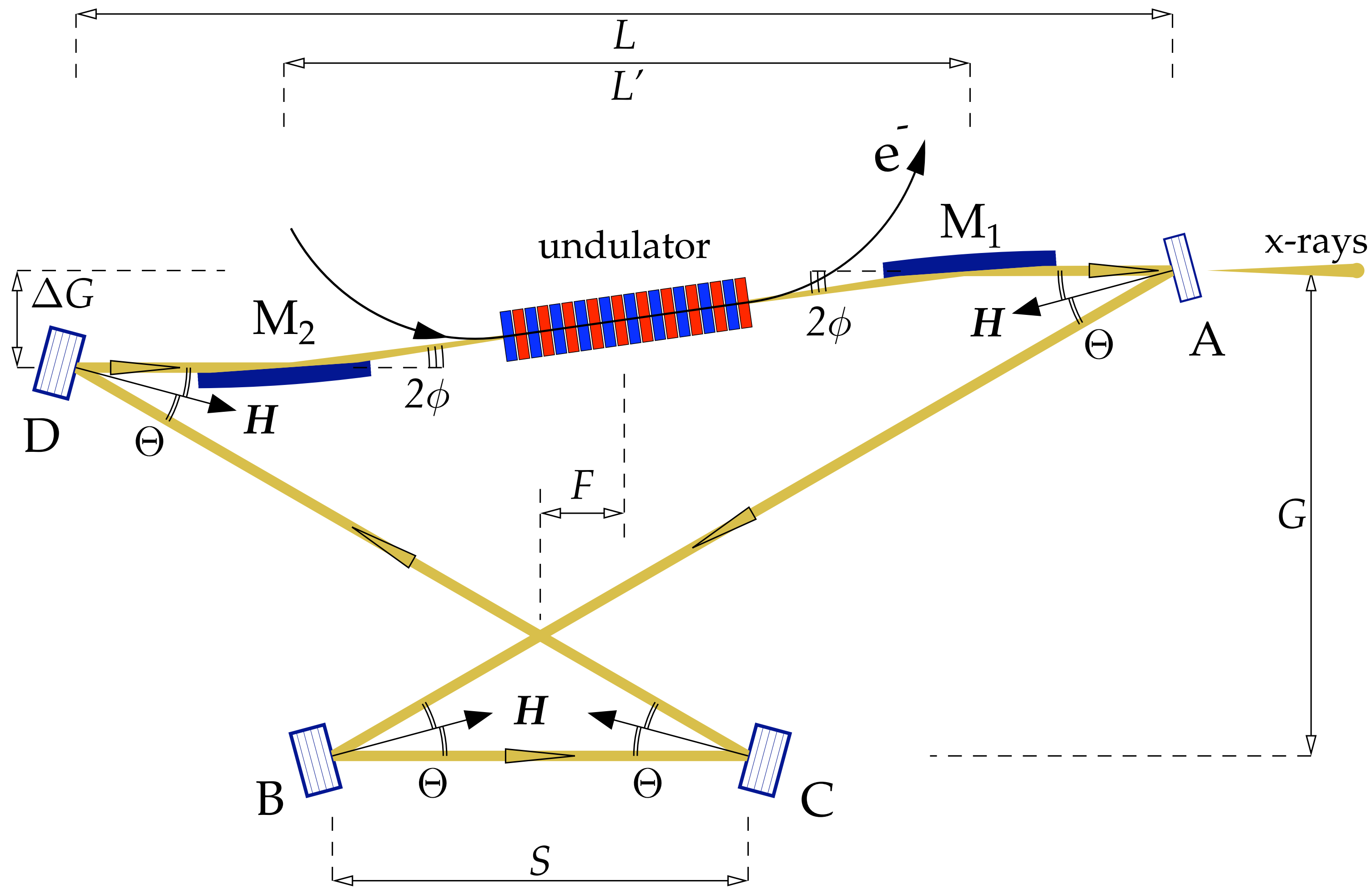




rms angular divergence 9.9961×10^{-7}

rms angular divergence 8.9905×10^{-7}

4-crystal configuration



$$M_4 = L_1 F L_2 C_4 L_3 C_3 L_4 L_4 C_2 L_3 C_1 L_2 F L_1$$

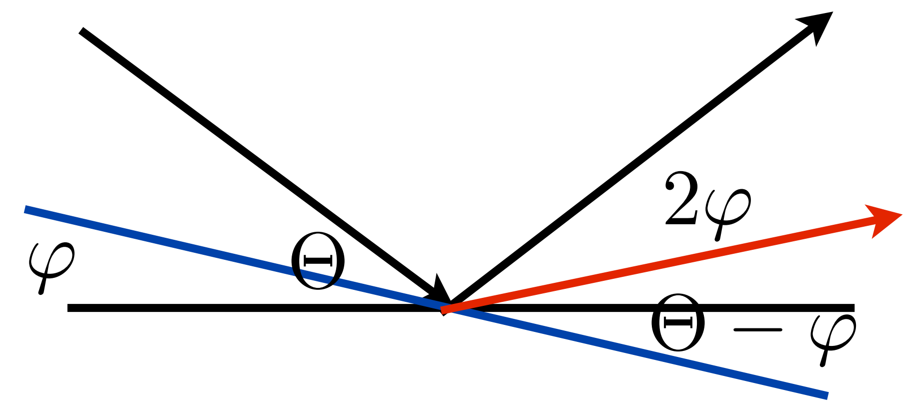
$$= \begin{bmatrix} 2Y_1Y_2 - 1 & 2fY_1(1 - Y_1Y_2) & 0 & (1-b)(Y_1 b - 1/b^2) l_3 \tan\theta_B \\ -2Y_2/f & 2Y_1Y_2 - 1 & 0 & -(1-b)(b - 1/b^2) l_3 \tan\theta_B / f \\ a(b^2 - 1/b) l_3 / f & -Y_1 a(b^2 - 1/b) l_3 & 1 & -a(1-b)(b + 1/b) l_3 \tan\theta_B \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Y_1 = 1 - \frac{l_1}{f}, \quad Y_2 = 1 - \frac{1}{f}(l_2 + \frac{l_3}{2}(b^2 + \frac{1}{b^2}) + l_4)$$

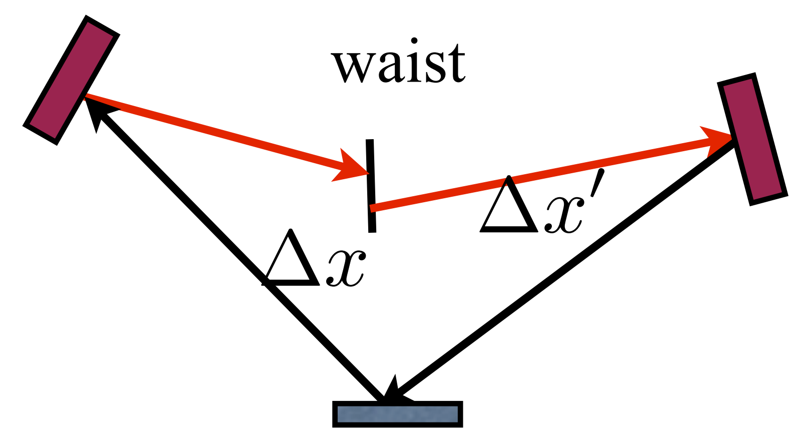
3. Errors in Optical Element and their Tolerances

- Misalignments

For vertical deviation in the orientation of the misaligned optical element given by φ, reference ray deviates by 2φ.



For stability, we require a periodicity of optical axis deviation



$$\begin{pmatrix} \Delta x \\ \Delta x' \\ \Delta y \\ \Delta y' \\ \Delta \xi \end{pmatrix} = M \begin{pmatrix} \Delta x \\ \Delta x' \\ \Delta y \\ \Delta y' \\ \Delta \xi \end{pmatrix} + \sum_k M_k \begin{pmatrix} 0 \\ 2\varphi \\ 0 \\ 2\varphi \\ 0 \end{pmatrix}$$

Its solution is given as $\Delta x = 300.84\varphi, \quad \Delta x' = 8.28\varphi$

$$\Delta x = 10^{-6}[m], \quad \Delta x' = 10^{-7} \rightarrow \varphi = 3.3 \times 10^{-9}, 1.21 \times 10^{-8}$$

- Focal Length Errors

Errors in focal length of focusing mirror leads to unmatching of waist with stability issue.

With error by ε, transfer matrix F is modified to

$$F' = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1/f+\varepsilon & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f+\varepsilon & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} = F(1+\varepsilon R)$$

$$\text{where } R = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

With some manipulations, total transform matrix is modified to

$$\mathcal{M}' = \mathcal{M} + \varepsilon(L_1 R L_1^{-1}\mathcal{M} + \mathcal{M}L_1^{-1}RL_1) + \varepsilon^2(L_1RL_1^{-1}\mathcal{M}L_1^{-1}RL_1)$$

Tune should avoid half-integer value for stability

We find tune by taking trace of one-turn matrix:

- ideal

$$Q_x = 4.68 \times 10^{-1}$$
$$Q_y = 4.67 \times 10^{-1}$$

For 1% error, we have

$$\Delta Q_x(0.01) = 6.07 \times 10^{-3}, \quad \Delta Q_y(0.01) = 6.27 \times 10^{-3}$$

But for 5% errors, we have unstable betatron motion.

4. Conclusion

- We found optical cavity configuration that has asymmetric crystal and allows radiation with constant pulse length.
- We evaluated tolerance limit for mislignment and focal length error