Extraction of resonance parameters from meson production reactions

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Data of Meson Production Reaction

$$\pi N, \gamma^* N \quad \rightarrow \quad \pi N, \eta N, \pi\pi N, KY, \omega N$$

B. Julia-Diaz

Partial Wave Amplitudes

$$f_{I,lj}(W), M_{\pm L}(W,Q^2), E_{\pm L}(W,Q^2)$$

This talk

Resonance Parameters

$$J^\pi, I, S \qquad M - i\Gamma/2 \quad F_{N^*,N}(Q^2)$$

Extraction of Resonance Parameters

	πN	$\gamma^* N$
VPI/GW	BW,Pole(AC)	BW
CMB,Pitt-ANL	BW,Pole(AC)	
Giessen	BW,Pole(SP)	BW
Jlab/Yerevan		BW
MAID		BW
DMT	BW,Pole(SP)	
Juelich	BW	
EBAC	Pole(AC)	BW,Pole(AC)

BW Breit-Wigner, SP speed-plot, AC Analytic continuation

BW and Pole

$$T \sim \frac{1}{W - m(W)}$$

'BW' $$M = Re(m(M)), \quad \Gamma = -2Im(m(M))$$

$$m(W) = M_{BW} - i\Gamma(W)/2$$

Pole $$M - i\Gamma/2 = m(M - i\Gamma/2)$$

Pole:

Z radiative correction Gauge Inv. A. Sirlin (91)

Δ_{33} ChPT independent of chosing field D. Djukanovic et at. (07)

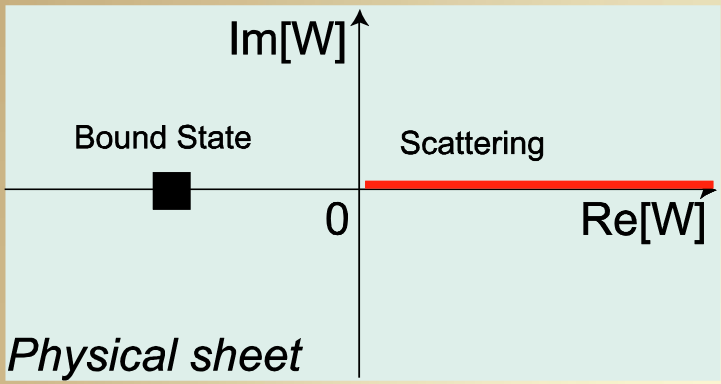
- Characterize Resonance
- Extracting Resonance parameters
  - ✓ mass
  - ✓ electromagnetic form factors
- Summary

Resonance: pole of S/T-matrix

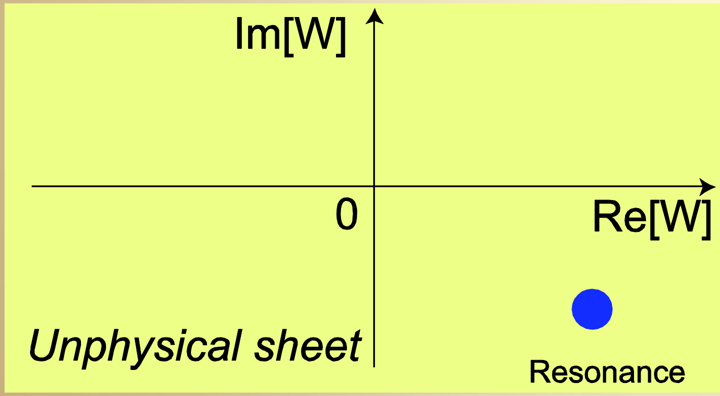
$$T(W) \sim \frac{T_{residue}}{W - M + i\Gamma/2}$$

- Resonances appear as poles of the scattering amplitudes
- characterize resonances

Pole position → mass, Residue → Form factors

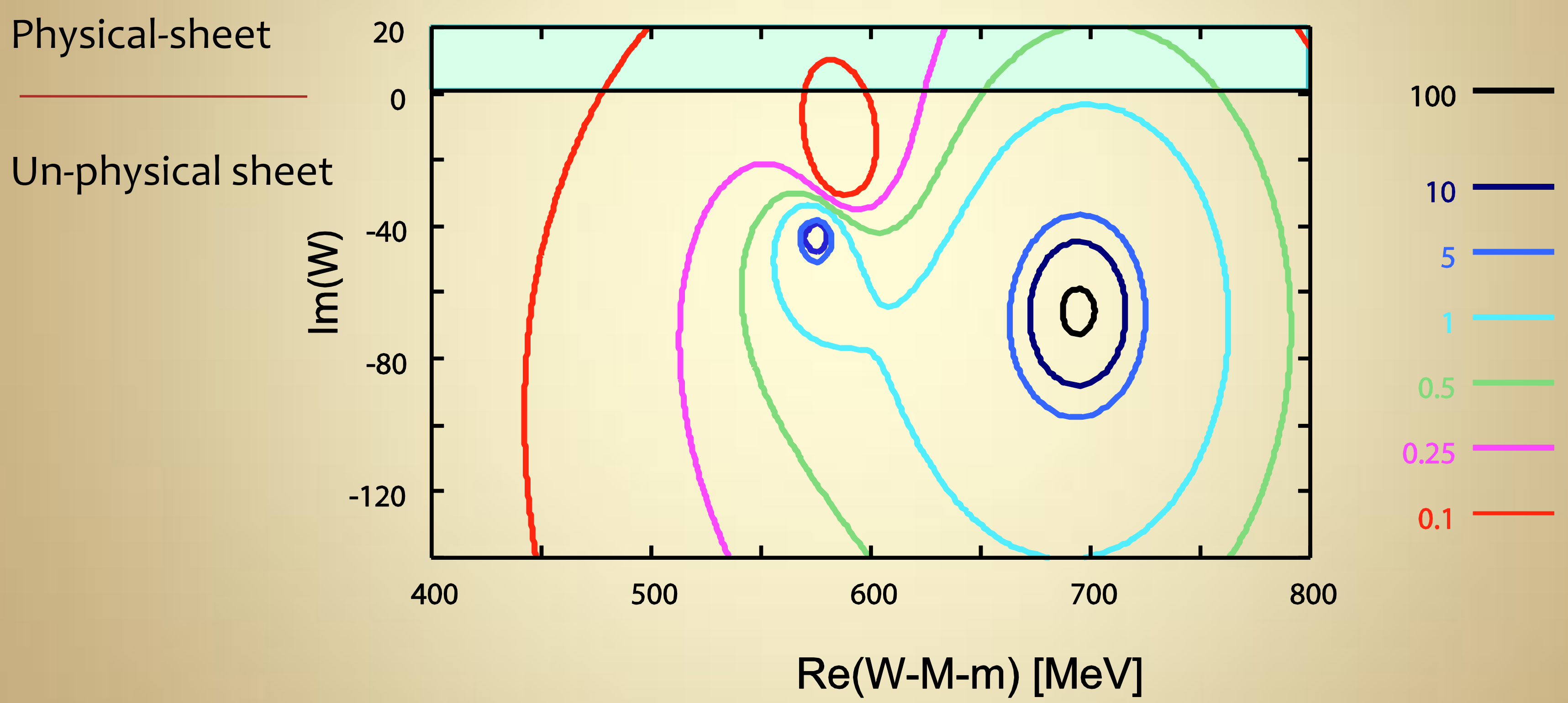


$$\frac{1}{W-H} = \sum_n \frac{|\phi_n><\phi_n|}{W-\epsilon_n} + \int_{\epsilon_{th}} d\epsilon \frac{|\phi_\epsilon><\phi_\epsilon|}{W-\epsilon}$$



$$\frac{1}{W-H} \sim \frac{|\phi_R><\tilde{\phi}_R|}{W-M+i\Gamma/2}$$

$|T|^2(W)$ toy model of πN-ηN coupled channel



Residue of T-matrix at resonance pole

$$< f|T(W)|i > \sim \frac{< f|V|\phi_R >< \tilde{\phi}_R|V|i >}{W - M + i\Gamma/2}$$

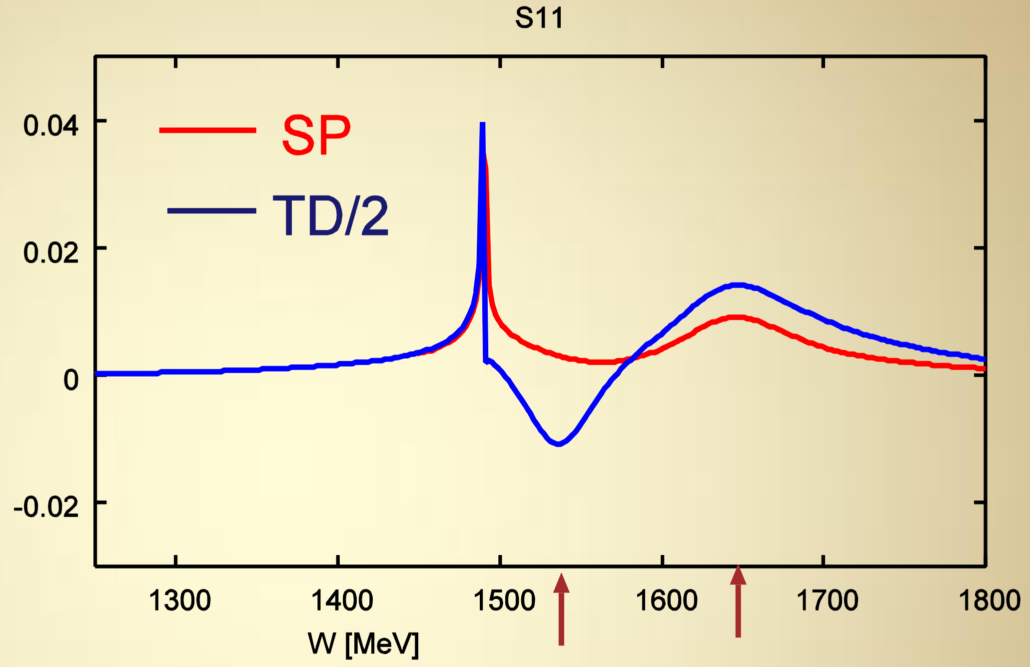
Form Factor $F_{N^*N}(Q^2) =< \tilde{\phi}_R|j_{em}|N >$ $V \to j_{em}$

Resonance Masses

- Speed Plot
- Analytic continuation of amplitude
  (Dynamical model)

Speed Plot

$$\left|\frac{dT(W)_{\pi N-\pi N}}{dW}\right|$$



S11(1535) ---------------

S11(1650) 1644 – 89i [MeV] (1642 – 41i Pole)

Improved speed plot (S. Ceci et al. PRD77 (08))

analytic continuation using polynomial expansion:

$$f(W) = (W_{res} - W)T(W) = \sum_n \frac{f^{(n)}(W_0, W_{res})}{n!}(W - W_0)^n$$

$$W, W_0 : Real \quad W_{res} = M - i\Gamma/2$$

$$W \to Complex$$

$$f(W_{res}) = Residue \sim \frac{T^{(N)}(W_0)}{N!}(W_{res} - W_0)^{N-1}$$

SP' 1522 – 146i (N=7) <- - - > Pole 1517 – 190 i

K-matrix Pole from Tr(K) S.Ceci et al. PLB 659(2008)

-> R. Workman, R. Arndt 0808.2176

Dynamical model of meson production reactions

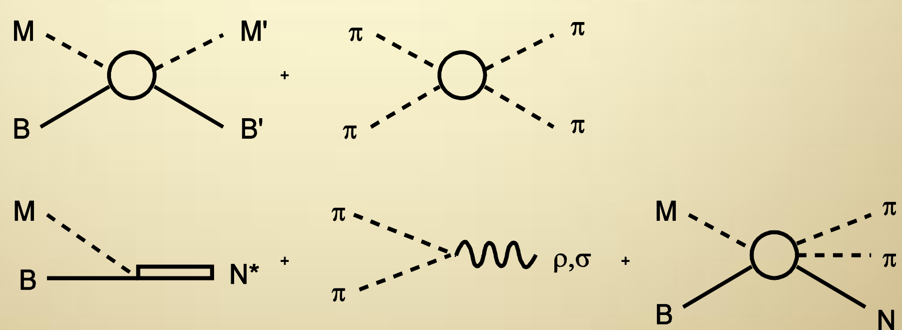
Coupled channels model with ππN three-body unitarity

$$\pi N, \eta N, \pi\pi N(\pi\Delta, \sigma N, \rho N) KY, \omega N$$

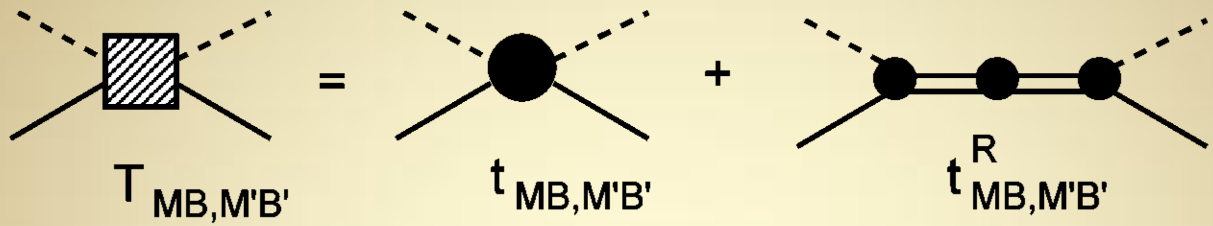
Simultaneous analysis of π and γ induced reactions

Extract resonance parameters and study hadronic structure of resonances

$$V =$$



MB-M'B' T-matrix



$$T_{\alpha,\beta}(W) = t^{non-res}_{\alpha,\beta}(W) + \sum_{i,j} \bar{\Gamma}_{\alpha,i}(W)[\frac{1}{W - m_0 - \Sigma(W)}]_{ij}\bar{\Gamma}_{\beta,j}(W)$$

α, β Meson-Baryon channel

i, j Resonances

$$\bar{\Gamma}(W) = (1 + t^{non-res}(W)G^0(W))\Gamma$$

$$< p_\alpha | T(W) | p_\beta > \; = \; t^{non-res}(W, p_\alpha, p_\beta) + \frac{\bar{\Gamma}(W, p_\alpha)\bar{\Gamma}(W, p_\beta)}{W - m_0 - \Sigma(W)}$$

- Analytic continuation of T(W) on unphysical sheet by using contour deformation
- Pole can be in non-resonant and resonant amplitudes
- Pole of T as a function of W, p's are arbitrary

Resonance Mass

$$M - i\Gamma/2 = m_0 + \Sigma(M - i\Gamma/2)$$

Resonance Form Factor (complex number)

$$< \tilde{\phi}_R | j_{em} | N > = \frac{1}{\sqrt{1 - d\Sigma/dW}} \bar{\Gamma}(M - i\Gamma/2)$$

$$|\phi_R > = \frac{1}{\sqrt{1 - d\Sigma/dW}} [1 + G_0^{+}(1 + t^{non-res})\Gamma] | N^* >$$

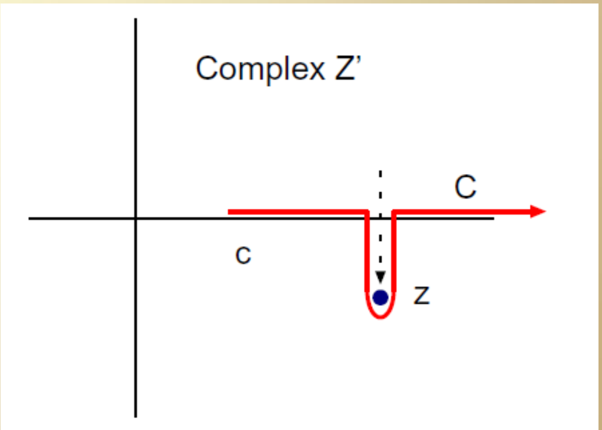
T(W) on unphysical sheet (Analytic continuation)

T(W) : obtained by solving Lippman-Schwinger Equation

$$T(W) = V + \int dn\ \ V|n><n|\frac{1}{W - H_0 + i\epsilon}|n><n|T(W)$$

AC : deformation of momentum integration contour to take care of two and three body singularities of Green function

$$F(z) = \int dx \frac{g(x)}{x-z} = \int_C dz' \frac{g(z')}{z'-z}$$



Mass from analytic continuation of piN amplitude

(model prc-07)

	Our Model	PDG
S11	$(1535, -157)$	$(1490 \sim 1530,\ -45 \sim -125)$
	$(1642, -41)$	$(1640 \sim 1670,\ -75 \sim -90)$
D13	$(1521, -58)$	$(1505 \sim 1515,\ -62 \sim -75)$
P33	$(1211,\ -50)$	$(1209 \sim 1211,\ -49 \sim -51)$

Electromagnetic Form Factor

Helicity amplitudes : defined from radiative decay of N*

(L.A. Copley et al . 69)

$$A_\lambda = N < N^*|\vec{j}_{em} \cdot \epsilon_{\lambda_\gamma}|N_{\lambda_N} >$$

Residue at resonance pole

$$A_\lambda = N < \phi_R|\vec{j}_{em} \cdot \epsilon_{\lambda_\gamma}|N_{\lambda_N} >$$

(Dynamical Model)

$$= \frac{N}{\sqrt{1 - d\Sigma/dW}}\bar{\Gamma}_\gamma(W)$$

'BW' parametrization

simulate 'BW' within dynamical model

$$T_{\pi\gamma}(W) = t_{\pi\gamma}(Non{-}res)(W) + \frac{\bar{\Gamma}_{\pi}(W)\bar{\Gamma}_{\gamma}(W)}{W - m_0 - \Sigma(W)}$$

mass $M_{BW} = m_0 + Re(\Sigma(M_{BW}))$

Helicity amplitude $A_{\lambda} = N\bar{\Gamma}(M_{BW})$

Comparison with other works : need to identify t(non-res)

Residue of M1+(3/2)

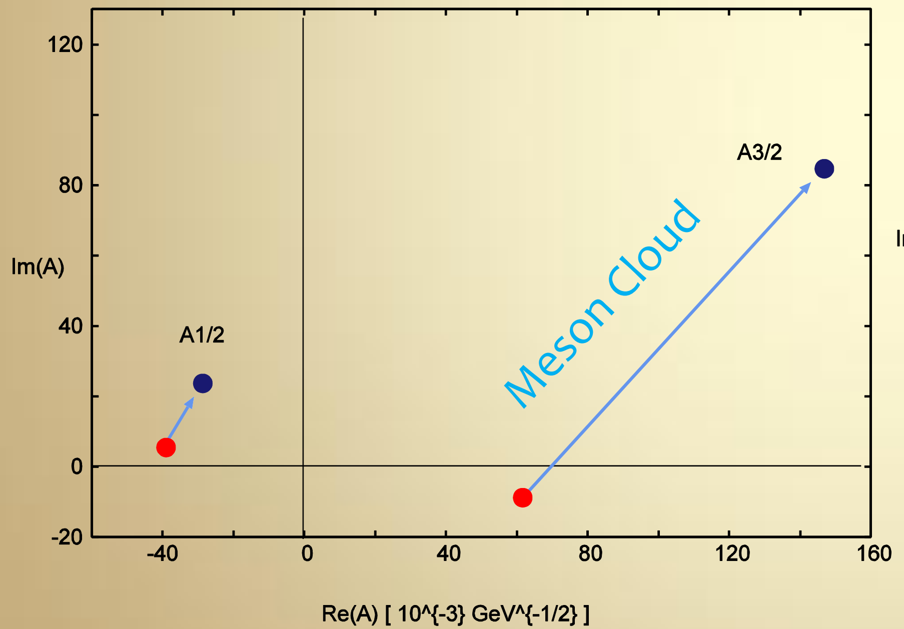
M1+(3/2) at Q^2=0   at P33 pole (SL model)

Ours	-0.041	- i 0.042
Hanstein et al.	-0.035	- i 0.046
Arndt et al.	-0.034 ± 0.005	$- i\ 0.055 \pm 0.005$

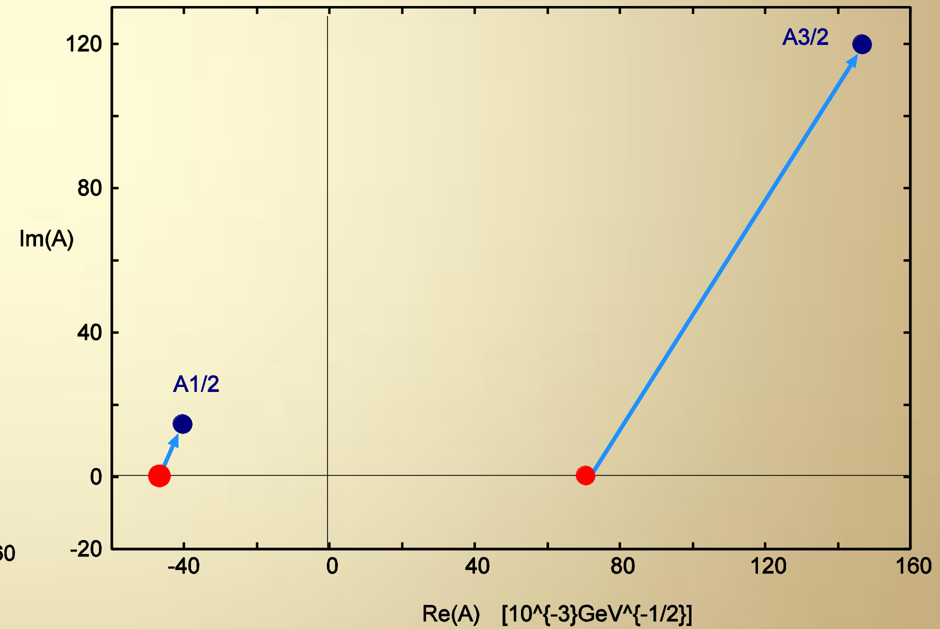
Transition form factor (D13)

Helicity amplitude at Q^2=0 (preliminary results)

$$M_{pole} = (1527 - 59i) \quad \leftrightarrow \quad M_{BW} = (1550 - 83i)$$



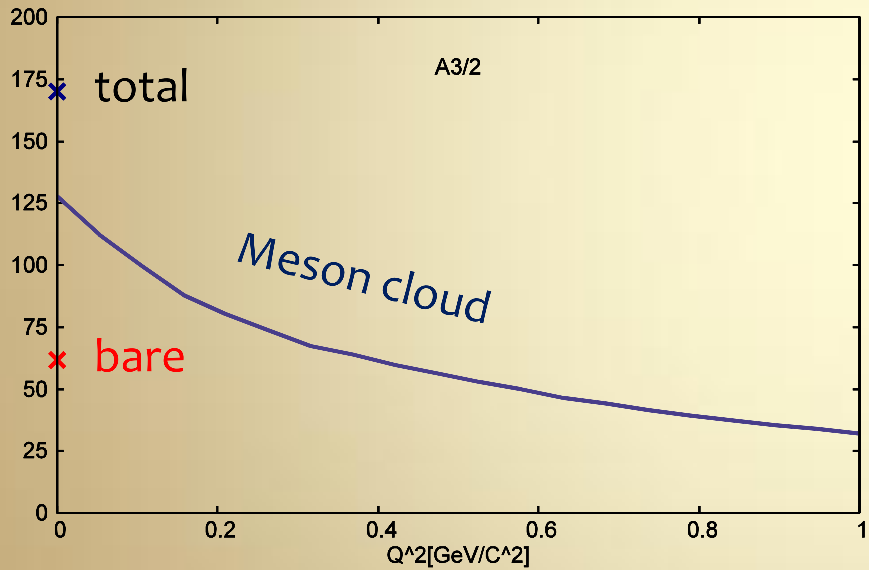
Residue at Pole

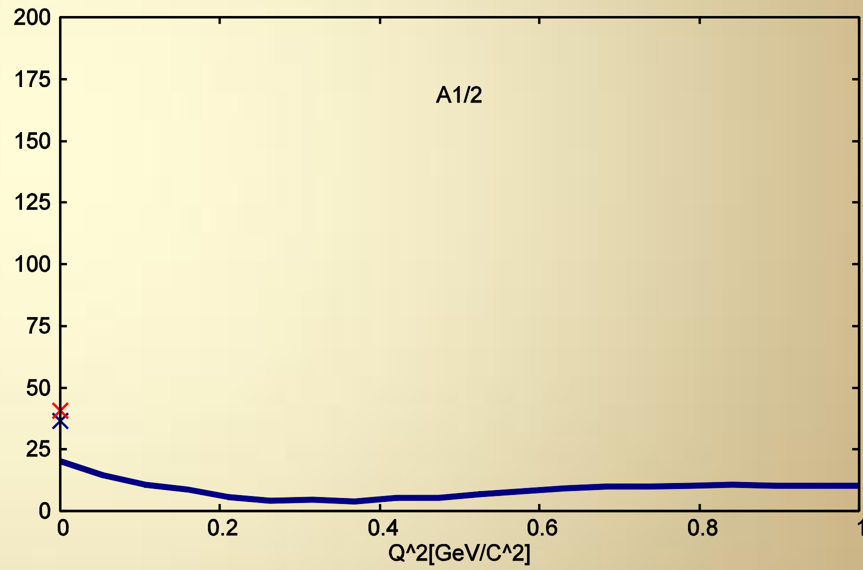


‘BW’

Q^2 dependence of helicity amplitudes (preliminary results)

$$|A_\lambda(Q^2)|$$





Summary

- Pole of T matrix gives resonance information
- Mass and width from pole position, form factors from residue
- A method to extract resonance parameters using analytic continuation of the scattering amplitude is developed.
- Extracting resonance parameters is in progress.

Resonance state

Resonance associated with 'eigen state' of hamiltonian
(A. Donnachie(72), R.H. Dalitz R. G. Moorhouse(70),Siegert(39))

Asymptotic form of wave function

$$\psi_j \sim A_j e^{-iq_j r} + \sum_i S_{ji}(W) A_i e^{iq_i r}$$

Outgoing boundary condition for all channel

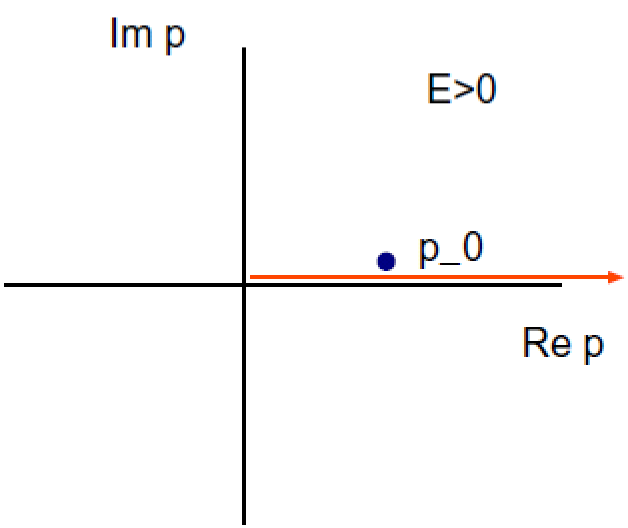
$$(d\psi_j/dr - iq_j\psi_j)_{r\to\infty} = 0$$

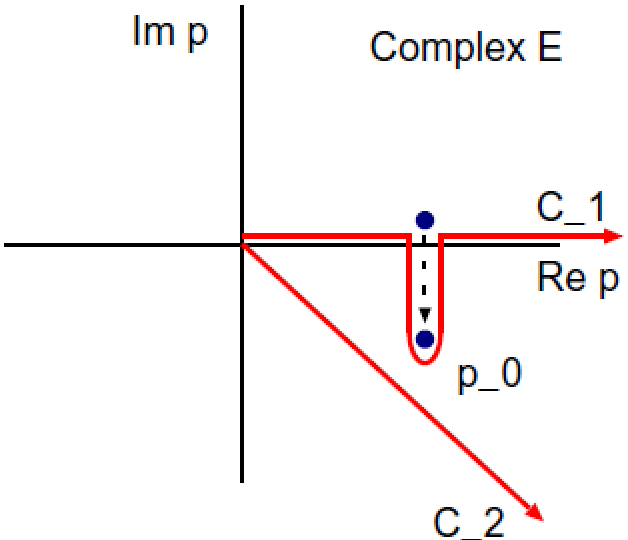
eigen value W must be singular point of S.
Only complex eigenvalue W can satisfy boundary condition

Analytic Continuation of T-matrix

Stable Meson-Baryon states $G_{MB}(E) = \frac{1}{E - E_B(p) - E_M(p) + i\eta}$

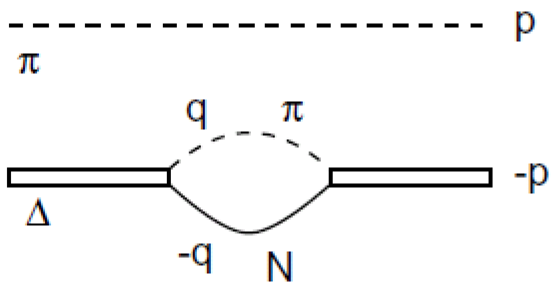
$$T(p_f, p_i, E) = V(p_f, p_i) + \int_C dp p^2 V(p_f, p) G_{MB}(E) T(p, p_i, E)$$





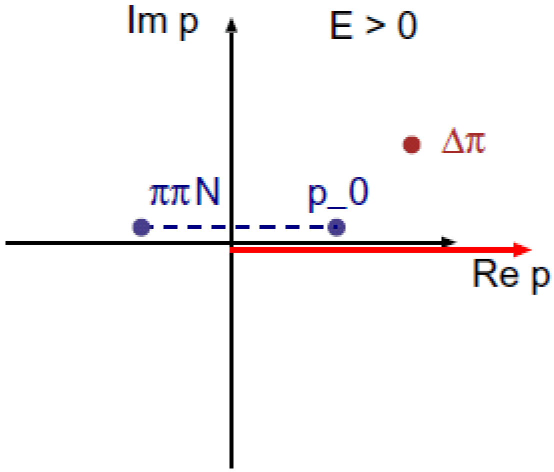
(Pearce-Gibson, A. Bohm)

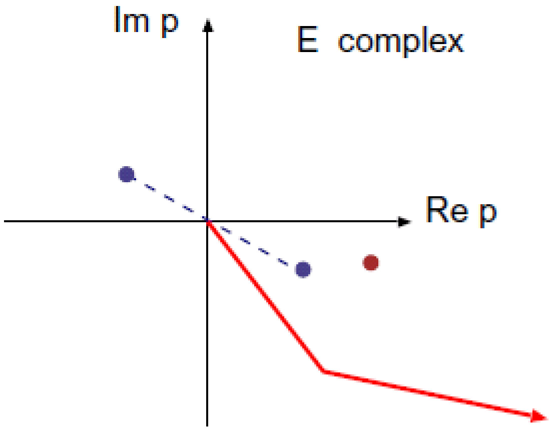
$\pi\Delta, \sigma N, \rho N$(Un-stable particle $(\pi\pi N)$)



$$G_{MB} = \frac{1}{E - E_\pi(p) - E_\Delta(p) - \Sigma_\Delta(E,p)}$$

$\pi\pi N$ cut for $p < p_0$: $E = E_\pi(p_0) + \sqrt{(m_\pi + m_N)^2 + p_0^2}$

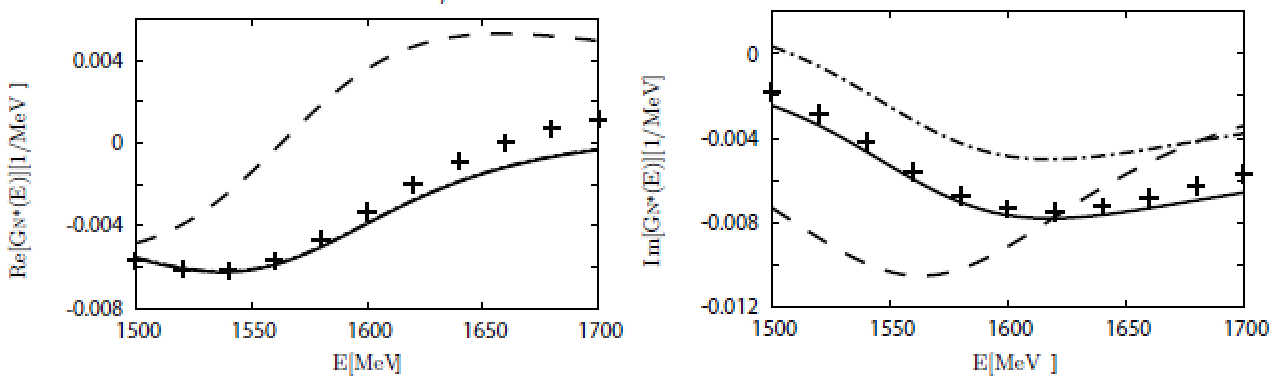




How the methods works (example S_{31})

$$
\begin{aligned}
G_{N^*}(E) &= \frac{1}{E - m_0 - \Sigma(E)} \\
&\sim \frac{1}{E - E_R}(1 - \Sigma'(E_R)) + \frac{\Sigma''(E_R)}{2(1 - \Sigma'(E_R))^2}
\end{aligned}
$$

with $E_R = M - i\Gamma/2$



- $T_{\pi\gamma} = (1 + iT^{on}_{\pi N})T_{\pi\gamma}(non - res) + T_{BW}$

$$T_{BW} \sim \frac{1}{W - M + i\Gamma} \quad \rightarrow \quad A_\lambda = X Im(T_{BW})$$

- $T^{ours}_{\pi\gamma} = (1 + iT^{on}_{\pi N})T_{\pi\gamma}(non - res) + \delta T(non - res) + \frac{\bar{\Gamma}_\pi(\Gamma_\gamma + \delta\Gamma_\gamma)}{W - m_0 - \Sigma}$

$$\delta T(non - res) = t_{\pi,\pi} G^P v_{\pi\gamma} + \sum_{n \neq \pi N} t_{\pi,n} G^0 v_{n,\gamma}$$

$$\delta\Gamma_\gamma = \bar{\Gamma}_\pi G^P v_{\pi\gamma} + \sum_{n \neq \pi N} \bar{\Gamma}_n G^0 v_{n,\gamma}$$