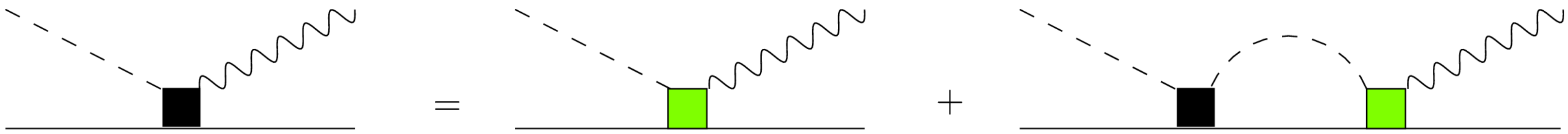
Dynamical coupled channel calculation of pion and omega meson production



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EmNN* Workshop
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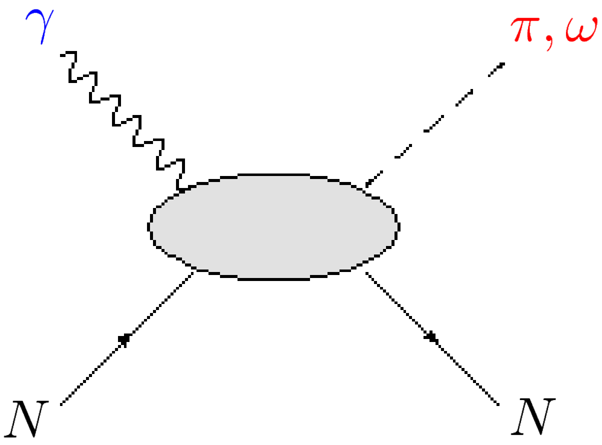
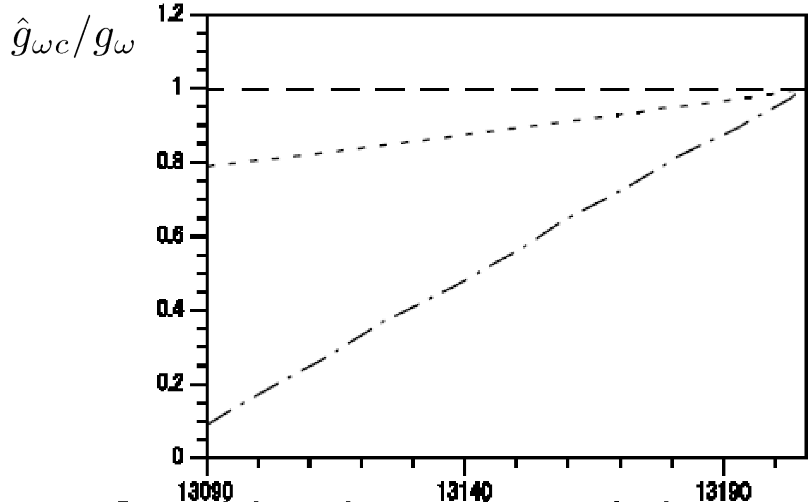
Outline

- Model
  - 5 channel “core” [B.Julia-Diaz et.al. Phys. Rev. C 76: 065201, 2007] + ωN
- Fitting
  - πN→πN, πN→ωN, YN→πN,YN→ωN
- Predictions
  - Σ_ω photon beam asymmetry
  - $\rho^0_{\lambda\lambda'}$ spin density matrix elements
  - ωN scattering length
- Conclusion



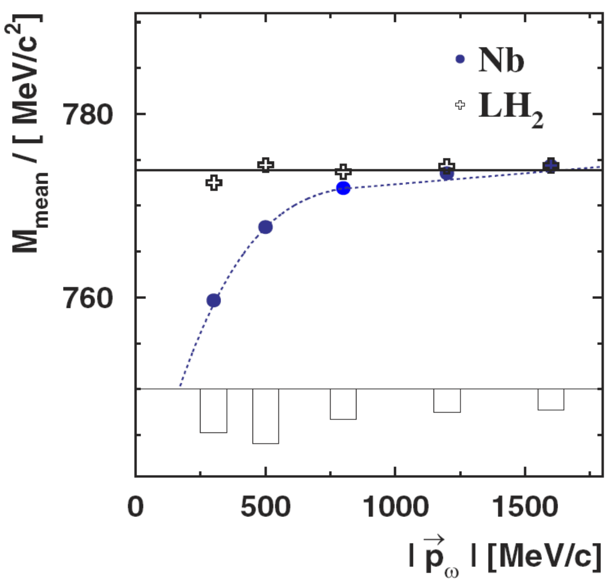
Motivation: ωN interactions

- ω interactions are poorly determined
  - SU(3):
    - → $g_{\omega NN}/g_{\rho NN}=3/2$
  - CD-Bonn realistic NN potential
    - → $g_{\omega NN}=8.$ $\kappa_{\omega NN}=0$
  - Sato-Lee (Δ)
    - → $g_{\omega NN}=10.5$ $\kappa_{\omega NN}=0$
  - Giessen
    - → $g_{\omega NN}\sim4.6$ $\kappa_{\omega NN}\sim-1$



[Tuchitani, et al. nucl-th/0407004]

- Possible "in-medium" modification



CBELSA/TAPS Collaboration
[Trnka PRL 192303(05)]

$$\delta m_\omega^{*2} \approx 0.1 m_\omega^2 \implies \delta R_{NS} \approx 1 \text{ km}$$

→$T_\omega=0$ means only T=1/2 N* contribute → simplicity

→can we fit unpol./pol. data and accurately predict other unfitted observables?

Model: dynamical equation

Starting model space: neglect $\pi\pi N$

$$V_{\alpha\beta} = v_{\alpha\beta} + v^R_{\alpha\beta}$$

$$v^R(E) = \sum_{N^*_i} \frac{\Gamma_i^\dagger \Gamma_i}{E - M_i^0}$$

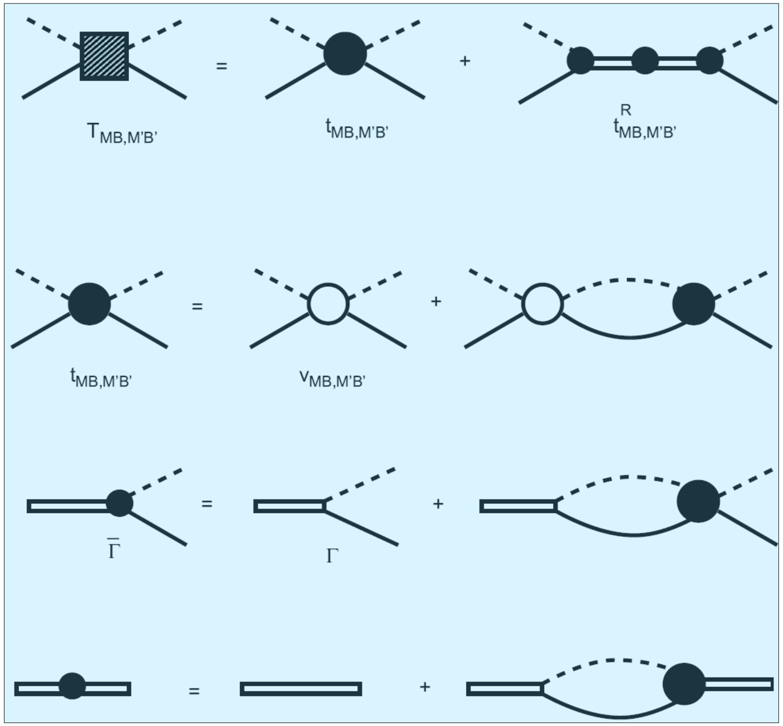
$$T_{\alpha\beta}(E) = t_{\alpha\beta}(E) + t^R_{\alpha\beta}(E)$$

$$t^R_{\alpha\beta}(E) = \sum_{i,j} \overline{\Gamma}_{\alpha,i}[E - H_0 - \Sigma]^{-1}_{ij}\overline{\Gamma}_{j,\beta}$$

$$t_{\alpha\beta}(E) = v_{\alpha\beta} + \sum_\gamma v_{\alpha\gamma} G_{0,\gamma}(E) t_{\gamma\beta}(E)$$

$$\overline{\Gamma}_{i,\beta}(E) = \Gamma_{i,\beta}(E) + \sum_\gamma \Gamma_{i,\gamma} G_{0,\gamma}(E) t_{\gamma\beta}(E)$$

$$\Sigma_{ij} = \sum_\gamma \Gamma_{i,\gamma} G_{0,\gamma} \overline{\Gamma}_{\gamma,j}$$



$$G_{0,\gamma} = \frac{1}{E - \sqrt{k^2 + m_B^2} - \sqrt{k^2 + m_M^2} - \Sigma_\gamma(E)}$$

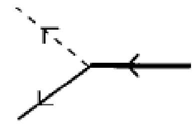
NB sums:

$\sum_\gamma : \gamma = \pi N, \eta N, \pi\Delta, \sigma N, \rho N$

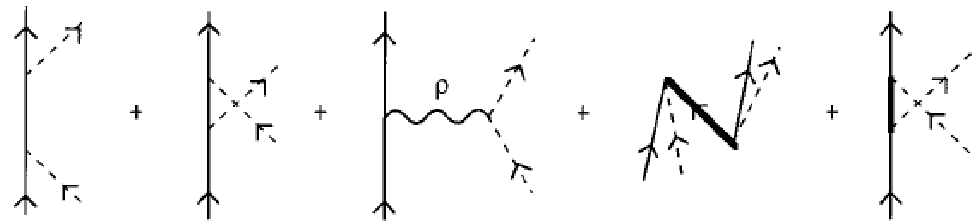
$\sum_{i,j} : i, j =$ set of resonances

model: interaction
$MB \to M'B'$

$$V = \Gamma_V + v_{22} + v'$$
$$\Gamma_V = \sum_{N^*,MB} \Gamma_{N^*,MB}$$
$$v_{22} = \sum_{M'B',MB} v_{M'B',MB}$$
$$v' = v_{23} + v_{33}$$



[Hermitian conjugate implied]



For 6 channels: $\pi N, \eta N, \pi\Delta, \sigma N, \rho N, \omega N \implies$ **45 Feynman amplitudes**

Interaction evaluation:
- evaluate diagrams
- project into partial wave basis
- test against plane wave code

Propagators:
- unitary transform modifies Feynman form
- off-shell→on-shell: Feynman

$$\overline{V}^{16}_a = i g_{\omega NN} \frac{f_{\pi NN}}{m_\pi} \Gamma_{\omega'} \frac{1}{2} \left[\frac{1}{\not p + \not k - m} + \frac{1}{\not p' + \not k' - m} \right] \tau^i \not k \gamma_5$$

$$\overline{V}^{16}_b = i g_{\omega NN} \frac{f_{\pi NN}}{m_\pi} \tau^i \not k \gamma_5 \frac{1}{2} \left[\frac{1}{\not p - \not k - m} + \frac{1}{\not p' - \not k' - m} \right] \Gamma_{\omega'}$$

$$\overline{V}^{16}_e = g_{\rho NN} \frac{g_{\omega\pi\rho}}{m_\omega} \frac{\tau^i}{2} \Big[\frac{1}{2} \frac{1}{t - m_\rho^2} \epsilon_{\alpha\beta\gamma\delta} \epsilon^{\alpha *}_{\lambda_\omega} (p - p')^\beta k^\gamma \Gamma^\delta_\rho (p - p')$$
$$+ \frac{1}{2} \frac{1}{t' - m_\rho^2} \epsilon_{\alpha\beta\gamma\delta} \epsilon^{\alpha *}_{\lambda_\omega} (k' - k)^\beta k^\gamma \Gamma^\delta_\rho (k' - k) \Big]$$

Fitting procedure

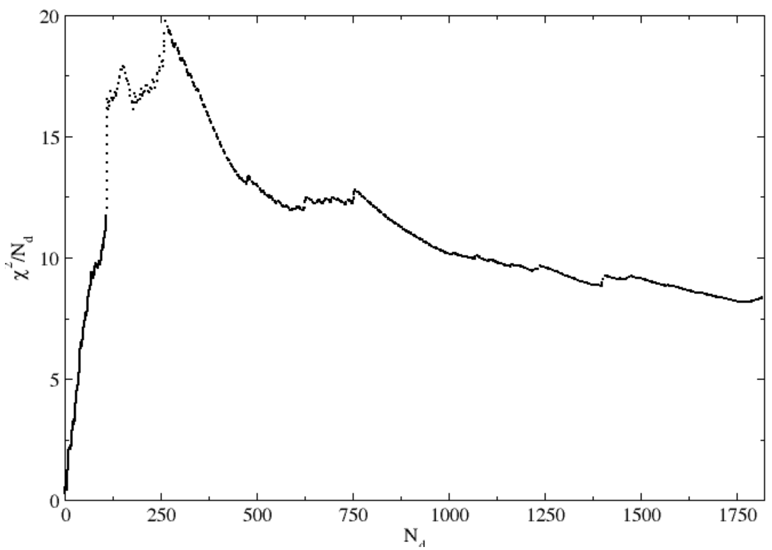
- Fit χ^2 via conjugate gradient/simplex methods

$$\chi^2 = \sum_i \left(\frac{y_{\text{calc}}(\alpha_p; \theta_i, E_i) - y_{\text{obs}}(\theta_i; E_i)}{\delta y(\theta_i; E_i)} \right)^2$$

- Simultaneous fit of πN and ωN data
  - πN→πN: PWA from SAID [energy dep. SP06 solution]
  - $\pi^- p$→ωn: unp. DCS from Nimrod [Karami et.al. '79]
  - Υp→ωp: unp. DCS from SAPHIR [Barth et.al. '03]
  - ΥN→πN: unp. DCS world data (from SAID)
  - ΥN→πN: PBA Σ(E) world data (from SAID)
- Complete data set
  - ~1800 data points

$$\text{DCS} = \frac{d\sigma}{d\Omega}$$

$$\text{PBA} = \Sigma = \frac{\frac{d\sigma_\perp}{d\Omega} - \frac{d\sigma_{||}}{d\Omega}}{\frac{d\sigma_\perp}{d\Omega} + \frac{d\sigma_{||}}{d\Omega}}$$



Fit parameters (bare non-resonant)

m_N	938.5
m_π	138.5
m_η	547.5
m_Δ	1300.0
m_σ	898.6
m_ρ	811.7
m_ω	782.6

$G_0(k;E)$

$f^2_{\pi NN}/4\pi$	0.08
$f_{\pi N\Delta}$	2.206
$f_{\eta NN}$	3.889
$g_{\rho NN}$	8.721
κ_ρ	2.654
$g_{\omega NN}$	8.100
κ_ω	1.020
$g^t_{\omega NN}$	1.298
κ^t_ω	1.002

$g_{\sigma NN}$	6.815
$g_{\rho\pi\pi}$	4.000
$f_{\pi\Delta\Delta}$	1.000
$f_{\rho N\Delta}$	7.516
$g_{\sigma\pi\pi}$	2.353
$g_{\omega\pi\rho}$	6.956
$g_{\rho\Delta\Delta}$	3.302
$\kappa_{\rho\Delta\Delta}$	2.000
m_σ	500.1 MeV

$\Lambda_{\pi NN}$	810
$\Lambda_{\pi N\Delta}$	829
$\Lambda_{\rho NN}$	1087
$\Lambda_{\rho\pi\pi}$	1094
$\Lambda_{\omega NN}$	1523
$\Lambda^t_{\omega NN}$	589
$\Lambda_{\eta NN}$	624
$\Lambda_{\sigma NN}$	781
$\Lambda_{\rho N\Delta}$	1200
$\Lambda_{\pi\Delta\Delta}$	600
$\Lambda_{\sigma\pi\pi}$	1200
$\Lambda_{\omega\pi\rho}$	600
$\Lambda_{\rho\Delta\Delta}$	600

$v_{M'B',MB}$

Fixed in 5 channel fits: Phys.Rev.C76:065201,2007

Fit parameters (bare resonance)

New resonance required to fit D_{15}

compare to PDG $D_{15}(2200)$

#	$L_{TJ}^{(\pi N)}$	$M^{(0)}$	k_{N^*}	πN	ηN	$\pi\Delta$		σN	ρN			ωN			$A_{\frac{1}{2}}$	$A_{\frac{3}{2}}$
1	S_{11}	1800	99.9	7.049	9.100	−1.853		−2.795	2.028	0.027		−3.761	0.405		83.8	
2	S_{11}	1880	100.0	9.824	0.600	0.045		1.139	−9.518	−3.014		−0.516	0.366		−40.3	
3	S_{31}	1850	20.7	5.275		−6.175			−4.299	5.638					129.4	
4	P_{11}	1763	76.1	3.912	2.621	−9.905		−7.162	−5.157	3.456		−3.362	5.231		−21.8	
5	P_{11}	2037	22.1	9.998	3.661	−6.952		8.629	−2.955	−0.945		−2.095	1.043		−27.5	
6	P_{13}	1711	76.4	3.270	−0.999	−9.988	−5.038	1.015	−0.003	2.000	−0.081	5.737	−0.548	−0.204	−12.4	−63.8
7	P_{31}	1900	100.0	6.803		2.118			9.915	0.153					54.1	
8	P_{33}	1603	83.9	1.312		1.078	1.524		2.012	−1.249	0.379				−78.6	−131.2
9	P_{33}	1391	−93.3	1.319		2.037	9.538		−0.317	1.036	0.766				−6.7	5.3
10	D_{13}	1899	−35.3	0.445	−0.017	−1.950	0.978	−0.482	1.133	−0.314	0.179	−0.081	3.740	0.230	88.8	−71.4
11	D_{13}	1988	−41.7	0.465	0.357	9.919	3.876	−5.499	0.289	9.628	−0.141	7.883	9.900	3.386	−54.5	46.8
12	D_{15}	1898	0.0	0.312	−0.096	4.792	0.020	−0.455	−0.179	1.249	−0.101	0.625	1.086	−0.156	33.0	40.3
13	$\boldsymbol{D_{15}}$	2334	9.7	0.167	−0.106	0.190	−0.098	−0.075	−0.530	0.228	0.099	−0.150	−1.990	0.199	12.6	87.4
14	D_{33}	1976	36.7	0.945		3.999	3.997		0.162	3.949	−0.856				95.9	−6.1
15	F_{15}	2187	92.1	0.062	0.000	1.040	0.005	1.527	−1.035	1.607	−0.026	−0.046	2.212	0.078	−99.8	−68.1
16	F_{35}	2162	−84.2	0.174		−2.961	−1.093		−0.076	8.034	−0.061				−61.0	−103.4
17	F_{37}	2137	−100.0	0.254		−0.316	−0.023		0.100	0.100	0.100				45.9	47.7

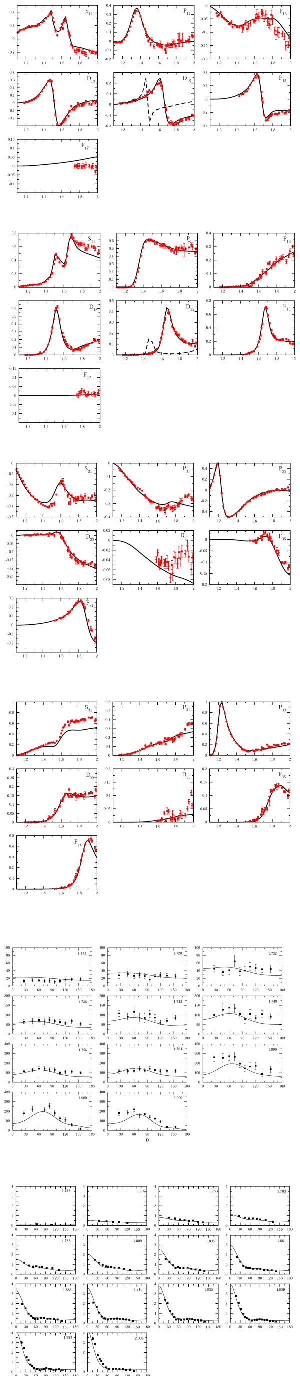
fixed in 5 channel πN→πN fit

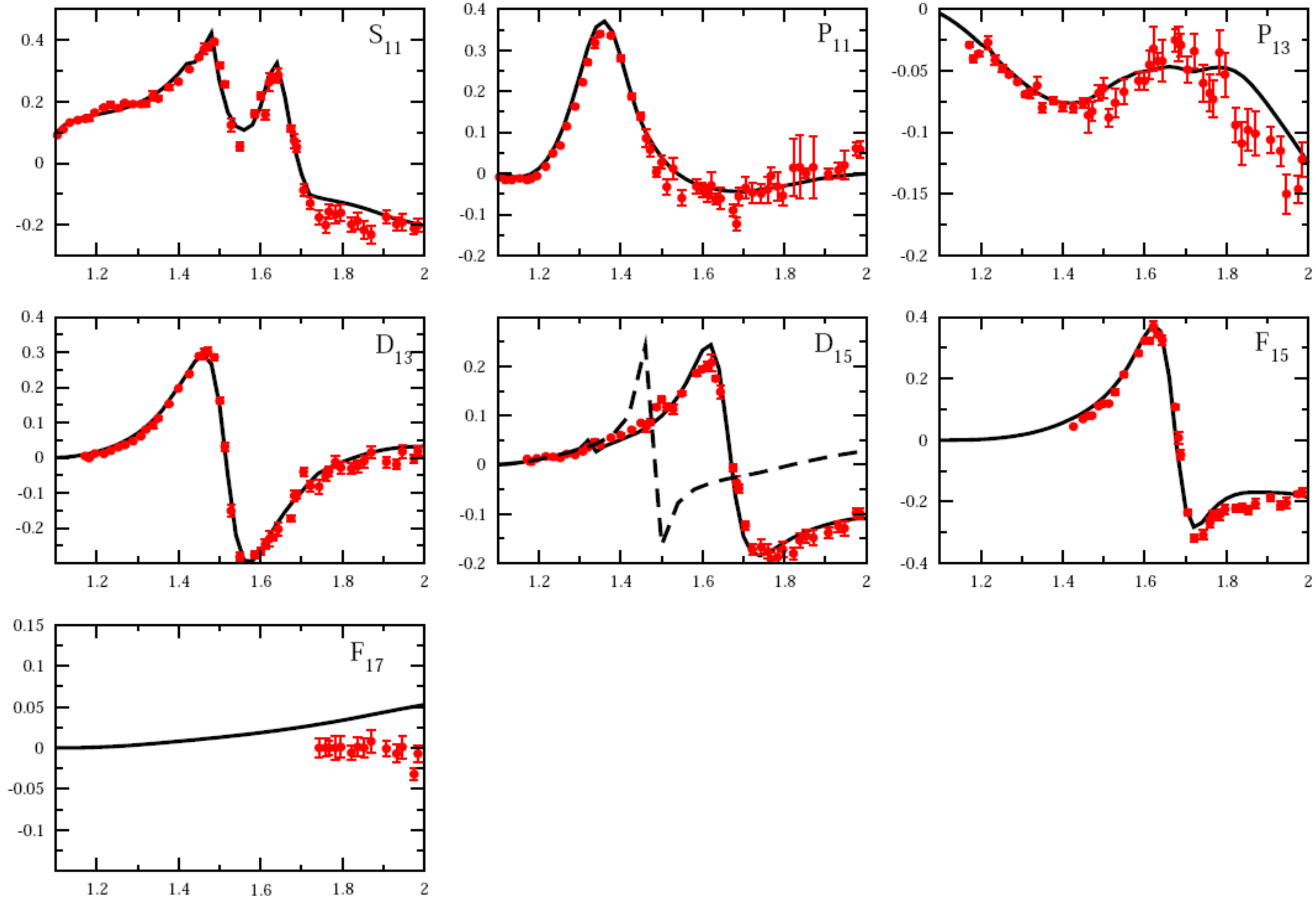
results of present study

$$\Gamma^{JT}_{LSMB,N^*}(k) = \zeta_{MB}\frac{1}{(2\pi)^{3/2}}\frac{1}{\sqrt{m_N}}C^{JT}_{N^*LSMB}\left(\frac{k}{m_\pi}\right)^L f^{JT}_{N^*LSMB}(k)$$

$$\Gamma^{JT}_{N^*,\lambda_\gamma\lambda_N T_{N,z}}(q) = \frac{1}{(2\pi)^{3/2}}\sqrt{\frac{m_N}{E_N(q_R)}}A^{JT}_{\lambda T_{N,z}}\sqrt{\frac{q_R}{q}}\,g^{JT}_{N^*\lambda T_{N,z}}(q)\,\delta_{\lambda,\lambda_\gamma-\lambda_N}$$

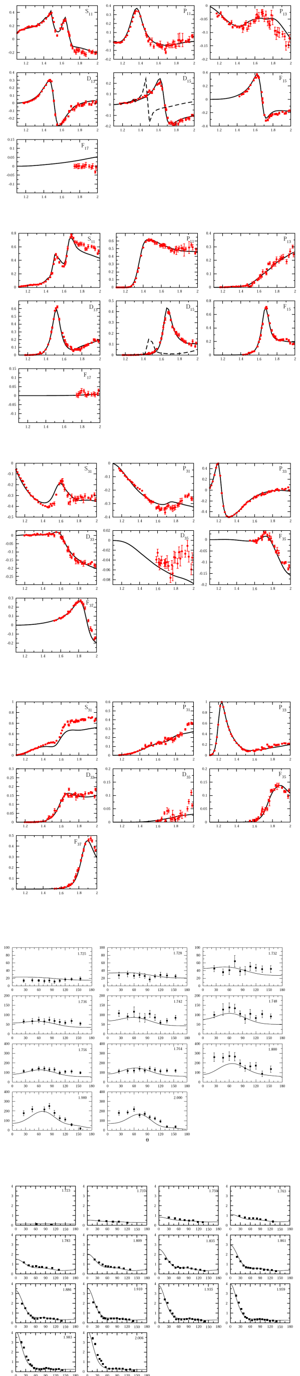
Stage 1: πN→πN & πN→ωN & γN→ωN

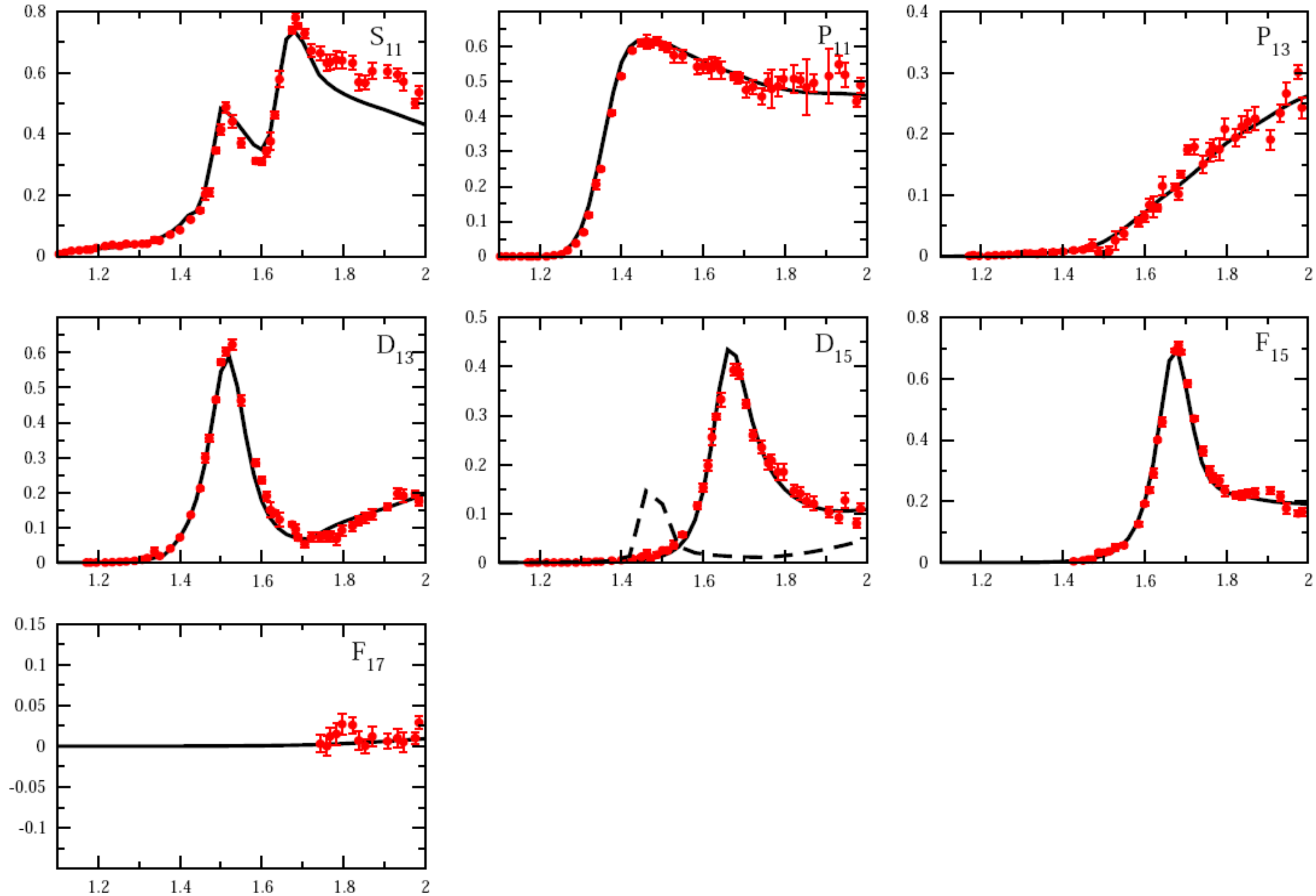




Real Part: T=1/2 cf. SAID PRC74(06)

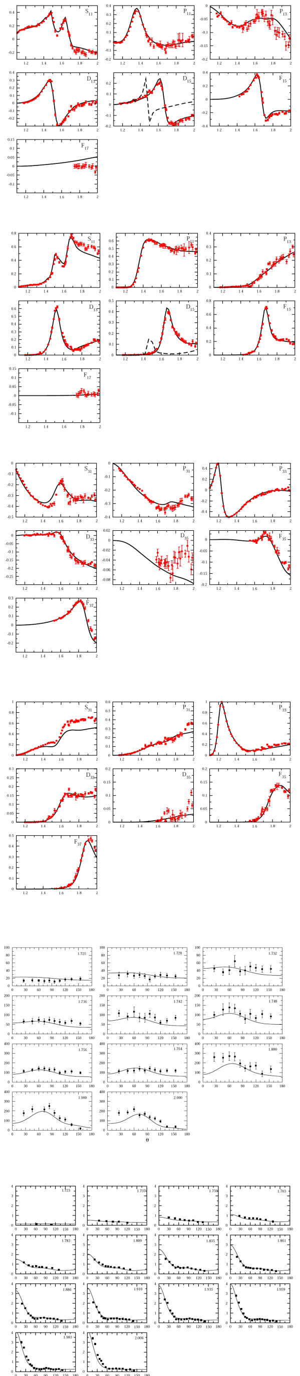
Stage 1: πN→πN & πN→ωN & γN→ωN

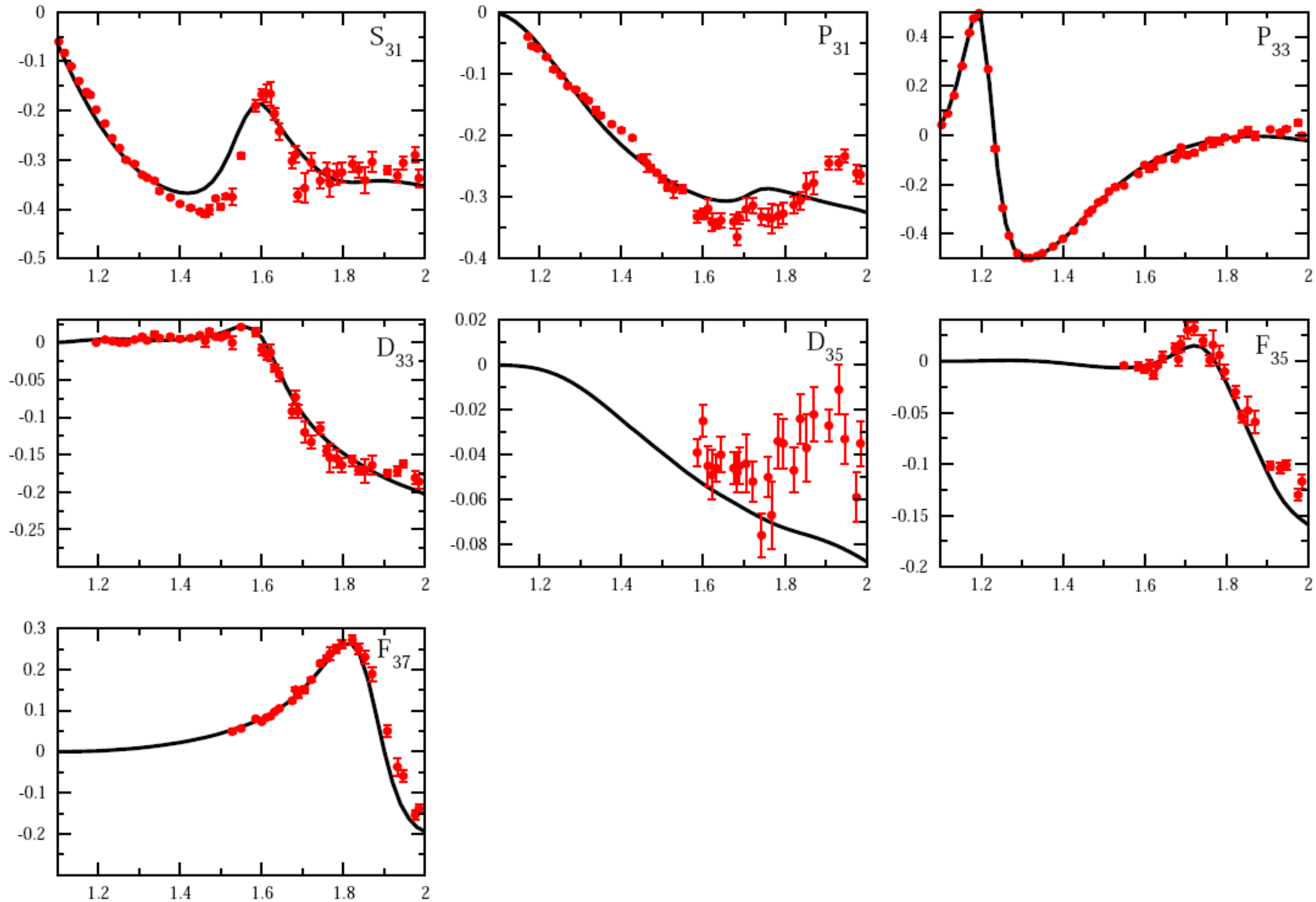




Imag. Part: T=1/2 cf. SAID PRC74(06)

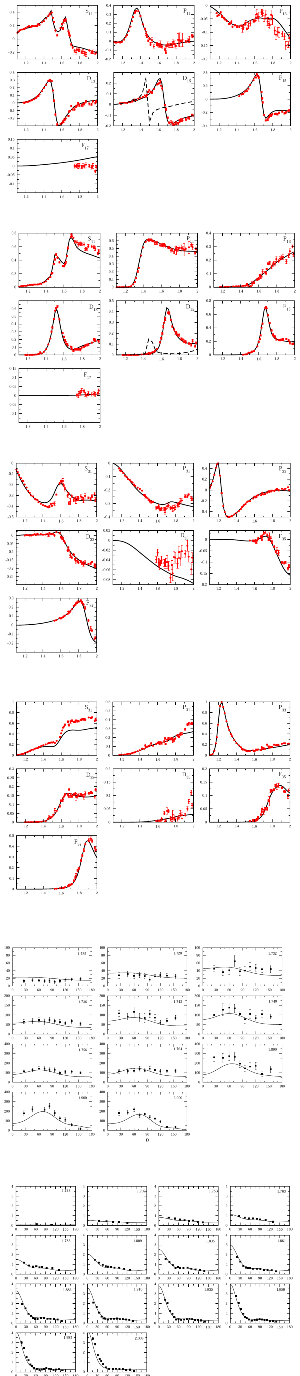
Stage 1: πN→πN & πN→ωN & γN→ωN

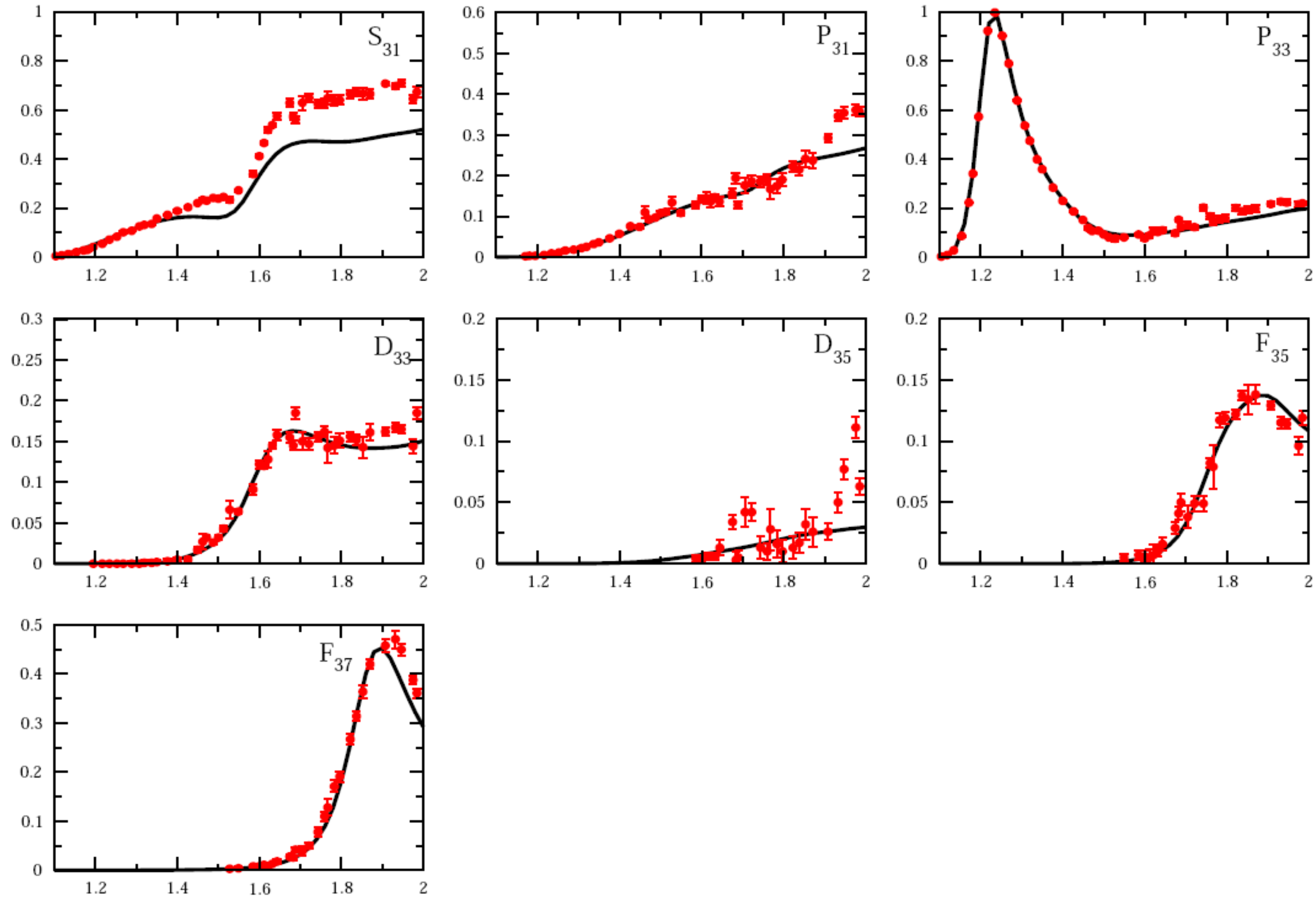




Real Part: T=3/2 cf. SAID PRC74(06)

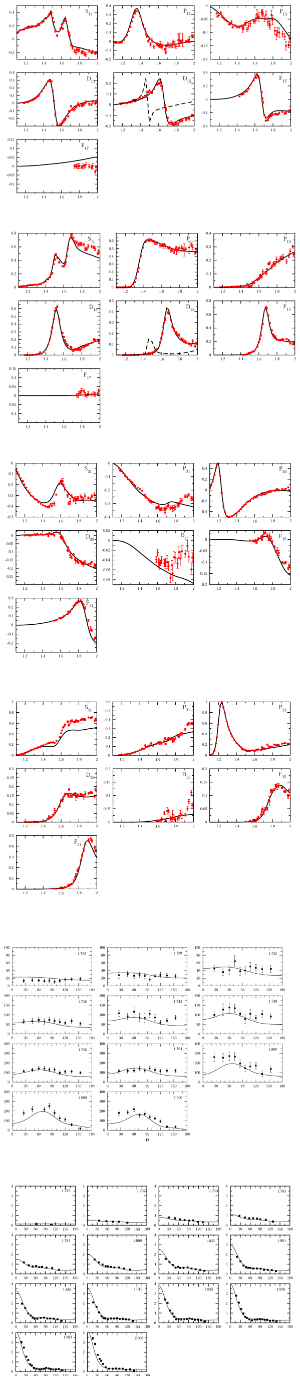
Stage 1: πN→πN & πN→ωN & γN→ωN

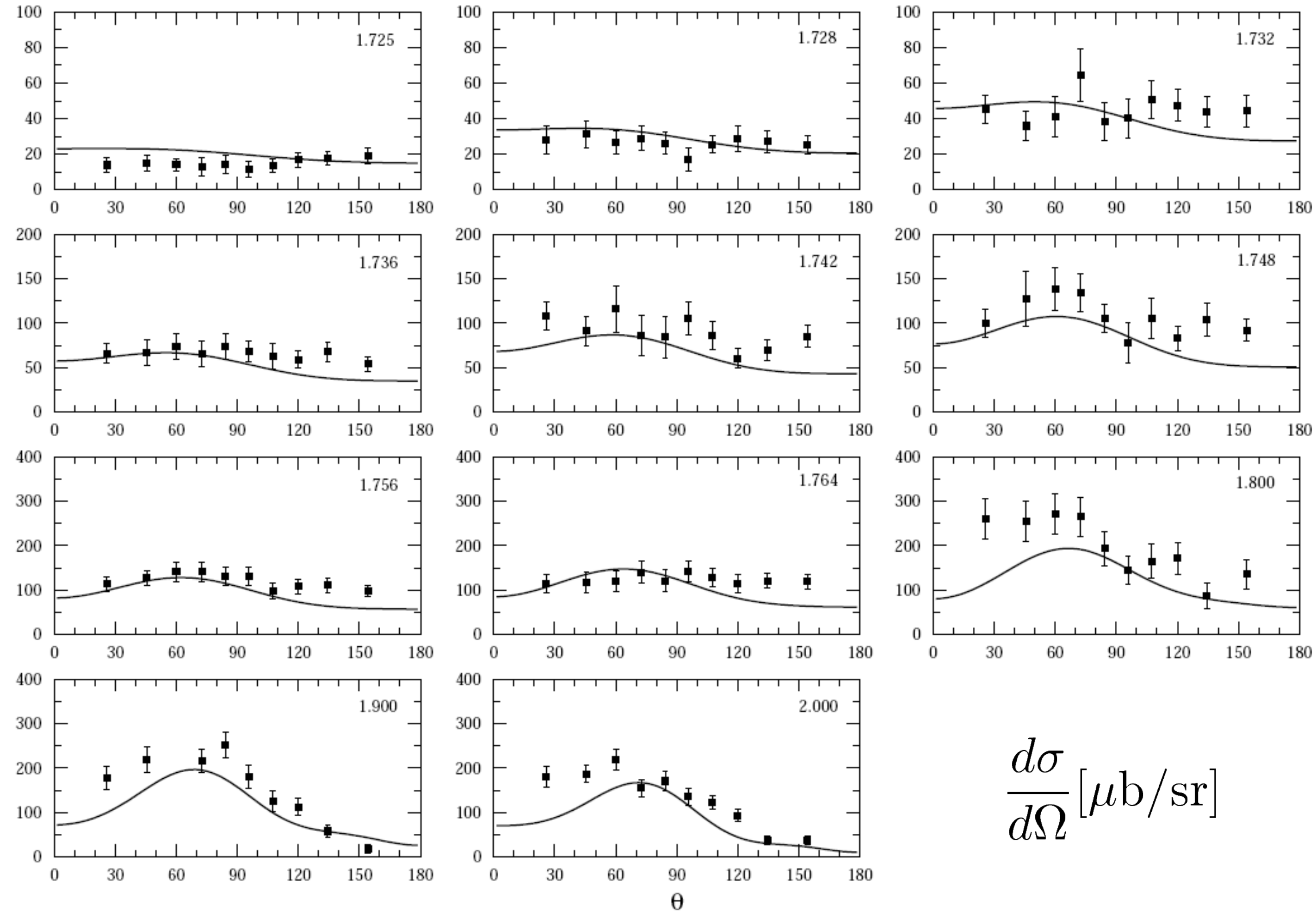




Imag. Part: T=3/2 cf. SAID PRC74(06)

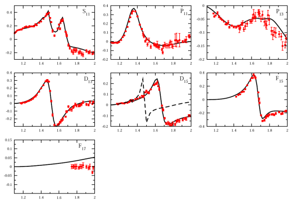
Stage 1: πN→πN & πN→ωN & γN→ωN

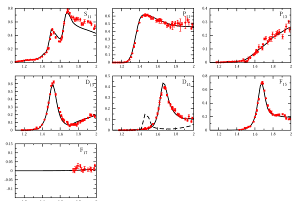


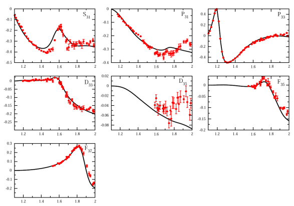


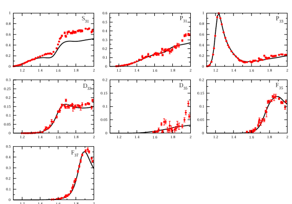
$$\frac{d\sigma}{d\Omega}[\mu\mathrm{b/sr}]$$

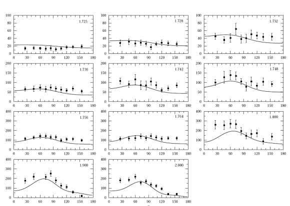
Stage 1: πN→πN & πN→ωN & γN→ωN

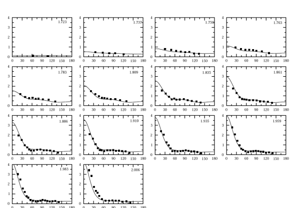


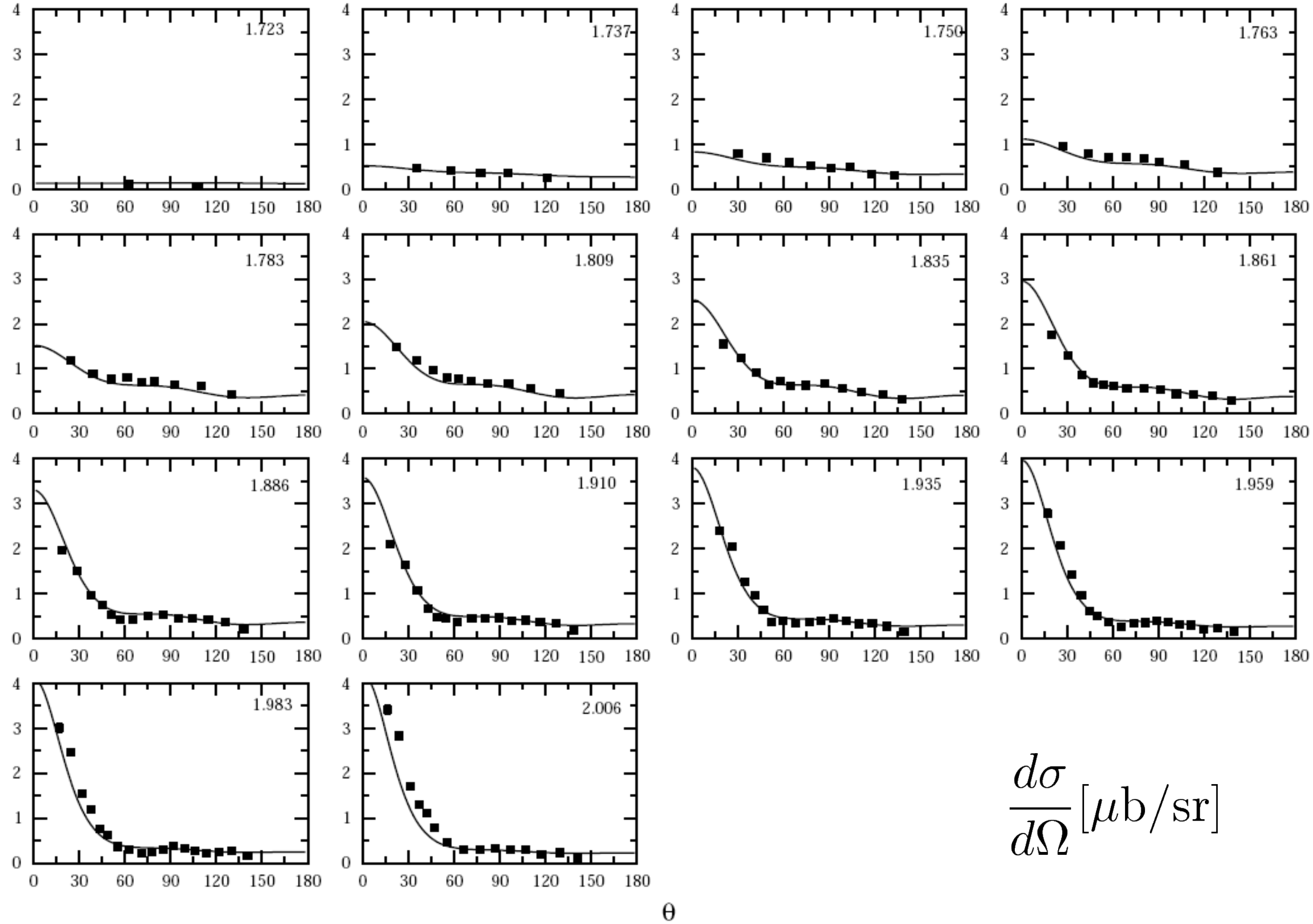




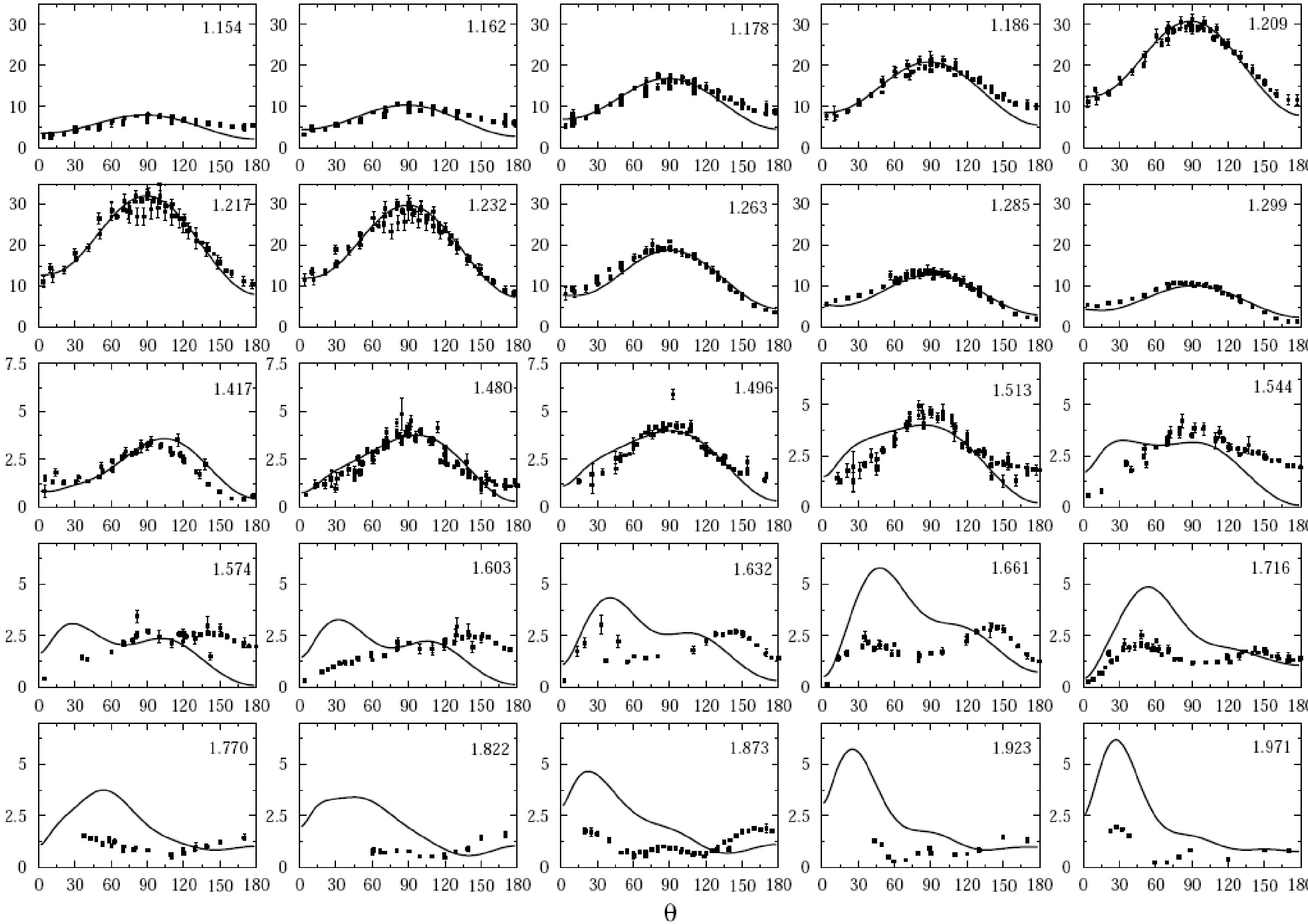






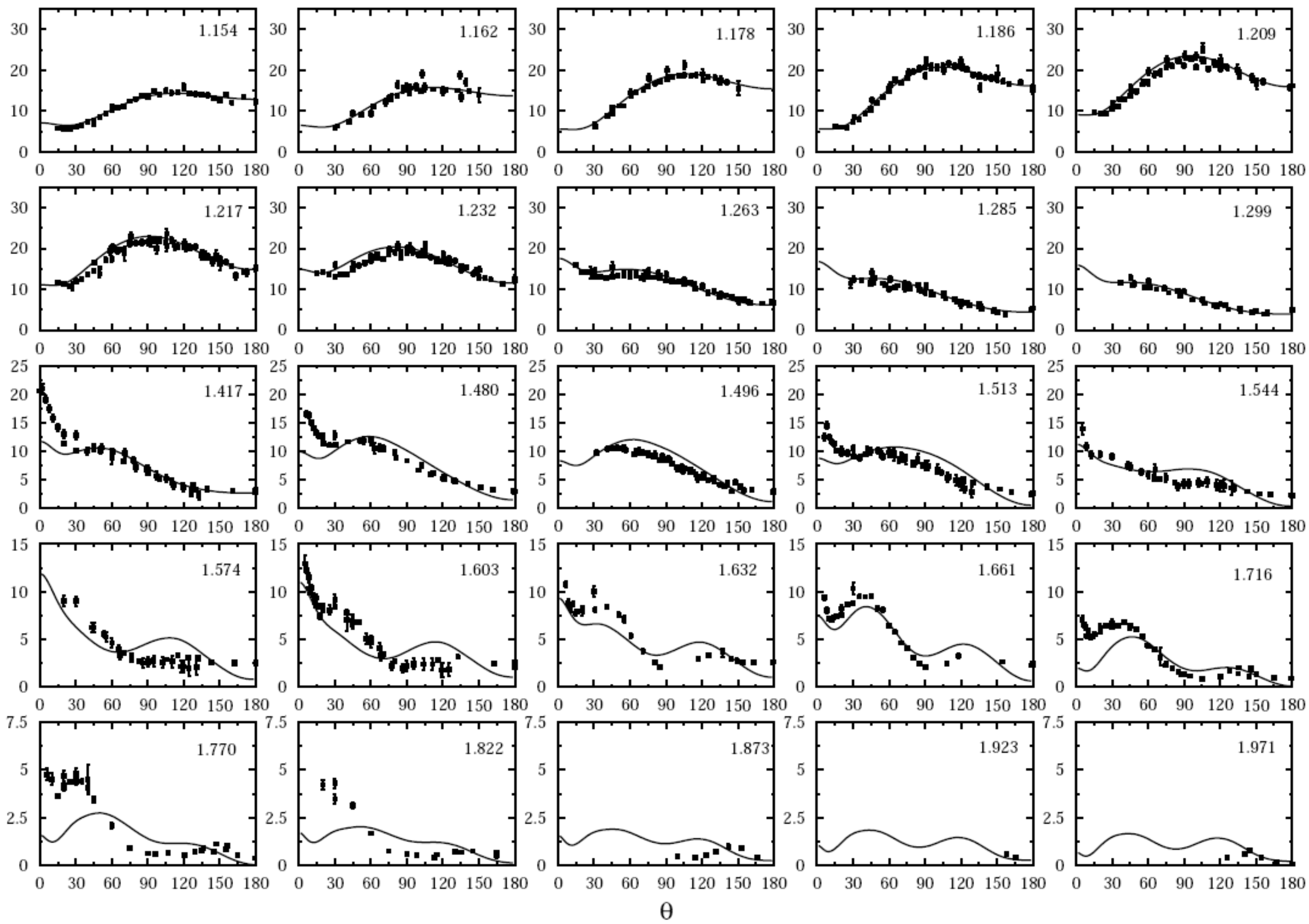


Stage 2: DCS Yp→π^0p



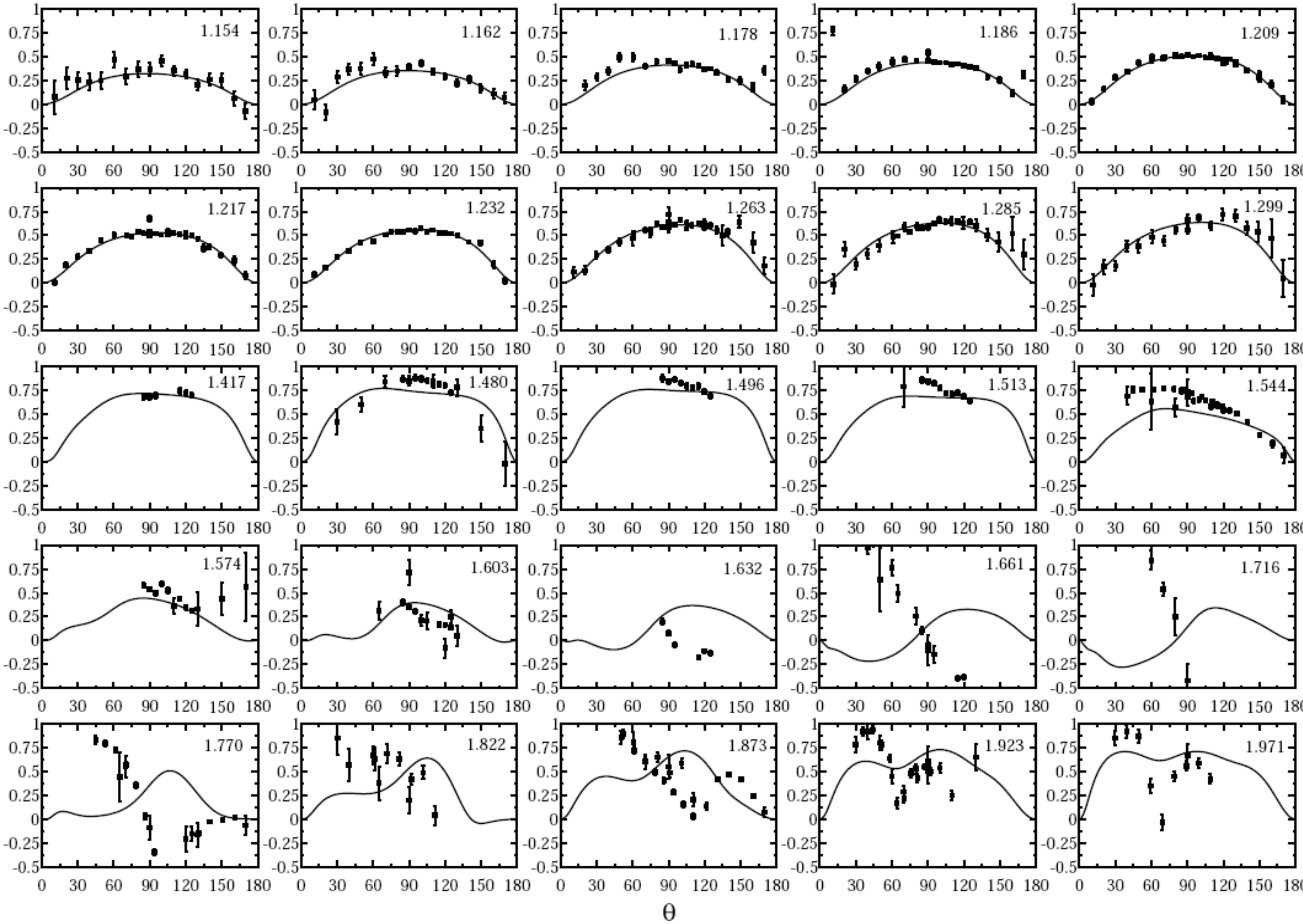
- Quality: χ^2/N~1 for E<~1.5 GeV
- Possible improvements
  - global fit; other non-res mechanisms; more resonances; ππN

Stage 2: DCS $\gamma p \to \pi^+ n$



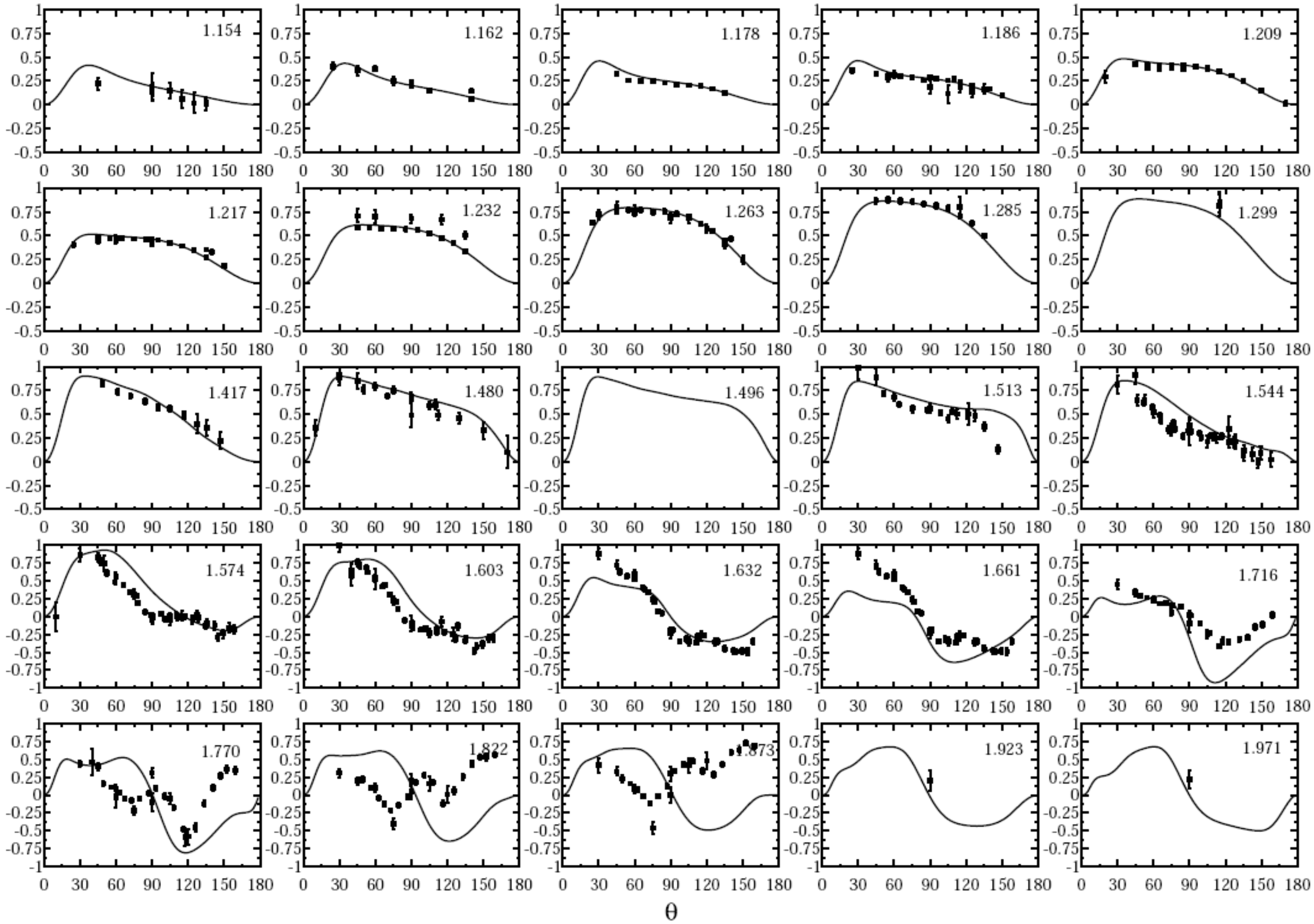
- Quality: $\chi^2/N \sim 1$ for E<~1.7 GeV
  - ➔ angular dependence okay at high E

Stage 2: PBA Σ_0 $\gamma p \to \pi^0 p$



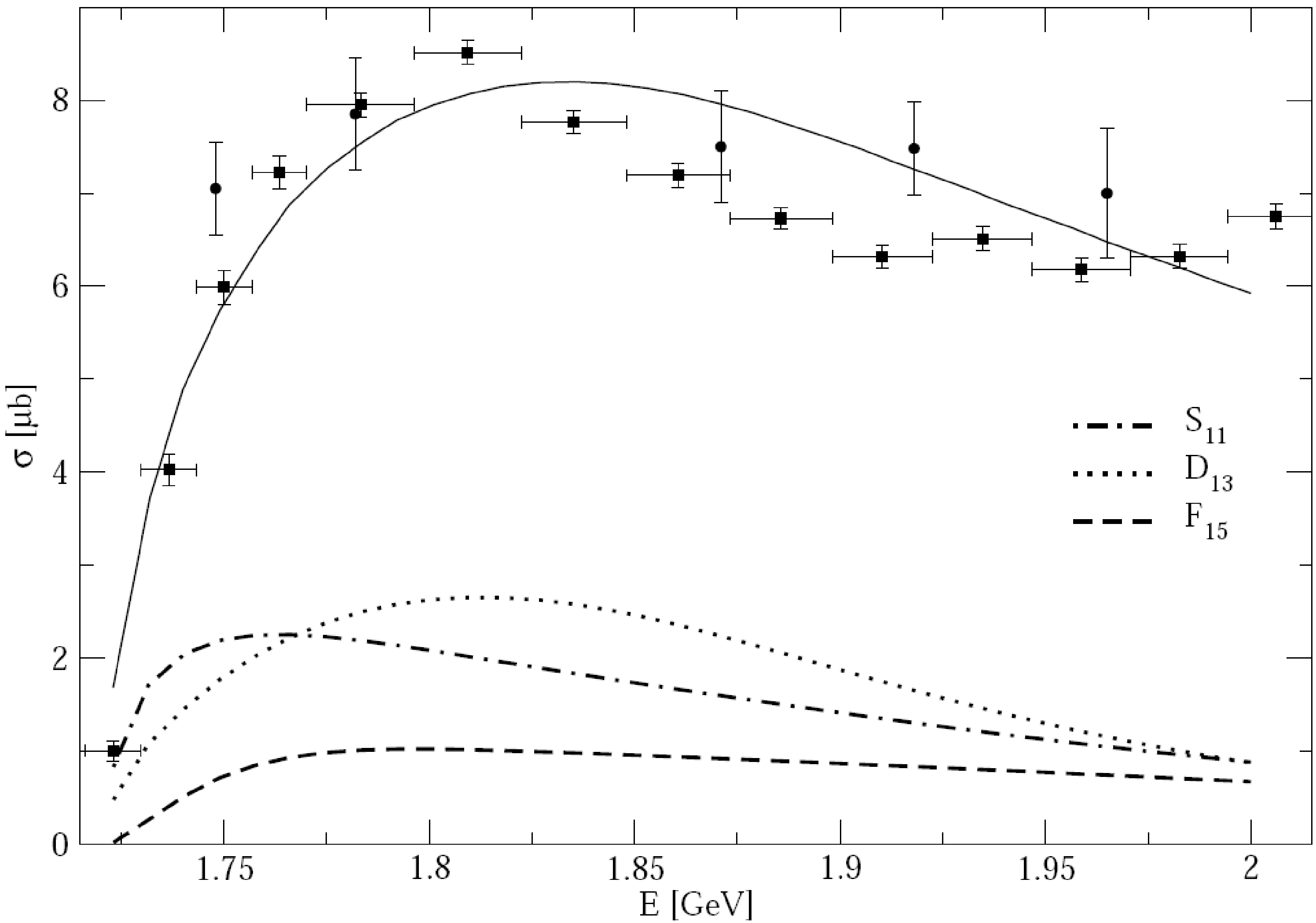
- Quality: $\chi^2/N \sim 1$ for E<~1.6 GeV

Stage 2: PBA Σ_+ $\gamma p \to \pi^+ n$

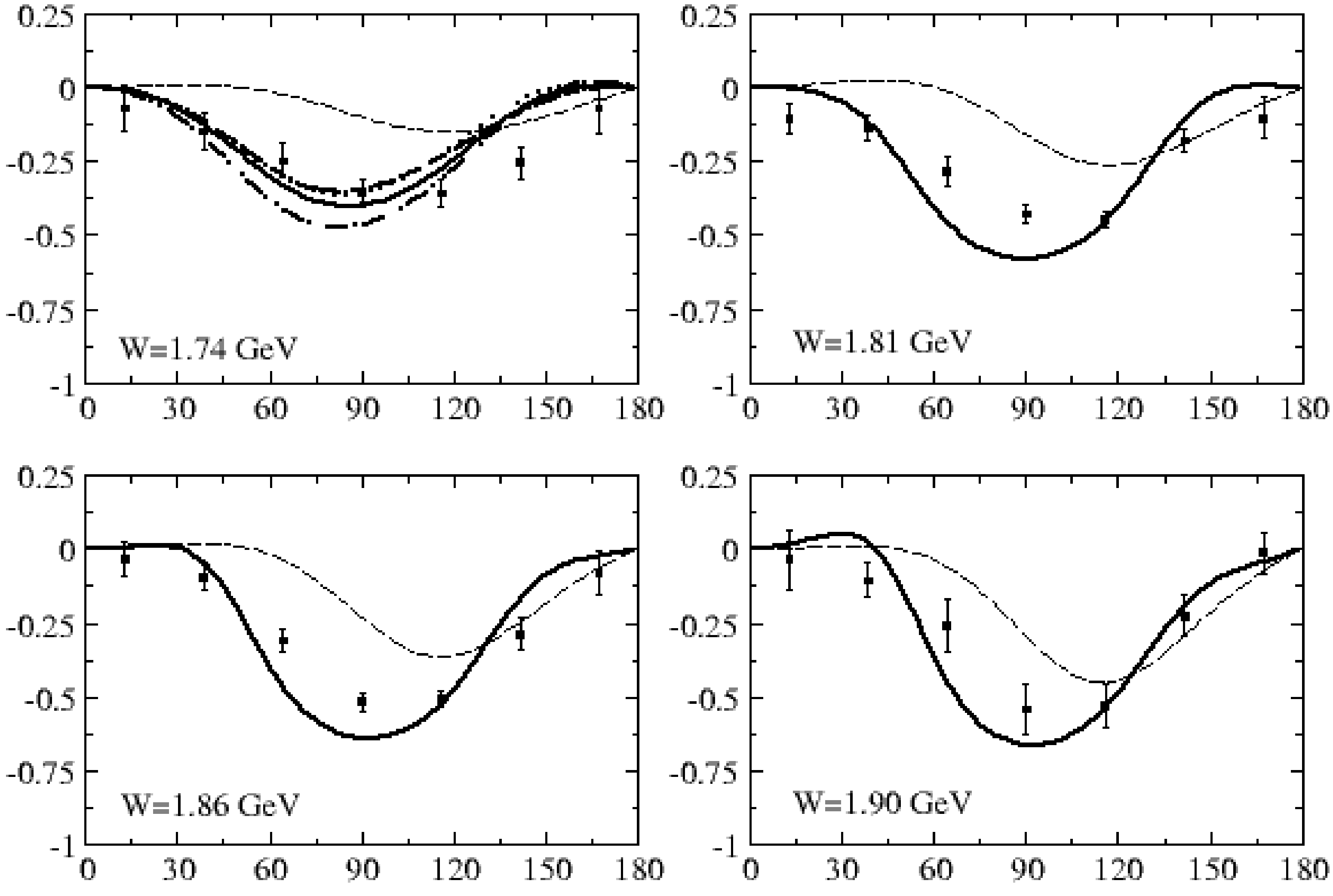


- Quality: $\chi^2/N \sim 1$ for E<~1.7 GeV
  - ➔ angular dependence poor at high E

Total cross section $\sigma_{\gamma p \to \omega p}$



Prediction for Σ_ω



Spin density matrix elements: $\rho^0_{\lambda\lambda'}$

- ω decay amplitude

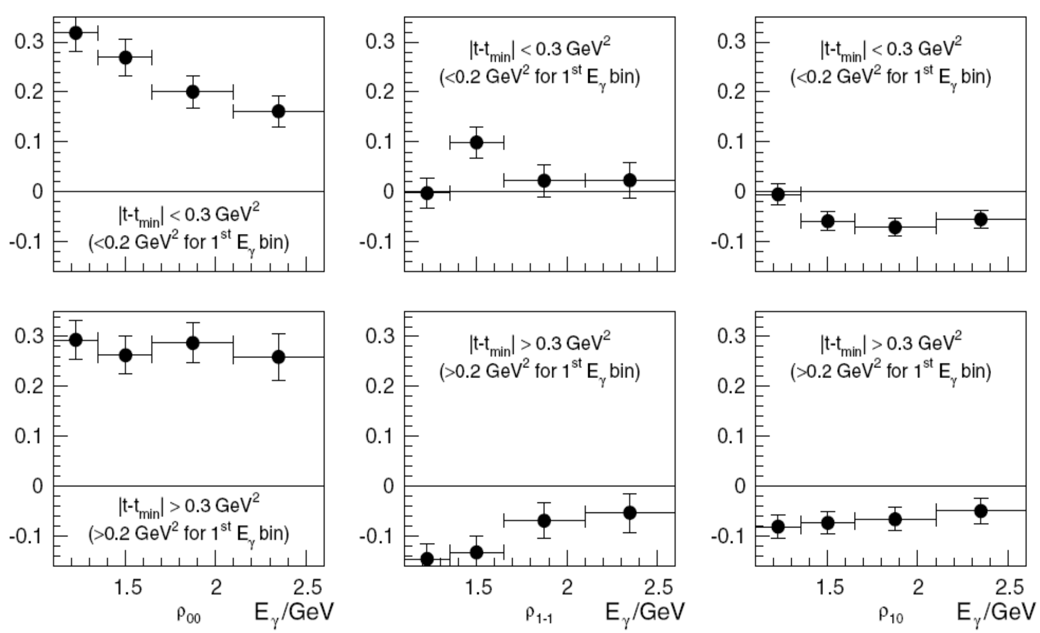
$$
\begin{aligned}
A_{\omega\to\pi^+\pi^-\pi^0} &= iN\epsilon_{\mu\nu\alpha 0}p^{\nu}_{\pi^+}p^{\alpha}_{\pi^-}w_\omega\epsilon^{\mu}(q,m_\omega)\\
&= iNw_\omega\epsilon^{i}(q,m_\omega)\epsilon_{ijk}p^{j}_{\pi^+}p^{k}_{\pi^-}\\
&= iNw_\omega\left(\vec{p}_{\pi^+}\times\vec{p}_{\pi^-}\right)\cdot\vec{\epsilon}(q,m_\omega)
\end{aligned}
$$

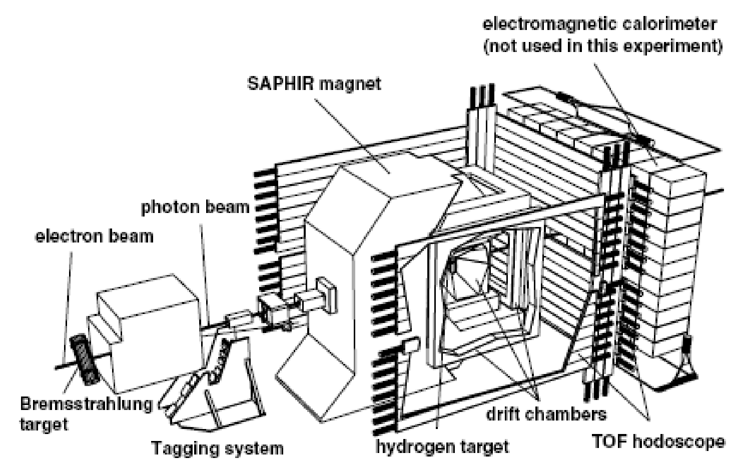
- ω decay angular distribution (unpolarized photons)

$$\rho(V) = T\,\rho(\gamma)\,T^{\dagger}$$

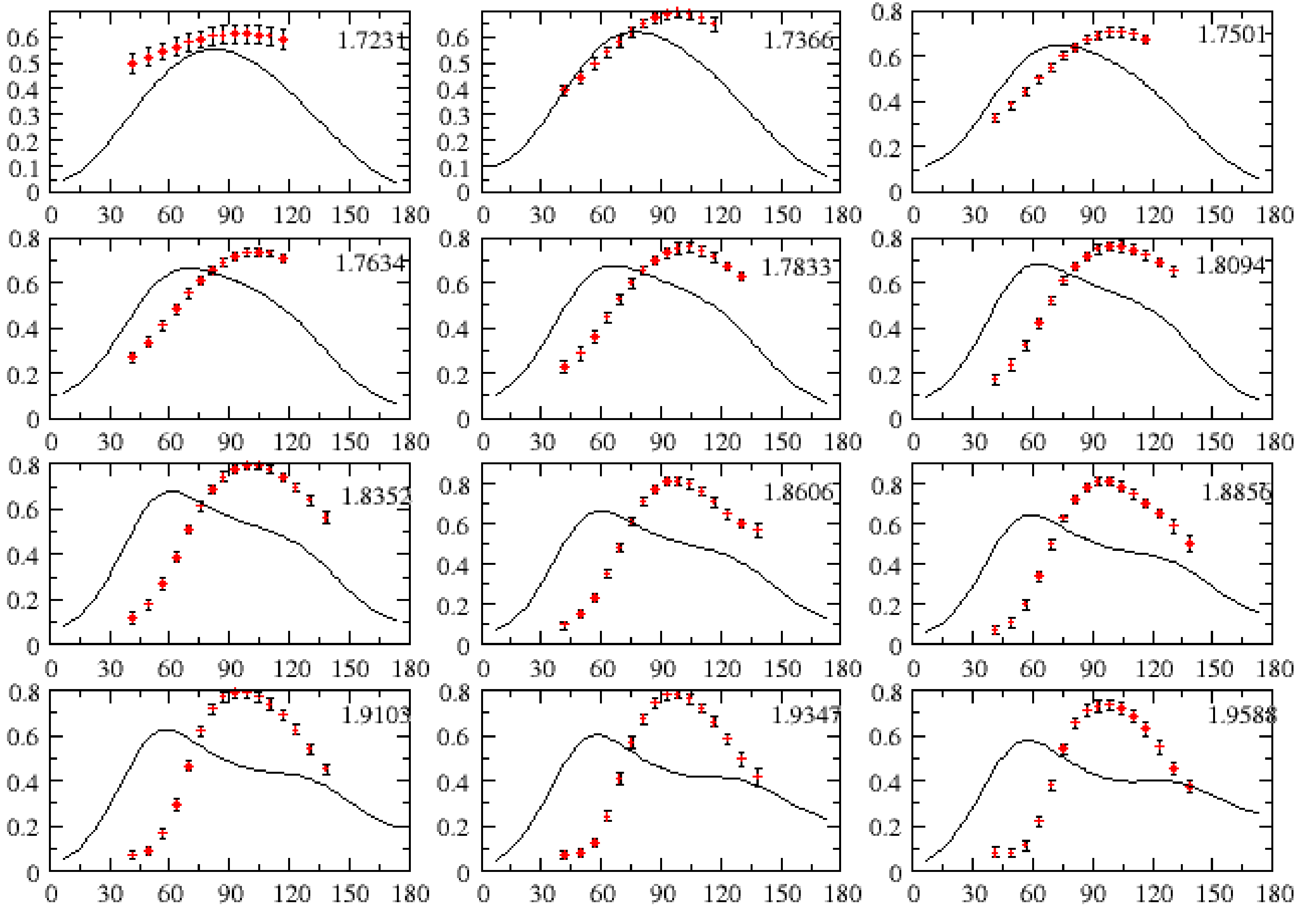
$$W^0(\cos\theta,\phi) = \frac{3}{4\pi}\left(\tfrac{1}{2}(1-\rho^0_{00}) + \tfrac{1}{2}(3\rho^0_{00}-1)\cos^2\theta\right.$$

$$\left. - \sqrt{2}\,\mathrm{Re}\,\rho^0_{10}\sin 2\theta\cos\phi - \rho^0_{1-1}\sin^2\theta\cos 2\phi\right)$$





Prediction for ρ^0_{00}



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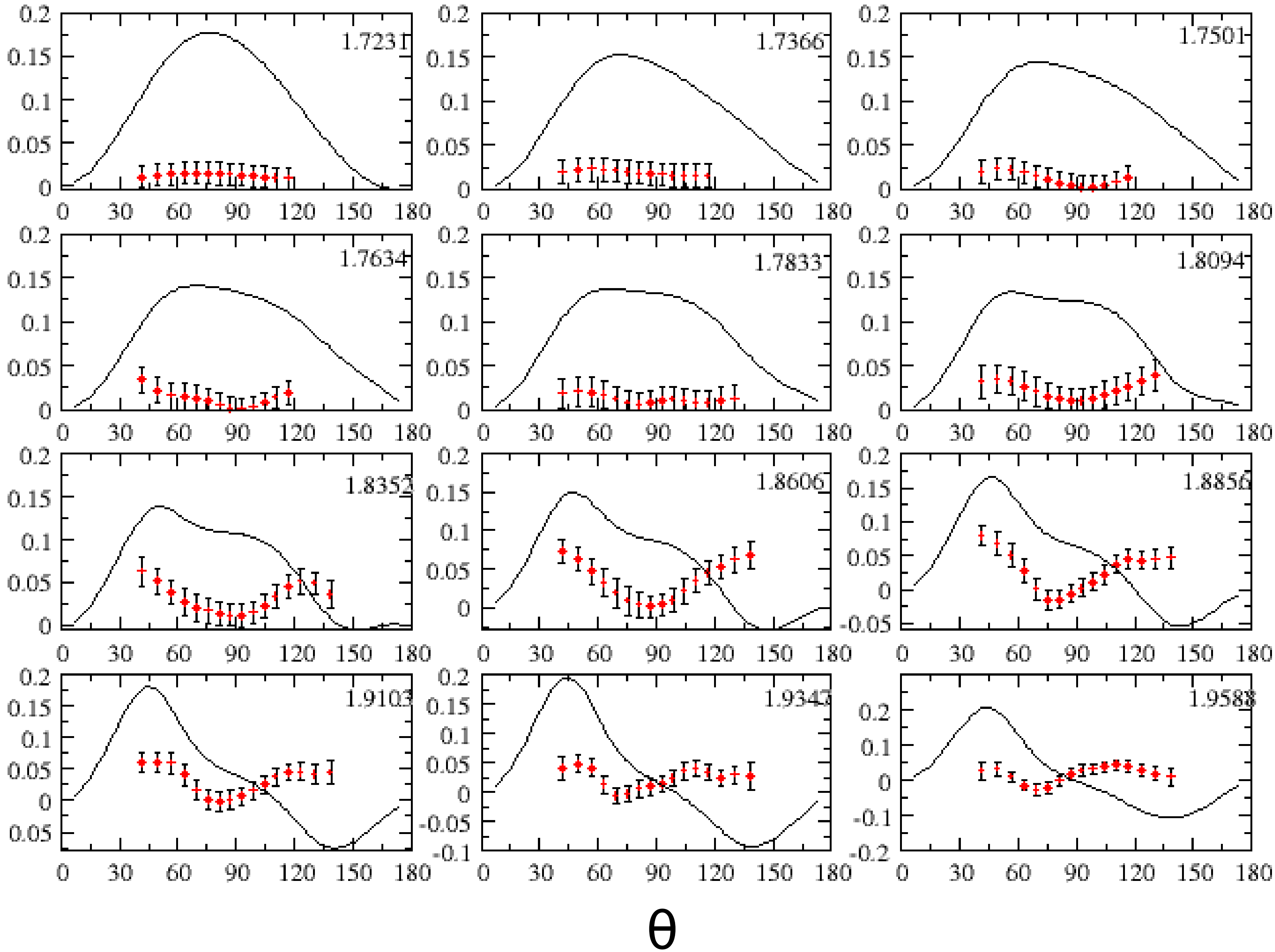
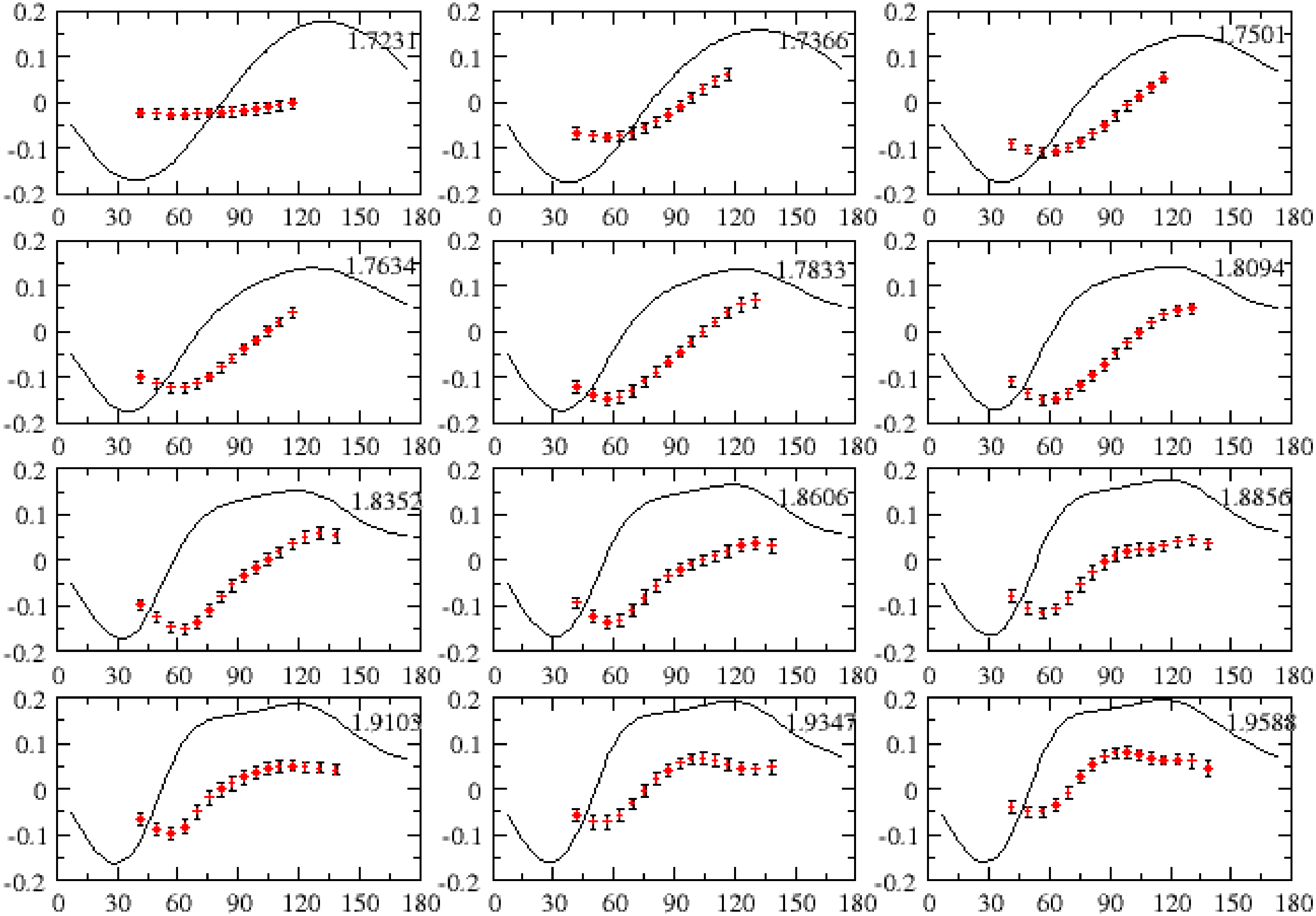
Prediction for $\rho^0_{1,-1}$



Prediction for ρ^0_{10}

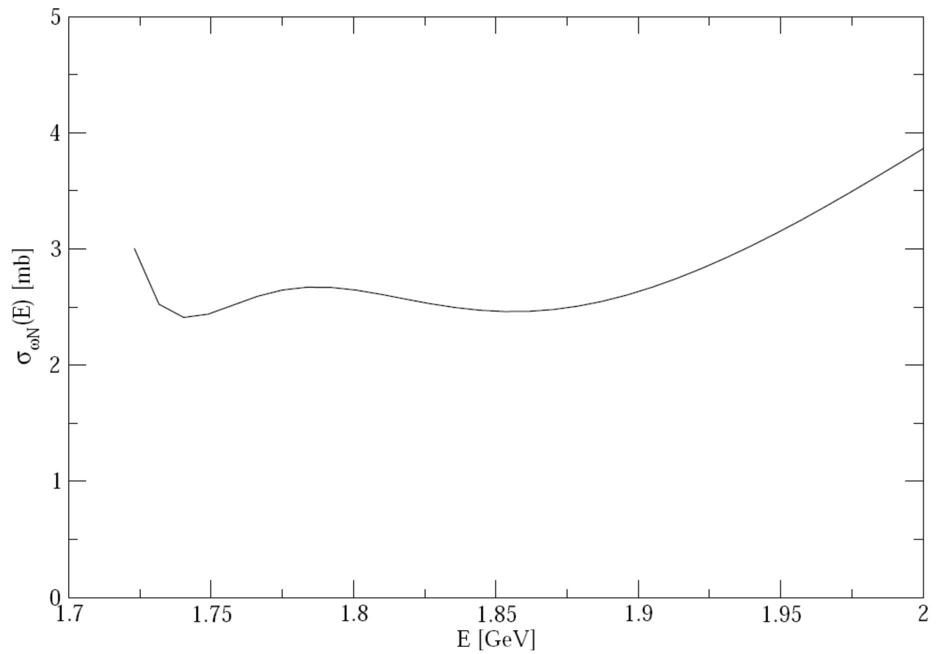


θ

ωN interactions

Average scattering length:

$$\bar{a} \;=\; \tfrac{1}{3}\bar{a}(J=\tfrac{1}{2}) + \tfrac{2}{3}\bar{a}(J=\tfrac{3}{2})$$



Klingl/Weise: $a = 1.6 + \mathrm{i}0.30$ fm

Lutz: $\bar{a} = -0.44 + \mathrm{i}0.20$

Giessen: $\bar{a} \;=\; -0.026 + \mathrm{i}0.28$

$$a_J = \lim_{E \to m_\omega + m_N} \frac{\pi m_\omega m_N}{m_\omega + m_N} T^J_{0J\omega N, 0J\omega N}(E)$$

$$a_{\frac{1}{2}} = [-0.0454 - i0.0695]\ \mathrm{fm},$$

$$a_{\frac{3}{2}} = [0.180 - i0.0597]\ \mathrm{fm},$$

$$\sigma_{\omega N}(E \to m_\omega + m_N) = 4\pi(|a_{\frac{1}{2}}|^2 + 2|a_{\frac{3}{2}}|^2)/3$$

$$\overline{a} = 0.12 - i0.07\mathrm{fm}$$

Conclusion

- Coupled channel approach
  - need improvements COM E>~1.6 GeV
    - outlined possible solutions; most likely $\pi\pi N$ is important
  - prediction of polarized observables appear only loosely constrained by fits to unpolarized data
- Outstanding questions pertaining to modeling
  - can model dependencies be identified & controlled?
  - connection to model independent results of χPT?
  - can contact with Lattice QCD be made?
    - “valence” QCD (K.-F. Liu et. al.)

FIN