Covariant Faddeev calculation of N-Δ(1232) form factors

Ian Cloët
(University of Washington)

Collaborators

Wolfgang Bentz (Tokai Uni) & Anthony Thomas (JLab)

[Craig Roberts (ANL)]

Electromagnetic N-N Transition Form Factors Workshop*

14th October 2008

Themes

❖ Themes
❖ NJL model
❖ Baryons . . .
❖ $N \to \Delta$ FFs
❖ Constituents
❖ Nucleon FFs
❖ Scalar Diquark FF
❖ Axial-Vector FF
❖ Nucleon FFs
❖ Delta FFs
❖ $N \to \Delta$ FFs
❖ Off-Shell
❖ Conclusion

- Demonstrate utility of Nambu–Jona-Lasinio model
  - Nucleon, Delta, $N \to \Delta$ form factors
  - quark distributions, etc
- Highlight important challenges in modelling form factors
  - in particular the pion cloud
- Results: Nucleon, Delta form factors
- Preliminary Results: $N \to \Delta$ Transition Form Factors
- Examine Off-Shell Form Factors

Nambu–Jona-Lasinio Model

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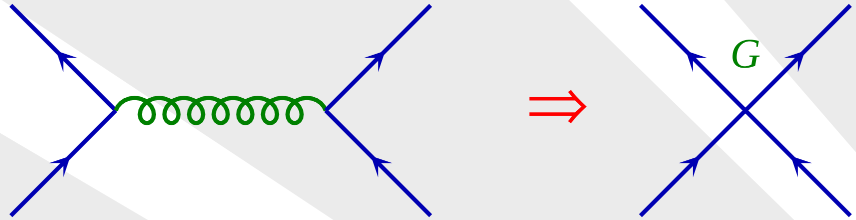
- Low energy chiral effective theory of QCD



- Investigate the role of quark degrees of freedom.
- Much in common with DSE
- Lagrangian has same symmetries as QCD:
  - Importantly chiral symmetry and DχSB,
    $\longrightarrow$ Dynamically generated quark masses,
    $\longrightarrow$ Non-zero chiral condensate.
- Lagrangian (Γ = Dirac, colour, isospin matrices)

$$\mathcal{L}_{NJL} = \overline{\psi}\left(i\partial\!\!\!/ - m\right)\psi + G\left(\overline{\psi}\Gamma\psi\right)^2$$

Baryons in the NJL model

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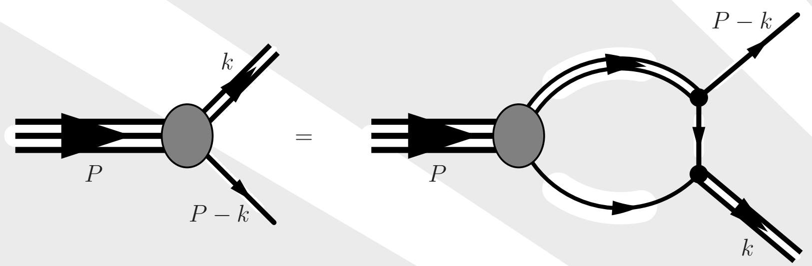
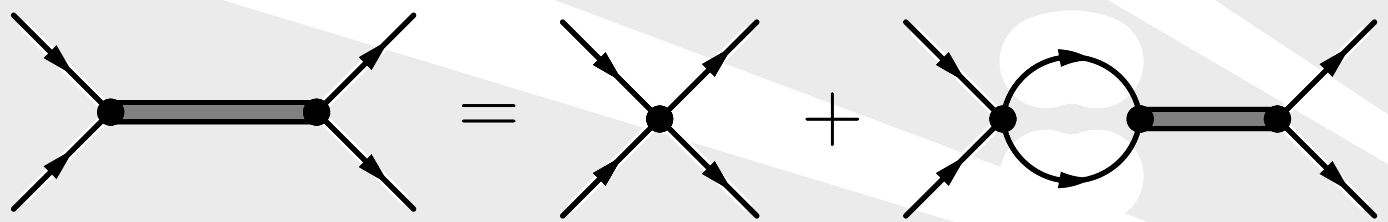
- Baryons approximated as quark-diquark bound states.
- Use relativistic Faddeev approach:



- Diquark - bound state of two quarks:
- Solve Bethe-Salpeter equation for diquark.



- We include scalar and axial-vector diquarks.
- Static Approximation: $S_{ex}(k) \to -1/M$.

Regularization

❖ Themes
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- Proper-time regularization

$$\frac{1}{X^n} = \frac{1}{(n-1)!} \int_0^\infty d\tau\, \tau^{n-1}\, e^{-\tau X}$$
$$\longrightarrow \quad \frac{1}{(n-1)!} \int_{1/(\Lambda_{UV})^2}^{1/(\Lambda_{IR})^2} d\tau\, \tau^{n-1}\, e^{-\tau X} .$$

- Λ_{IR} eliminates unphysical thresholds for the nucleon to decay into quarks: $\to$ simulates confinement.

  ❖ G. Hellstern, R. Alkofer and H. Reinhardt, Nucl. Phys. A **625**, 697 (1997).

- Needed for: nuclear matter saturation, Δ baryon.

  ❖ W. Bentz, A.W. Thomas, Nucl. Phys. A **696**, 138 (2001)

Model Parameters

- ❖ Themes
- ❖ NJL model
- ❖ Baryons . . .
- ❖ $N \to \Delta$ FFs
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- Free Parameters:

  $\Lambda_{IR},\ \Lambda_{UV},\ M_0,\ G_\pi,\ G_s,\ G_a,\ G_\omega$ and G_ρ

- Constraints:
  - $f_\pi = 93$ MeV, $m_\pi = 140$ MeV & $M_N = 940$ MeV
  - $\int_0^1 dx\ (\Delta u_v(x) - \Delta d_v(x)) = g_A = 1.267$
  - $(\rho, E_B/A) = (0.16\,\text{fm}^{-3}, -15.7\ \text{MeV})$
  - $a_4 = 32$ MeV
  - $\Lambda_{IR} = 240$ MeV
- We obtain [MeV]:
  - $\Lambda_{UV} = 644$
  - $M_0 = 400, \quad M_s = 690, \quad M_a = 990, \quad \ldots$
- Can now model a very large array of observables

Nucleon–Delta Transition Form Factors

- ❖ Themes
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- ❖ Baryons . . .
- ❖ $N \longrightarrow \Delta$ FFs
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- ❖ $N \longrightarrow \Delta$ FFs
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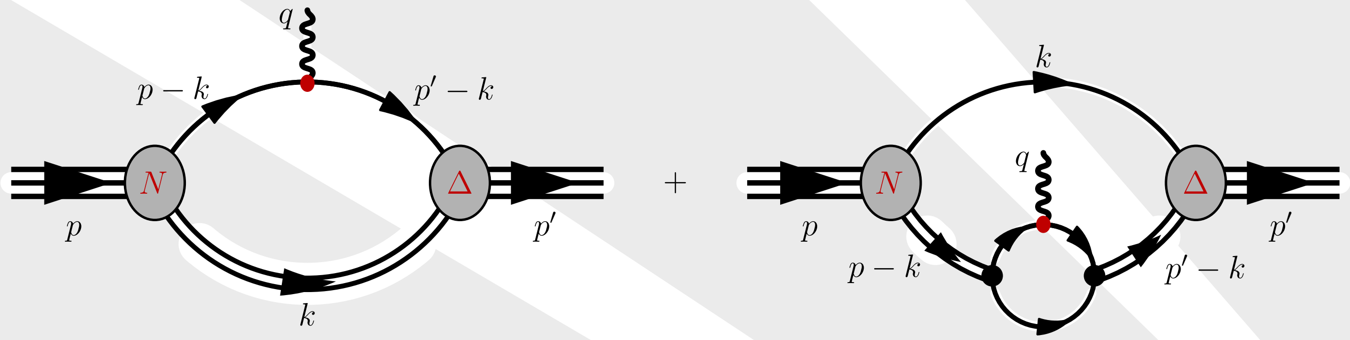
- ❖ Conclusion

- Transition Form Factor Feynman diagrams



❖ $\bullet = \gamma^\mu\, F_{1q}(Q^2) + \frac{i\,\sigma^{\mu\nu} q_\nu}{2M}\, F_{2q}(Q^2)$

- Approach is completely covariant
- No frame is assumed & Current is conserved
- Diagrams are expressed in form (Jones & Scadron):

$$J^\mu = \overline{u}_{\Delta,\alpha} \left[H_1^{\alpha\mu}\, G_1(Q^2) + H_2^{\alpha\mu}\, G_2(Q^2) + H_3^{\alpha\mu}\, G_3(Q^2)\right] u_N$$

- Many Model ingredients necessary

Constituent Quark Form Factors

❖ Themes
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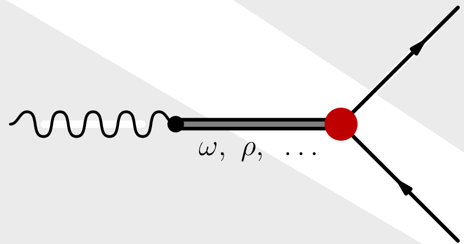
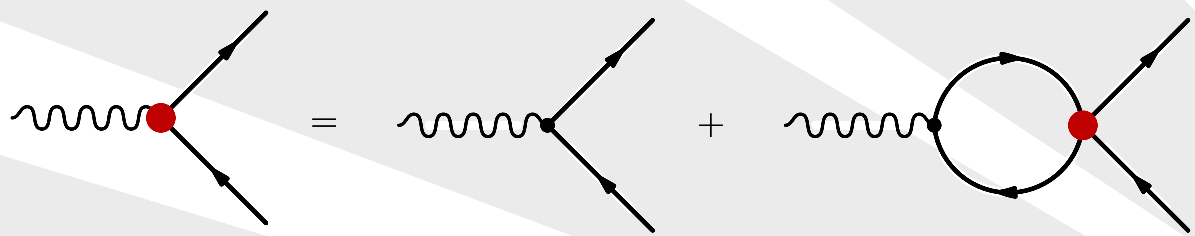
- Vector Meson Dominance – traditional view



$$\propto \quad \frac{M_\rho^2}{M_\rho^2 + Q^2}$$

- We solve integral equation for vertex



- Vertex becomes

$$\left(\frac{1}{6} + \frac{\tau_3}{2}\right) \gamma^\mu \ \rightarrow \ \left[\frac{1}{6}\, F_\omega + \frac{\tau_3}{2}\, F_\rho\right] \gamma^\mu$$

VMD Results

❖ Themes
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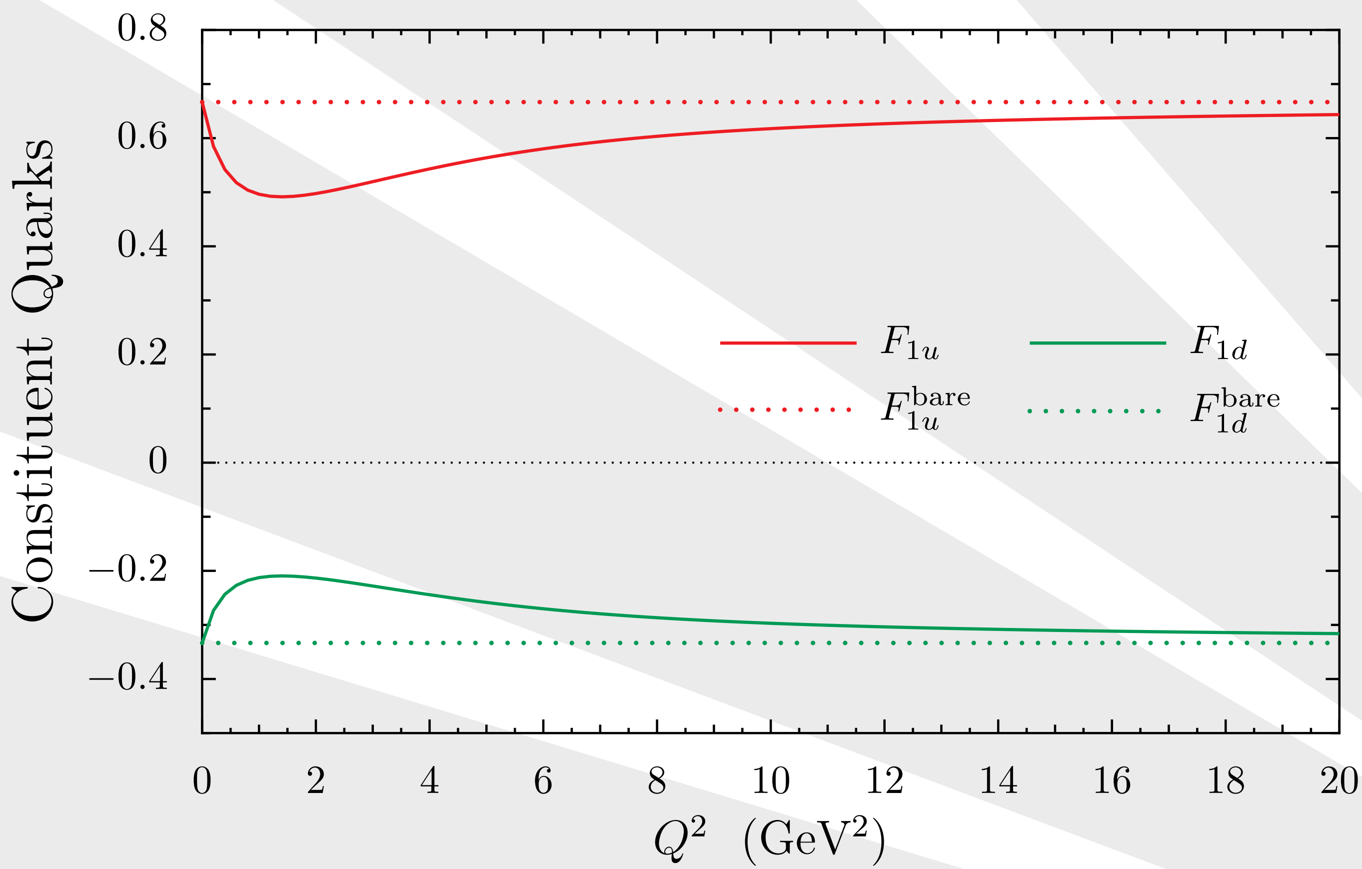

- Expanding about $Q^2 = 0$ gives

$$\left[\tfrac{1}{6}\,F_\omega + \tfrac{\tau_3}{2}\,F_\rho\right]\gamma^\mu \quad\sim\quad \left[\frac{1}{6}\,\frac{M_\omega^2}{M_\omega^2+Q^2} + \frac{\tau_3}{2}\,\frac{M_\rho^2}{M_\rho^2+Q^2}\right]\gamma^\mu$$

Constituent Quarks – Pion

❖ Themes
❖ NJL model
❖ Baryons . . .
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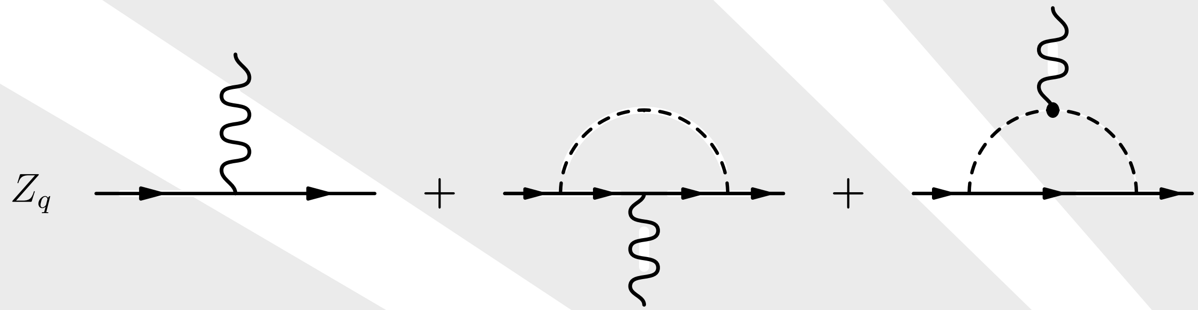

- Probability for find bare quark: $Z_q = 1 + \dfrac{\partial \Sigma_q}{\partial \not{p}}$
- Pion cloud $\to$ anomalous m.m for constituent quarks.

$$F_{1q}(Q^2) = Z_q \left(\frac{1}{6}\, F_\omega + \frac{1}{2}\, \tau_3\, F_\rho\right) + (F_\omega - \tau_3\, F_\rho)\ F_{1q}^{(q)} + \tau_3\, F_\rho\ F_{1q}^{(\pi)}$$

$$F_{2q}(Q^2) = (F_\omega - \tau_3\, F_\rho)\ F_{2q}^{(q)} + \tau_3\, F_\rho\ F_{2q}^{(\pi)}$$

- Self-consistent pion cloud
- However no pion exchange between quarks
- Better to add pion at nucleon level

Constituent Up Quark Results

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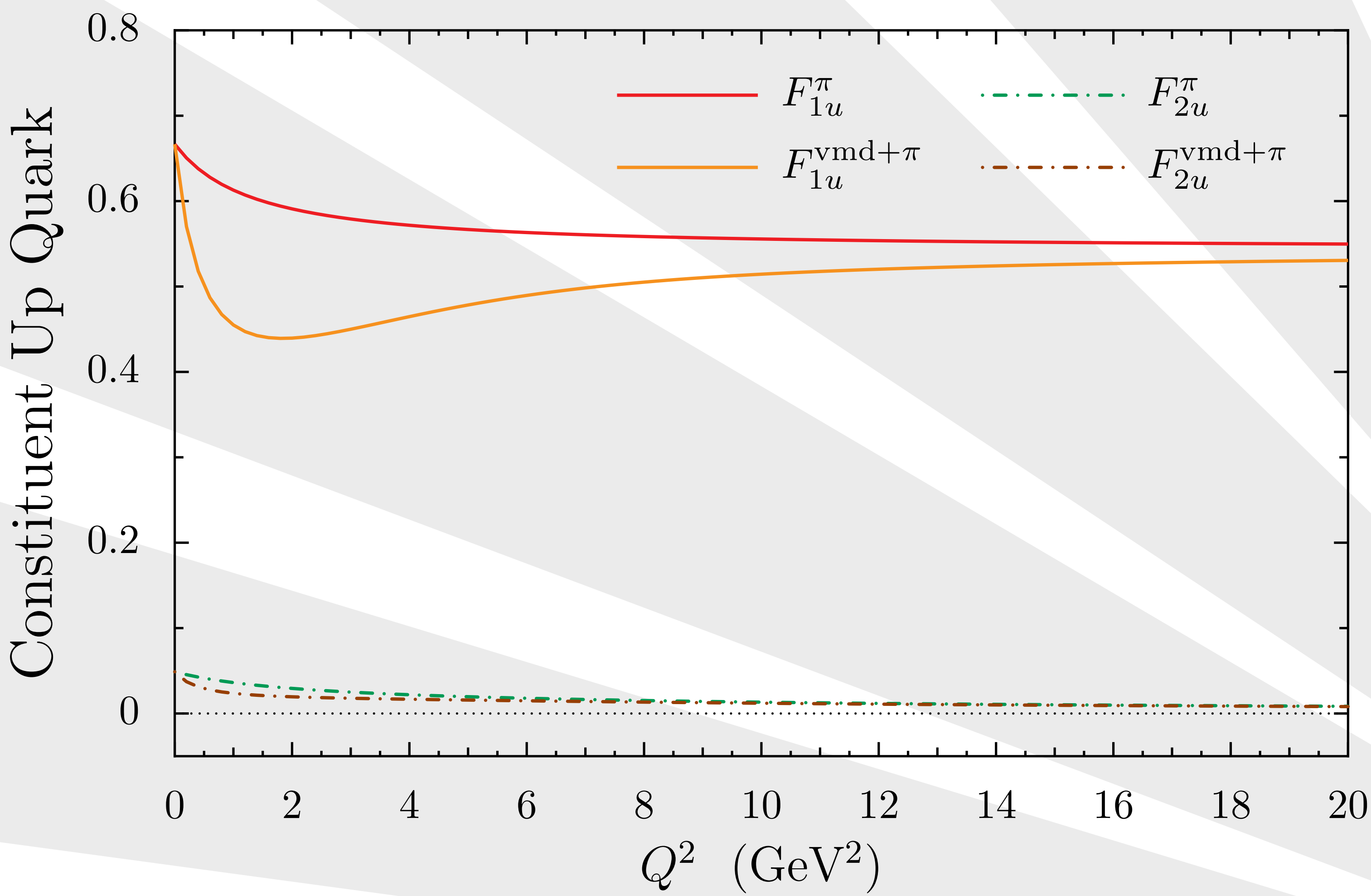
- ❖ Conclusion



Constituent Down Quark Results

- ❖ Themes
- ❖ NJL model
- ❖ Baryons . . .
- ❖ $N \rightarrow \Delta$ FFs
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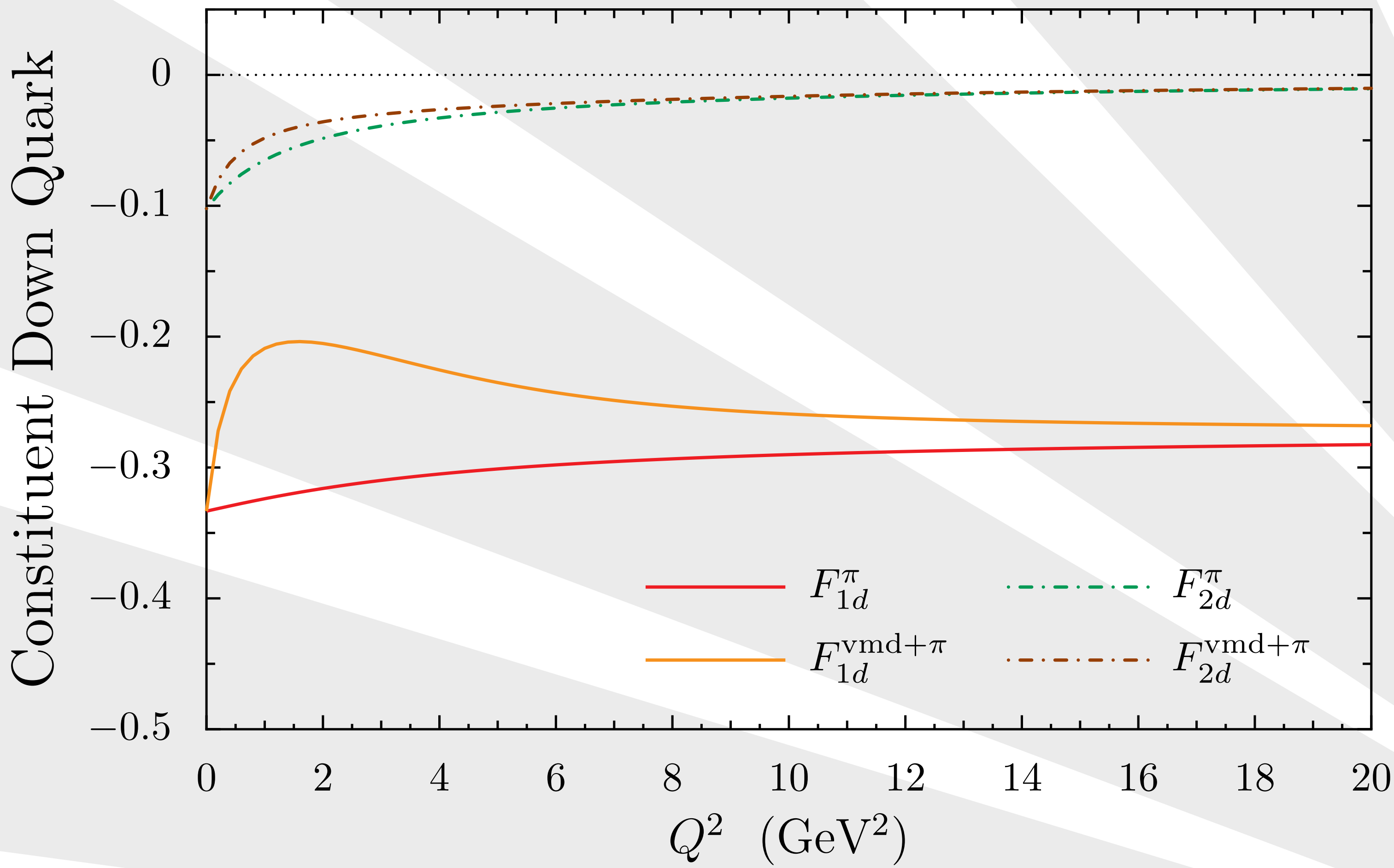
- ❖ Conclusion



Nucleon Form Factors

- Themes
- NJL model
- Baryons . . .
- $N \rightarrow \Delta$ FFs
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- $N \rightarrow \Delta$ FFs
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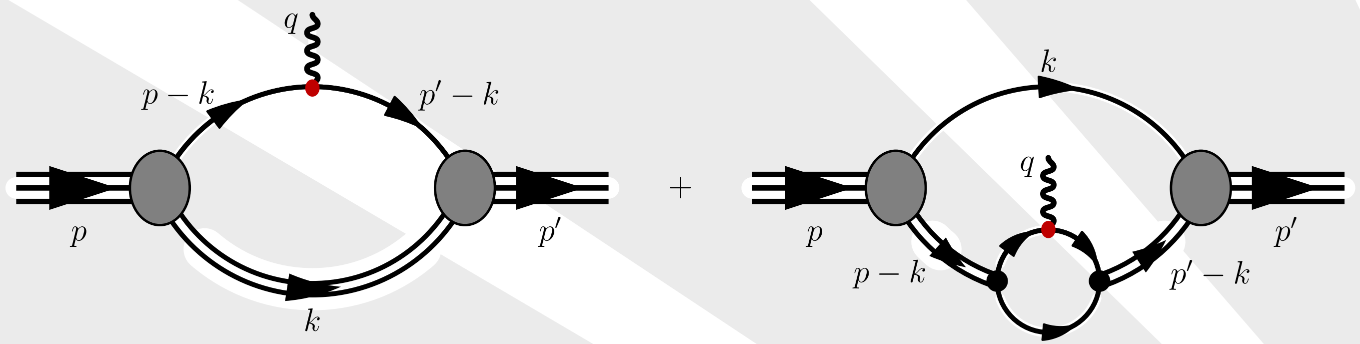
- Form Factor Feynman diagrams



$$\bullet = \gamma^\mu F_{1q}(Q^2) + \frac{i\,\sigma^{\mu\nu}q_\nu}{2M} F_{2q}(Q^2)$$

- Approach is completely covariant
- No frame is assumed
- Charge is conserved automatically
- Diagrams are expressed in form:

$$\langle J^\mu \rangle = \overline{u}_N(p') \left[\gamma^\mu F_{1N}(Q^2) + \frac{i\,\sigma^{\mu\nu}q_\nu}{2M_N} F_{2N}(Q^2)\right] u_N(p)$$

Scalar Diquark & Pion Form Factors

❖ Themes
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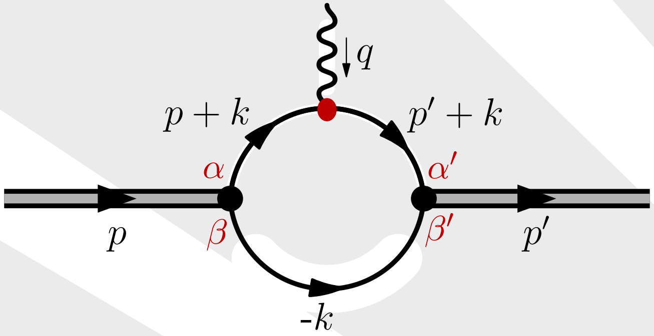

- Scalar diquark: $(\gamma_5\, C\, \tau_2\, \beta^{A'})(C^{-1}\gamma_5\, \tau_2\, \beta^{A})$, Pion: $(\tau\, \gamma_5)$
- Form Factor expressions same: $g_\pi \leftrightarrow g_s,\ m_\pi \leftrightarrow M_s$
- Two form factors in general

$$\langle J^\mu_\pi \rangle = (p' + p)^\mu\, F_\pi(Q^2) + \underbrace{(p' - p)^\mu\, F^{\text{os}}_\pi(Q^2)}_{\to 0}$$

- Result charge radius: $\langle r^2_E \rangle_\pi = 0.46\,\text{fm}^2$
- Experiment: $\langle r^2_E \rangle_\pi = 0.45\, \pm 0.01\text{fm}^2$

Pion Form Factor

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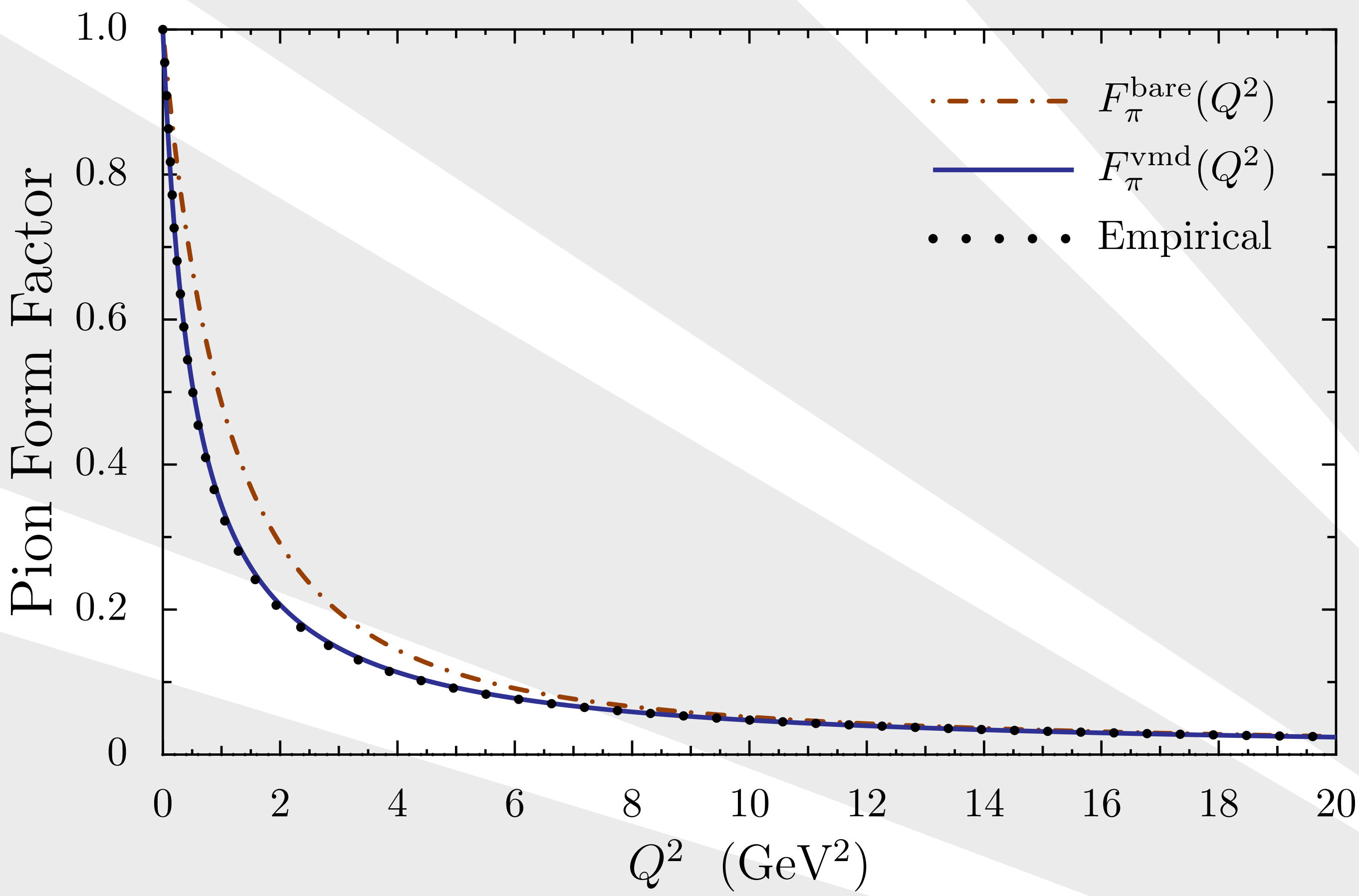

- $F_\pi(Q^2) = \left[1 + Q^2/\Lambda^2\right]^{-1}$ $\quad \Lambda^2 = 0.5$ GeV^2
- No pion dressing on quarks $\leftrightarrow$ ρ excitation

Pion Form Factor

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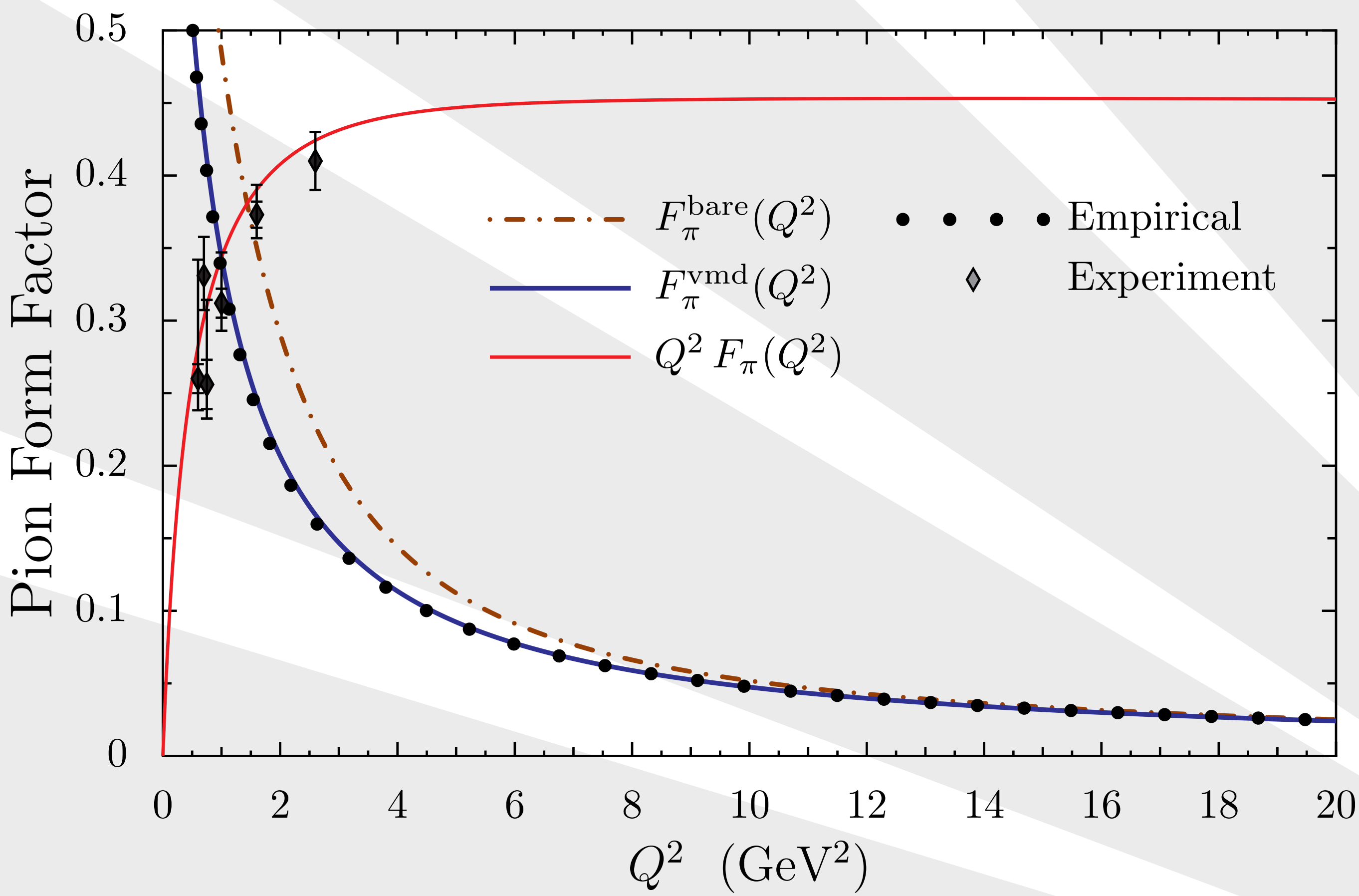

- V. Tadevosyan *et al.* [Jefferson Lab F(pi) Collaboration], Phys. Rev. C **75**, 055205 (2007)
- $Q^2\, F_\pi(Q^2) \rightarrow 16\pi\, f_\pi^2\, \alpha_s(Q^2) \implies \alpha_{NJL} = 0.94 \implies Q^2 \sim 0.46$ GeV^2

Axial-Vector Diquark & Rho Form Factors

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❖ Conclusion

- AV diquark: $(\gamma^{\beta}\, C\, \tau_i \tau_2\, \beta^{A'})(C^{-1}\gamma^{\alpha}\, \tau_2 \tau_j\, \beta^{A})$, Rho: $(\tau_j\, \gamma^{\mu})$
- Form Factor expressions same: $g_\rho \leftrightarrow g_a, \;\; m_\rho \leftrightarrow M_a$
- 3 on shell form factors

$$J^{\mu}_{\rho} = \left[g^{\alpha\beta}\, F_1(Q^2) - \frac{q^{\alpha} q^{\beta}}{2M_a^2}\, F_2(Q^2)\right](p+p')^{\mu} - \left(q^{\alpha}\, g^{\mu\beta} - q^{\beta}\, g^{\mu\alpha}\right) F_3(Q^2)$$

- Sachs Form Factors

$$G_C(Q^2) = F_1(Q^2) + \frac{2}{3}\,\frac{Q^2}{4\,M^2}\, G_Q(Q^2), \qquad G_M(Q^2) = F_3(Q^2)$$

$$G_Q(Q^2) = F_1(Q^2) + \left(1 + \frac{Q^2}{4\,M^2}\right) F_2(Q^2) - F_3(Q^2)$$

- NJL Results: $\langle r_E^2 \rangle_\rho = 0.52, \;\; \mu_\rho = 2.08, \;\; \mathcal{Q}_\rho = -0.52$
- DSE Results: $\langle r_E^2 \rangle_\rho = 0.54, \;\; \mu_\rho = 2.01, \;\; \mathcal{Q}_\rho = -0.41$

❖ M. S. Bhagwat and P. Maris, Phys. Rev. C **77**, 025203 (2008)

Axial-Vector Diquark Form Factors

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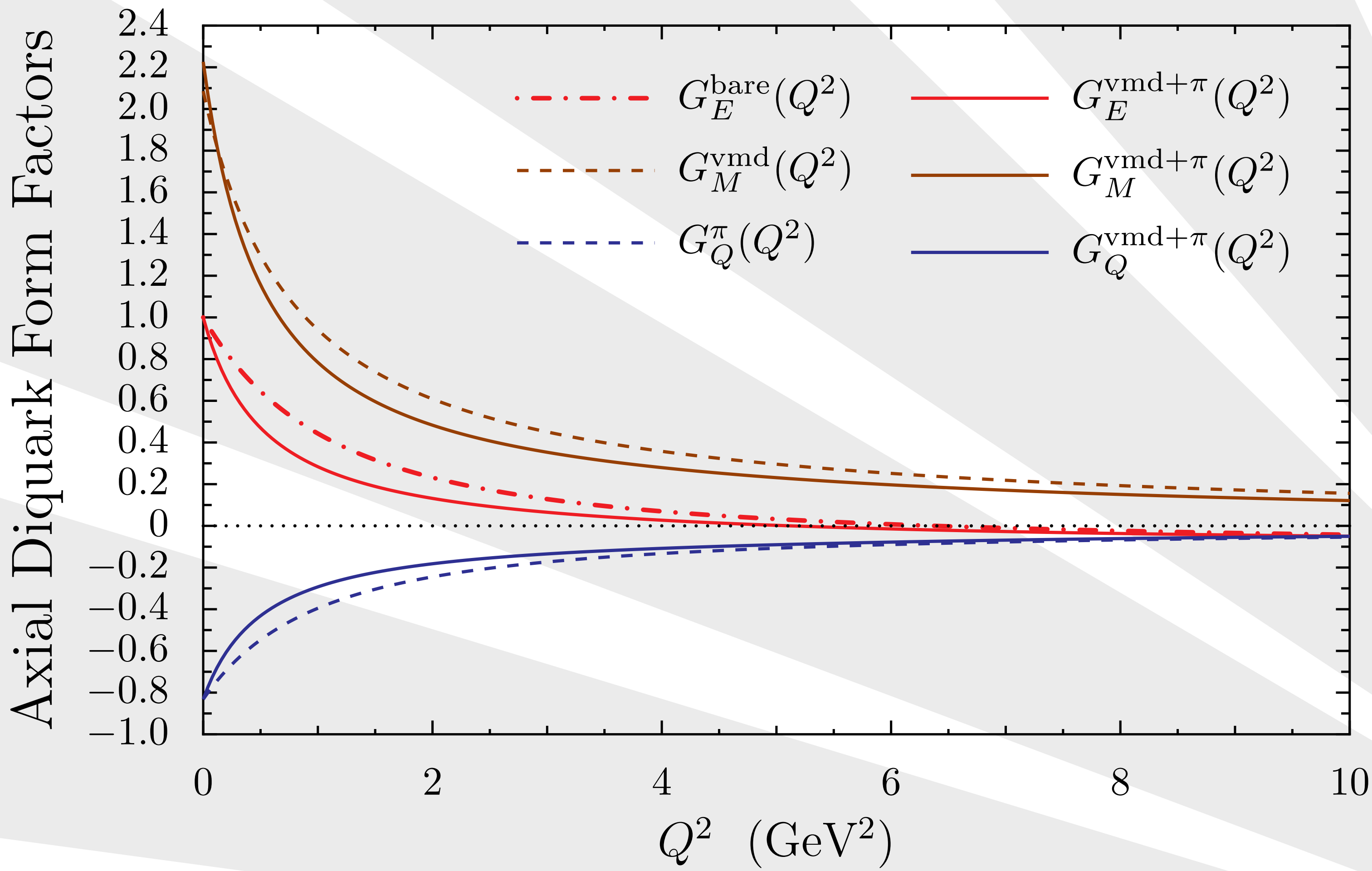

- For large Q^2 we find $G_Q \to G_E$.

Nucleon Form Factors – Recall

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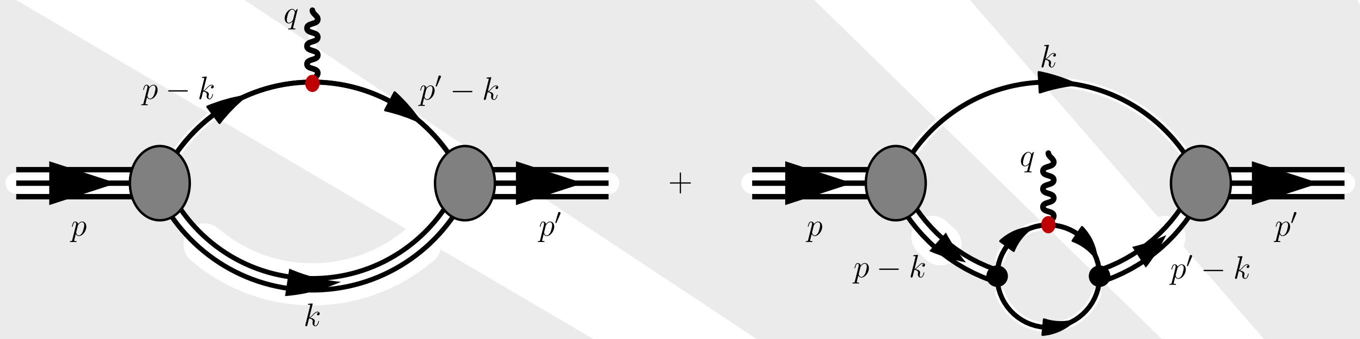
- Form Factor Feynman diagrams



❖ $\bullet = \gamma^\mu F_{1q}(Q^2) + \frac{i\,\sigma^{\mu\nu} q_\nu}{2M} F_{2q}(Q^2)$

- Diagrams are expressed in form:

$$\langle J^\mu \rangle = \overline{u}_N(p') \left[\gamma^\mu F_{1N}(Q^2) + \frac{i\,\sigma^{\mu\nu} q_\nu}{2M_N} F_{2N}(Q^2) \right] u_N(p)$$

Proton Form Factors: Results

- Themes
- NJL model
- Baryons . . .
- $N \to \Delta$ FFs
- Constituents
- Nucleon FFs
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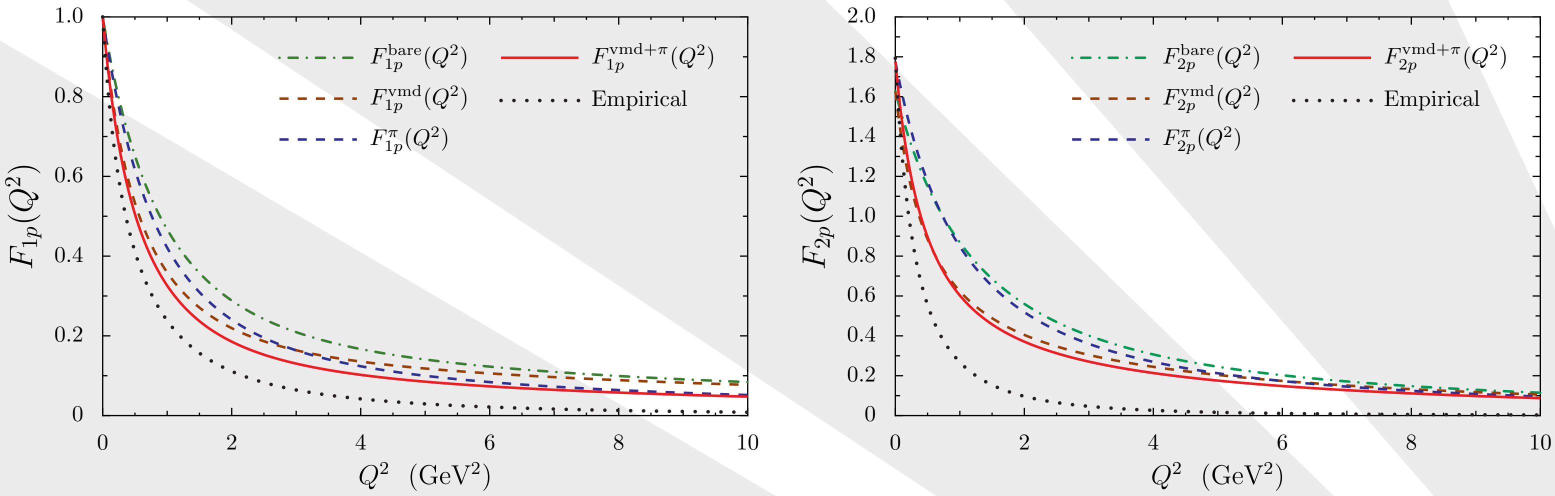
- Nucleon FFs
- Delta FFs
- $N \to \Delta$ FFs
- Off-Shell
- Conclusion



- Arrington, et al, Phys. Rev. C **76**, 035205 (2007)
- NJL: $\kappa_p = 1.77$, Experiment: $\kappa_p = 1.79$
- NJL: $\langle r_E^2\rangle_p = 0.58\,\text{fm}^2$, Experiment: $\langle r_E^2\rangle_p = 0.72\,\text{fm}^2$
- NJL: $\langle r_M^2\rangle_p = 0.56\,\text{fm}^2$, Experiment: $\langle r_M^2\rangle_p = 0.71\,\text{fm}^2$
- NJL$_{\text{bare}}$: $\kappa_p = 1.61$, $\langle r_E^2\rangle_p = 0.36\,\text{fm}^2$, $\langle r_M^2\rangle_p = 0.38\,\text{fm}^2$
- Need extra $\sim 1/Q^2$ factor $\leftrightarrow$ Static Approximation

Neutron Form Factors: Results

❖ Themes
❖ NJL model
❖ Baryons . . .
❖ $N \longrightarrow \Delta$ FFs
❖ Constituents
❖ Nucleon FFs
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❖ $N \longrightarrow \Delta$ FFs
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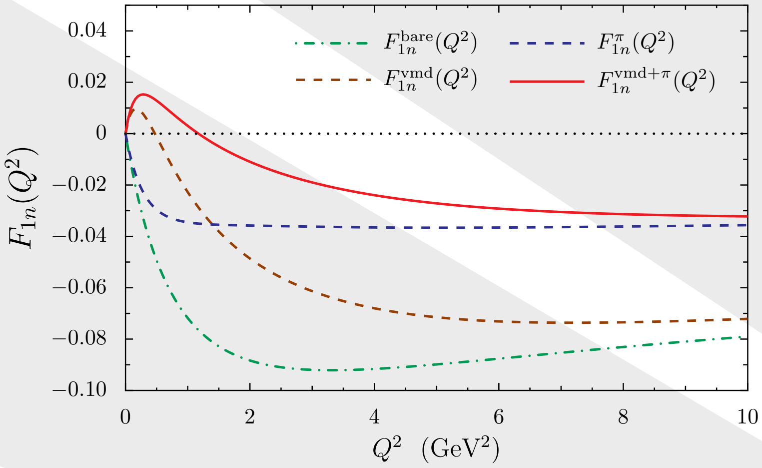
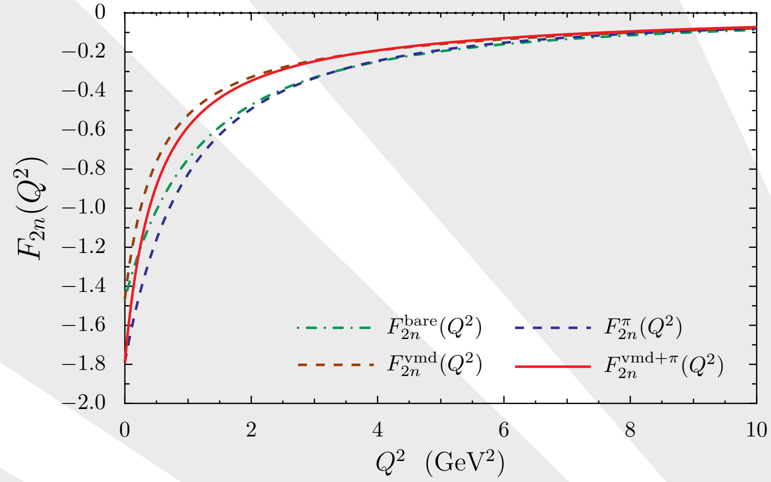
❖ Conclusion





- NJL: $\kappa_n = 1.79$, Experiment: $\kappa_n = 1.91$
- NJL: $\langle r_E^2 \rangle_n = -0.15\,\text{fm}^2$, Experiment: $\langle r_E^2 \rangle_n = -0.12\,\text{fm}^2$
- NJL: $\langle r_M^2 \rangle_n = 0.54\,\text{fm}^2$, Experiment: $\langle r_M^2 \rangle_n = 0.79\,\text{fm}^2$
- bare: $\kappa_n = -1.46$, $\langle r_E^2 \rangle_n = -0.07\,\text{fm}^2$, $\langle r_M^2 \rangle_p = 0.38\,\text{fm}^2$
- Need extra $\sim 1/Q^2$ factor $\leftrightarrow$ Static Approximation

Delta Form Factors

- Themes
- NJL model
- Baryons . . .
- $N \to \Delta$ FFs
- Constituents
- Nucleon FFs
- Scalar Diquark FF
- Axial-Vector FF
- Nucleon FFs
- **Delta FFs**
- $N \to \Delta$ FFs
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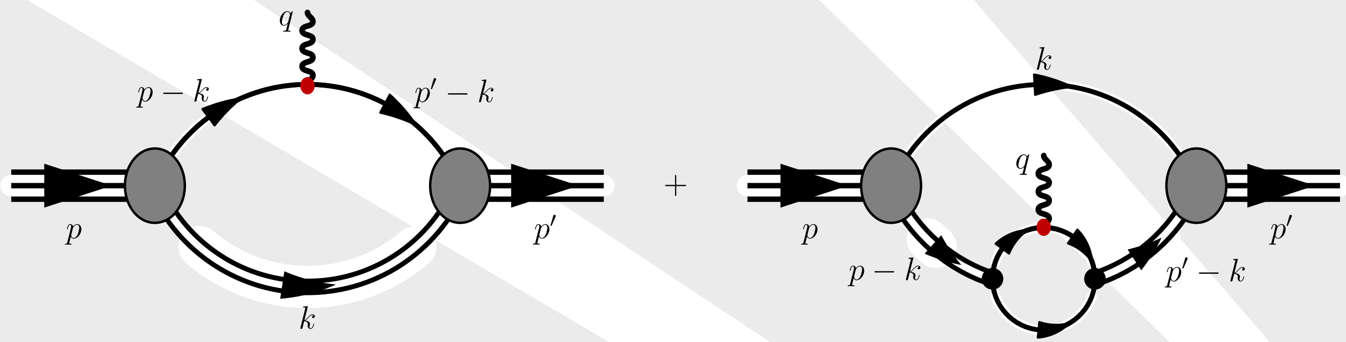
- Conclusion

- Form Factor Feynman diagrams



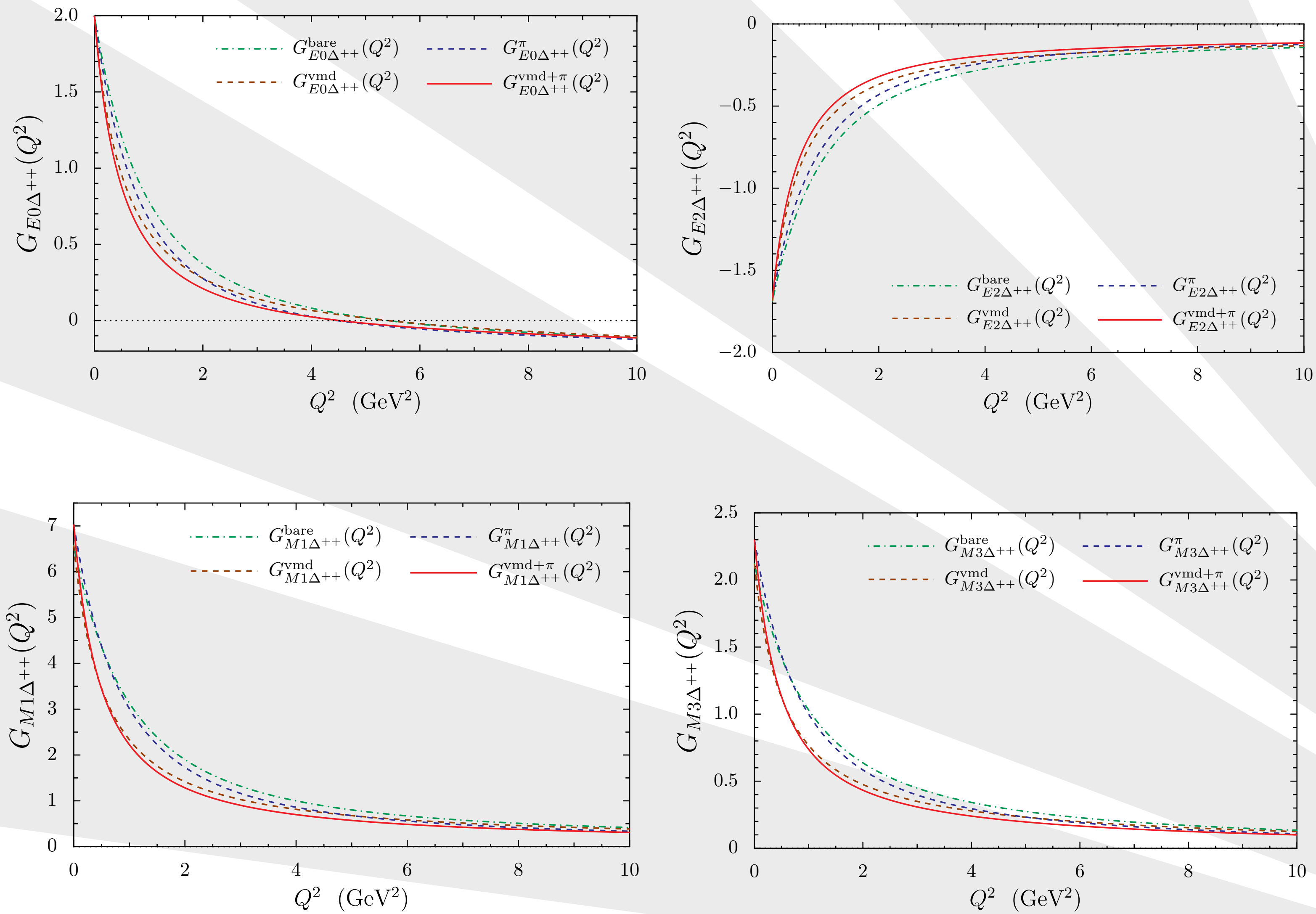
- Only axial-vector diquarks
- completely covariant, current conservation, no frame
- Diagrams are expressed in form:

$$\Gamma^{\mu,\alpha\beta} = H_1^{\alpha\beta,\mu} a_1 + H_2^{\alpha\beta,\mu} a_2 + H_3^{\alpha\beta,\mu} c_1 + H_4^{\alpha\beta,\mu} c_2$$

- Multipole Form Factors: $G_{E0}, G_{E2}, G_{M1}, G_{M3}$

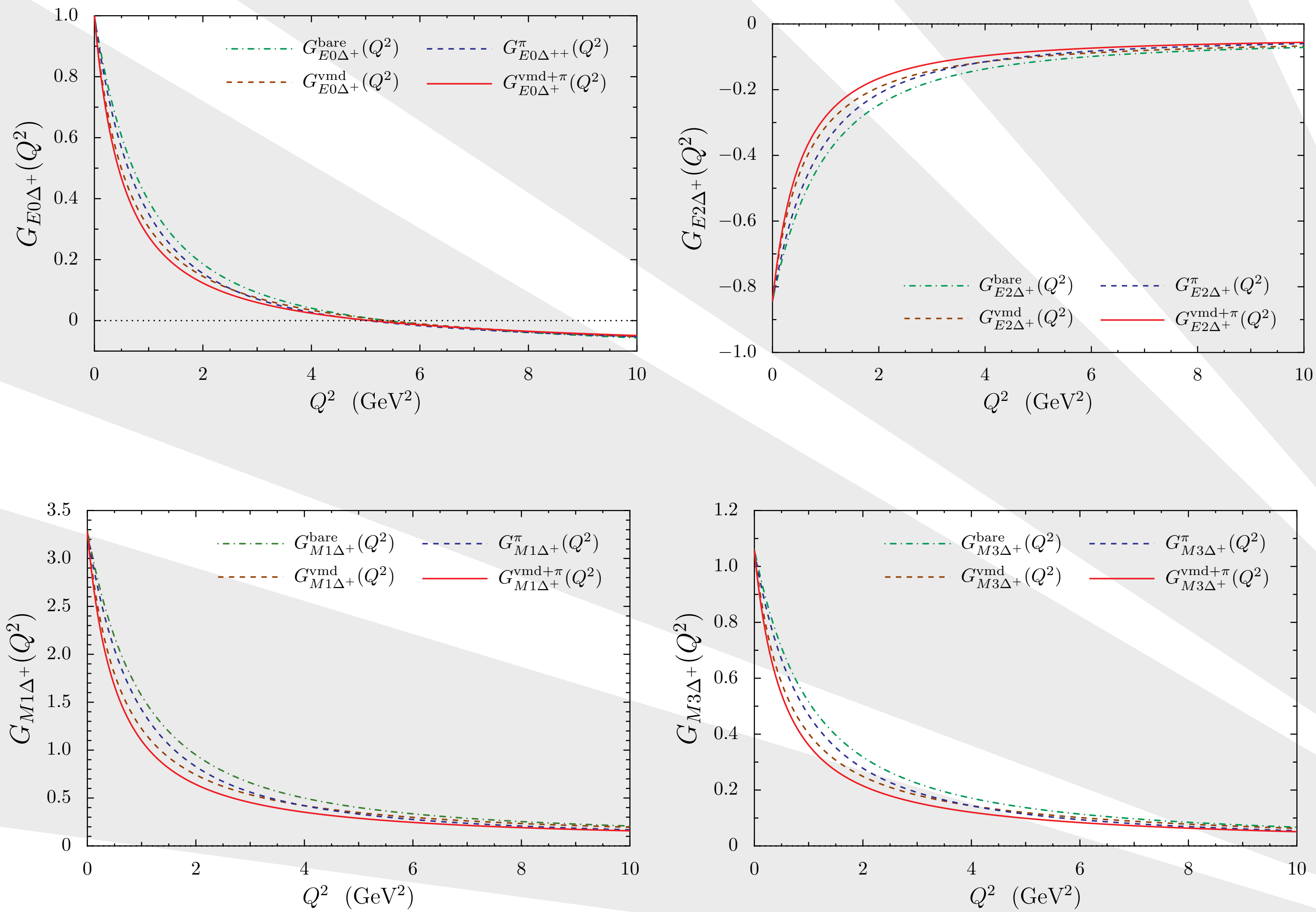
Δ^{++} Form Factors Results

- Themes
- NJL model
- Baryons . . .
- $N \to \Delta$ FFs
- Constituents
- Nucleon FFs
- Scalar Diquark FF
- Axial-Vector FF
- Nucleon FFs
- **Delta FFs**
- $N \to \Delta$ FFs
- Off-Shell
- Conclusion



Δ^+ Form Factors Results

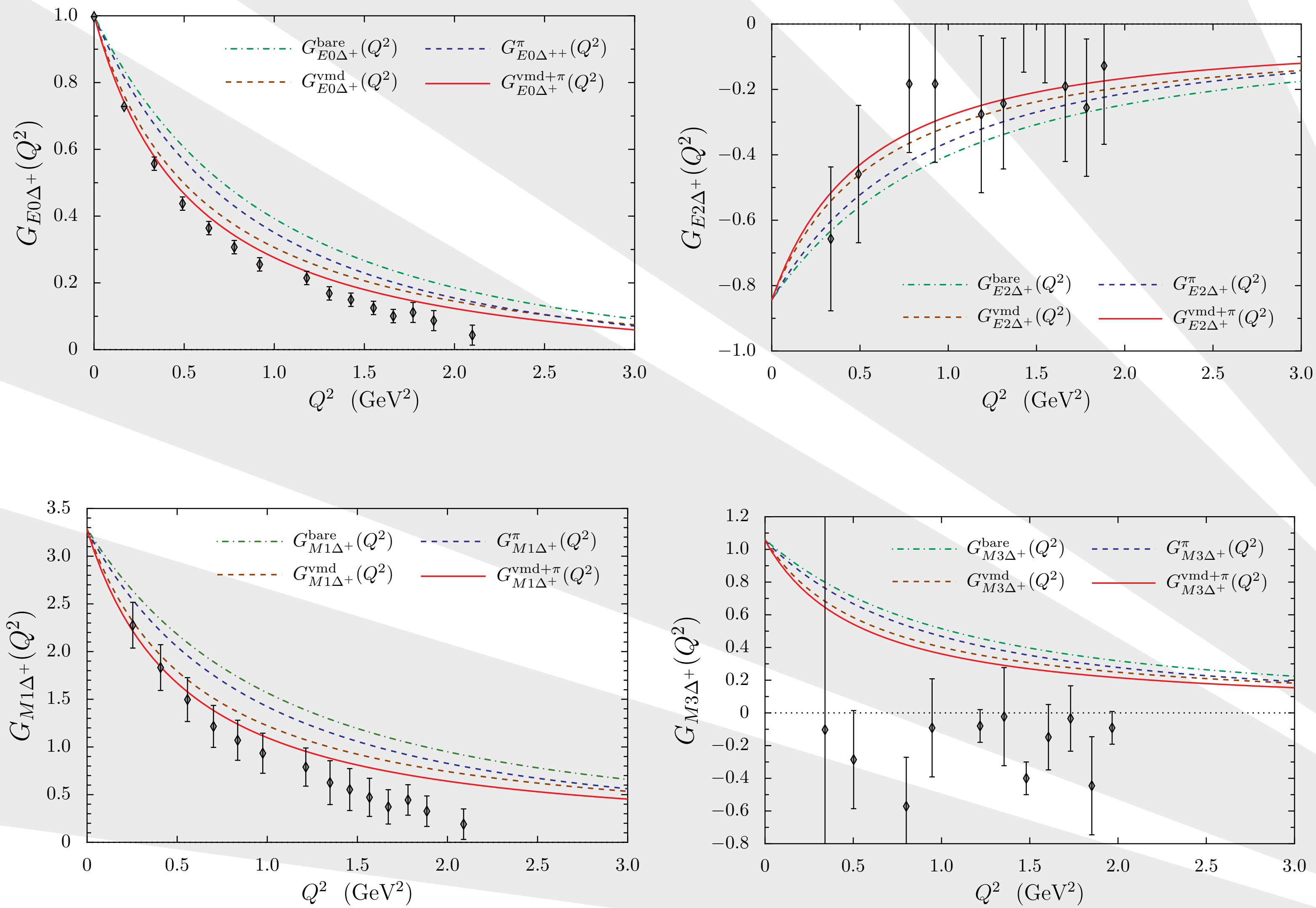
- ❖ Themes
- ❖ NJL model
- ❖ Baryons . . .
- ❖ $N \to \Delta$ FFs
- ❖ Constituents
- ❖ Nucleon FFs
- ❖ Scalar Diquark FF
- ❖ Axial-Vector FF
- ❖ Nucleon FFs
- ❖ **Delta FFs**
- ❖ $N \to \Delta$ FFs
- ❖ Off-Shell
- ❖ Conclusion



- $G_{\Delta^+} \sim \frac{1}{2} G_{\Delta^{++}}$

Δ^+ *Form Factors with Lattice Results*

❖ Themes
❖ NJL model
❖ Baryons . . .
❖ $N \rightarrow \Delta$ FFs
❖ Constituents
❖ Nucleon FFs
❖ Scalar Diquark FF
❖ Axial-Vector FF
❖ Nucleon FFs
❖ **Delta FFs**
❖ $N \rightarrow \Delta$ FFs
❖ Off-Shell
❖ Conclusion



● C. Alexandrou, *et al*, PoS **LAT2007**, 149 (2007)

Delta Moments

❖ Themes
❖ NJL model
❖ Baryons . . .
❖ $N \to \Delta$ FFs
❖ Constituents
❖ Nucleon FFs
❖ Scalar Diquark FF
❖ Axial-Vector FF
❖ Nucleon FFs
❖ Delta FFs
❖ $N \to \Delta$ FFs
❖ Off-Shell
❖ Conclusion

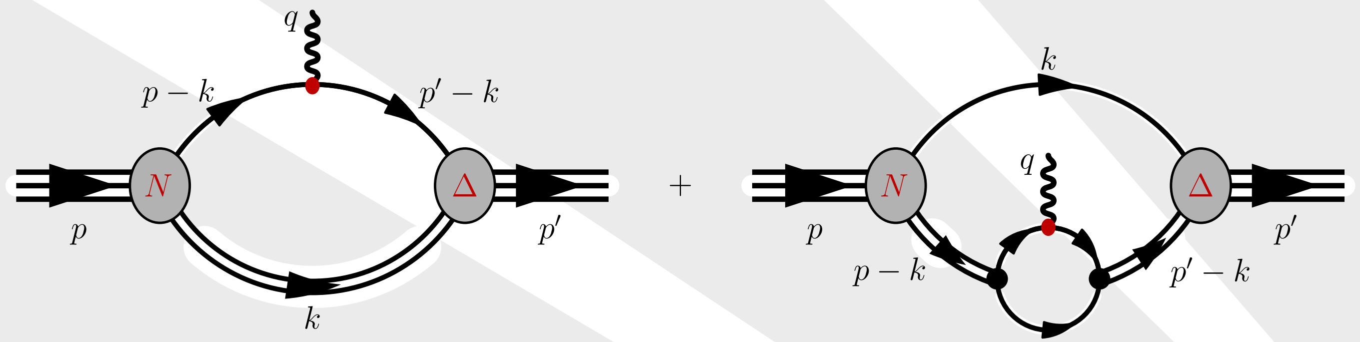
- Largely unexplored experimentally: Summary
  - $\mu_{\Delta^{++}} = 4.52 \pm 0.51 \pm 0.45\ \mu_N$ or $\mu_{\Delta^{++}} = 6.14 \pm 0.51 \mu_N$

    Bosshard, 1991 — Lopez Castro, 2002
  - $\mu_{\Delta^{+}} = 2.7^{+1.0}_{-1.3}(\text{stat.}) \pm 1.5(\text{syst.}) \pm 3(\text{theory})\ \mu_N$ Kotulla,2002
- Chiral Extrapolated Lattice QCD Cloët, 2003
  - $\mu_{\Delta^{++}} = 4.99 \pm 0.56\ \mu_N$, $\mu_{\Delta^{+}} = 2.49 \pm 0.29\ \mu_N$
  - $\mu_{\Delta^{0}} \sim 0.06\ \mu_N$, $\mu_{\Delta^{-}} = 2.45 \pm 0.27\ \mu_N$
- NJL results (μ_N)
  - Pion: $\mu_{\Delta^{++}} = 5.59$, $\mu_{\Delta^{+}} = 2.60$, $\mu_{\Delta^{0}} = -0.38$, $\mu_{\Delta^{-}} = -3.36$
  - Bare: $\mu_{\Delta^{++}} = 5.39$, $\mu_{\Delta^{+}} = 2.61$, $\mu_{\Delta^{0}} = 0.0$, $\mu_{\Delta^{-}} = -2.61$
- Lattice: $\mu_{\Delta^{+}} < \mu_p$, NJL agrees.

$N \rightarrow \Delta$ Transition Form Factors

- Themes
- NJL model
- Baryons . . .
- $N \rightarrow \Delta$ FFs
- Constituents
- Nucleon FFs
- Scalar Diquark FF
- Axial-Vector FF
- Nucleon FFs
- Delta FFs
- $N \rightarrow \Delta$ FFs
- Off-Shell
- Conclusion

- Form Factor Feynman diagrams



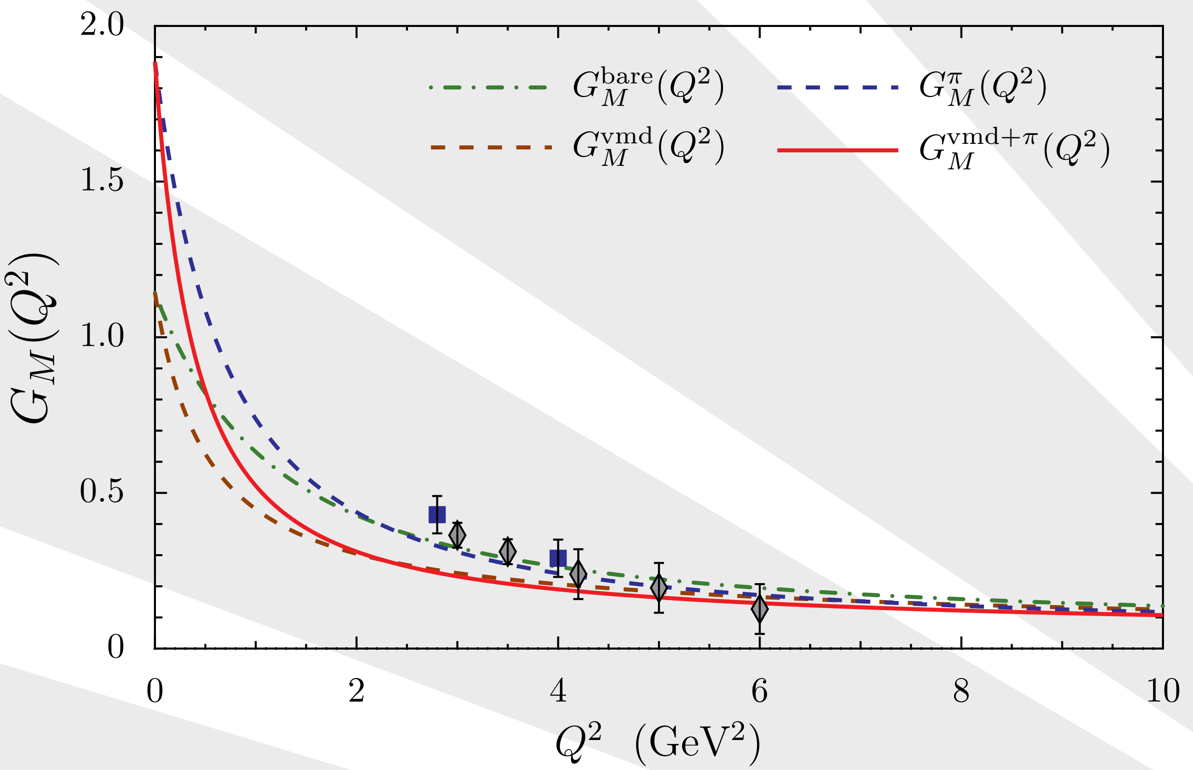
- Necessary ingredients are in place
- Scalar and axial-vector diquarks
- completely covariant, current conservation, no frame
- Diagrams are expressed in form:

$$J^{\mu} = \overline{u}_{\Delta,\alpha} \left[H_1^{\alpha\mu}\, G_1(Q^2) + H_2^{\alpha\mu}\, G_2(Q^2) + H_3^{\alpha\mu}\, G_3(Q^2) \right] u_N$$

- Jones & Scadron multipole form factors: G_E, G_M, G_C

Form Factor Results: G_M

- Themes
- NJL model
- Baryons . . .
- $N \to \Delta$ FFs
- Constituents
- Nucleon FFs
- Scalar Diquark FF
- Axial-Vector FF
- Nucleon FFs
- Delta FFs
- $N \to \Delta$ FFs
- Off-Shell
- Conclusion

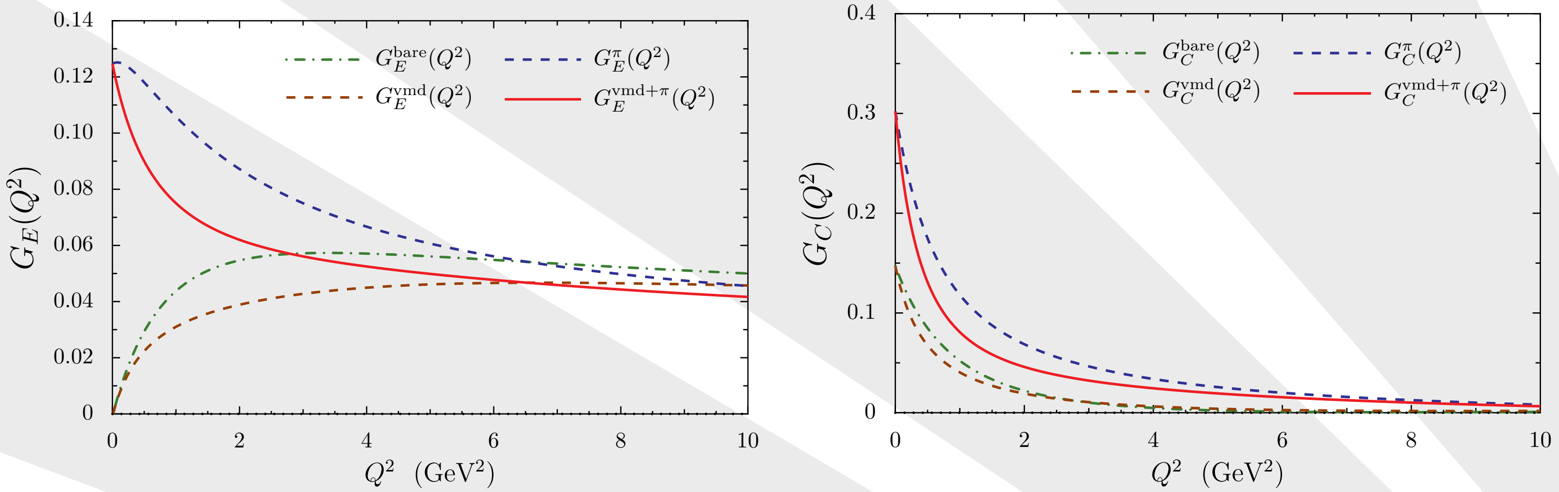


- V.V. Frolov, *et al.*, Phys. Rev. Lett. 82 (1999) 45.

  M. Ungaro, *et al.*, Phys. Rev. Lett. 97 (2006) 112003.
- Pion playing very important role
- We have a lot of missing strength at $G_M(Q^2) \sim 0$.

Form Factor Results: G_E & G_C

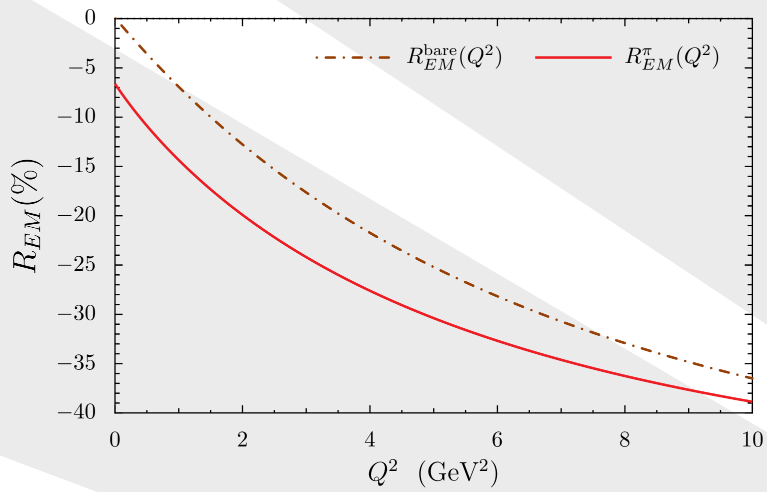
- Themes
- NJL model
- Baryons . . .
- $N \longrightarrow \Delta$ FFs
- Constituents
- Nucleon FFs
- Scalar Diquark FF
- Axial-Vector FF
- Nucleon FFs
- Delta FFs
- $N \longrightarrow \Delta$ FFs
- Off-Shell
- Conclusion

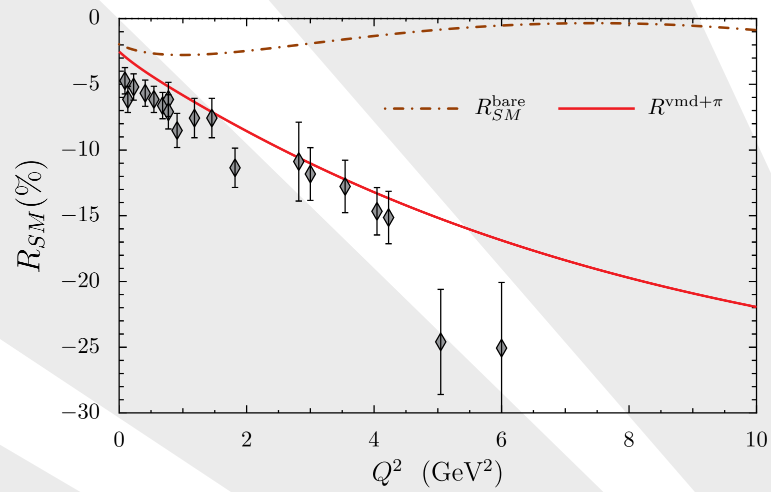


- Form factors are small
- Pion cloud playing important large role
- Significant pion cloud effects for G_E at large Q^2.

Form Factor Ratios

- ❖ Themes
- ❖ NJL model
- ❖ Baryons . . .
- ❖ $N \longrightarrow \Delta$ FFs
- ❖ Constituents
- ❖ Nucleon FFs
- ❖ Scalar Diquark FF
- ❖ Axial-Vector FF
- ❖ Nucleon FFs
- ❖ Delta FFs
- ❖ $N \longrightarrow \Delta$ FFs
- ❖ Off-Shell
- ❖ Conclusion



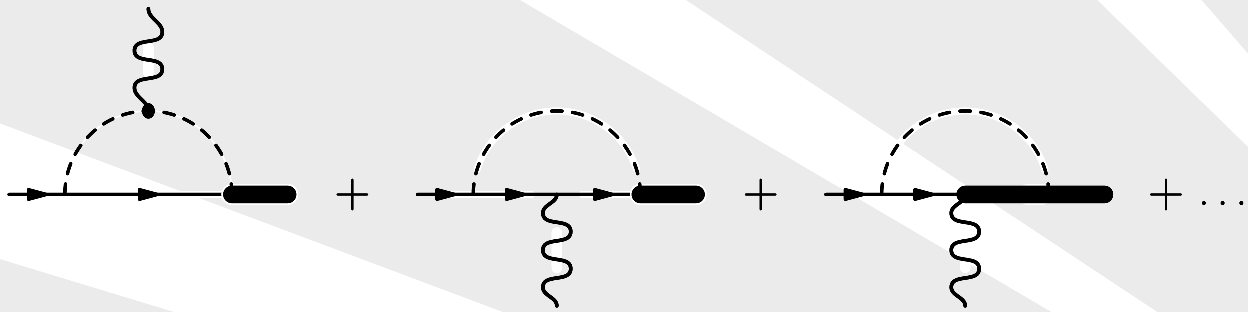


- R_{EM} is roughly a fact 10 too large
- G_E is very sensitive to cancellations between diagrams
  - compare neutron charge radius
- Nice result for R_{SM}
  - Data from MAMI, LEGS, MIT-Bates and JLab

What's Missing

- Themes
- NJL model
- Baryons . . .
- $N \to \Delta$ FFs
- Constituents
- Nucleon FFs
- Scalar Diquark FF
- Axial-Vector FF
- Nucleon FFs
- Delta FFs
- $N \to \Delta$ FFs
- Off-Shell
- Conclusion

- Need Faddeev calculation without static approx.
- Detailed description of pion cloud
- Need diagrams like:



- Important to quantify these effects
- Possible to calculate diagrams self-consistently in NJL

Off-Shell Effects

❖ Themes
❖ NJL model
❖ Baryons . . .
❖ $N \to \Delta$ FFs
❖ Constituents
❖ Nucleon FFs
❖ Scalar Diquark FF
❖ Axial-Vector FF
❖ Nucleon FFs
❖ Delta FFs
❖ $N \to \Delta$ FFs
❖ Off-Shell
❖ Conclusion

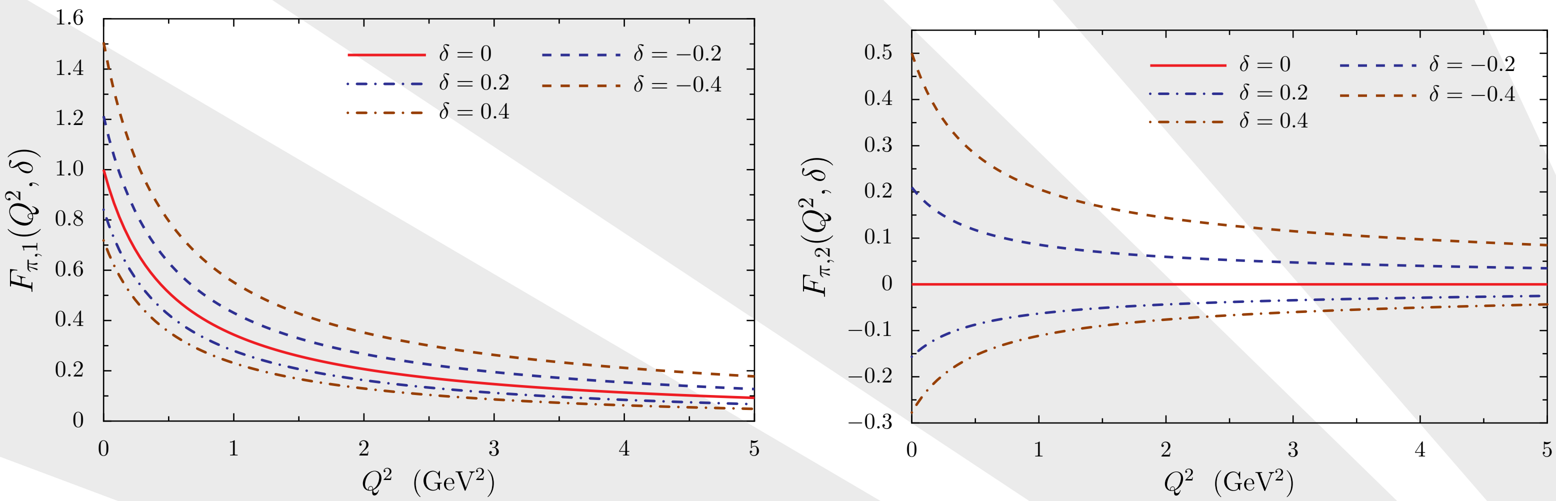
- Largely unexplored in literature
- Relax constraint: $p'^2 = p^2 = M^2$.
- Very difficult, or impossible, in many model approaches
  - Dynamical model in terms of elementary d.o.f.
  - Solve dynamical equations.
- Potentially important: in-medium quark distributions, form factors, etc
- Pion has two off-shell form factors

$$j_\pi^\mu = \left(p'^\mu + p^\mu\right)\, F_{\pi,1} + \left(p'^\mu - p^\mu\right)\, F_{\pi,2}$$

- For $p'^2 = p^2 = M^2$, we have $F_{\pi,2} = 0$ & $F_{\pi,1} \to F_\pi$.

Pion Off-Shell Form Factors

❖ Themes
❖ NJL model
❖ Baryons . . .
❖ $N \to \Delta$ FFs
❖ Constituents
❖ Nucleon FFs
❖ Scalar Diquark FF
❖ Axial-Vector FF
❖ Nucleon FFs
❖ Delta FFs
❖ $N \to \Delta$ FFs
❖ Off-Shell
❖ Conclusion



- Define $\delta = p'^2 - p^2$, put final state on-shell, i.e. $p'^2 = m_\pi^2$
- Effects maybe large
- Off-shell effects may be important for experimental extraction of $N \to \Delta$ transition.
- Also for in-medium form factors

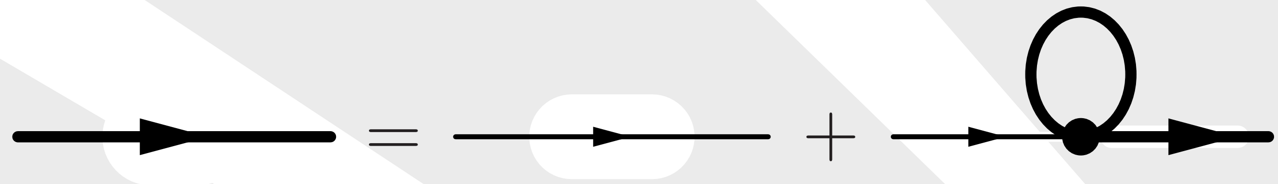
Conclusion & Outlook

- Themes
- NJL model
- Baryons . . .
- $N \to \Delta$ FFs
- Constituents
- Nucleon FFs
- Scalar Diquark FF
- Axial-Vector FF
- Nucleon FFs
- Delta FFs
- $N \to \Delta$ FFs
- Off-Shell
- Conclusion

- Covariant, Confining, Faddeev formalism:
  - Form Factors, Quark Distributions, etc
- Good description of "$Q^2 = 0$" observables
  - e.g. magnetic moments, quark distributions
  - has been long standing problem
- Transition form factors require detailed understanding of pion contributions
  - Important challenge for quark models
- Size of off-shell effects may be studied
- Results so far suggest further investigation warranted.

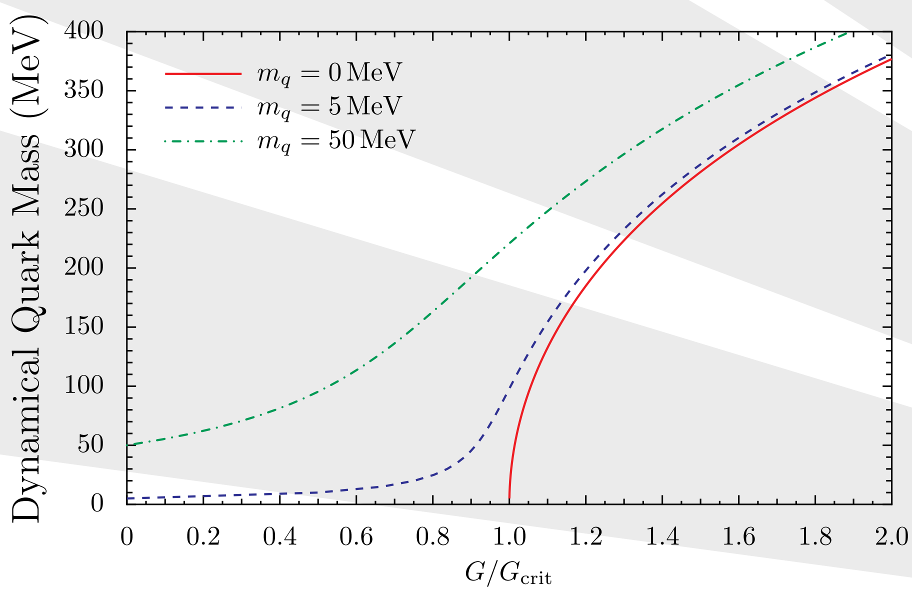
Gap Equation

- Themes
- NJL model
- Baryons . . .
- $N \to \Delta$ FFs
- Constituents
- Nucleon FFs
- Scalar Diquark FF
- Axial-Vector FF
- Nucleon FFs
- Delta FFs
- $N \to \Delta$ FFs
- Off-Shell
- Conclusion



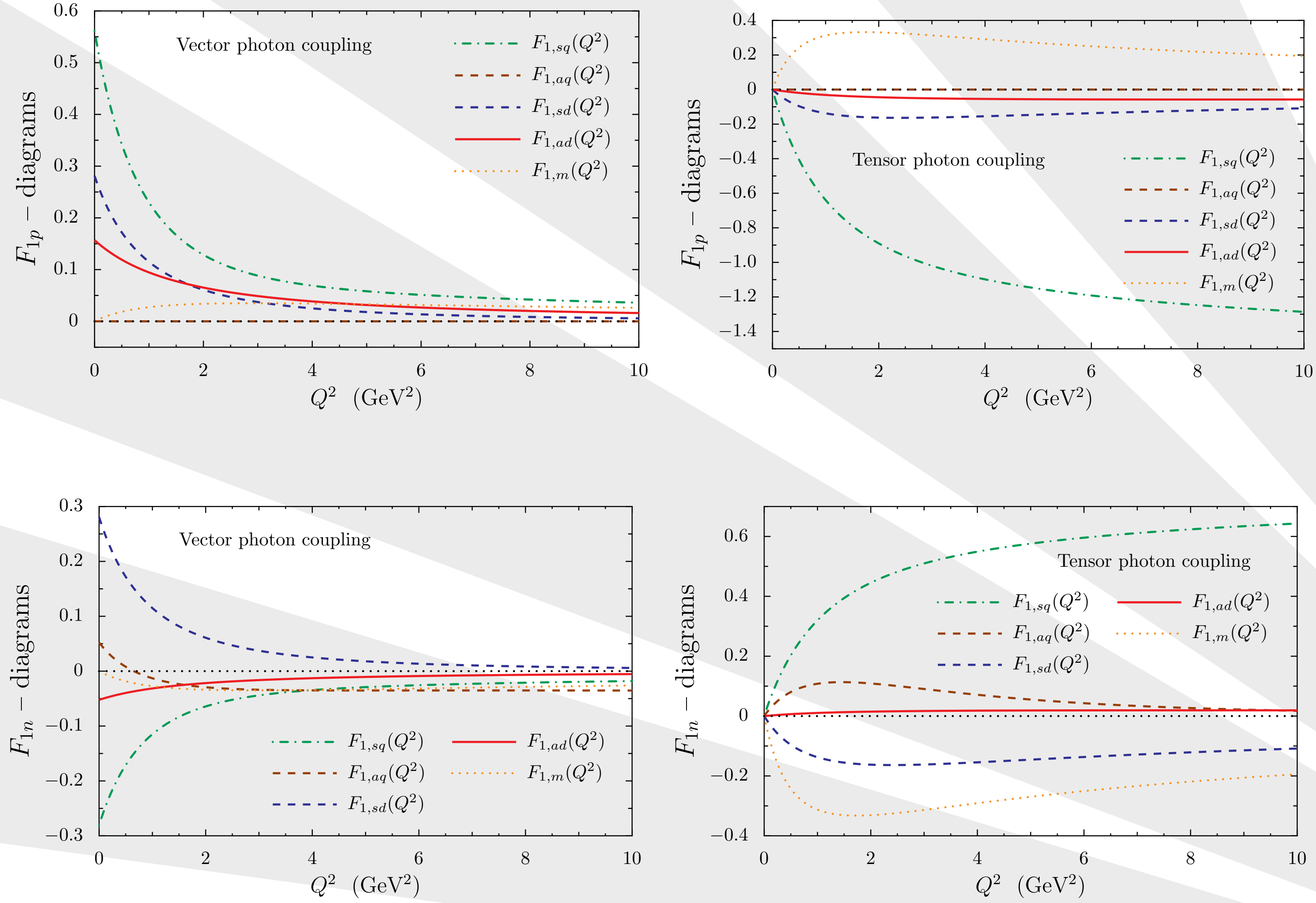
- Self-consistent solution – gives Quark Propagator

$$\frac{1}{\not{p} - m + i\varepsilon} \longrightarrow \frac{1}{\not{p} - M + i\varepsilon}$$



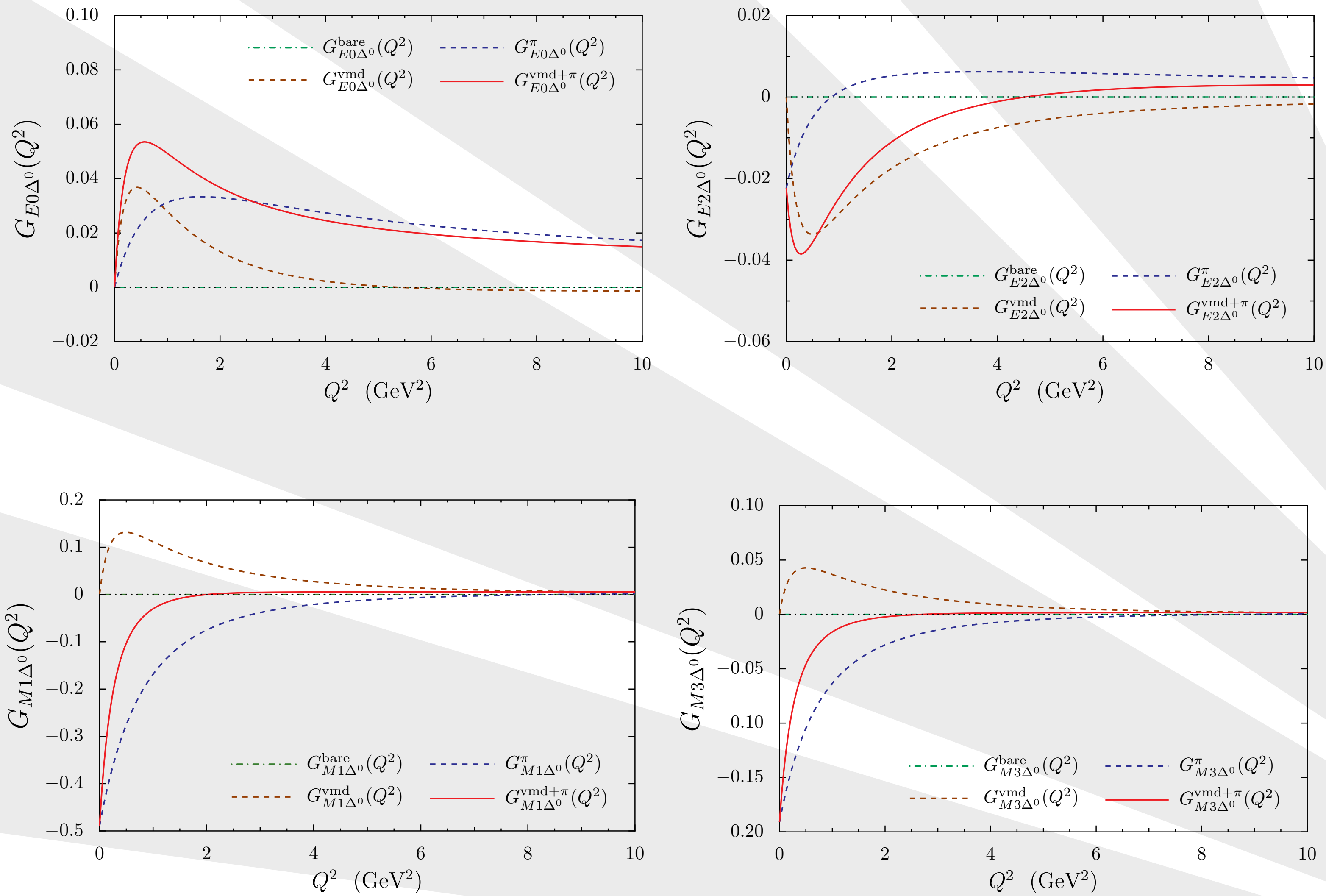
F_1 Form Factor Diagrams

- Themes
- NJL model
- Baryons . . .
- $N \longrightarrow \Delta$ FFs
- Constituents
- Nucleon FFs
- Scalar Diquark FF
- Axial-Vector FF
- Nucleon FFs
- Delta FFs
- $N \longrightarrow \Delta$ FFs
- Off-Shell
- Conclusion



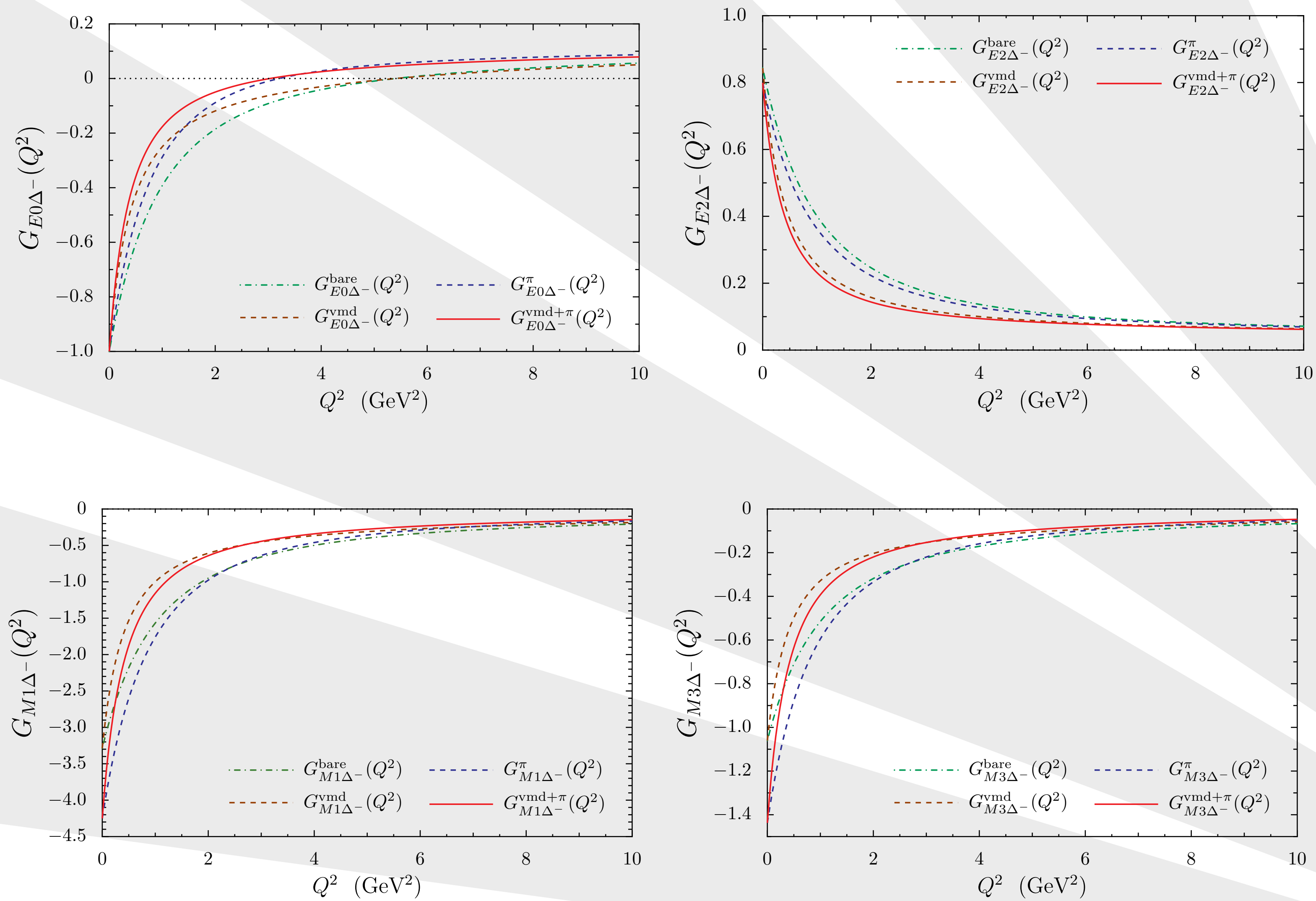
Δ^0 *Form Factors Results*

- ❖ Themes
- ❖ NJL model
- ❖ Baryons . . .
- ❖ $N \rightarrow \Delta$ FFs
- ❖ Constituents
- ❖ Nucleon FFs
- ❖ Scalar Diquark FF
- ❖ Axial-Vector FF
- ❖ Nucleon FFs
- ❖ Delta FFs
- ❖ $N \rightarrow \Delta$ FFs
- ❖ Off-Shell
- ❖ Conclusion



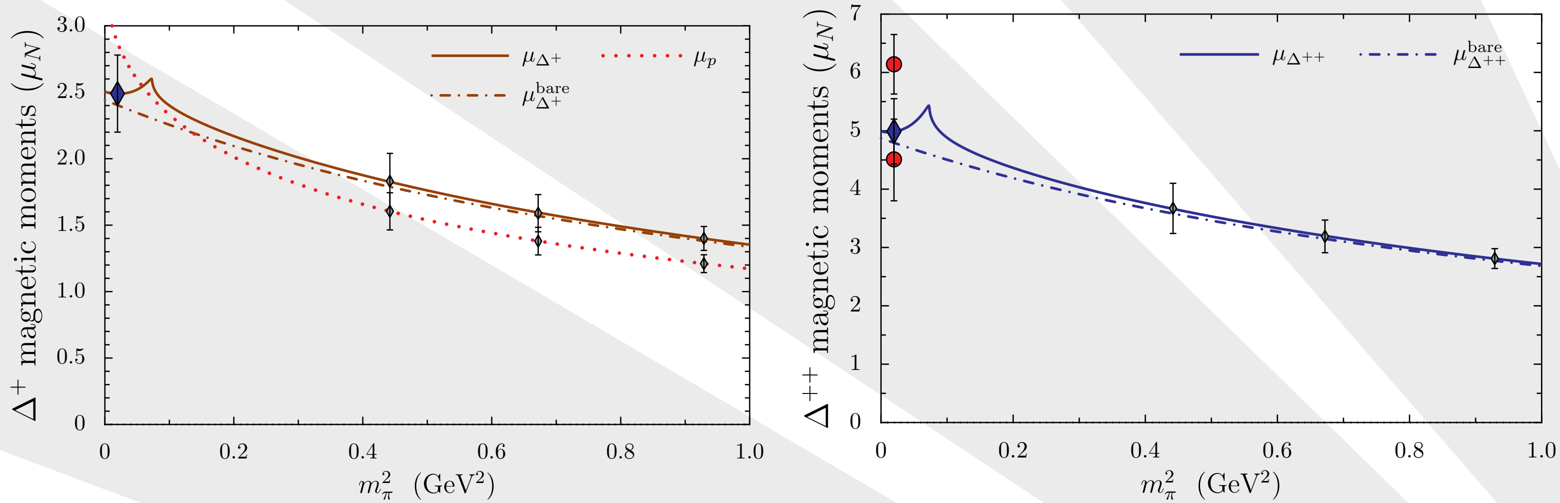
Δ^- Form Factors Results

- ❖ Themes
- ❖ NJL model
- ❖ Baryons . . .
- ❖ $N \longrightarrow \Delta$ FFs
- ❖ Constituents
- ❖ Nucleon FFs
- ❖ Scalar Diquark FF
- ❖ Axial-Vector FF
- ❖ Nucleon FFs
- ❖ Delta FFs
- ❖ $N \longrightarrow \Delta$ FFs
- ❖ Off-Shell
- ❖ Conclusion



Delta Moments: Pion Cloud

❖ Themes
❖ NJL model
❖ Baryons . . .
❖ $N \rightarrow \Delta$ FFs
❖ Constituents
❖ Nucleon FFs
❖ Scalar Diquark FF
❖ Axial-Vector FF
❖ Nucleon FFs
❖ Delta FFs
❖ $N \rightarrow \Delta$ FFs
❖ Off-Shell
❖ Conclusion



- Pion Cloud contribution surprisingly small
- NJL model consistent with this result
- Lattice: $\mu_{\Delta^+} < \mu_p$, NJL agrees.

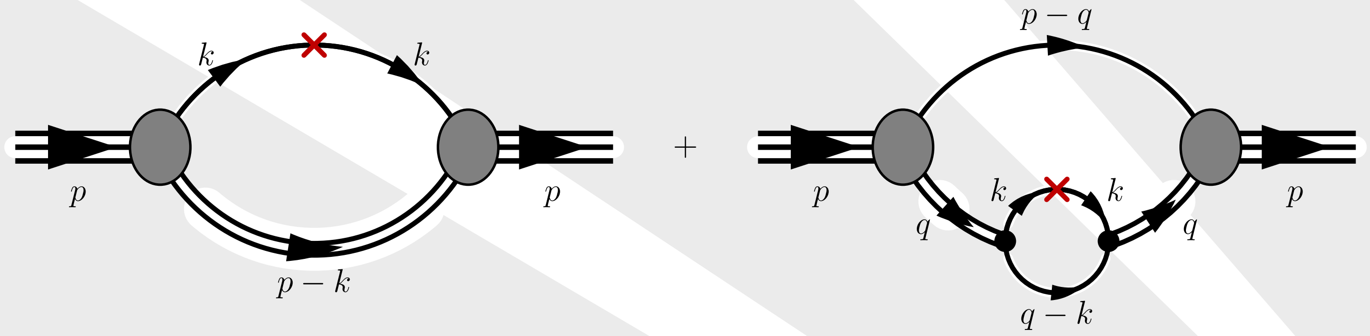
❖ C. Alexandrou, *et al*, Phys. Rev. D **74**, 034508 (2006)

❖ C. Alexandrou, *et al*, PoS **LAT2007**, 149 (2007)

Nucleon quark distributions

- Themes
- NJL model
- Baryons . . .
- $N \to \Delta$ FFs
- Constituents
- Nucleon FFs
- Scalar Diquark FF
- Axial-Vector FF
- Nucleon FFs
- Delta FFs
- $N \to \Delta$ FFs
- Off-Shell
- Conclusion

- Associated with a Feynman diagram calculation.

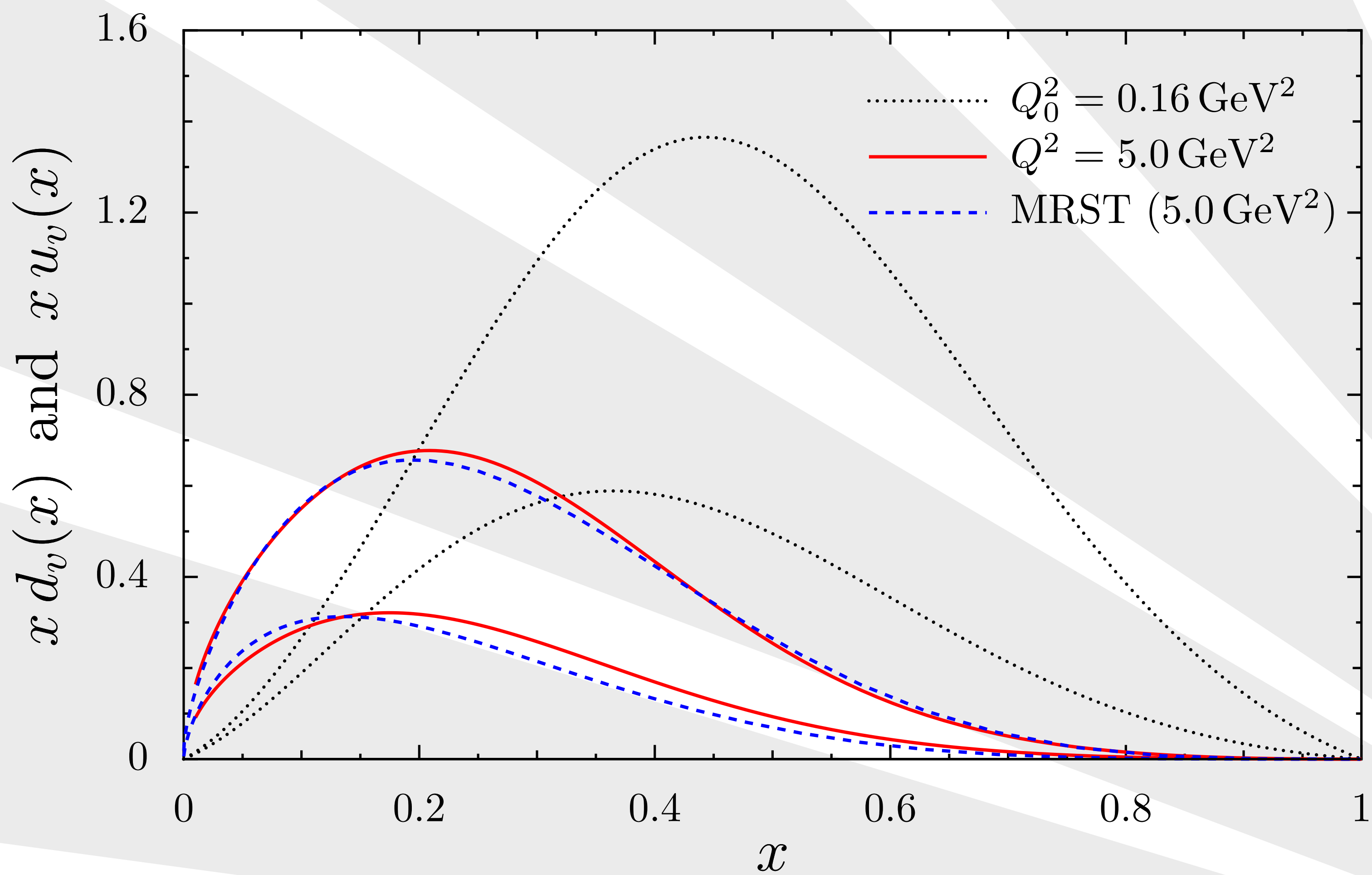


- $[q(x), \Delta q(x), \Delta_T q(x)]$
  $\to \; \mathrm{X} = \; \delta(x - \frac{k^+}{p^+}) \, [\gamma^+, \gamma^+\gamma_5, \gamma^+\gamma^1\gamma_5]$

- Satisfies baryon and momentum sum rules.
- Satisfies positivity constraints and Soffer bound.
- Covariant and gives correct support
- Model testing ground

$u_v(x)$ and $d_v(x)$ distributions

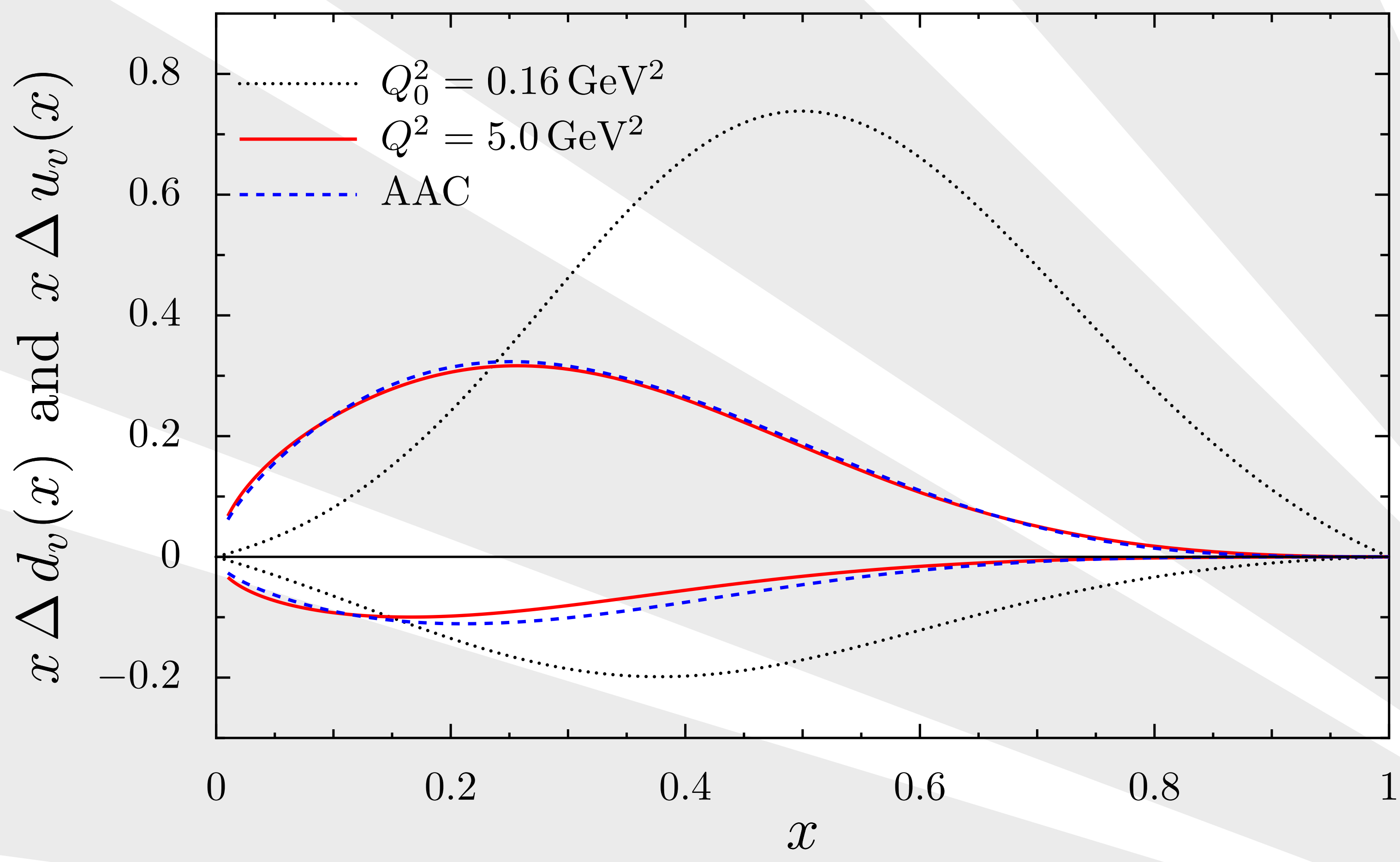
- Themes
- NJL model
- Baryons . . .
- $N \to \Delta$ FFs
- Constituents
- Nucleon FFs
- Scalar Diquark FF
- Axial-Vector FF
- Nucleon FFs
- Delta FFs
- $N \to \Delta$ FFs
- Off-Shell
- Conclusion



- MRST, Phys. Lett. B **531**, 216 (2002).

$\Delta u_v(x)$ and $\Delta d_v(x)$ distributions

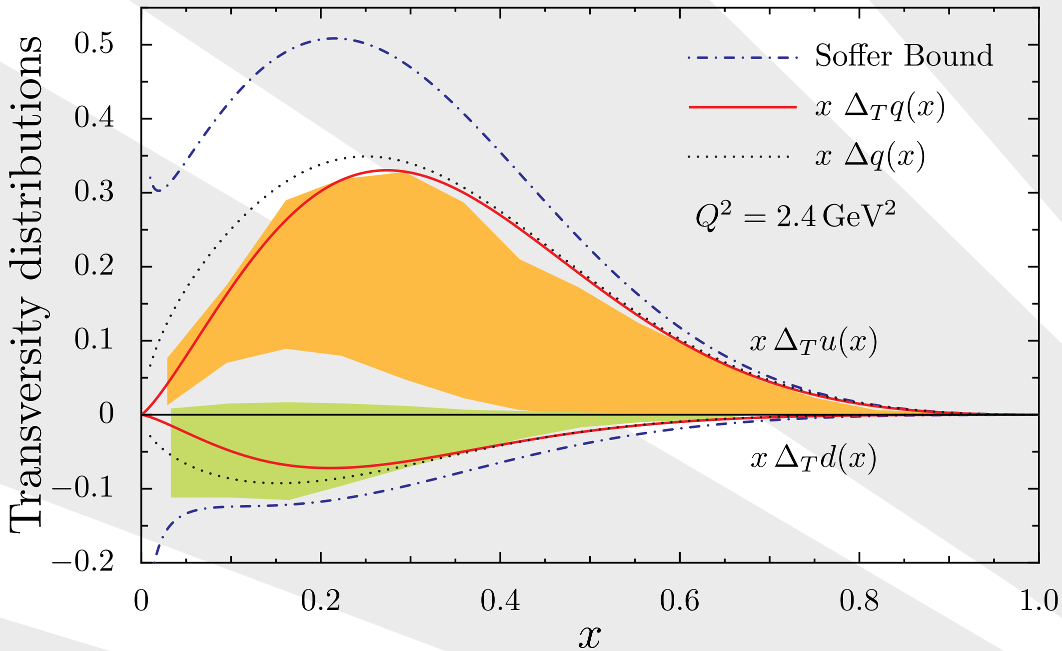
- ❖ Themes
- ❖ NJL model
- ❖ Baryons . . .
- ❖ $N \to \Delta$ FFs
- ❖ Constituents
- ❖ Nucleon FFs
- ❖ Scalar Diquark FF
- ❖ Axial-Vector FF
- ❖ Nucleon FFs
- ❖ Delta FFs
- ❖ $N \to \Delta$ FFs
- ❖ Off-Shell
- ❖ Conclusion



- M. Hirai, S. Kumano and N. Saito, Phys. Rev. D **69**, 054021 (2004).

$\Delta_T u_v(x)$ and $\Delta_T d_v(x)$ distributions

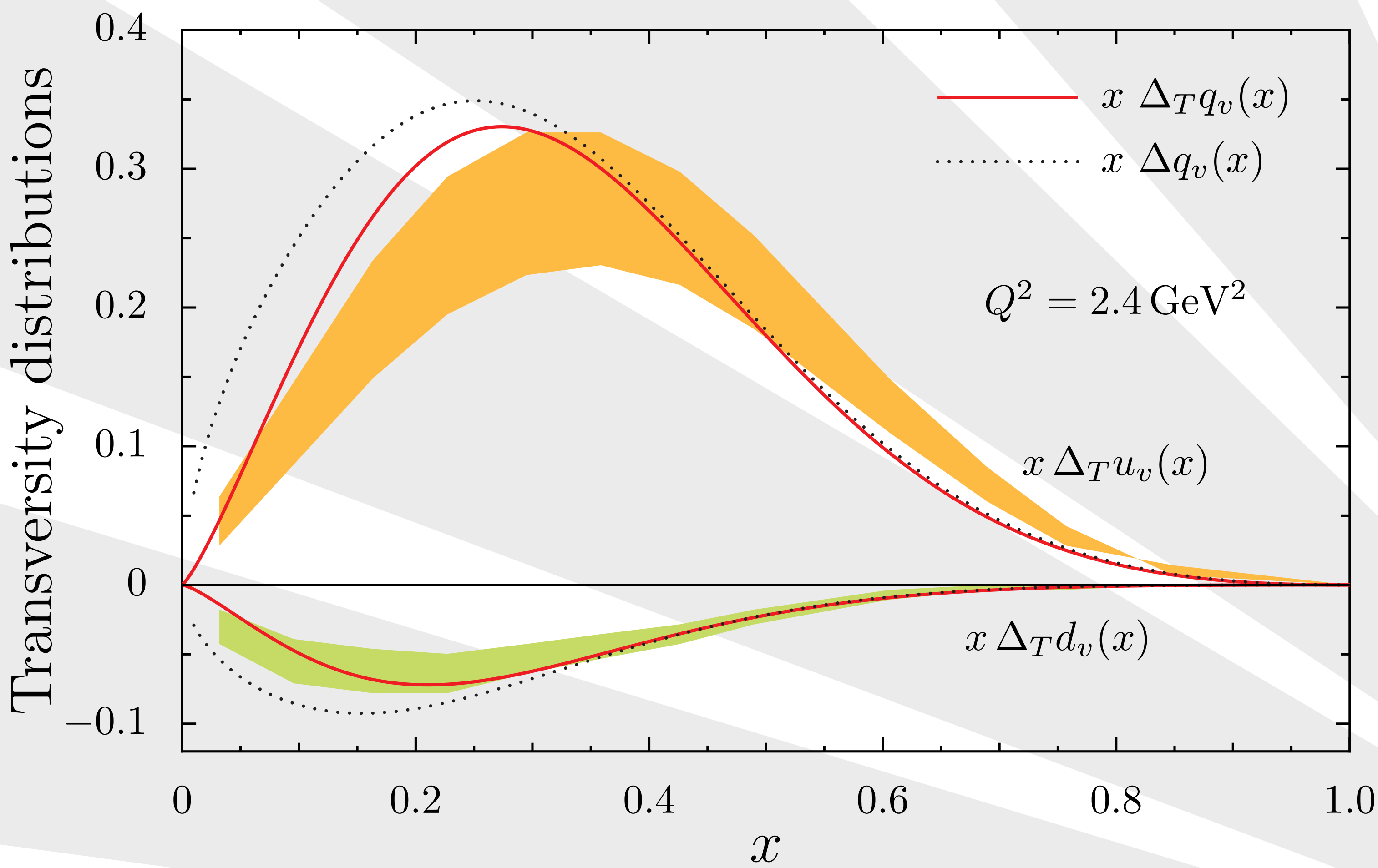
- ❖ Themes
- ❖ NJL model
- ❖ Baryons . . .
- ❖ $N \to \Delta$ FFs
- ❖ Constituents
- ❖ Nucleon FFs
- ❖ Scalar Diquark FF
- ❖ Axial-Vector FF
- ❖ Nucleon FFs
- ❖ Delta FFs
- ❖ $N \to \Delta$ FFs
- ❖ Off-Shell
- ❖ Conclusion



- M. Anselmino *et. al.*,Phys. Rev. D **75**, 054032 (2007).
- Non-relativistic limit: $\Delta_T q(x) = \Delta q(x)$
- $M \sim 400$ MeV, large relat. corrections unexpected
- Potential problem for models based concept of "constituent quarks" – maybe running mass

Transversity: Reanalysis

- Themes
- NJL model
- Baryons . . .
- $N \to \Delta$ FFs
- Constituents
- Nucleon FFs
- Scalar Diquark FF
- Axial-Vector FF
- Nucleon FFs
- Delta FFs
- $N \to \Delta$ FFs
- Off-Shell
- Conclusion



- Anselmino *et al* DIS 08
- I. C. Cloët, W. Bentz and A. W. Thomas, Phys. Lett. B **659**, 214 (2008)

Transversity Moments

- Themes
- NJL model
- Baryons . . .
- $N \rightarrow \Delta$ FFs
- Constituents
- Nucleon FFs
- Scalar Diquark FF
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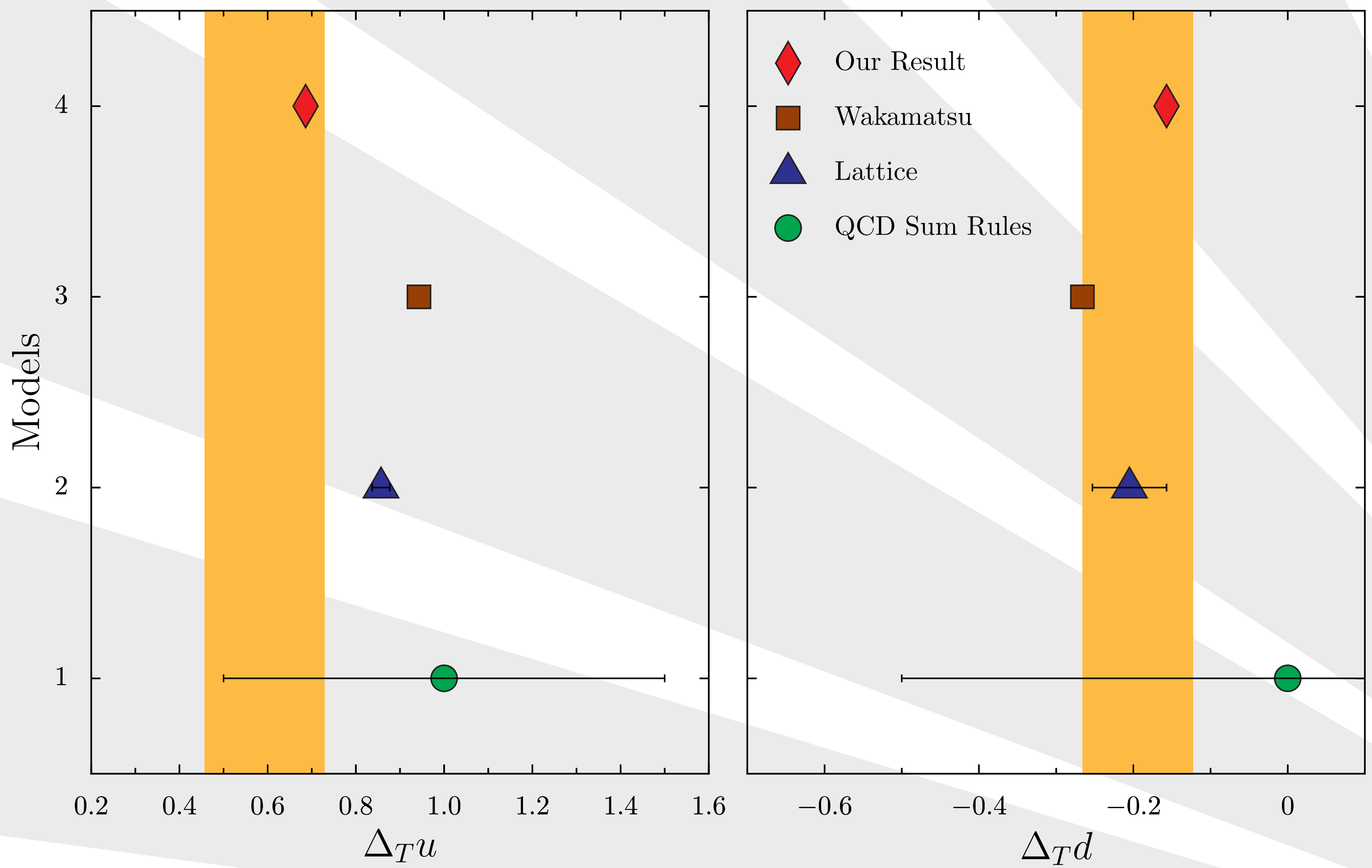
- Nucleon FFs
- Delta FFs
- $N \rightarrow \Delta$ FFs
- Off-Shell
- Conclusion



- Anselmino *et al* DIS 08
- I. C. Cloët, W. Bentz and A. W. Thomas, Phys. Lett. B **659**, 214 (2008)