Lattice Calculations of *N*-*N** Form Factors

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Electromagnetic N-N* Transition Form Factors Workshop

Oct. 14, 2008

Jefferson Lab
Exploring the Nature of Matter

In collaboration with

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Kostas Orginos
(W&M; JLab)

Lattice Baryon Form Factors

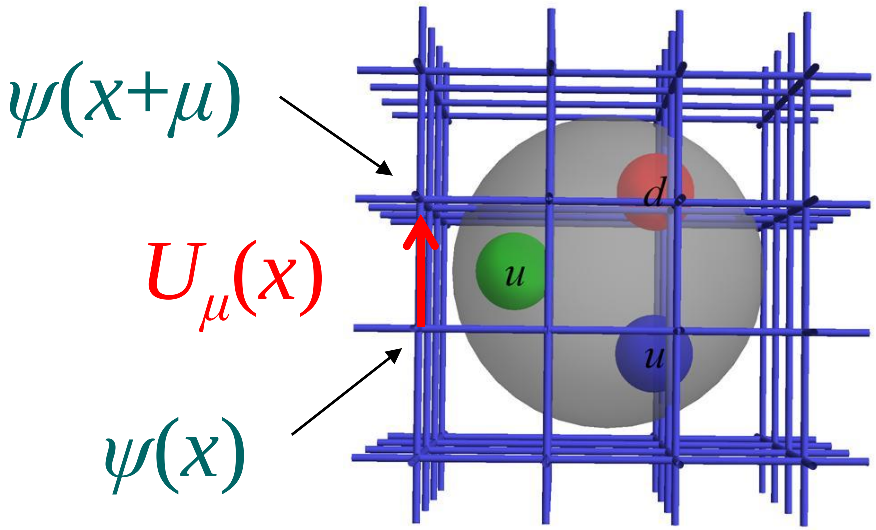
- Lattice QCD group at JLab
  - main physics directions in support of (JLab) hadronic physics experimental program
- Baryon form factors proposals
  - Strange baryon (transition) form factors
  - Radically excited transition form factors
    - Challenge: never been done in lattice calculations before
    - Starting with first excited state of nucleon: Roper(?)
    - Experiments at Jefferson Laboratory (**CLAS**), MIT-Bates, LEGS, Mainz, Bonn, GRAAL, and Spring-8
    - Long term goal: extend calculation to more *N**

Lattice QCD

- Physical observables are calculated from the path integral

$$\langle 0|O(\overline{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int [dA][d\overline{\psi}][d\psi]\, O(\overline{\psi},\psi,A)e^{i\int d^4x\,\mathcal{L}^{\text{QCD}}(\overline{\psi},\psi,A)}$$

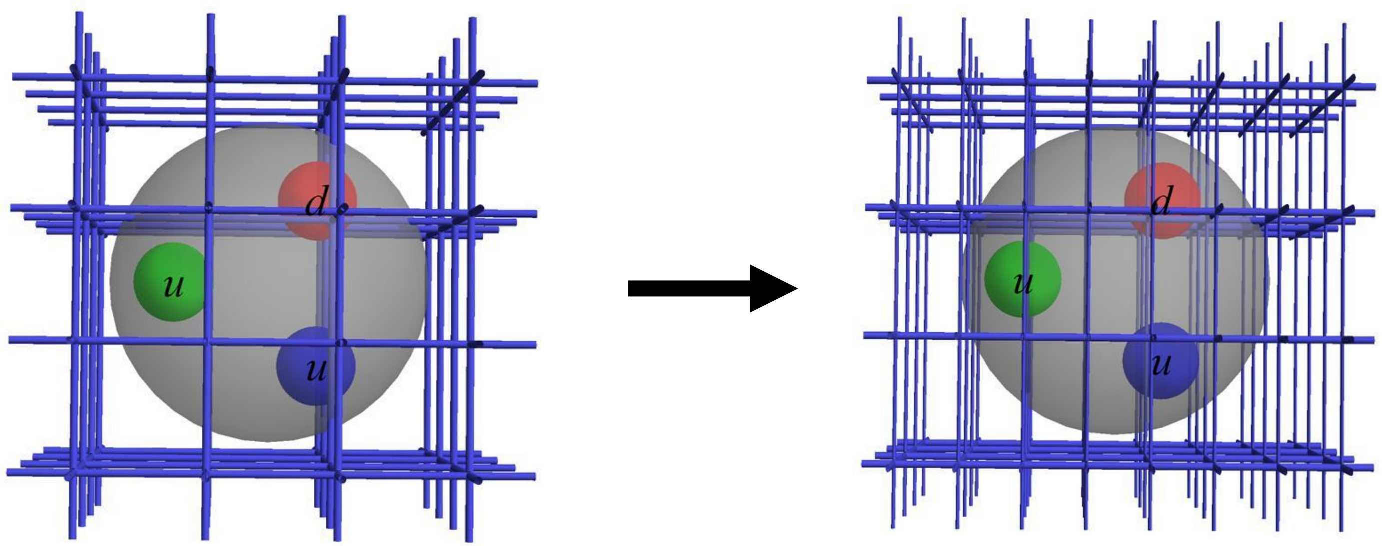
- Strong-coupling regions: expansions no longer converge
- Lattice QCD is a discrete version of continuum QCD theory



- Numerical integration to calculate the path integral
- Take $a \to 0$ and $V \to \infty$ in the continuum limit

Lattice Challenge

- Euclidean space:
  obtain correlators with time-dependent form $\sum_n Z_{n,B} e^{-E_n(\vec{P})t}$
- Signal falls exponentially with time
  dominated by ground state; challenge for excited states
- Solution: increase resolution

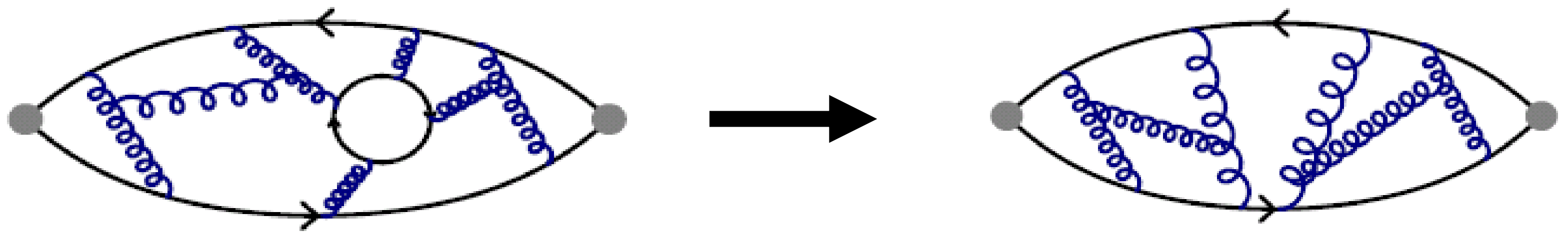


***An*isotropic lattice**

more complications to generate 2+1f lattice ensembles

Lattice Setup

- "Quenched" for exploratory study



no sea quark contributions

  - Bad: Uncontrollable systematic error
  - Good?
    - Preserve nice features:
      confining, asymptotically free, spontaneously broken chiral symmetry
    - Cheap exploratory studies to develop new methods

- Some detailed Parameters
  - $16^3 \times 64$ **an**isotropic lattice, $\xi = 3$
  - Wilson gauge action + clover fermion action
  - $a_t^{-1} \approx 6$ GeV and $a_s \approx 0.125$ fm ($L < 2$ fm)
  - $m_\pi \approx 720$ (480 and 1100) MeV
  - 200 configurations

Green Functions

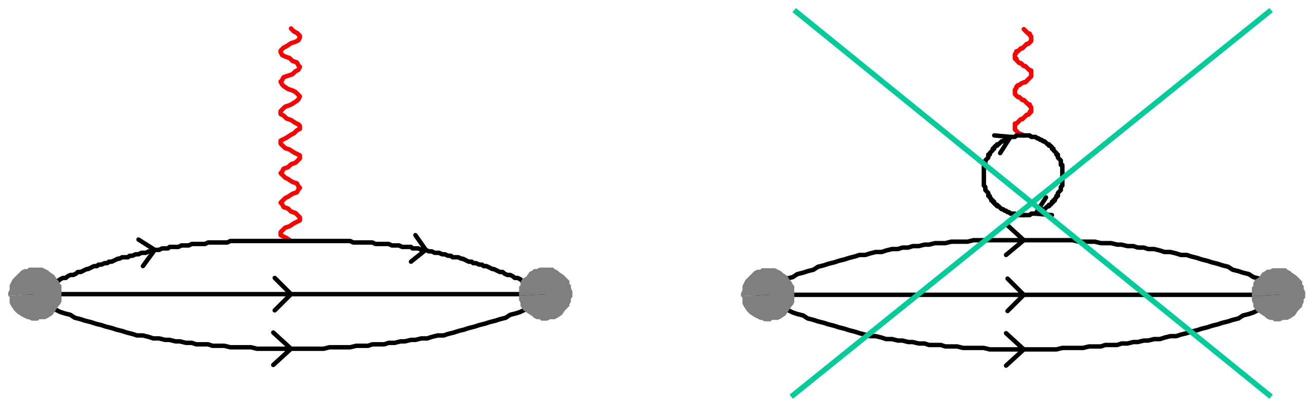
- Three-point function with interpolation operator J

$$C_{\text{3pt}}^{\Gamma,\mathcal{O}}\left(\vec{p},t,\tau\right)=\sum_{\alpha,\beta}\Gamma^{\alpha,\beta}\langle J_\beta\left(\vec{p},t\right)\mathcal{O}(\tau)\overline{J}_\alpha\left(\vec{p},0\right)\rangle$$

- Baryon interpolating field

$$J_\alpha\left(\vec{p},t\right)=\sum_{\vec{x},a,b,c}e^{i\vec{p}\cdot\vec{x}}\epsilon^{abc}\left[u_a^T(y_1,t)C\gamma_5 d_b(y_2,t)\right]u_{c,\alpha}(y_3,t)\phi(y_1-x)\phi(y_2-x)\phi(y_3-x)$$

- Two contraction categories:



- We use only the “connected” construction for this work
- Ongoing investigation into “disconnected” contribution

Form Factors

- The form factors are buried in the amplitudes

$$\Gamma^{(3),T}_{\mu,AB}(t_i, t, t_f, \vec{p}_i, \vec{p}_f)$$
$$= a^3 \sum_n \sum_{n'} \frac{1}{Z_j} \frac{Z_{n',B}(p_f) Z_{n,A}(p_i)}{4E'_n(\vec{p}_f) E_n(\vec{p}_i)} e^{-(t_f - t)E'_n(\vec{p}_f)} e^{-(t-t_i)E_n(\vec{p}_i)}$$
$$\times \sum_{s,s'} T_{\alpha\beta} u_{n'}(\vec{p}_f, s')_\beta \langle N_{n'}(\vec{p}_f, s') | j_\mu(0) | N_n(\vec{p}_i, s)\rangle \overline{u}_n(\vec{p}_i, s)_\alpha$$

- Nucleon form factor ($n = n' = 0$)

$$\langle N | V_\mu | N \rangle(q) = \overline{u}_N(p') \left[\gamma_\mu F_1(q^2) + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{2m} \right] u_N(p) e^{-iq\cdot x}$$

- Nucleon-Roper form factor ($n = 0$, $n' = 1$ or $n = 1$, $n' = 0$)

$$\langle N_2 | V_\mu | N_1 \rangle_\mu(q) = \overline{u}_{N_2}(p') \left[F_1(q^2) \left(\gamma_\mu - \frac{q_\mu}{q^2} \not{q} \right) + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{M_{N_1} + M_{N_2}} \right] u_{N_1}(p) e^{-iq\cdot x}$$

- Need best possible input from two-point correlators

Variational Method

- Generalized eigenvalue problem:

[C. Michael, Nucl. Phys. B 259, 58 (1985)]
[M. Lüscher and U. Wolff, Nucl. Phys. B 339, 222 (1990)]

  - Construct the matrix

$$C_{i\,j}(t) = \langle 0 \mid \mathcal{O}_i(t)^\dagger \mathcal{O}_j(0) \mid 0 \rangle$$

  - Solve for the generalized eigensystem of

$$C(t_0)^{-1/2} C(t) C(t_0)^{-1/2} v = \lambda(t, t_0) v$$

with eigenvalues

$$\lambda_n(t, t_0) = e^{-(t-t_0)E_n}\left(1 + \mathcal{O}(e^{-|\delta E|(t-t_0)})\right)$$

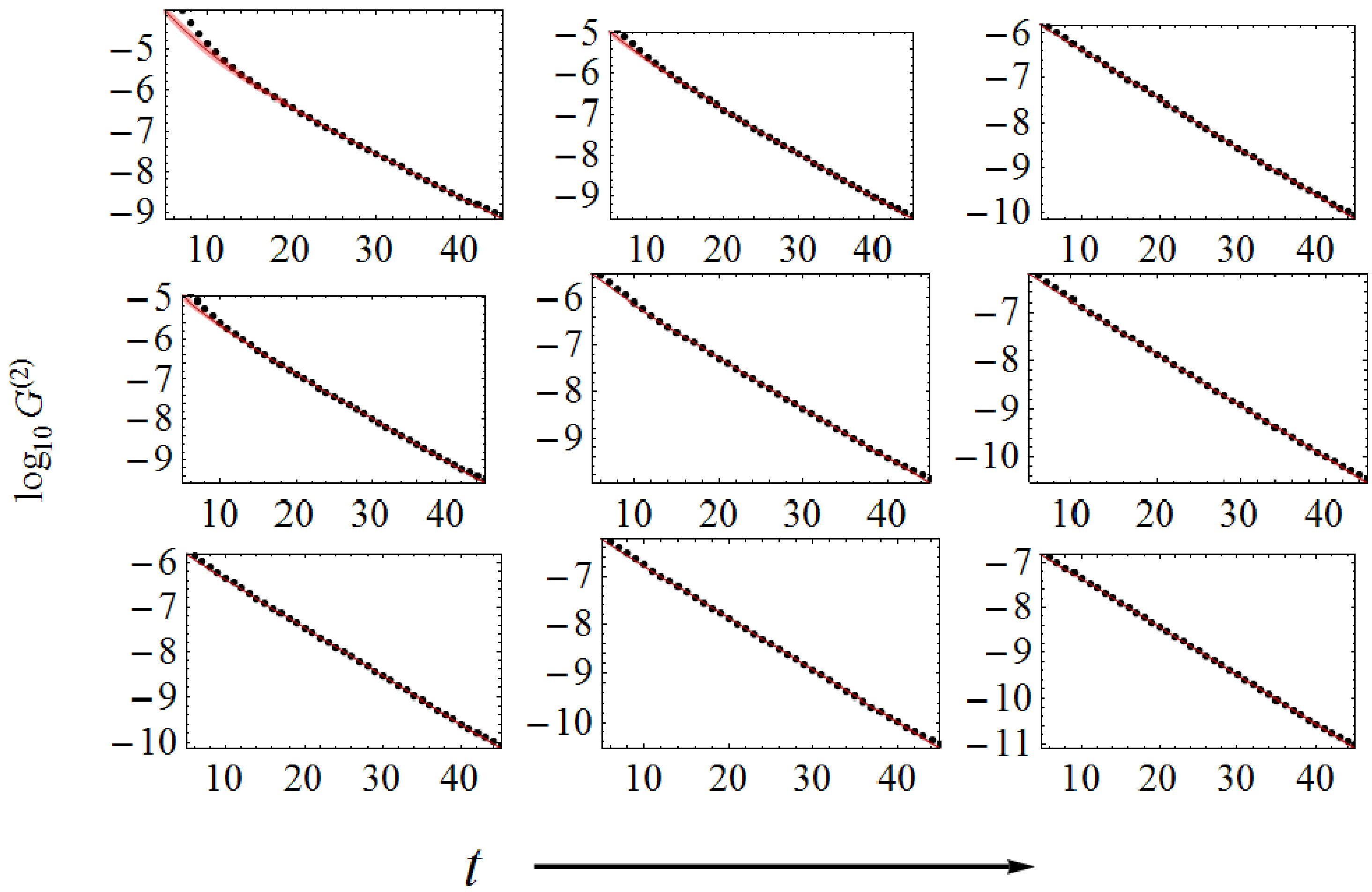
Now the original correlator matrix can be approximated by

$$C_{ij} = \sum_{n=1}^{r} (C(t_0)^{1/2} v_n^*)_i (v_n C(t_0)^{1/2})_j \lambda_n(t, t_0) = \sum_n \frac{E_n + m}{2E_n} Z_{i,n} Z_{j,n} e^{-E_n t}$$

  - Three smearings (*i*,*j*) are chosen for this work
    - 2nd excited state is contaminated by remaining states

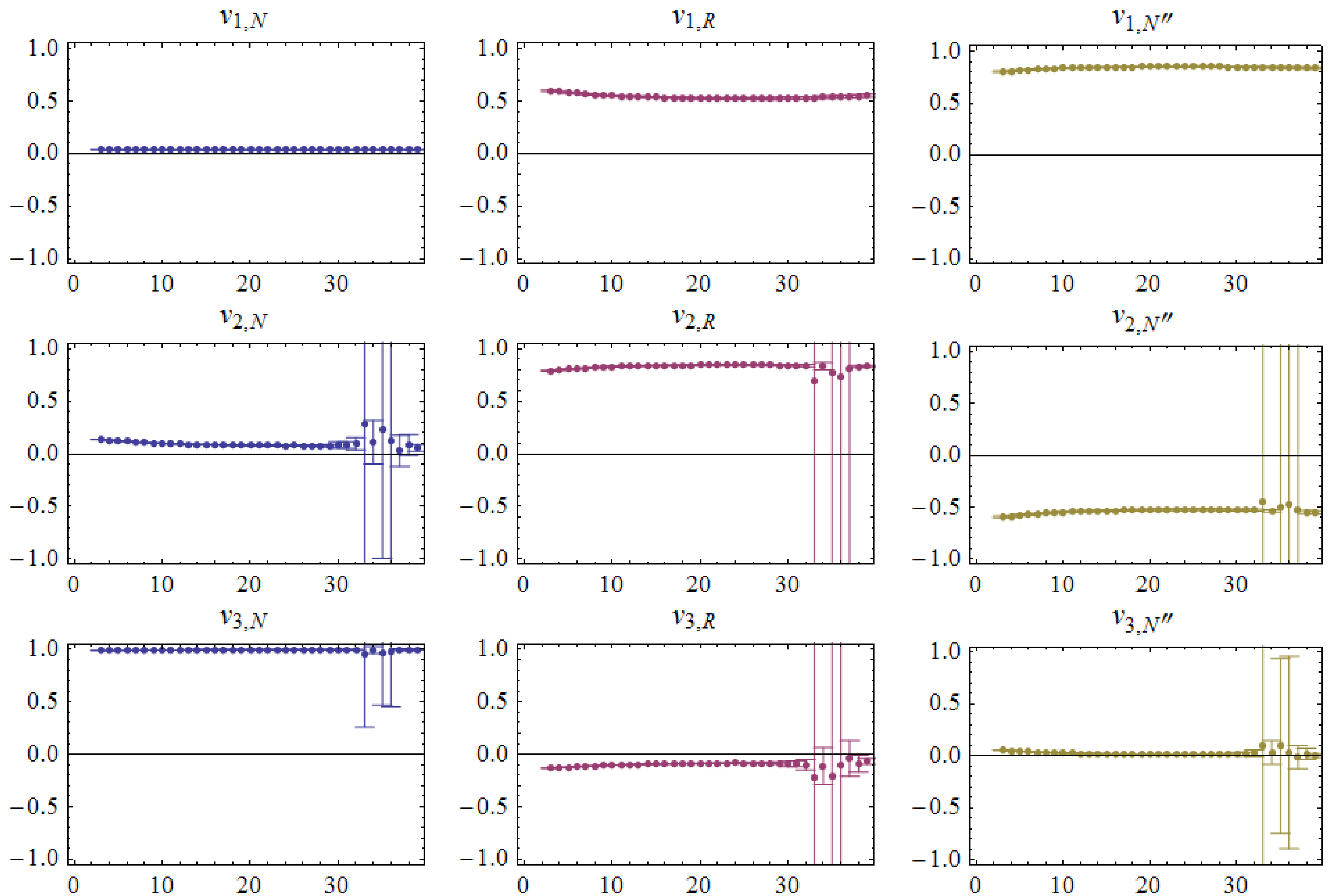
Variational Method

- Reconstruct two-point correlators from Z and λ



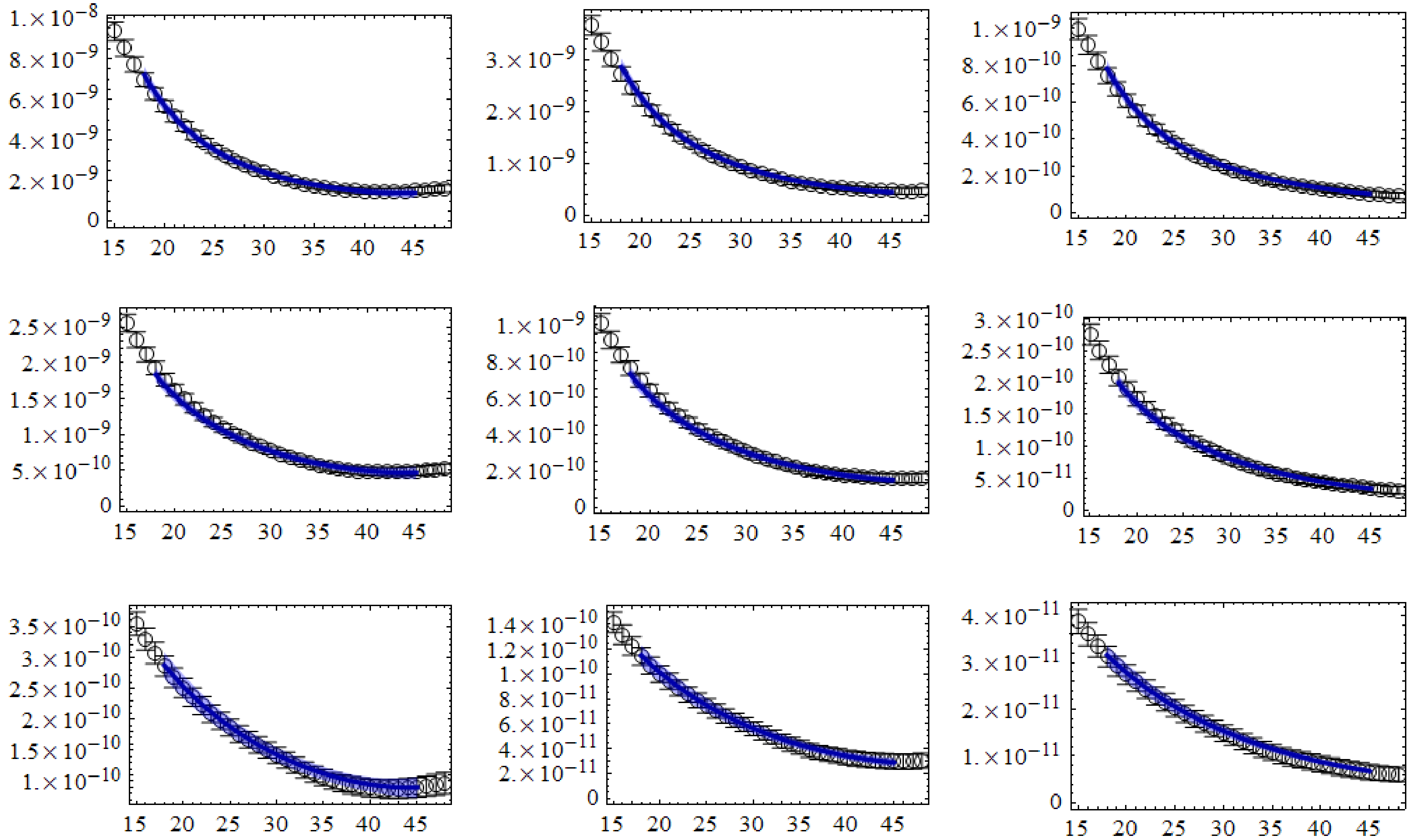
Variational Method

- Eigenvectors (at $p = 0$) show overlap of smearings with states



Three-Point Fitting

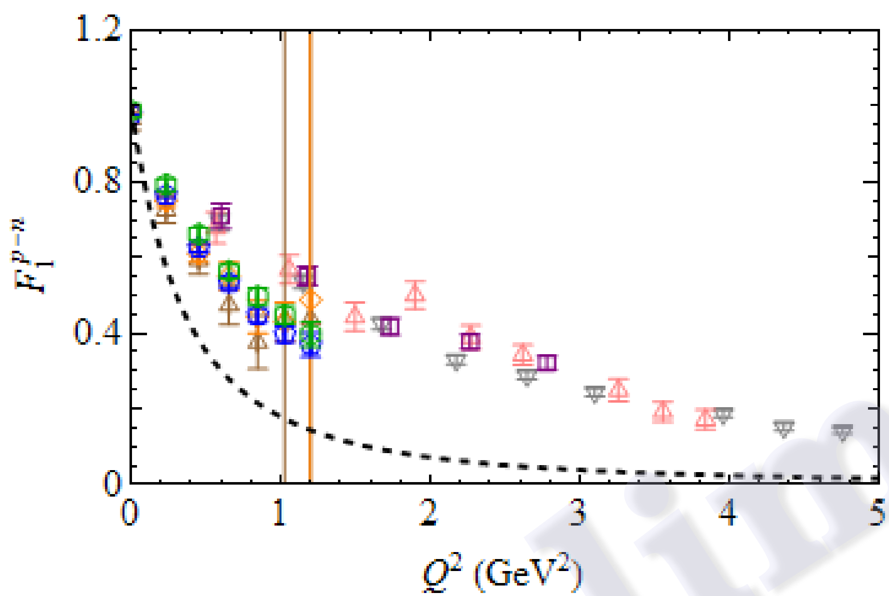
- Example: $P_f = \{0,0,0\}$, $P_i = \{0,1,1\}$, V_4



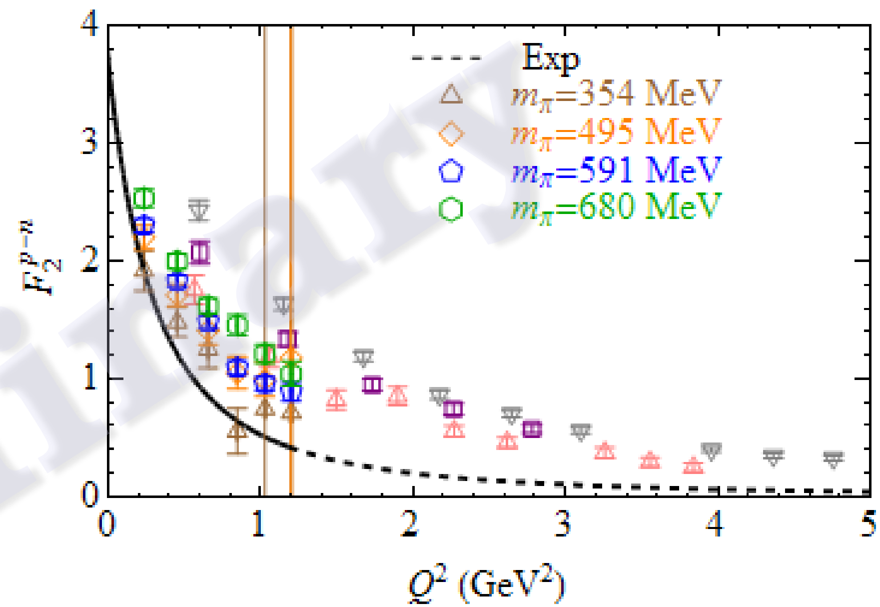
Nucleon Form Factors

- Pion masses around 480, 720 and 1100 MeV

Isovector F_1



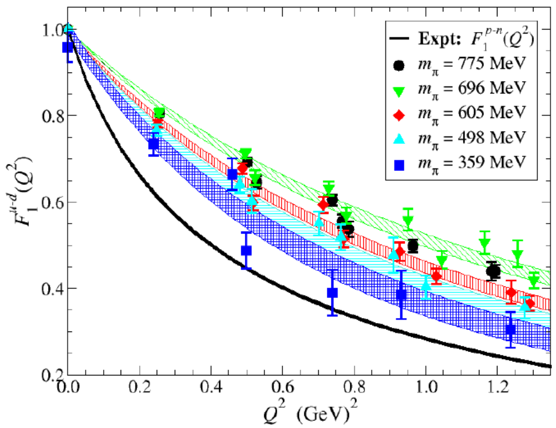
Isovector F_2

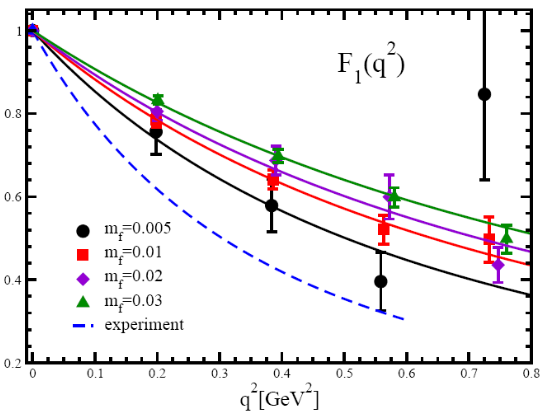


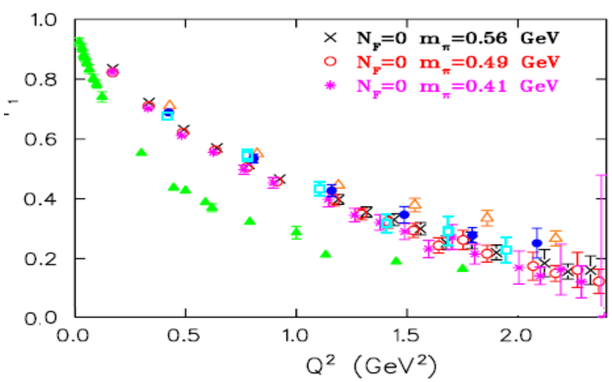
- Compare with $N_f = 2+1$ mixed action (DWF+ asqtad) calculation with conventional approach
- **Clean signal for momentum region of $3 < Q^2 < 5$ GeV^2**

Lattice Form Factors

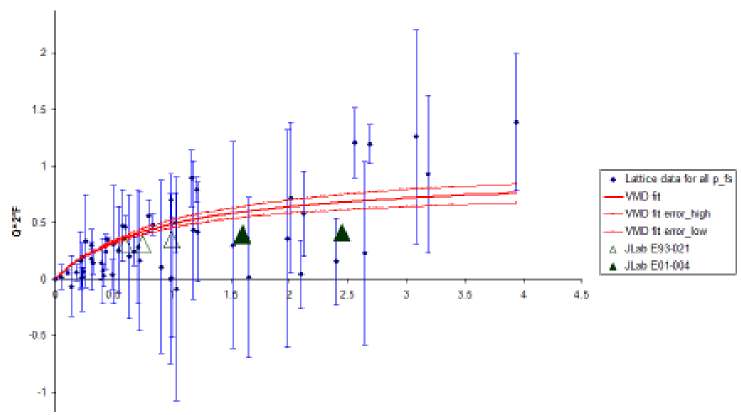
- Large-Q^2 calculations
  - Typical Q^2 range for nucleon form factors is < 3.0 GeV^2







  - Higher-Q^2 calculations suffer from poor noise-to-signal ratios even for pion



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Nucleon-Roper Form Factors

- Converting experimental data ($\gamma^* N_1 \to N_2$)

  $A_{½}(Q^2)/\kappa_A = G_M(Q^2) = F_1^*(Q^2) + F_2^*(Q^2)$

  $S_{½}(Q^2)/\kappa_S = G_E(Q^2) = F_1^*(Q^2) - F_2^*(Q^2)\, Q^2 / (M_{N_1} + M_{N_2})^2$

  with

$$k_A(Q^2) \equiv \sqrt{2\pi\alpha \frac{Q^2 + (M_{N_1} - M_{N_2})^2}{M_{N_1}(M_{N_1}^2 - M_{N_2}^2)}}$$

$$k_S(Q^2) \equiv k_A(Q^2) \frac{M_{N_1} + M_{N_2}}{2\sqrt{2}Q^2 M_{N_2}} \sqrt{Q^2 + (M_{N_1} - M_{N_2})^2} \times \sqrt{Q^2 + (M_{N_1} + M_{N_2})^2}$$

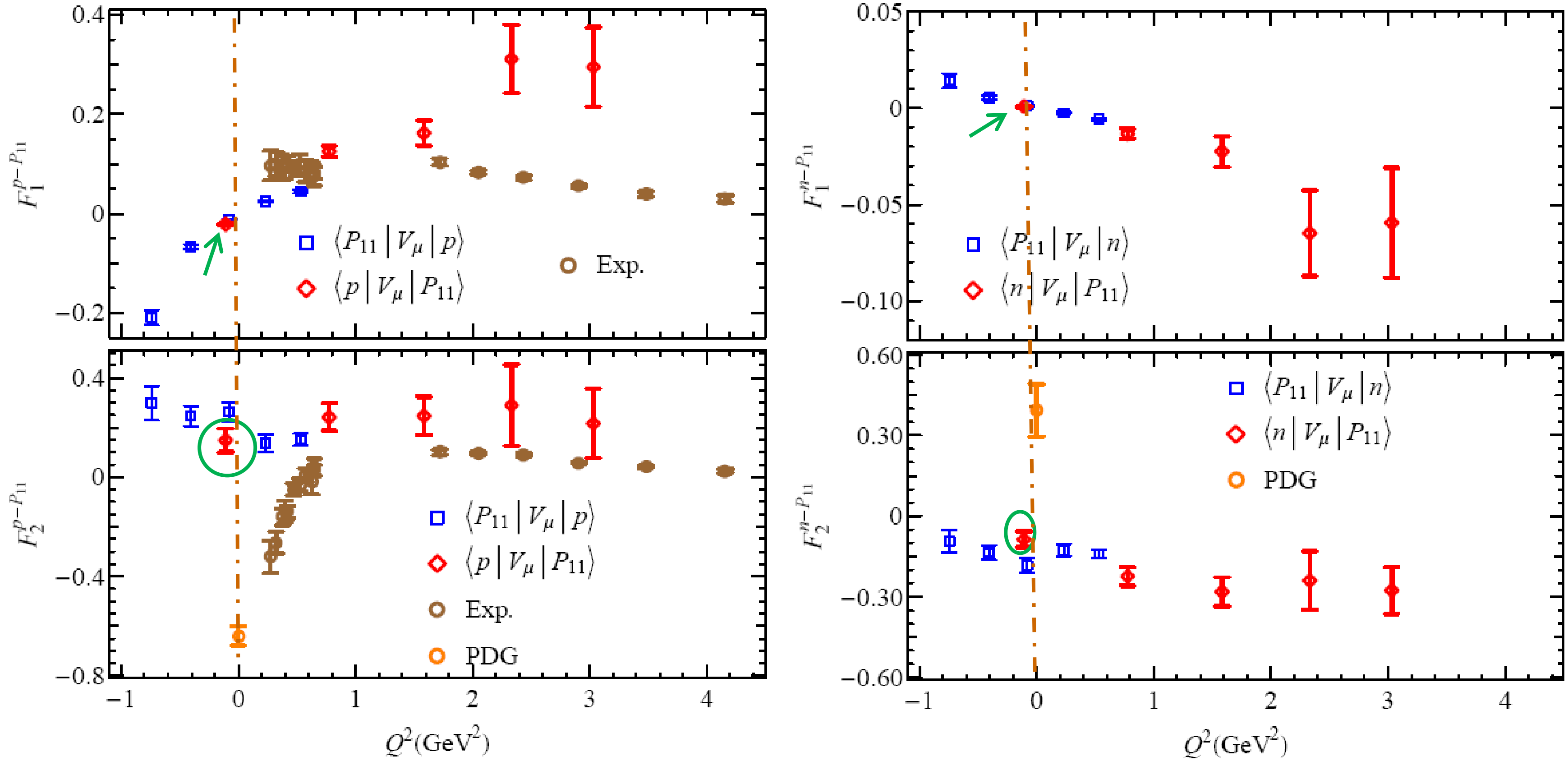
- Use CLAS analysis and PDG results to solve for $F_{1,2}^*(Q^2)$

I. Aznauryan et al., arXiv:0804.0447[nucl-ex], arXiv:0711.1120[nucl-th];
K. Park et al., Phys. Rev. C77, 015208 (2008);
V. I. Mokeev et. al, AIP Conf. Proc. 842, 339 (2006).

Nucleon-Roper Form Factors

- Completed exploratory study on quenched lattices arXiv:0803.3020

Proton-P_{11} **720 MeV Pion** **Neutron-P_{11}**

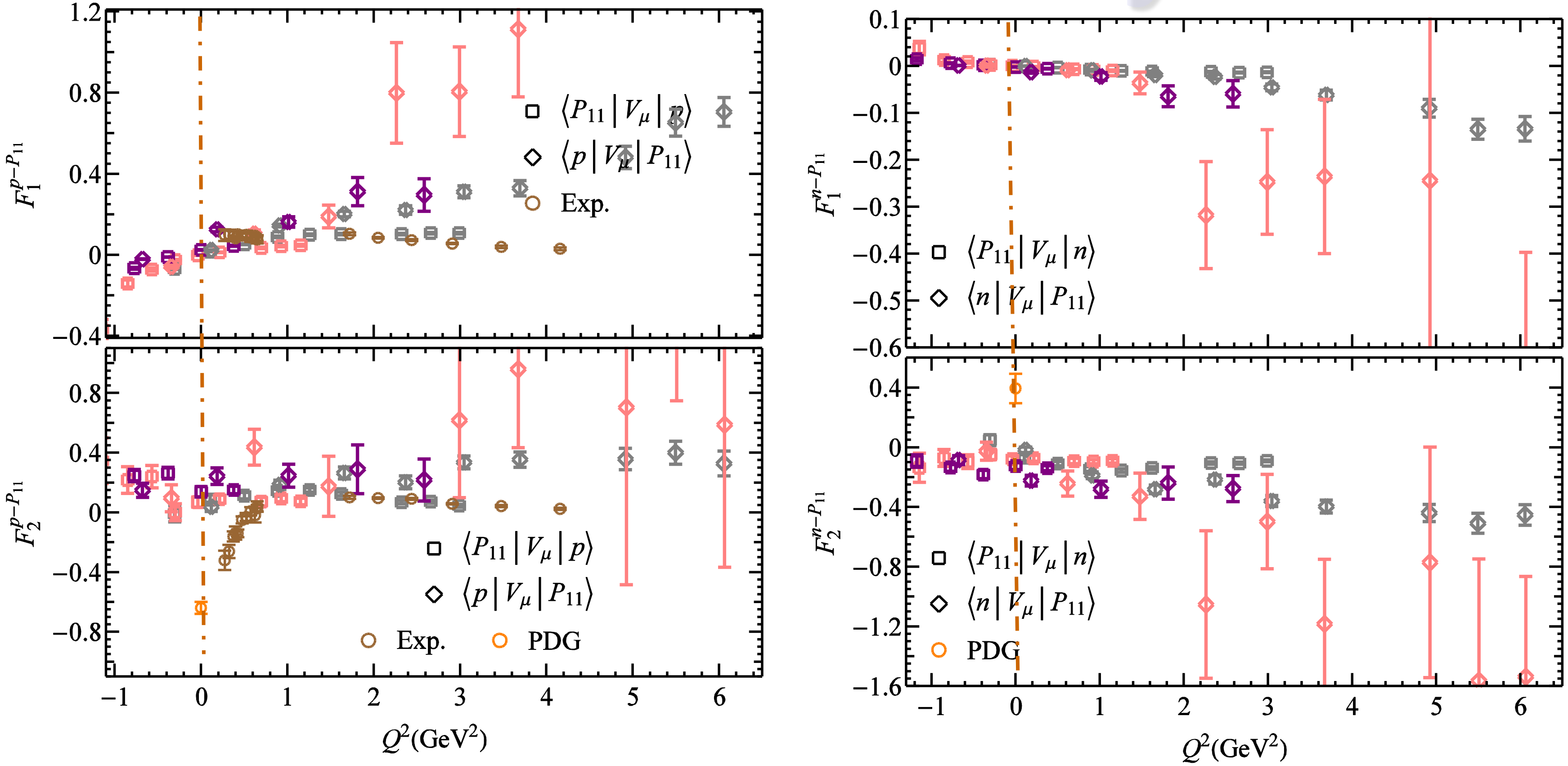


- Possible decaying state (circled above)
- 200 configurations give us reasonable signal
- Lower pion mass will shift the time-like region to space-like region

Nucleon-Roper Form Factors

- Add two more mass points at $m_\pi \sim$ 480 and 1100 MeV

Proton-P_{11} Preliminary **Neutron-P_{11}**



- Need to remove the points with potential decay kinematics
- May want to calculate on a second volume

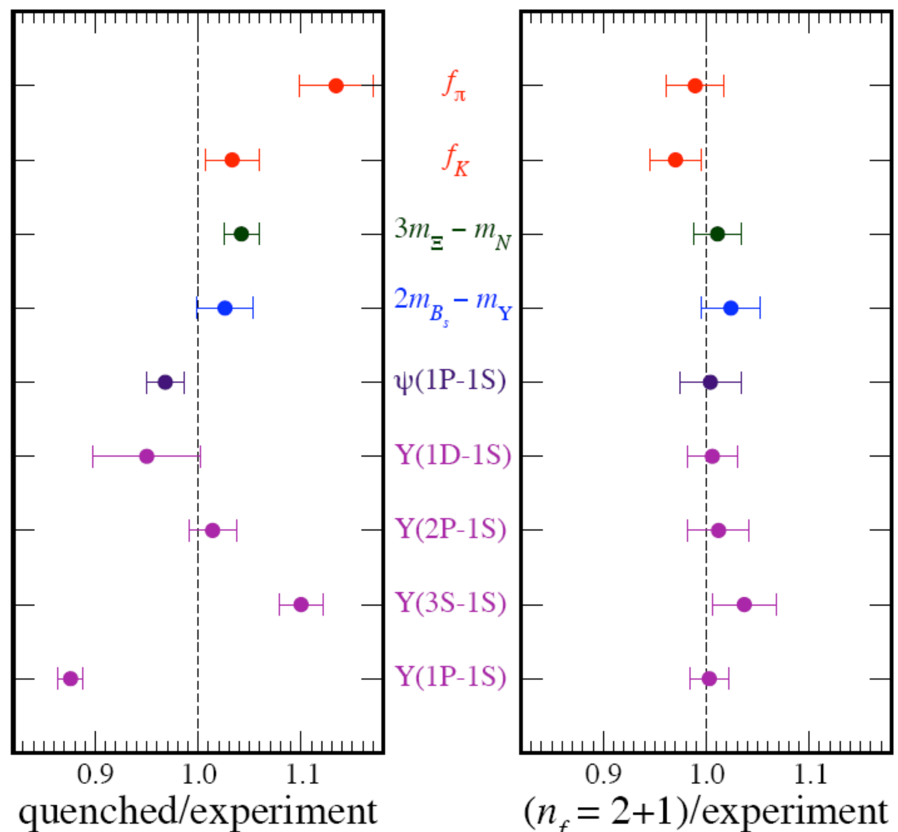
Full-QCD Ensembles

Why Dynamical?

Lattice QCD spectrum

- Successfully calculates many ground states (Nature, ...)

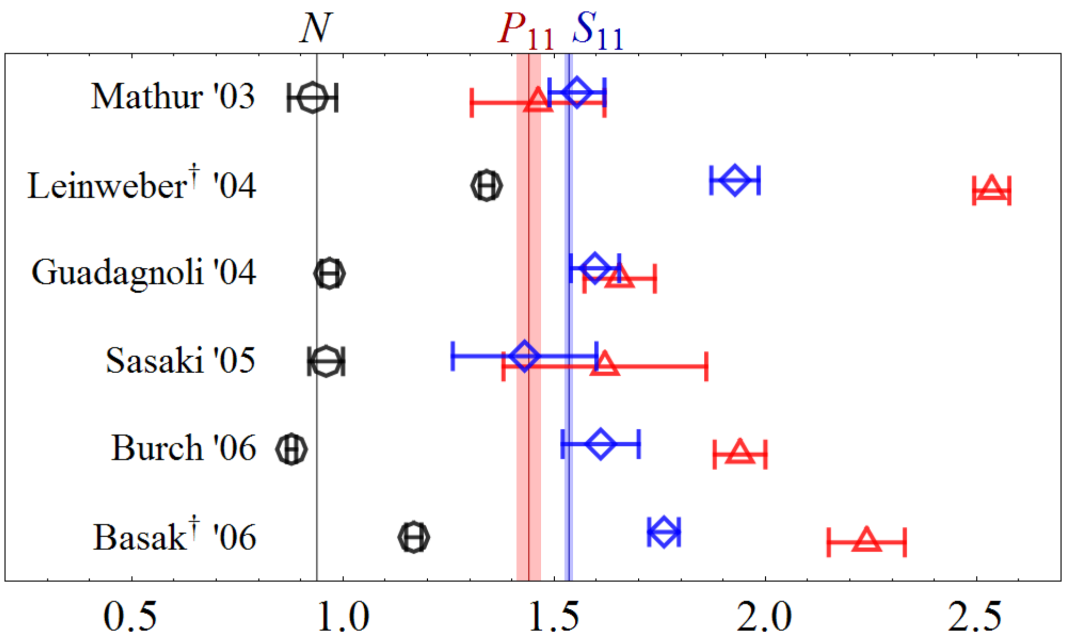
HPQCD



- Predictions: B_c mass, D and D_s decay constants, $D \rightarrow Kl\nu$ form factors

Roper Resonance on the Lattice

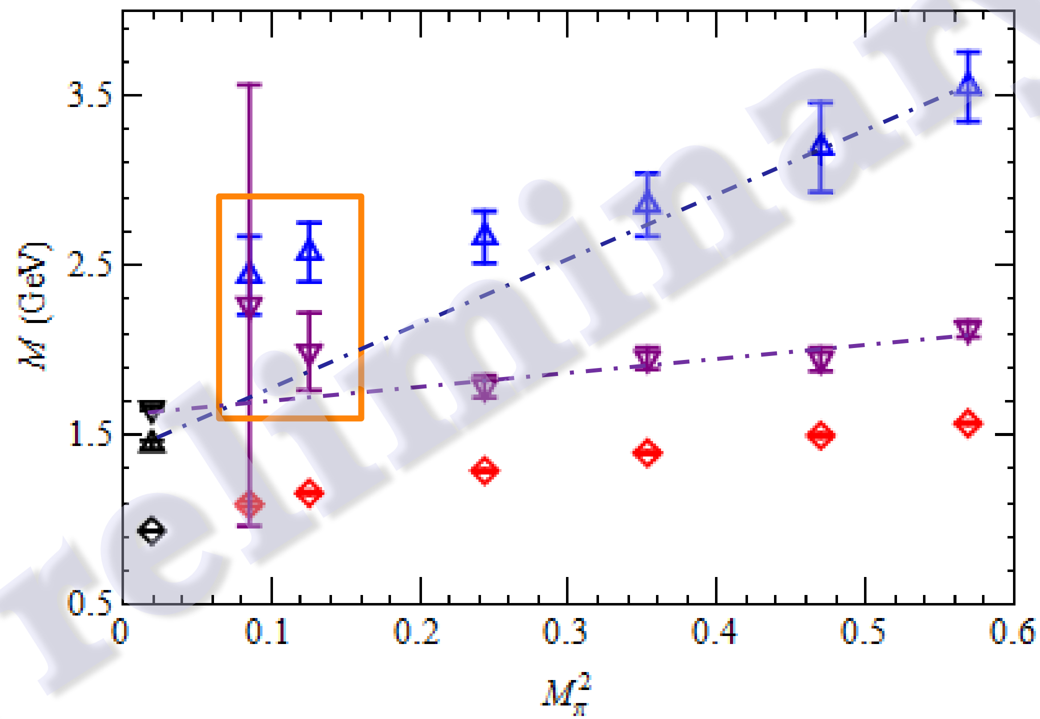
- Mostly done in "quenched" approx. N, P_{11}, S_{11} spectrum



Group	N_f	S_f	a_t^{-1} (GeV)	M_π (GeV)	L (fm)	Method	Extrapolation
Basak et al. [12]	0	Wilson	6.05	0.49	2.35	VM	N/A
Burch et al. [11]	0	CIDO	1.68,1.35	0.35–1.1	2.4	VM	$a + bm_\pi^2$
Sasaki et al. [9]	0	Wilson	2.1	0.61–1.22	1.5,3.0	MEM	$\sqrt{a + bm_\pi^2}$
Guadagnoli et al. [7]	0	Clover [13]	2.55	0.51–1.08	1.85	SBBM	$a + bm_\pi^2 + cm_\pi^4$
Leinweber et al. [8]	0	FLIC	1.6	0.50–0.91	2.0	VM	N/A
Mathur et al. [6]	0	Overlap [14]	1.0	0.18–0.87	2.4,3.2	CCF	$a + bm_\pi + cm_\pi^2$

Roper in Full QCD

- $N_f = 2+1$ mixed action (DWF+ asqtad) calculation ($L \sim 2.5$ fm)
- Symbols: J^P
  - ◇ $1/2^+$ ***N***
  - ▽ $1/2^-$ $\boldsymbol{S_{11}}$
  - △ $1/2^+$ $\boldsymbol{P_{11}}$

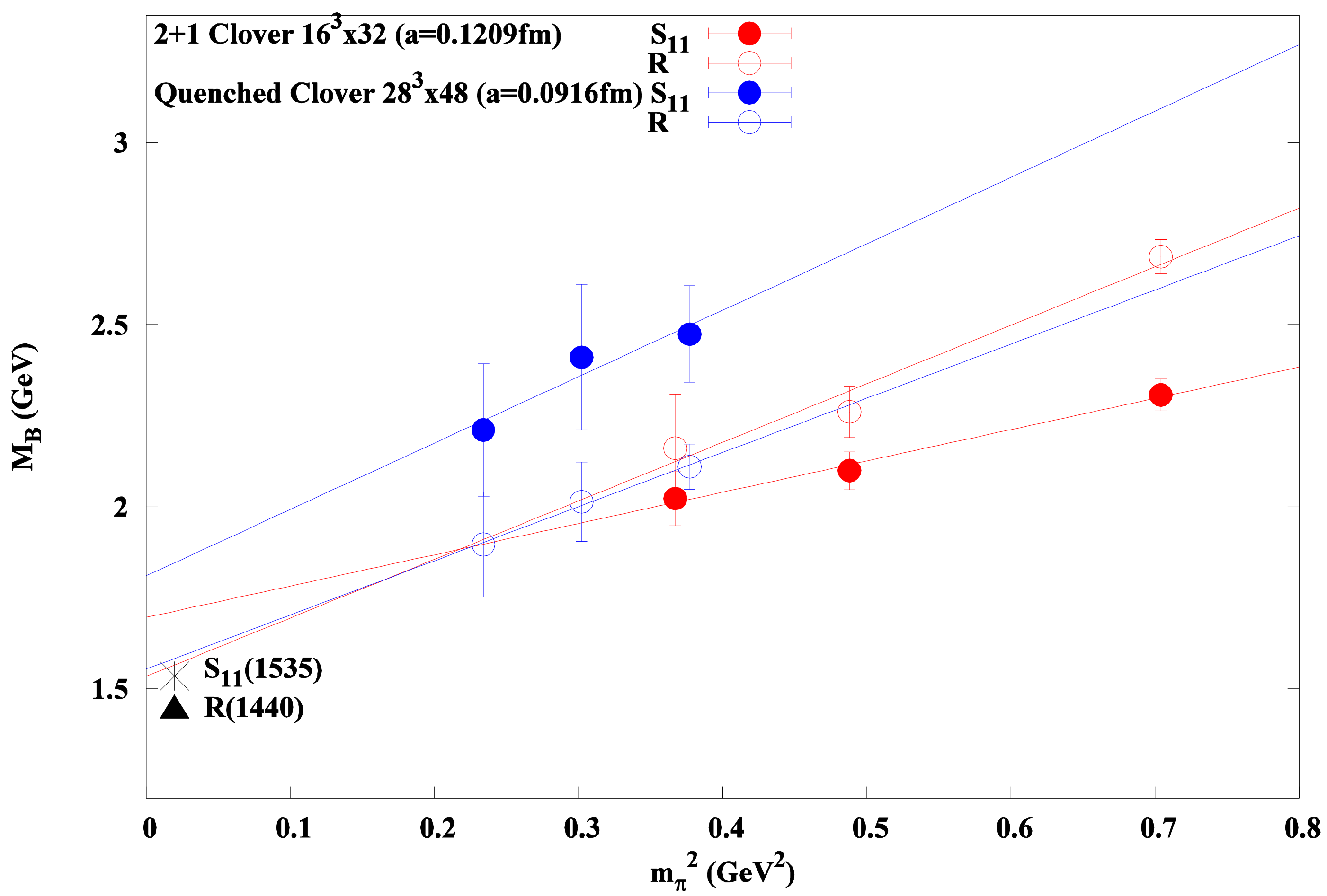


- Finite-volume effects starting at 350 MeV pion?
- Not a crazy possibility (see the hand-drawn extrapolation lines)
- Stay tuned for future $N_f = 2+1$ lattice calculations

Roper in Full QCD

- N_f = 2+1 isotropic clover action calculation (L ~ 2 fm)

[K.F. Liu, LAT2008]



Dynamical Anisotropic Lattices

- N_f = 2+1 **an**isotropic clover action available ensemble list

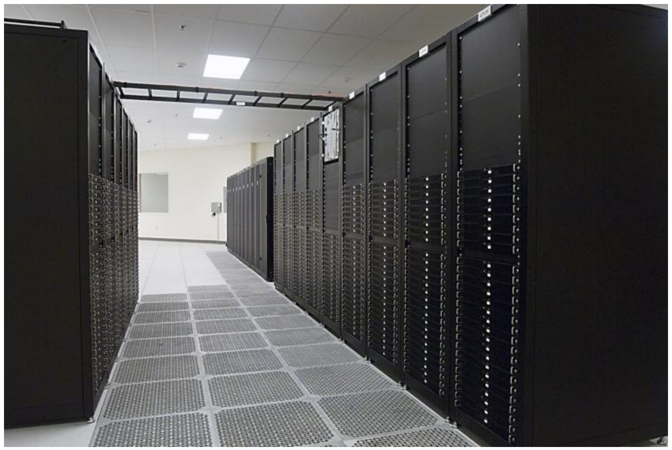
[R. Edwards et al., Phys. Rev. D 78, 014505 (2008)]

[R. Edwards and M. Peardon, LAT2008]

L_x	L_t	m_l	m_s	L (fm)	m_π L	m_π(MeV)
12	96	−0.0540	−0.0540	1.44	11.6	~ 1600
12	96	−0.0699	−0.0540	1.44	8.3	
12	96	−0.0794	−0.0540	1.44	5.9	
12	96	−0.0826	−0.0540	1.44	9.6	
16	96	−0.0826	−0.0540	1.92	6.3	~ 660
12	96	−0.0618	−0.0618	1.44	9.7	~ 1340
16	128	−0.0743	−0.0743	1.92	8.1	~ 850
16	128	−0.0808	−0.0743	1.92	5.6	
16	128	−0.0830	−0.0743	1.92	4.5	
16	128	−0.0840	−0.0743	1.92	3.8	
24	128	−0.0840	−0.0743	2.88	5.7	~ 360

Computational Resources

USQCD facilities: JLab, Fermilab, BNL



Non-lattice resources open to USQCD: ORNL, LLNL, ANL



NSF supercomputer and world-wide increase in computation facilities

Gauge Generation

- Most of the major 2+1-flavor gauge ensembles:

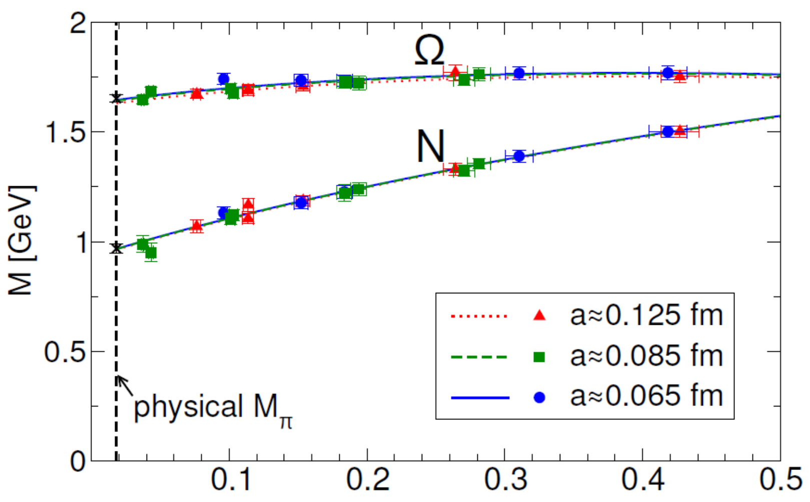
  $M_\pi < 300$ MeV (Most of them have multiple lattice spacings and volumes)

  - MILC (staggered): $M_\pi \sim 217$ MeV
  - PACS-CS (Clover action): $M_\pi \sim 156$ MeV (but small volume)
  - The Budapest-Marseille-Wuppertal (BMW) Collaboration: $M_\pi \sim 193$ MeV
  - RBC/UKQCD (DWF), $M_\pi \sim 210$ MeV (on-going)
  - The Hadron Spectrum Collaboration (anisotropic clover): $M_\pi \sim 175$ MeV (on-going)

- Small range of chiral extrapolation:

  significantly reduces the systematic uncertainties

  Example: BMW Collaboration, LAT2008

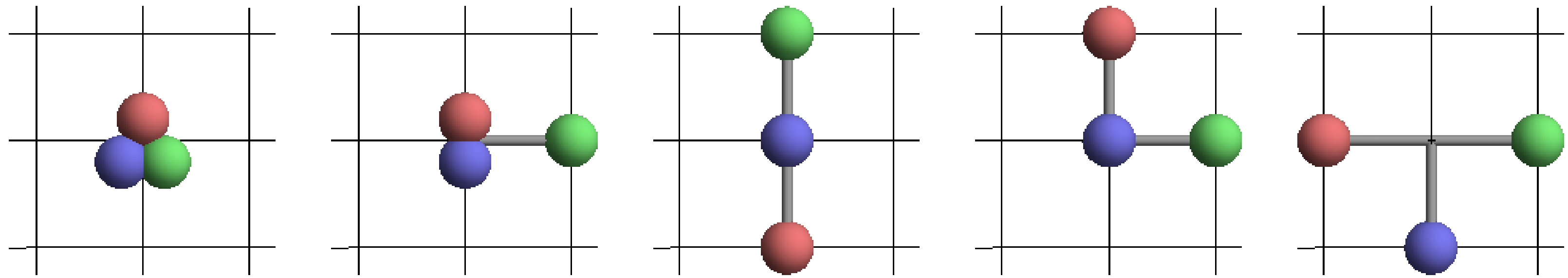


Extended Operators

on-going project

Orthogonal Operators

- Baryon field $\Phi^{ABC}_{\alpha\beta\gamma,ijk}(x) = \epsilon_{abc}[\tilde{D}^{(3)}_i\tilde{\psi}]_{Aa\alpha}(x)[\tilde{D}^{(3)}_j\tilde{\psi}]_{Bb\beta}(x)[\tilde{D}^{(3)}_k\tilde{\psi}]_{Cc\gamma}(x)$



D. Richards, this workshop

- Classify states according to symmetry properties
- Projection onto irreducible representations of finite groups
- Number of operators:

N^+ Operator type	G_{1g}	H_g	G_{2g}
Single-Site	3	1	0
Singly-Displaced	24	32	8
Doubly-Displaced-I	24	32	8
Doubly-Displaced-L	64	128	64
Triply-Displaced-T	64	128	64
Total	**179**	**321**	**144**

S. Basak et al., Phys. Rev. D72, 094506 (2005)

Summary and Outlook

Lattice QCD calculations of N-P_{11} form factors…

- Test case is in a small "quenched" box with large pion mass
- We demonstrate a method to determine N-N^* form factors with reasonable signal on anisotropic lattices
- Clean signal for large Q^2 momentum N-N form factors

Further along our roadmap…

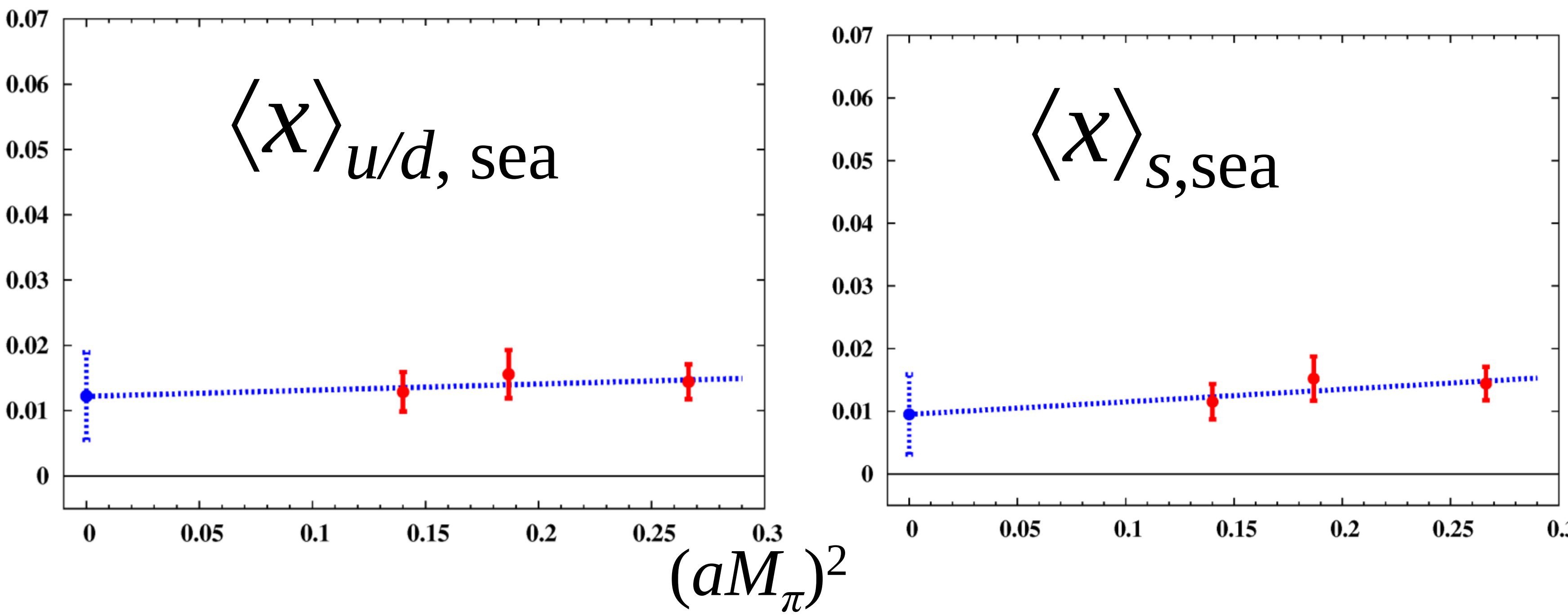
- $g_{\pi NN^*}$ from axial coupling and Goldberger-Treiman relation
- Moving forward to full-QCD anisotropic lattice calculations
- Implement group theory operators for baryons
- Other N-N^* form factors. The methodology developed can be applied to many other excited-nucleon form factors.

Backup Slides

Including Disconnected Diagrams

- Example: Sea-quark contribution to (unrenormalized) quark momentum fraction
  - 2+1f clover lattices, M_π~ 610–840 MeV

T. Doi (χQCD), Lattice 2008



Challenges for the Future

As one goes to lighter pion-mass regions…

- Need more statistics to get sufficient single-to-noise ratio

$$\frac{\text{Signal}}{\text{Noise}} = \frac{\langle J(t)J(0)\rangle}{\frac{1}{\sqrt{N}}\sqrt{\langle |J(t)J(0)|^2\rangle - \langle J(t)J(0)\rangle^2}}$$
$$\sim \frac{Ae^{-M_n t}}{\frac{1}{\sqrt{N}}\sqrt{Be^{-3m_\pi t} - Ce^{-2M_n t}}}$$
$$\sim \sqrt{N}De^{-(M_n - \frac{3}{2}m_\pi)t}$$

Challenges for the Future

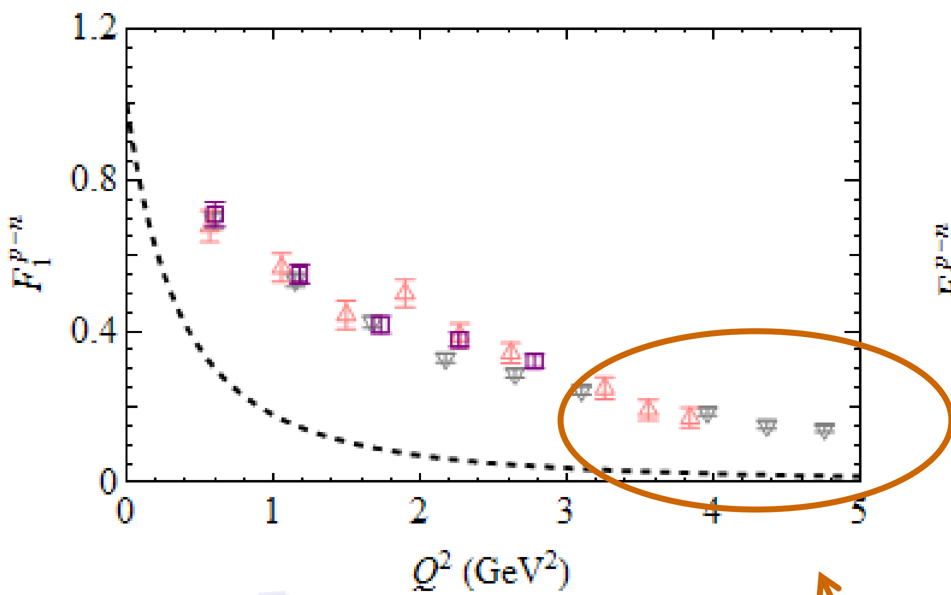
As one goes to lighter pion-mass regions…

- Need more statistics to get sufficient single-to-noise ratio
- Decay channels open up…
  - Conservative approach: using multiple volumes to identify single- and multiple-particle states
- Further improvement:
  - Symmetry-breaking on the lattice:
    - construct more operators that would generate the same quantum numbers in $a = 0$ world
  - More data input
  - Better discrimination among different states

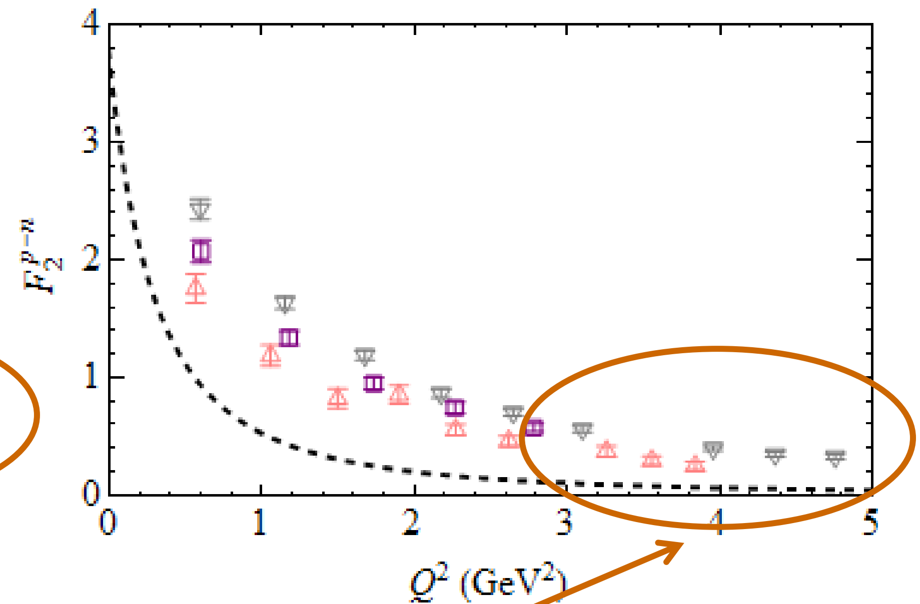
Nucleon Form Factors

- Pion masses around 480, 720 and 1100 MeV

Isovector F_1



Isovector F_2



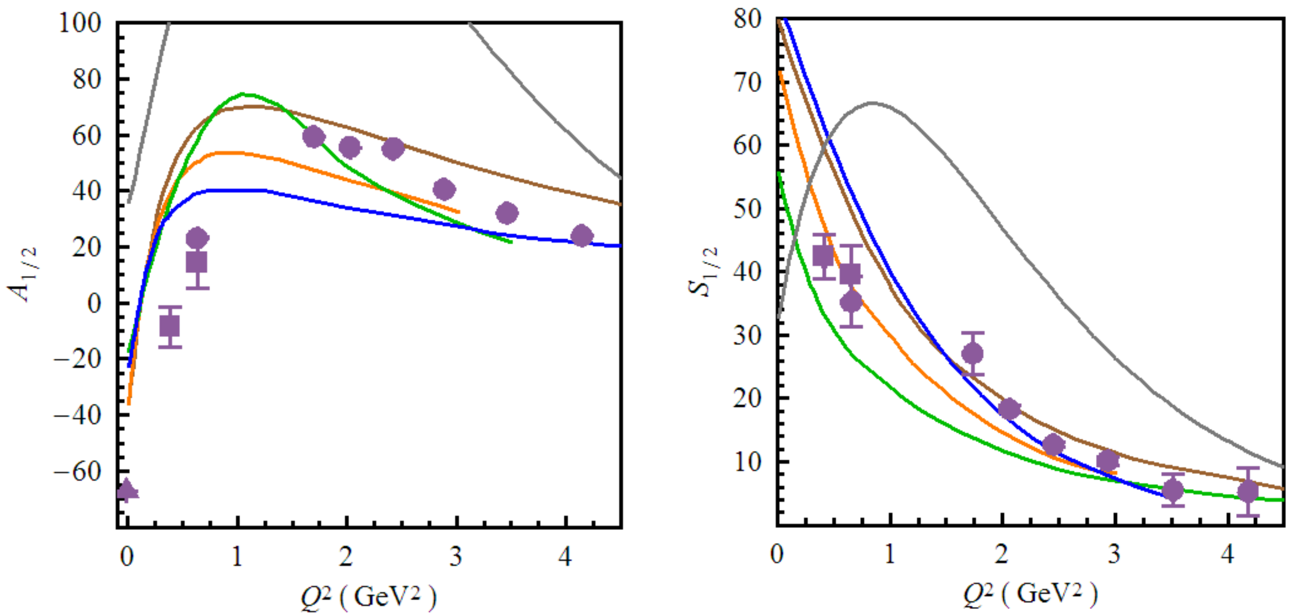
Preliminary

Clean signal for momentum region of $3 < Q^2 < 5$ GeV^2

- Working on larger-Q^2 regions with non-zero p_f
- $N_f = 3$ and 2+1 data are on the way

N-P_{11} Form Factor

- Experiments at Jefferson Laboratory (**CLAS**), MIT-Bates, LEGS, Mainz, Bonn, GRAAL, and Spring-8
- Helicity amplitudes are measured (in 10^{-3} GeV$^{-1/2}$ units)
- Many models disagree (a selection are shown below)



- If the Roper is the first radially excited state of the nucleon, this is the data to compare with