Extraction of Total Amplitudes from Complete Photoproduction Experiments

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Outline

- Observables & Helicity Amplitudes
- Observables & Transversity Amplitudes
- Bilinear Form and SU(4) Γ Matrices
- Discrete Ambiguities
- Fierzing The Observables
- Fierz Theorem Consequences
- Sample case: $\pi - Nucleon$
- BDS versus CT Theorems
- Limitations
- Conclusions

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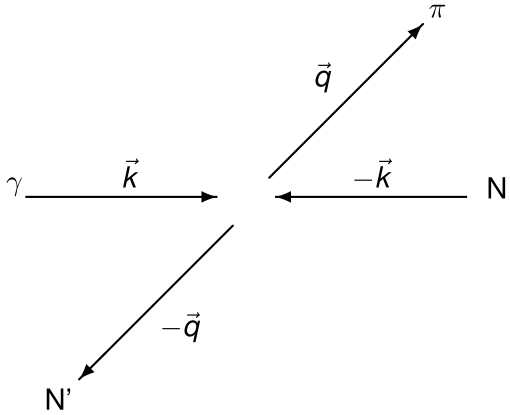
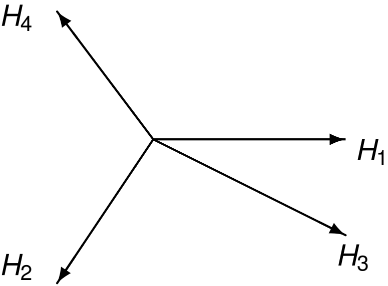
Kinematics

$$\sigma(\theta, \phi) = \frac{q}{k}\, \mathcal{I}(\theta, \phi)$$



The Four Complex Amplitudes
In Helicity Space

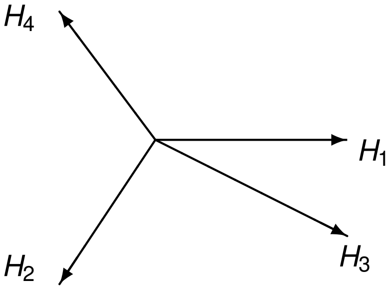


$$H_i(E, \theta, \phi) = |H_i| \; e^{i\phi_i}$$

$$4 + 3 = 7$$

Discrete ambiguities $7 + ?$

The Four Complex Amplitudes In Helicity Space



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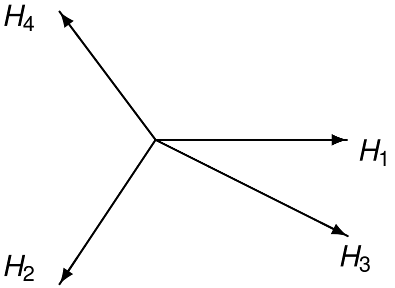

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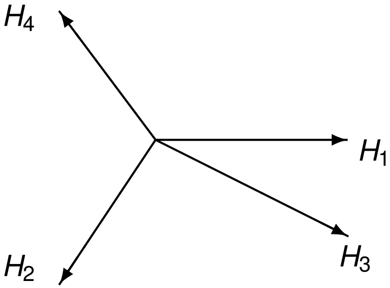

$$H_i(E,\theta,\phi) = |H_i|\ e^{i\phi_i}$$

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The Bilinear Helicity Product (BHP) Form

$$\check{\Omega}^\alpha = \Omega^\alpha \, \mathcal{I}(\theta)$$

$$= \frac{1}{2} H_i^* \; \Gamma_{ij}^\alpha \; H_j$$

$$\equiv \frac{1}{2} \langle H | \Gamma^\alpha | H \rangle \; ,$$

$$\alpha = 1, \cdots \qquad 16$$

Spin Observables: Sixteen spin observables are expressed in helicity representation and BHP forms.
Classified into four sets:

- $\mathcal{S}$ for the differential cross section and single spin observables,
- $\mathcal{BT}$ beam-target,
- $\mathcal{BR}$ beam-recoil,
- $\mathcal{TR}$ target-recoil .

Helicity Representation $\mathcal{S}$

Spin Observable	Helicity Representation	BHP
$\check{\Omega}^1 \equiv \mathcal{I}(\theta)$	$\frac{1}{2}(\lvert H_1\rvert^2 + \lvert H_2\rvert^2 + \lvert H_3\rvert^2 + \lvert H_4\rvert^2)$	$\frac{1}{2}\langle H\lvert\Gamma^1\rvert H\rangle$
$\check{\Omega}^4 \equiv \check{\Sigma}$	$\mathrm{Re}(-H_1 H_4^* + H_2 H_3^*)$	$\frac{1}{2}\langle H\lvert\Gamma^4\rvert H\rangle$
$\check{\Omega}^{10} \equiv -\check{T}$	$\mathrm{Im}(H_1 H_2^* + H_3 H_4^*)$	$\frac{1}{2}\langle H\lvert\Gamma^{10}\rvert H\rangle$
$\check{\Omega}^{12} \equiv \check{P}$	$\mathrm{Im}(-H_1 H_3^* - H_2 H_4^*)$	$\frac{1}{2}\langle H\lvert\Gamma^{12}\rvert H\rangle$

Helicity Representation $\mathcal{BT}$

Spin Observable		Helicity Representation	BHP
$\check{\Omega}^3 \equiv$	$\check{G}$	$\mathrm{Im}(H_1 H_4^* - H_3 H_2^*)$	$\frac{1}{2}\langle H\|\Gamma^3\|H\rangle$
$\check{\Omega}^5 \equiv$	$\check{H}$	$\mathrm{Im}(-H_2 H_4^* + H_1 H_3^*)$	$\frac{1}{2}\langle H\|\Gamma^5\|H\rangle$
$\check{\Omega}^9 \equiv$	$\check{E}$	$\frac{1}{2}(\|H_1\|^2 - \|H_2\|^2 + \|H_3\|^2 - \|H_4\|^2)$	$\frac{1}{2}\langle H\|\Gamma^9\|H\rangle$
$\check{\Omega}^{11} \equiv$	$\check{F}$	$\mathrm{Re}(-H_2 H_1^* - H_4 H_3^*)$	$\frac{1}{2}\langle H\|\Gamma^{11}\|H\rangle$

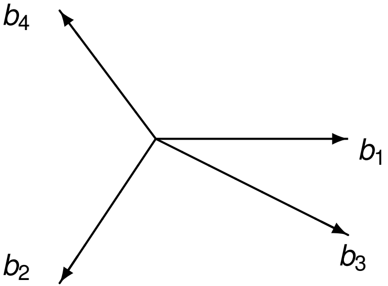
Helicity Representation $\mathcal{BR}$

Spin Observable	Helicity Representation	BHP
$\check{\Omega}^{14} \equiv \check{O}_x$	$\mathrm{Im}(-H_2 H_1^* + H_4 H_3^*)$	$\frac{1}{2}\langle H \vert \Gamma^{14} \vert H\rangle$
$\check{\Omega}^{7} \equiv -\check{O}_z$	$\mathrm{Im}(H_1 H_4^* - H_2 H_3^*)$	$\frac{1}{2}\langle H \vert \Gamma^{7} \vert H\rangle$
$\check{\Omega}^{16} \equiv -\check{C}_x$	$\mathrm{Re}(H_2 H_4^* + H_1 H_3^*)$	$\frac{1}{2}\langle H \vert \Gamma^{16} \vert H\rangle$
$\check{\Omega}^{2} \equiv -\check{C}_z$	$\frac{1}{2}(\vert H_1\vert^2 + \vert H_2\vert^2 - \vert H_3\vert^2 - \vert H_4\vert^2)$	$\frac{1}{2}\langle H \vert \Gamma^{2} \vert H\rangle$

Helicity Representation $\mathcal{TR}$

Spin Observable	Helicity Representation	BHP
$\check{\Omega}^6 \equiv -\check{T}_x$	$\mathrm{Re}(-H_1 H_4^* - H_2 H_3^*)$	$\frac{1}{2}\langle H\|\Gamma^6\|H\rangle$
$\check{\Omega}^{13} \equiv -\check{T}_z$	$\mathrm{Re}(-H_1 H_2^* + H_4 H_3^*)$	$\frac{1}{2}\langle H\|\Gamma^{13}\|H\rangle$
$\check{\Omega}^8 \equiv \check{L}_x$	$\mathrm{Re}(H_2 H_4^* - H_1 H_3^*)$	$\frac{1}{2}\langle H\|\Gamma^8\|H\rangle$
$\check{\Omega}^{15} \equiv \check{L}_z$	$\frac{1}{2}(-\|H_1\|^2 + \|H_2\|^2 + \|H_3\|^2 - \|H_4\|^2)$	$\frac{1}{2}\langle H\|\Gamma^{15}\|H\rangle$

The Four Complex Amplitudes In Transversity Space



$$b_i(E, \theta, \phi) = |b_i| \, e^{i\phi_i}$$

$$4 + 3 = 7$$

Discrete ambiguities $7 + ?$

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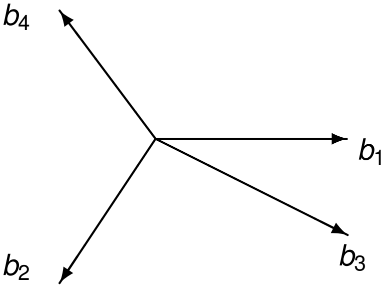

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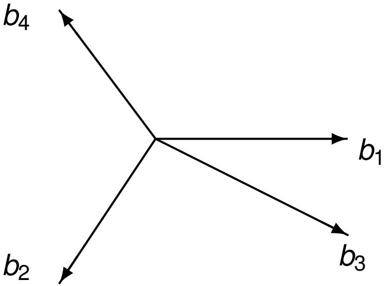

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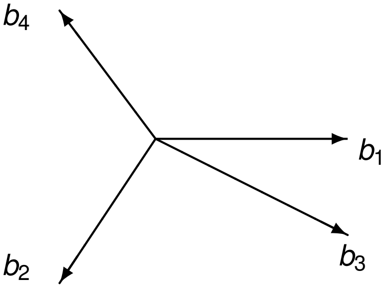

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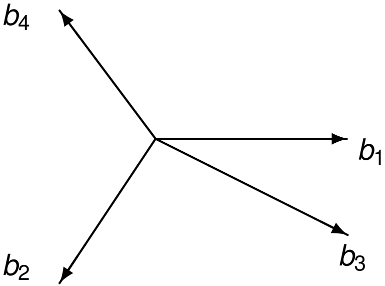

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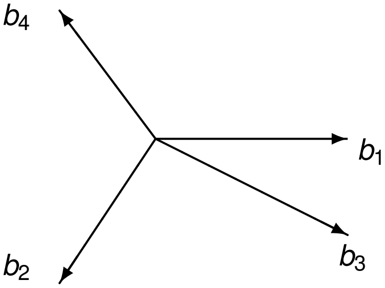

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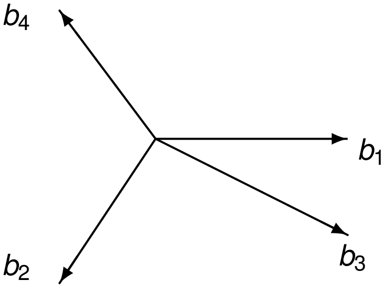

$$b_i(E, \theta, \phi) = |b_i| \; e^{i\phi_i}$$

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Discrete ambiguities $7 + ?$

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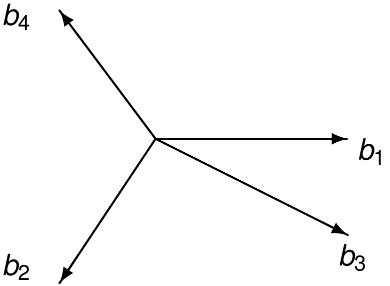

$$b_i(E, \theta, \phi) = |b_i|\ e^{i\phi_i}$$

$$4 + 3 = 7$$

Discrete ambiguities $7 + ?$

Transversity Representation $\mathcal{S}$

Spin Observable	Transversity Representation	BTP
$\check{\Omega}^1 \equiv \mathcal{I}(\theta)$	$\frac{1}{2}(\lvert b_1\rvert^2 + \lvert b_2\rvert^2 + \lvert b_3\rvert^2 + \lvert b_4\rvert^2)$	$\frac{1}{2}\langle b\lvert\widetilde{\Gamma}^1\rvert b\rangle$
$\check{\Omega}^4 \equiv \check{\Sigma}$	$\frac{1}{2}(\lvert b_1\rvert^2 + \lvert b_2\rvert^2 - \lvert b_3\rvert^2 - \lvert b_4\rvert^2)$	$\frac{1}{2}\langle b\lvert\widetilde{\Gamma}^4\rvert b\rangle$
$\check{\Omega}^{10} \equiv -\check{T}$	$\frac{1}{2}(-\lvert b_1\rvert^2 + \lvert b_2\rvert^2 + \lvert b_3\rvert^2 - \lvert b_4\rvert^2)$	$\frac{1}{2}\langle b\lvert\widetilde{\Gamma}^{10}\rvert b\rangle$
$\check{\Omega}^{12} \equiv \check{P}$	$\frac{1}{2}(-\lvert b_1\rvert^2 + \lvert b_2\rvert^2 - \lvert b_3\rvert^2 + \lvert b_4\rvert^2)$	$\frac{1}{2}\langle b\lvert\widetilde{\Gamma}^{12}\rvert b\rangle$

Transversity Representation $\mathcal{BT}$

Spin Observable		Transversity Representation	BTP
$\check{\Omega}^3 \equiv$	$\check{G}$	$\mathrm{Im}(-b_1 b_3^* - b_2 b_4^*)$	$\frac{1}{2}\langle b\vert\widetilde{\Gamma}^3\vert b\rangle$
$\check{\Omega}^5 \equiv$	$\check{H}$	$\mathrm{Re}(b_1 b_3^* - b_2 b_4^*)$	$\frac{1}{2}\langle b\vert\widetilde{\Gamma}^5\vert b\rangle$
$\check{\Omega}^9 \equiv$	$\check{E}$	$\mathrm{Re}(b_1 b_3^* + b_2 b_4^*)$	$\frac{1}{2}\langle b\vert\widetilde{\Gamma}^9\vert b\rangle$
$\check{\Omega}^{11} \equiv$	$\check{F}$	$\mathrm{Im}(b_1 b_3^* - b_2 b_4^*)$	$\frac{1}{2}\langle b\vert\widetilde{\Gamma}^{11}\vert b\rangle$

Spin Observable	Transversity Representation	BTP
$\check{\Omega}^{14} \equiv \check{O}_x$	$\mathrm{Re}(-b_1 b_4^* + b_2 b_3^*)$	$\frac{1}{2}\langle b \vert \widetilde{\Gamma}^{14} \vert b\rangle$
$\check{\Omega}^{7} \equiv -\check{O}_z$	$\mathrm{Im}(-b_1 b_4^* - b_2 b_3^*)$	$\frac{1}{2}\langle b \vert \widetilde{\Gamma}^{7} \vert b\rangle$
$\check{\Omega}^{16} \equiv -\check{C}_x$	$\mathrm{Im}(b_1 b_4^* - b_2 b_3^*)$	$\frac{1}{2}\langle b \vert \widetilde{\Gamma}^{16} \vert b\rangle$
$\check{\Omega}^{2} \equiv -\check{C}_z$	$\mathrm{Re}(b_1 b_4^* + b_2 b_3^*)$	$\frac{1}{2}\langle b \vert \widetilde{\Gamma}^{2} \vert b\rangle$

Transversity Representation $\mathcal{TR}$

Spin Observable	Transversity Representation	BTP
$\check{\Omega}^6 \equiv -\check{T}_x$	$\mathrm{Re}(-b_1 b_2^* + b_3 b_4^*)$	$\frac{1}{2}\langle b \| \widetilde{\Gamma}^6 \| b\rangle$
$\check{\Omega}^{13} \equiv -\check{T}_z$	$\mathrm{Im}(b_1 b_2^* - b_3 b_4^*)$	$\frac{1}{2}\langle b \| \widetilde{\Gamma}^{13} \| b\rangle$
$\check{\Omega}^8 \equiv \check{L}_x$	$\mathrm{Im}(-b_1 b_2^* - b_3 b_4^*)$	$\frac{1}{2}\langle b \| \widetilde{\Gamma}^8 \| b\rangle$
$\check{\Omega}^{15} \equiv \check{L}_z$	$\mathrm{Re}(-b_1 b_2^* - b_3 b_4^*)$	$\frac{1}{2}\langle b \| \widetilde{\Gamma}^{15} \| b\rangle$

The Bilinear Helicity Product (BHP) Summary

$\mathcal{S}: \quad (\mathcal{I}, \check{\Sigma}, -\check{T}, \check{P})$ $\quad \Gamma^1, \Gamma^4, \Gamma^{10}, \Gamma^{12}$

$\mathcal{BT}: (\check{G}, \check{H}, \check{E}, \check{F})$ $\quad \Gamma^3, \Gamma^5, \Gamma^9, \Gamma^{11}$

$\mathcal{BR}: (\check{O}_x, -\check{O}_z, -\check{C}_x, -\check{C}_z)$ $\quad \Gamma^{14}, \Gamma^7, \Gamma^{16}, \Gamma^2$

$\mathcal{TR}: (-\check{T}_x, -\check{T}_z, \check{L}_x, \check{L}_z)$ $\quad \Gamma^6, \Gamma^{13}, \Gamma^8, \Gamma^{15}$

Ambiguities: $\Gamma^4, \Gamma^{10}, \Gamma^{12}$
$\Gamma^{15}\ K_0,$
$\Gamma^6\ K_0, \Gamma^8\ K_0, \Gamma^{13}\ K_0$

Sixteen Hermitian Gamma Matrices

$$\Gamma^{\alpha=1\cdots5} = 1, \gamma^0, i\vec{\gamma}$$

$$\Gamma^{\alpha=6\cdots11} = \sigma^{0x}, i\sigma^{0y}, i\sigma^{0z}, \\ i\sigma^{xy}, i\sigma^{xz}, i\sigma^{zy}$$

$$\Gamma^{\alpha=12\cdots16} = i\gamma^5\gamma^0, \gamma^5\vec{\gamma}, \gamma^5.$$

Γ^α Properties

- Γ^α are Hermitian and unitary.
- $\mathrm{Tr}(\Gamma^\alpha \Gamma^\beta) = 4\delta_{\alpha\beta}$.
- Γ^α are linearly independent. and form a complete set (a basis) for 4×4 matrices. Any 4×4 matrices X can be expanded as $X = \sum_\alpha C_\alpha U^4$ with $C_\alpha = \frac{1}{4}\mathrm{Tr}(\Gamma^\alpha X)$.
- $\sum_\alpha \Gamma^\alpha_{ba} \Gamma^\alpha_{st} = 4\delta_{as}\delta_{bt}$.
- $\Gamma^\alpha \Gamma^\beta = \rho_{\alpha\beta\gamma}\Gamma^\gamma$ with $\rho_{\alpha\beta\gamma} = \frac{1}{4}\mathrm{Tr}(\Gamma^\alpha \Gamma^\beta \Gamma^\gamma)$.
- $\frac{1}{4}\rho_{\alpha\gamma\delta}\rho_{\beta\gamma\eta} = \frac{1}{16}\mathrm{Tr}(\Gamma^\delta \Gamma^\alpha \Gamma^\eta \Gamma^\beta) \equiv C^{\alpha\beta}_{\delta\eta}$: used for the Fierz transformation

Transversity Γ^α

Transversity transformation: $|b\rangle = U^4\ |H\rangle \quad \widetilde{\Gamma}^\alpha = U^4 \Gamma^\alpha U^{\dagger 4}$

$$U^{(4)} = \frac{1}{2} \begin{pmatrix} 1 & -i & i & 1 \\ 1 & i & -i & 1 \\ 1 & i & i & -1 \\ 1 & -i & -i & -1 \end{pmatrix} ,$$

which involves rotating the helicity quantization axis to the direction normal to the scattering plane. The sixteen spin observables can be expressed in this *transversity basis* by

$$\check{\Omega}^\alpha = \Omega^\alpha \, \mathcal{I}(\theta) = \frac{1}{2}\, b_i^* \, \widetilde{\Gamma}^\alpha_{ij} \, b_j = \frac{1}{2} \langle b | \widetilde{\Gamma}^\alpha | b \rangle \ , \qquad \alpha = 1, \cdots 16 \ .$$

The transversity $\widetilde{\Gamma}$ matrices form four classes with four members in each class according to their "shape:" diagonal (*D*); right parallelogram (*PR*); antidiagonal (*AD*); and left parallelogram (*PL*) correspond to $\mathcal{S}$, $\mathcal{BT}$, $\mathcal{BR}$ and $\mathcal{TR}$ type experiments.

$$\widetilde{\Gamma}_D = \widetilde{\Gamma}_{\mathcal{S}} = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix} ;$$

	a	b	c	d
$\widetilde{\Gamma}_1$	$+1$	$+1$	$+1$	$+1$
$\widetilde{\Gamma}_4$	$+1$	$+1$	-1	-1
$\widetilde{\Gamma}_{10}$	-1	$+1$	$+1$	-1
$\widetilde{\Gamma}_{12}$	-1	$+1$	-1	$+1$

$$|b_i|$$

$$\widetilde{\Gamma}_{PR} = \widetilde{\Gamma}_{\mathcal{BT}} = \begin{bmatrix} 0 & 0 & a & 0 \\ 0 & 0 & 0 & b \\ c & 0 & 0 & 0 \\ 0 & d & 0 & 0 \end{bmatrix} \; ; \qquad \begin{array}{ccccc} & a & b & c & d \\ \widetilde{\Gamma}_3 & -i & -i & +i & +i \\ \widetilde{\Gamma}_5 & +1 & -1 & +1 & -1 \\ \widetilde{\Gamma}_9 & +1 & +1 & +1 & +1 \\ \widetilde{\Gamma}_{11} & +i & -i & -i & +i \end{array}$$

$$\phi_{13}, \phi_{24}$$

$$\widetilde{\Gamma}_{AD} = \widetilde{\Gamma}_{\mathcal{BR}} = \begin{bmatrix} 0 & 0 & 0 & a \\ 0 & 0 & b & 0 \\ 0 & c & 0 & 0 \\ d & 0 & 0 & 0 \end{bmatrix} \; ;$$

	a	b	c	d
$\widetilde{\Gamma}_{14}$	-1	$+1$	$+1$	-1
$\widetilde{\Gamma}_{7}$	$-i$	$-i$	$+i$	$+i$
$\widetilde{\Gamma}_{16}$	$+i$	$-i$	$+i$	$-i$
$\widetilde{\Gamma}_{2}$	$+1$	$+1$	$+1$	$+1$

$$\phi_{14}, \phi_{23}$$

$$\widetilde{\Gamma}_{PL} = \widetilde{\Gamma}_{\mathcal{TR}} = \begin{bmatrix} 0 & a & 0 & 0 \\ b & 0 & 0 & 0 \\ 0 & 0 & 0 & c \\ 0 & 0 & d & 0 \end{bmatrix} \; ; \qquad \begin{array}{ccccc} & a & b & c & d \\ \widetilde{\Gamma}_6 & -1 & -1 & +1 & +1 \\ \widetilde{\Gamma}_{13} & +i & -i & -i & +i \\ \widetilde{\Gamma}_8 & -i & +i & -i & +i \\ \widetilde{\Gamma}_{15} & -1 & -1 & -1 & -1 \end{array}$$

$$\phi_{12}, \phi_{34}$$

Discrete Ambiguities in Helicity Basis

G. Keaton & R. Workman Phys.Rev. C53, 1434 (1996) gave discrete ambiguity relations associated with transformations of helicity amplitudes:

Ambiguity I: $H_1 \longleftrightarrow H_4 \quad H_2 \longleftrightarrow -H_3$

$$\begin{bmatrix} H_1' \\ H_2' \\ H_3' \\ H_4' \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & +1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ +1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} = \Gamma^4 \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix}.$$

Discrete Ambiguities in Helicity Basis

Ambiguity II:

$H_1 \longrightarrow H_2 \quad H_2 \longrightarrow -H_1 \quad H_3 \longrightarrow H_4 \quad H_4 \longrightarrow -H_3$

$$\begin{bmatrix} H_1' \\ H_2' \\ H_3' \\ H_4' \end{bmatrix} = \begin{bmatrix} 0 & +1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} = -i\, \Gamma^{10} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} .$$

Discrete Ambiguities in Helicity Basis

Ambiguity III:

$H_1 \longrightarrow H_3 \quad H_2 \longrightarrow H_4 \quad H_3 \longrightarrow -H_1 \quad H_4 \longrightarrow -H_2$

$$\begin{bmatrix} H_1' \\ H_2' \\ H_3' \\ H_4' \end{bmatrix} = \begin{bmatrix} 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} = i\,\Gamma^{12} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} .$$

Ambiguity IV:

$H_1 \longrightarrow -H_1^* \quad H_2 \longrightarrow H_2^* \quad H_3 \longrightarrow H_3^* \quad H_4 \longrightarrow -H_4^*$

$$\begin{bmatrix} H_1' \\ H_2' \\ H_3' \\ H_4' \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} H_1^* \\ H_2^* \\ H_3^* \\ H_4^* \end{bmatrix} = \Gamma^{15} \begin{bmatrix} H_1^* \\ H_2^* \\ H_3^* \\ H_4^* \end{bmatrix}.$$

Ambiguities in the transversity basis

Ambiguity I:

$b_1 \longrightarrow +b_1 \quad b_2 \longrightarrow +b_2 \quad b_3 \longrightarrow -b_3 \quad b_4 \longrightarrow -b_4$

$$\begin{bmatrix} b'_1 \\ b'_2 \\ b'_3 \\ b'_4 \end{bmatrix} = \begin{bmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \widetilde{\Gamma}^4 \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} .$$

Ambiguity II:

$b_1 \longrightarrow -b_1 \quad b_2 \longrightarrow +b_2 \quad b_3 \longrightarrow +b_3 \quad b_4 \longrightarrow -b_4$

$$\begin{bmatrix} b'_1 \\ b'_2 \\ b'_3 \\ b'_4 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = -i\, \widetilde{\Gamma}^{10} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} .$$

Ambiguity III:

$b_1 \longrightarrow -b_1 \quad b_2 \longrightarrow +b_2 \quad b_3 \longrightarrow -b_3 \quad b_4 \longrightarrow +b_4$

$$\begin{bmatrix} b_1' \\ b_2' \\ b_3' \\ b_4' \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & +1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = i\widetilde{\Gamma}^{12} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} .$$

Ambiguity IV:

$b_1 \longrightarrow -b_2^* \quad b_2 \longrightarrow -b_1^* \quad b_3 \longrightarrow -b_4^* \quad b_4 \longrightarrow -b_3^*$

$$\begin{bmatrix} b_1' \\ b_2' \\ b_3' \\ b_4' \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} b_1^* \\ b_2^* \\ b_3^* \\ b_4^* \end{bmatrix} = \widetilde{\Gamma}^{15} \begin{bmatrix} b_1^* \\ b_2^* \\ b_3^* \\ b_4^* \end{bmatrix} . \quad (1)$$

Linear ambiguity I, II and III $L = \widetilde{\Gamma}^4, \widetilde{\Gamma}^{10}$, and $\widetilde{\Gamma}^{12}$

Antilinear ambiguity IV ambiguity $A = \widetilde{\Gamma}^{15} K_0$.

Other three antilinear ambiguities $A = \widetilde{\Gamma}^6 K_0$, $\widetilde{\Gamma}^{13} K_0$, and $\widetilde{\Gamma}^8 K_0$, can be constructed by Ambiguity IV and the three linear ambiguities I to III. I

$$
\begin{aligned}
\widetilde{\Gamma}^6 &= \widetilde{\Gamma}^4 \, \widetilde{\Gamma}^{15} \\
\widetilde{\Gamma}^{13} &= i \, \widetilde{\Gamma}^{10} \, \widetilde{\Gamma}^{15} \\
\widetilde{\Gamma}^8 &= -i \, \widetilde{\Gamma}^{12} \, \widetilde{\Gamma}^{15} \, .
\end{aligned}
$$

See KW (Ref. [**?**]).

Spin	Linear Transformation L			Antilinear Transformation A				
Observable	$\widetilde{\Gamma}_4$	$\widetilde{\Gamma}_{10}$	$\widetilde{\Gamma}_{12}$	$\widetilde{\Gamma}_6$	$\widetilde{\Gamma}_8$	$\widetilde{\Gamma}_{13}$	$\widetilde{\Gamma}_{15}$	
$\sigma(\theta)$	+	+	+	+	+	+	+	
Σ	+	+	+	+	+	+	+	$\mathcal{S}$
T	+	+	+	+	+	+	+	
P	+	+	+	+	+	+	+	
G	$-$	$-$	+	$+$	$-$	$+$	$-$	
H	$-$	$-$	+	$-$	$+$	$-$	$+$	$\mathcal{BT}$
E	$-$	$-$	+	$-$	$+$	$-$	$+$	
F	$-$	$-$	+	$+$	$-$	$+$	$-$	

$$L: \quad b_i \longrightarrow b_i' = L_{ij} b_j$$

$$A: \quad b_i \longrightarrow b_i' = A_{ij} b_j^*$$

Spin	Linear Transformation L			Antilinear Transformation A			
Observable	$\widetilde{\Gamma}_4$	$\widetilde{\Gamma}_{10}$	$\widetilde{\Gamma}_{12}$	$\widetilde{\Gamma}_6$	$\widetilde{\Gamma}_8$	$\widetilde{\Gamma}_{13}$	$\widetilde{\Gamma}_{15}$
O_x	$-$	+	$-$	$-$	$-$	$+$	$+$
O_z	$-$	+	$-$	$+$	$+$	$-$	$-$
C_x	$-$	+	$-$	$+$	$+$	$-$	$-$
C_z	$-$	+	$-$	$-$	$-$	$+$	$+$
T_x	+	$-$	$-$	$+$	$-$	$-$	$+$
T_z	+	$-$	$-$	$-$	$+$	$+$	$-$
L_x	+	$-$	$-$	$-$	$+$	$+$	$-$
L_z	+	$-$	$-$	$+$	$-$	$-$	$+$

$$L: \quad b_i \longrightarrow b_i' = L_{ij} b_j$$
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Fierzing $\vec{\gamma} N \longrightarrow \pi + N'$ Case

Fierz for SU(4)

$$\Gamma^{\alpha}_{ij}\,\Gamma^{\beta}_{st} = \sum_{a,b=1}^{16} C^{\alpha,\beta}_{a,b}\,\Gamma^{a}_{it}\,\Gamma^{b}_{sj} \quad \longrightarrow \Gamma^{\alpha}\,\Gamma^{\beta} = \sum_{a,b=1}^{16} C^{\alpha,\beta}_{a,b}\,\Gamma^{a}\,\Gamma^{b}$$

$$C^{\alpha,\beta}_{a,b} \equiv \frac{1}{16} Tr[\Gamma^{a}\Gamma^{\alpha}\Gamma^{\beta}\Gamma^{b}]$$

Fierzing Observables:

$$\Omega^{\alpha}\,\Omega^{\beta} = \sum_{a,b=1}^{16} C^{\alpha,\beta}_{a,b}\,\Omega^{a}\,\Omega^{b}$$

$16 \times 16/2 = 128 \longrightarrow x$ relations; for example:

$$\Gamma^{1}_{ij}\,\Gamma^{1}_{st} = \frac{1}{x}[\Gamma^{1}_{it}\,\Gamma^{1}_{sj} + \Gamma^{2}_{it}\,\Gamma^{2}_{sj} + \cdots + \Gamma^{16}_{it}\,\Gamma^{16}_{sj}]$$

Linear-Quadratic Relations (16):

$$\begin{aligned}
\Omega_1 &= 1 = \frac{1}{4}\sum_{\alpha=1}^{16}(\Omega_\alpha)^2 \\
\Omega_4 &= \Omega_{10}\Omega_{12} + \Omega_6\Omega_{15} - \Omega_8\Omega_{13} \\
\Omega_{10} &= \Omega_4\,\Omega_{12} + \Omega_2\Omega_{14} + \Omega_7\Omega_{16} \\
\Omega_{12} &= \Omega_4\,\Omega_{10} + \Omega_3\Omega_{11} - \Omega_5\Omega_9
\end{aligned}$$

$$\begin{aligned}
\Omega_3 &= +\Omega_{11}\Omega_{12} - \Omega_7\Omega_{15} + \Omega_{14}\Omega_8 \\
\Omega_5 &= -\Omega_9\,\Omega_{12} + \Omega_7\Omega_{13} - \Omega_{14}\Omega_6 \\
\Omega_9 &= -\Omega_5\,\Omega_{12} - \Omega_2\Omega_{15} - \Omega_{16}\Omega_8 \\
\Omega_{11} &= +\Omega_3\,\Omega_{12} + \Omega_2\Omega_{13} + \Omega_{16}\Omega_6
\end{aligned}$$

Linear-Quadratic Relations:

$$
\begin{aligned}
\Omega_{14} &= \Omega_2\,\Omega_{10} + \Omega_3\Omega_8\ - \Omega_5\,\Omega_6 \\
\Omega_7 &= \Omega_{16}\Omega_{10} - \Omega_3\Omega_{15} + \Omega_5\,\Omega_{13} \\
\Omega_{16} &= \Omega_7\,\Omega_{10} - \Omega_9\Omega_8\ + \Omega_{11}\Omega_6 \\
\Omega_2 &= \Omega_{14}\Omega_{10} - \Omega_9\Omega_{15} + \Omega_{11}\Omega_{13}
\end{aligned}
$$

$$
\begin{aligned}
\Omega_6 &= +\Omega_{15}\Omega_4 - \Omega_5\,\Omega_{14} + \Omega_{11}\Omega_{16} \\
\Omega_{13} &= -\Omega_8\,\Omega_4 + \Omega_5\,\Omega_7\ + \Omega_{11}\Omega_2 \\
\Omega_8 &= -\Omega_{13}\Omega_4 + \Omega_3\,\Omega_{14} - \Omega_9\,\Omega_{16} \\
\Omega_{15} &= +\Omega_6\,\Omega_4 - \Omega_3\,\Omega_7\ - \Omega_9\,\Omega_2
\end{aligned}
$$

Linear-Quadratic Relations:

$$
\begin{aligned}
\mathcal{I} &= 1 = \frac{1}{4}\sum_{\alpha=1}^{16}(\Omega_\alpha)^2 \\
\Sigma &= -T\,P - T_x\,L_z + L_x\,T_z \\
T &= -\Sigma\,P + C_z\,O_x - O_z\,C_x \\
P &= -\Sigma\,T + G\,F - H\,E
\end{aligned}
$$

$$
\begin{pmatrix} 1 & 0 & 0 & -P \\ 0 & 1 & P & 0 \\ 0 & P & 1 & 0 \\ -P & 0 & 0 & 1 \end{pmatrix}
\begin{pmatrix} G \\ H \\ E \\ F \end{pmatrix}
=
\begin{pmatrix} 0 & 0 & O_x & O_z \\ O_x & O_z & 0 & 0 \\ 0 & 0 & C_x & C_z \\ C_x & C_z & 0 & 0 \end{pmatrix}
\begin{pmatrix} T_x \\ T_z \\ L_x \\ L_z \end{pmatrix}
$$

$$
\mathcal{S} \times \mathcal{BT} = \mathcal{BR} \times \mathcal{TR}
$$

$$\begin{pmatrix} 1 & 0 & 0 & -T \\ 0 & 1 & T & 0 \\ 0 & T & 1 & 0 \\ -T & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} O_x \\ O_z \\ C_x \\ C_z \end{pmatrix} = \begin{pmatrix} H & 0 & G & 0 \\ 0 & H & 0 & G \\ F & 0 & E & 0 \\ 0 & F & 0 & E \end{pmatrix} \begin{pmatrix} T_x \\ T_z \\ L_x \\ L_z \end{pmatrix}$$

$$\mathcal{S} \times \mathcal{BR} = \mathcal{BT} \times \mathcal{TR}$$

$$\begin{pmatrix} 1 & 0 & 0 & \Sigma \\ 0 & 1 & -\Sigma & 0 \\ 0 & -\Sigma & 1 & 0 \\ \Sigma & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} T_x \\ T_z \\ L_x \\ L_z \end{pmatrix} = \begin{pmatrix} H & 0 & F & 0 \\ 0 & H & 0 & F \\ G & 0 & E & 0 \\ 0 & G & 0 & E \end{pmatrix} \begin{pmatrix} O_x \\ O_z \\ C_x \\ C_z \end{pmatrix}$$

$$\mathcal{S} \times \mathcal{TR} = \mathcal{BT} \times \mathcal{BR}$$

Quadratic Relations (15):

$$\begin{aligned}
\Omega_2\,\Omega_7 - \Omega_{14}\Omega_{16} - \Omega_3\,\Omega_9 - \Omega_5\,\Omega_{11} &= 0\\
\Omega_3\,\Omega_5 + \Omega_9\,\Omega_{11} + \Omega_6\,\Omega_8 + \Omega_{13}\Omega_{15} &= 0\\
\Omega_2\,\Omega_{16} - \Omega_7\,\Omega_{14} - \Omega_6\,\Omega_{13} - \Omega_8\,\Omega_{15} &= 0\\
\Omega_4\,\Omega_3 - \Omega_{10}\Omega_{11} + \Omega_7\,\Omega_6 + \Omega_{14}\Omega_{13} &= 0\\
\Omega_4\,\Omega_5 + \Omega_{10}\Omega_9 + \Omega_7\,\Omega_8 + \Omega_{14}\Omega_{15} &= 0\\
\Omega_4\,\Omega_9 + \Omega_{10}\Omega_5 + \Omega_2\,\Omega_6 - \Omega_{16}\Omega_{13} &= 0\\
\Omega_4\,\Omega_{11} - \Omega_{10}\Omega_3 + \Omega_2\,\Omega_8 - \Omega_{16}\Omega_{15} &= 0\\
\Omega_4\,\Omega_{14} - \Omega_{12}\Omega_2 + \Omega_3\,\Omega_{13} + \Omega_5\,\Omega_{15} &= 0
\end{aligned}$$

Quadratic Relations:

$$
\begin{aligned}
\Omega_4\,\Omega_7 - \Omega_{12}\Omega_{16} + \Omega_3\,\Omega_6 + \Omega_5\,\Omega_8 &= 0\\
\Omega_4\,\Omega_{16} - \Omega_{12}\Omega_7 - \Omega_9\,\Omega_{13} - \Omega_{11}\Omega_{15} &= 0\\
\Omega_4\,\Omega_2 - \Omega_{12}\Omega_{14} + \Omega_9\,\Omega_6 + \Omega_{11}\Omega_8 &= 0\\
\Omega_{10}\Omega_6 - \Omega_{12}\Omega_{15} + \Omega_5\,\Omega_2 - \Omega_{11}\Omega_7 &= 0\\
\Omega_{10}\Omega_{13} + \Omega_{12}\Omega_8 - \Omega_5\,\Omega_{16} - \Omega_{11}\Omega_{14} &= 0\\
\Omega_{10}\Omega_8 + \Omega_{12}\Omega_{13} - \Omega_3\,\Omega_2 + \Omega_9\,\Omega_7 &= 0\\
\Omega_{10}\Omega_{15} - \Omega_{12}\Omega_6 + \Omega_3\,\Omega_{16} + \Omega_9\,\Omega_{14} &= 0
\end{aligned}
$$

Square Relations(6):

$$
\begin{aligned}
(\Omega_3)^2 + (\Omega_5)^2 + (\Omega_9)^2 + (\Omega_{11})^2 &= (\Omega_1)^2 - (\Omega_4)^2 - (\Omega_{10})^2 + (\Omega_{12})^2 \\
(\Omega_{14})^2 + (\Omega_7)^2 + (\Omega_{16})^2 + (\Omega_2)^2 &= (\Omega_1)^2 - (\Omega_4)^2 + (\Omega_{10})^2 - (\Omega_{12})^2 \\
(\Omega_6)^2 + (\Omega_{13})^2 + (\Omega_8)^2 + (\Omega_{15})^2 &= (\Omega_1)^2 + (\Omega_4)^2 - (\Omega_{10})^2 - (\Omega_{12})^2
\end{aligned}
$$

$$
\begin{aligned}
(\Omega_3)^2 + (\Omega_5)^2 - (\Omega_9)^2 - (\Omega_{11})^2 &= (\Omega_{14})^2 + (\Omega_7)^2 - (\Omega_{16})^2 - (\Omega_2)^2 \\
-(\Omega_3)^2 + (\Omega_5)^2 - (\Omega_9)^2 + (\Omega_{11})^2 &= (\Omega_6)^2 + (\Omega_{13})^2 - (\Omega_8)^2 - (\Omega_{15})^2 \\
(\Omega_{14})^2 - (\Omega_7)^2 + (\Omega_{16})^2 - (\Omega_2)^2 &= (\Omega_6)^2 - (\Omega_{13})^2 + (\Omega_8)^2 - (\Omega_{15})^2
\end{aligned}
$$

Bounds on measurements

From

$$\begin{aligned} & (\Omega_6)^2 + (\Omega_{13})^2 + (\Omega_8)^2 + (\Omega_{15})^2 \pm 2\,(\Omega_6\,\Omega_{15} - \Omega_8\,\Omega_{13}) \\ = \; & (\Omega_1)^2 + (\Omega_4)^2 - (\Omega_{10})^2 - (\Omega_{12})^2 \pm 2\,(\Omega_1\Omega_4 - \Omega_{10}\,\Omega_{12}) \end{aligned}$$

we obtain

$$(\Omega_6 \pm \Omega_{15})^2 + (\Omega_8 \mp \Omega_{13})^2 = (\Omega_1 \pm \Omega_4)^2 - (\Omega_{10} \pm \Omega_{12})^2 \,.$$

The left hand side of the equation is positive, so is the right hand side. Therefore

$$\Omega_1 \pm \Omega_4 \geq |\Omega_{10} \pm \Omega_{12}| \qquad \text{or} \qquad 1 \pm \Sigma \geq |T \pm P| \,.$$

Other bounds, within the set $\mathcal{S}$, can be derived in the same way:

$$1 \pm T \geq |P \pm \Sigma| \,, \qquad 1 \pm P \geq |\Sigma \pm T| \,.$$

We can deduce the bounds [1]

$$\begin{aligned} 1 + \Sigma^2 &\geq P^2 + T^2 \\ 1 + T^2 &\geq \Sigma^2 + P^2 \\ 1 + P^2 &\geq \Sigma^2 + T^2 \end{aligned}$$

Bounds on measurements

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$$(\Omega_6)^2 + (\Omega_{13})^2 + (\Omega_8)^2 + (\Omega_{15})^2 \pm 2\,(\Omega_6\,\Omega_{15} - \Omega_8\,\Omega_{13})$$
$$= \quad (\Omega_1)^2 + (\Omega_4)^2 - (\Omega_{10})^2 - (\Omega_{12})^2 \pm 2\,(\Omega_1\Omega_4 - \Omega_{10}\,\Omega_{12})$$

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Bounds on measurements

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$$(\Omega_6)^2 + (\Omega_{13})^2 + (\Omega_8)^2 + (\Omega_{15})^2 \pm 2\,(\Omega_6\,\Omega_{15} - \Omega_8\,\Omega_{13})$$
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$$
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= \quad & (\Omega_1)^2 + (\Omega_4)^2 - (\Omega_{10})^2 - (\Omega_{12})^2 \pm 2\,(\Omega_1\Omega_4 - \Omega_{10}\,\Omega_{12})
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$$

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1 + T^2 &\geq \Sigma^2 + P^2 \\
1 + P^2 &\geq \Sigma^2 + T^2
\end{aligned}
$$

The BDS rule: Barker, Donnachie, & Storrow, *Nucl. Phys.* **B95**, 347 (1975)

In BDS, the following rule was promulgated:

> *In order to determine all amplitudes without discrete ambiguities, one has to measure five double spin observables along with the four type $\mathcal{S}$ measurements, provided no four double spin observables are selected from the same set of $\mathcal{BT}$, $\mathcal{BR}$ and $\mathcal{TR}$.*

Thus, they say **nine** experiments are required.

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Thus, they say **eight** "carefully selected" experiments are required.

Pion Nucleon Elastic Case

Amplitude

$$T = f_1 + i\,\vec{\sigma}\cdot\hat{n}\,f_2$$

Transversity amplitudes

$$b_1 = \frac{f_1 + i\,f_2}{\sqrt{2}} \qquad b_2 = \frac{f_1 - i\,f_2}{\sqrt{2}}$$

Observables

$$\mathcal{I}(\theta) = |b_1|^2 + |b_2|^2 = < b|\sigma^0|b >$$

$$\mathcal{I}(\theta)P = \hat{P} = |b_1|^2 - |b_2|^2 = {A_N}^2 = < b|\sigma^z|b >$$

$$\mathcal{I}(\theta)A = \hat{A} = 2\,\Re(b_1^*\,b_2) = 2|b_1||b_2|\cos(\theta) = < b|\sigma^x|b >$$

$$\mathcal{I}(\theta)R = \hat{R} = 2\,\Im(b_1^*\,b_2) = 2|b_1||b_2|\sin(\theta) = < b|\sigma^y|b >$$

[2] $P = A_N$ TRI—-Ashkin & Wolfenstein

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Observables

$$\mathcal{I}(\theta) = |b_1|^2 + |b_2|^2 =< b|\sigma^0|b >$$

$$\mathcal{I}(\theta)P = \hat{P} = |b_1|^2 - |b_2|^2 = A_N{}^2 =< b|\sigma^z|b >$$

$$\mathcal{I}(\theta)A = \hat{A} = 2\,\Re(b_1^*\,b_2) = 2|b_1||b_2|\cos(\theta) =< b|\sigma^x|b >$$

$$\mathcal{I}(\theta)R = \hat{R} = 2\,\Im(b_1^*\,b_2) = 2|b_1||b_2|\sin(\theta) =< b|\sigma^y|b >$$

[2] $P = A_N$ TRI—-Ashkin & Wolfenstein

Fierzing Pion Nucleon Elastic Case

Fierz for SU(2)

$$\sigma^{\alpha}_{ij}\ \sigma^{\beta}_{st} = \sum_{a,b=0}^{3} C^{\alpha,\beta}_{a,b}\ \sigma^{a}_{it}\ \sigma^{b}_{sj} \quad \longrightarrow \sigma^{\alpha}\ \sigma^{\beta} = \sum_{a,b=0}^{3} C^{\alpha,\beta}_{a,b}\ \sigma^{a}\ \sigma^{b}$$

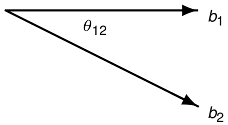
$4 \times 4/2 = 8 \longrightarrow 1$

$$\sigma^{0}_{ij}\ \sigma^{0}_{st} = \sigma^{1}_{it}\ \sigma^{1}_{sj} + \sigma^{2}_{it}\ \sigma^{2}_{sj} + \sigma^{3}_{it}\ \sigma^{3}_{sj}$$

$$P^2 + A^2 + R^2 = 1$$

Thus deduce bounds: $|P| \leq 1$ and $0 \leq A^2 + R^2 \leq 1$
Complete set: $\mathcal{I}, P, A$ =3 plus 1 for sign of R, for example. **The**

Two Complex Amplitudes In Transversity Space:



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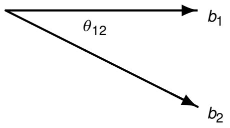

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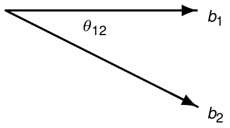

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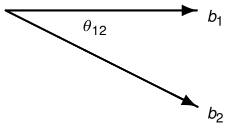

Complete Double spin measurements

'**X**'s' –> 3 selected measurements,
'**O**'s' –> possible 4th observable to resolve ambiguities.

G	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**
H	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**						
E									**X**	**X**	**X**	**X**	**X**	**X**
F														
O_x	**X**		**O**		**O**	**O**	**O**	**O**	**X**	**O**	**O**	**O**	**O**	**O**
O_z		**X**		**O**	**O**	**O**	**O**	**O**	**O**	**X**	**O**	**O**	**O**	**O**
C_x	**O**		**X**		**O**	**O**	**O**	**O**	**O**	**O**	**X**	**O**	**O**	**O**
C_z		**O**		**X**	**O**	**O**	**O**	**O**	**O**	**O**	**O**	**X**	**O**	**O**
T_x	**O**	**O**	**O**	**O**	**X**	**O**	**O**	**O**	**O**	**O**	**O**	**O**	**X**	
T_z	**O**	**O**	**O**	**O**	**O**	**X**	**O**	**O**	**O**	**O**	**O**	**O**		**X**
L_x	**O**	**O**	**O**	**O**	**O**	**O**	**X**	**O**	**O**	**O**	**O**	**O**	**O**	
L_z	**O**	**O**	**O**	**O**	**O**	**O**	**O**	**X**	**O**	**O**	**O**	**O**		**O**

Complete Double spin measurements

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G														
H	X	X	X	X	X	X	X	X	X	X	X	X	X	X
E	X	X	X	X	X	X	X	X						
F									X	X	X	X	X	X
O_x	X		O			O	O		X	O	O	O	O	O
O_z		X		O	O			O	O	X	O	O	O	O
C_x	O		X		O			O	O	O	X	O	O	O
C_z		O		X		O	O		O	O	O	X	O	O
T_x		O	O		X		O		O	O	O	O	X	
T_z	O			O		X		O	O	O	O	O		X
L_x	O			O	O		X		O	O	O	O	O	
L_z		O	O			O		X	O	O	O	O		O

Complete Double spin measurements

‘**X**’s’ –> 3 selected measurements,
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G	**X**		**O**		**O**	**O**	**O**	**O**	**X**	**O**	**O**	**O**	**O**	**O**
H		**X**		**O**	**O**	**O**	**O**	**O**	**O**	**X**	**O**	**O**	**O**	**O**
E	**O**		**X**		**O**	**O**	**O**	**O**	**O**	**O**	**X**	**O**	**O**	**O**
F		**O**		**X**	**O**	**O**	**O**	**O**	**O**	**O**	**O**	**X**	**O**	**O**
O_x	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**
O_z	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**						
C_x									**X**	**X**	**X**	**X**	**X**	**X**
C_z														
T_x	**O**	**O**	**O**	**O**	**X**	**O**	**O**	**O**	**O**	**O**	**O**	**O**	**X**	**O**
T_z	**O**	**O**	**O**	**O**	**O**	**X**	**O**	**O**	**O**	**O**	**O**	**O**	**O**	**X**
L_x	**O**	**O**	**O**	**O**	**O**	**O**	**X**	**O**	**O**	**O**	**O**	**O**		
L_z	**O**	**O**	**O**	**O**	**O**	**O**	**O**	**X**	**O**	**O**	**O**	**O**		

Complete Double spin measurements

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G	X		O		O			O	X	O	O	O	O	O
H		X		O		O	O		O	X	O	O	O	O
E	O		X			O	O		O	O	X	O	O	O
F		O		X	O			O	O	O	O	X	O	O
O_x														
O_z	X	X	X	X	X	X	X	X	X	X	X	X	X	X
C_x	X	X	X	X	X	X	X	X						
C_z									X	X	X	X	X	X
T_x	O			O	X	O			O	O	O	O	X	O
T_z		O	O		O	X			O	O	O	O	O	X
L_x		O	O				X	O	O	O	O	O		
L_z	O			O			O	X	O	O	O	O		

Complete Double spin measurements

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G	X	O			O	O	O	O	X	O	O	O	O	O
H	O	X			O	O	O	O	O	X	O	O	O	O
E			X	O	O	O	O	O	O	O	X	O	O	O
F			O	X	O	O	O	O	O	O	O	X	O	O
O_x	O	O	O	O	X	O	O	O	O	O	O	O	X	O
O_z	O	O	O	O	O	X	O	O	O	O	O	O	O	X
C_x	O	O	O	O	O	O	X	O	O	O	O	O		
C_z	O	O	O	O	O	O	O	X	O	O	O	O		
T_x	X	X	X	X	X	X	X	X	X	X	X	X	X	X
T_z	X	X	X	X	X	X	X	X						
L_x									X	X	X	X	X	X
L_z														

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G	**X**	**O**			**O**			**O**	**X**	**O**	**O**	**O**	**O**	**O**
H	**O**	**X**				**O**	**O**		**O**	**X**	**O**	**O**	**O**	**O**
E			**X**	**O**		**O**	**O**		**O**	**O**	**X**	**O**	**O**	**O**
F			**O**	**X**	**O**			**O**	**O**	**O**	**O**	**X**	**O**	**O**
O_x	**O**			**O**	**X**	**O**			**O**	**O**	**O**	**O**	**X**	**O**
O_z		**O**	**O**		**O**	**X**			**O**	**O**	**O**	**O**	**O**	**X**
C_x		**O**	**O**				**X**	**O**	**O**	**O**	**O**	**O**		
C_z	**O**			**O**			**O**	**X**	**O**	**O**	**O**	**O**		
T_x														
T_z	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**
L_x	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**						
L_z									**X**	**X**	**X**	**X**	**X**	**X**

Limitations

Partial Wave Extraction is made difficult by:

- Unknown Energy and Angle dependent Overall Phase
- Cusp and Threshold effects
- Error bars

Some Observations:

Determination of unique Underlining dynamics is almost always limited in QM. Examples: Phase equivalent potentials & Redundant Inverse scattering solutions

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Conclusions

- The 16 Measurements are related by a measurement algebra
- That algebra is equivalent to that of 16 Hermitian 4×4 SU(4) group
- More than 7 experiments are needed due to discrete ambiguities, which are also described using SU(4) algebra.
- Constraints and Bounds can be deduced from that algebra
- Constrains allow for one less experiment needed and is why CT say 8 and BDS say 9 needed for compete set.
- Procedures can be generalized to N amplitudes [3]

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Fierz —> explicit and rigorous relationships between observables. Of course, such relationships can be derived from the bilinear structure of the observables, with much effort. That effort is now replaced by simply invoking the well-known Fierz rules as a general property. That allows us to avoid much algebra and to find all relations in one step. There are direct physical consequences of these relations. If double spin observables in a type set are known, then the fourth member of that type is uniquely determined. The fourth measurement is thus redundant.

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Some Additional references

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Thanks for including me!