Prospects for N-N* Transitions from Lattice QCD

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(Jefferson Laboratory)

Outline

- Baryons as seen from the lattice - spectrum
- Anatomy of EM calculations
- Recent results
  - N Delta transition form factors
  - First transition form factor to P11 (H-W Lin)
  - GPDs
  - Axial vector charges
- Future plans

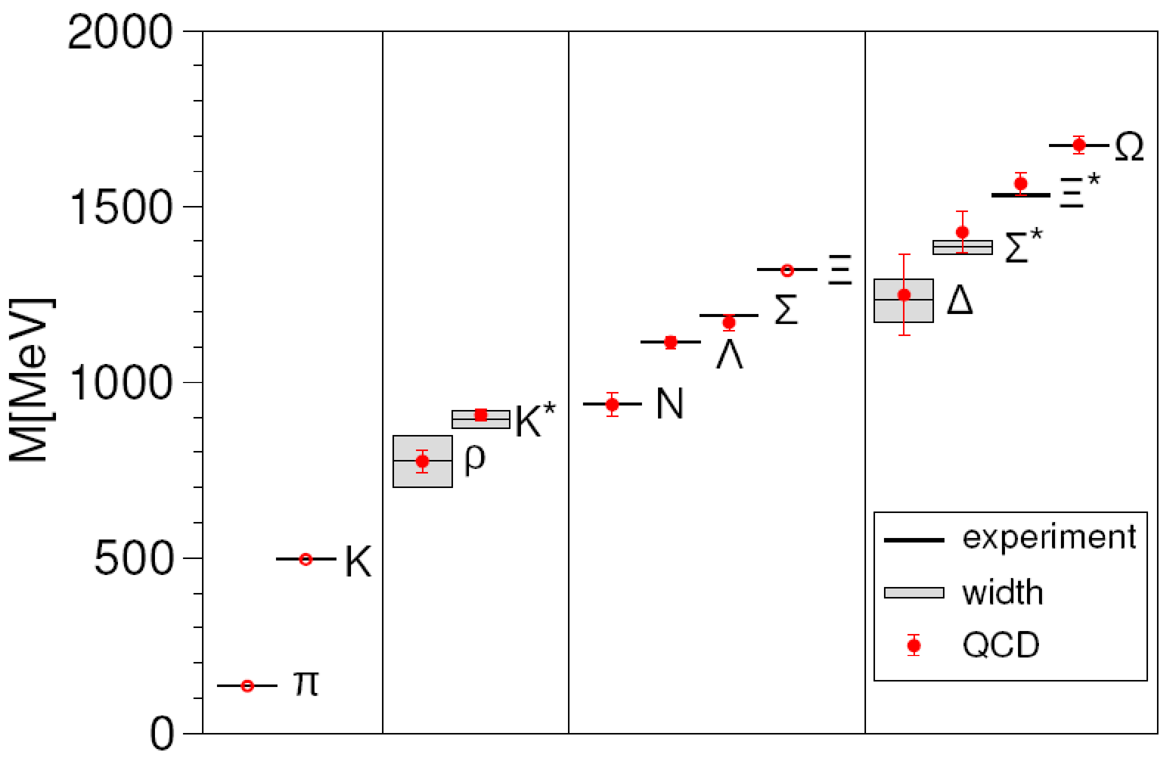
Spectrum from correlation functions

- **Euclidean space**: stationary state energies can be extracted from asymptotic decay rate of temporal correlations of the fields
- Spectral representation of a simple correlation function
  - assume transfer matrix, ignore temporal boundary conditions

$$C(t) = \langle 0 \mid N(t)\bar{N}(0) \mid 0 \rangle = \sum_n \langle 0 \mid e^{Ht} N(0) e^{-Ht} \mid n \rangle \langle n \mid \bar{N}(0) \mid 0 \rangle$$
$$= \mid \langle n \mid N(0) \mid 0 \rangle \mid^2 e^{-E_n t} = \sum_n A_n e^{-E_n t}$$

- *Extract lowest energy and amplitude as t $\to \infty$*

Low-lying Hadron Spectrum



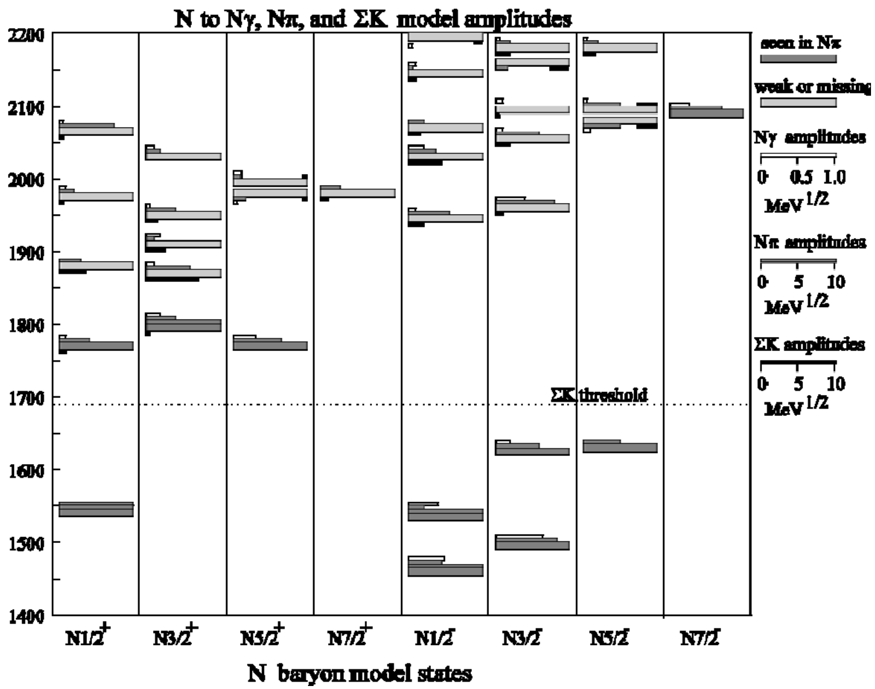
Ch. Hoelbling et al. (BMW Collaboration), Lattice 2008

Control over:

- ***Quark-mass dependence***
- ***Continuum extrapolation***
- ***finite-volume effects (pions, resonances)***

Spectroscopy - I

- **Quark model amplitudes – states classified by isospin, parity and spin.**



Capstick and Roberts, PRD58 (1998) 074011

Variational Method

- **Extracting excited-state energies described in C. Michael, NPB 259, 58 (1985) and Luscher and Wolff, NPB 339, 222 (1990)**
- **Can be viewed as exploiting the *variational method***
- **Given $N \times N$ correlator matrix $C_{\alpha\beta}(t) = \langle 0 \mid O_\alpha(t)\, O_\beta(0) \mid 0 \rangle$, one defines the N *principal correlators* $\lambda_i(t,t_0)$ as the eigenvalues of**

$$C^{-1/2}(t_0)C(t)C^{-1/2}(t_0)$$

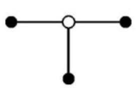
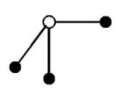
- **Principal effective masses defined from correlators plateau to lowest-lying energies**

$$\lambda_i(t, t_0) \rightarrow e^{-E_i(t-t_0)}\left(1 + O(e^{-\Delta E(t-t_0)})\right)$$

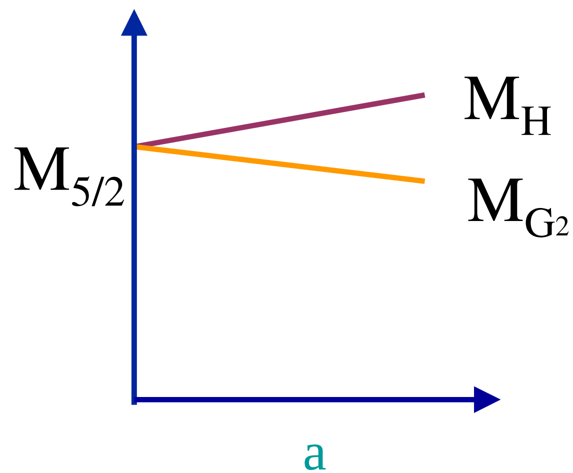
Eigenvectors, with metric $C(t_0)$, are orthonormal and project onto the respective states

Variational Method - II

- **Spectrum on lattice looks different – states at rest classified by isospin, parity and representation under cubic group**

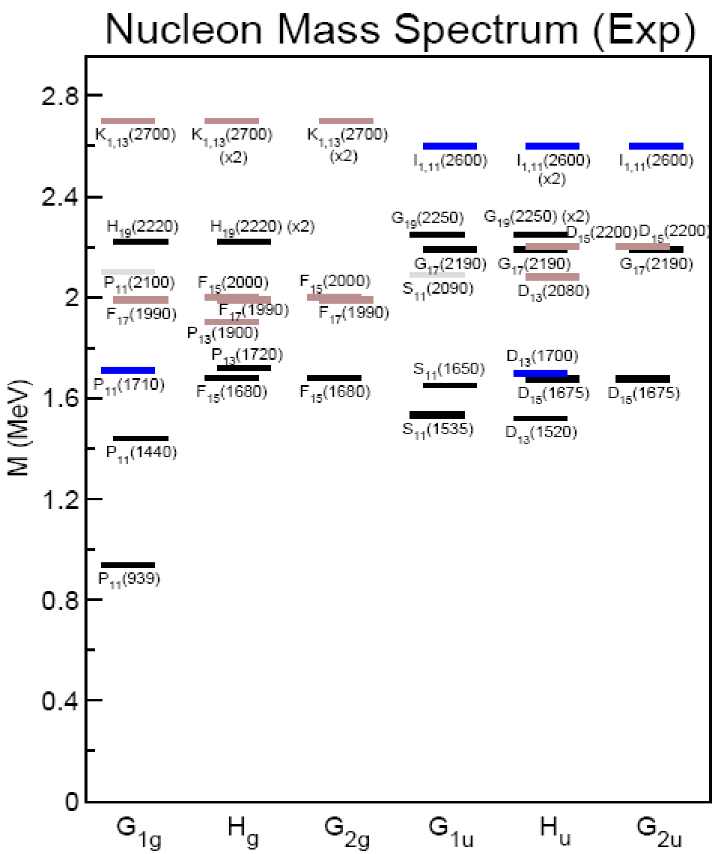
Illustration	Name	Explicit form ($\lvert i\rvert \neq \lvert j\rvert \neq \lvert k\rvert$)
	single-site	$\phi^F_{ABC}\ \varepsilon_{abc}\ \tilde\psi_{Aa\alpha}\ \tilde\psi_{Bb\beta}\ \tilde\psi_{Cc\gamma}$
	singly-displaced	$\phi^F_{ABC}\ \varepsilon_{abc}\ \tilde\psi_{Aa\alpha}\ \tilde\psi_{Bb\beta}\ \left(\tilde D^{(p)}_j\tilde\psi\right)_{Cc\gamma}$
	doubly-displaced-I	$\phi^F_{ABC}\ \varepsilon_{abc}\ \tilde\psi_{Aa\alpha}\ \left(\tilde D^{(p)}_{-j}\tilde\psi\right)_{Bb\beta}\left(\tilde D^{(p)}_j\tilde\psi\right)_{Cc\gamma}$
	doubly-displaced-L	$\phi^F_{ABC}\ \varepsilon_{abc}\ \tilde\psi_{Aa\alpha}\ \left(\tilde D^{(p)}_j\tilde\psi\right)_{Bb\beta}\left(\tilde D^{(p)}_k\tilde\psi\right)_{Cc\gamma}$
	triply-displaced-T	$\phi^F_{ABC}\ \varepsilon_{abc}\ \left(\tilde D^{(p)}_{-j}\tilde\psi\right)_{Aa\alpha}\left(\tilde D^{(p)}_j\tilde\psi\right)_{Bb\beta}\left(\tilde D^{(p)}_k\tilde\psi\right)_{Cc\gamma}$
	triply-displaced-O	$\phi^F_{ABC}\ \varepsilon_{abc}\ \left(\tilde D^{(p)}_i\tilde\psi\right)_{Aa\alpha}\left(\tilde D^{(p)}_j\tilde\psi\right)_{Bb\beta}\left(\tilde D^{(p)}_k\tilde\psi\right)_{Cc\gamma}$

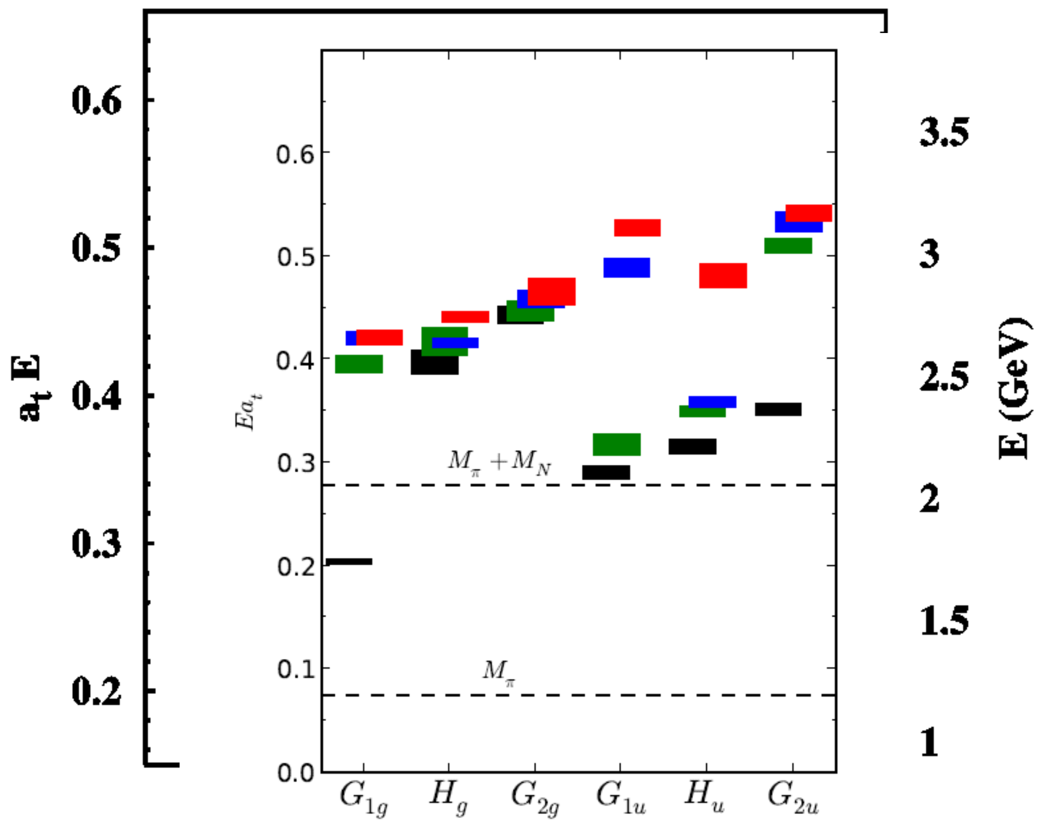
J	$n^J_{G_1}$	$n^J_{G_2}$	n^J_H
$\frac{1}{2}$	1	0	0
$\frac{3}{2}$	0	0	1
$\frac{5}{2}$	0	1	1
$\frac{7}{2}$	1	1	1
$\frac{9}{2}$	1	0	2
$\frac{11}{2}$	1	1	2
$\frac{13}{2}$	1	2	2
$\frac{15}{2}$	1	1	3
$\frac{17}{2}$	2	1	3



Extension to qqq $\overline{q}$q

Quenched Baryon Spectrum





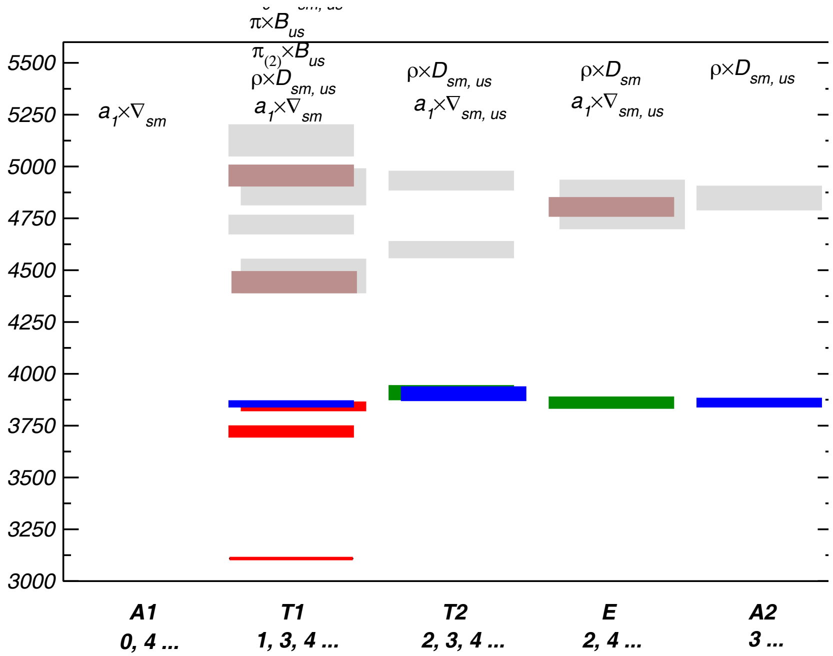
2-flavour calculation in progress: *Eric Engelson*

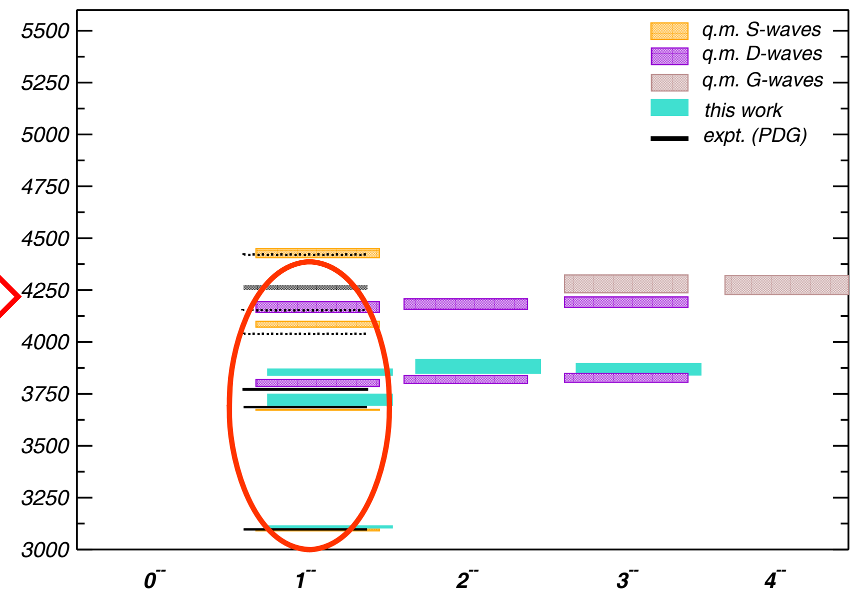
Spin dentification major goal! Continuum behavior of interpolating operators?

Precursor of studying electromagnetic properties

Charmonium Spectroscopy

- Use continuum overlap of operators to states of different spin – further handle on spin identification
- Apply to baryons?

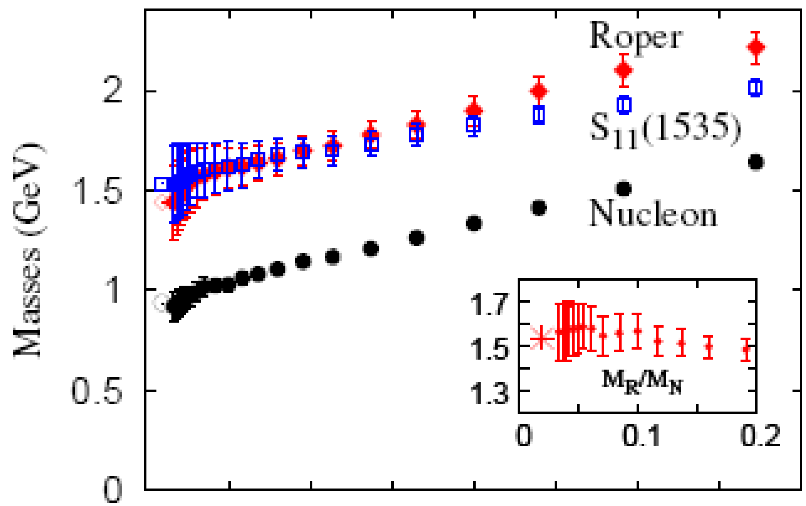


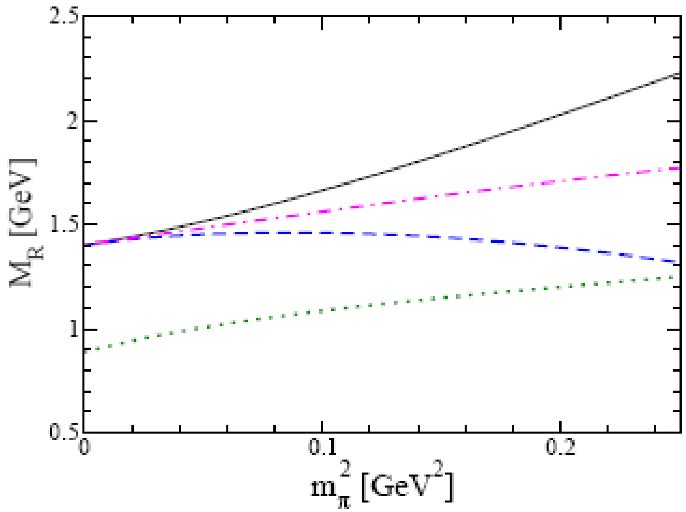


Resolve higher states

Dudek, Edwards, Mathur, DGR,
PRD77, 034501 (2008)

Roper Resonance





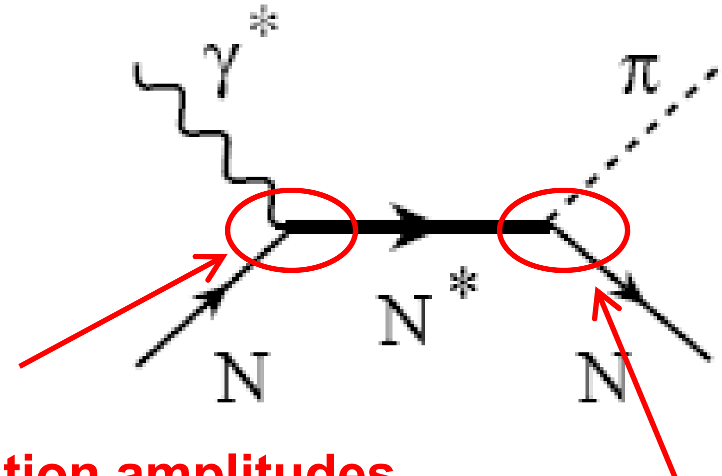
- Bayesian statistics and constrained curve fitting
- Used simple three-quark operator

Dong et al., PLB605, 137 (2005)

Borasoy et al., Phys.Lett. B641 (2006) 294-300

EM Transitions and Lattice QCD

Example: Single-pion photoproduction

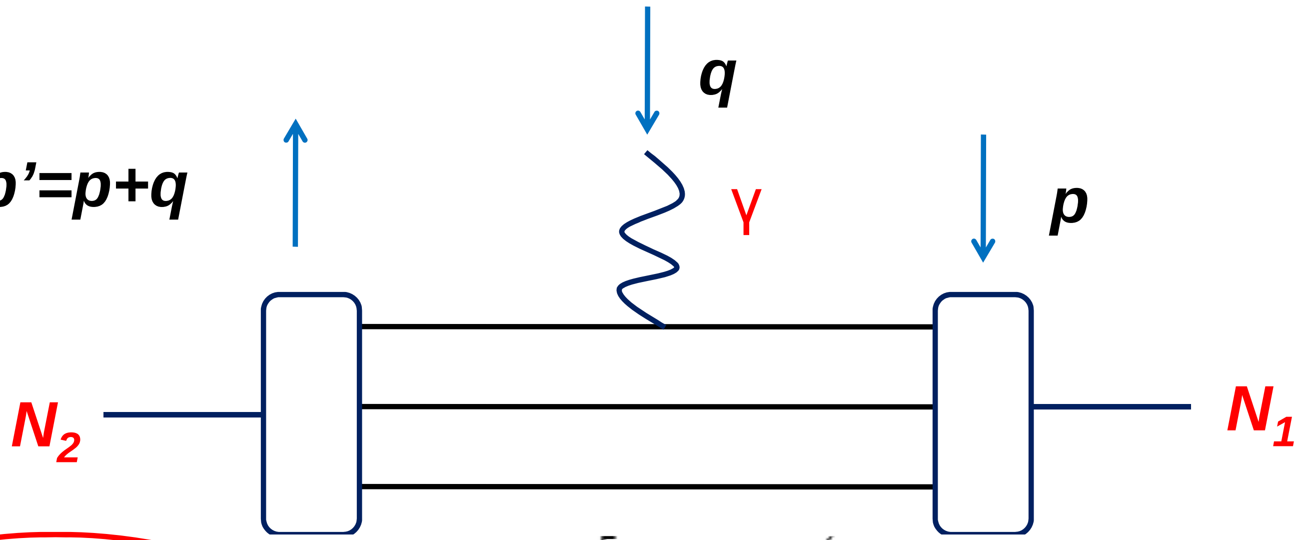


Radiative transition amplitudes

Axial-vector Couplings?

Anatomy of a Calculation - I

- Lattice QCD computes the transition between isolated states



$$\langle N_2 \left| V_\mu \right| N_1 \rangle_\mu(q) = \overline{u}_{N_2}(p') \left[F_1(q^2) \left(\gamma_\mu - \frac{q_\mu}{q^2} - (M_{N_2} - M_{N_1}) \not{q} \right) + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{M_{N_1} + M_{N_2}} \right] u_{N_1}(p) e^{-iq\cdot x},$$

Anatomy of a Calculation - II

$$\Gamma_{3\text{pt}}(\vec{p},\vec{q};t_f,t) = \sum_{\vec{x},\vec{y}} \langle N_1(\vec{x},t_f) V_\mu(\vec{y},t) N_2(0)\rangle e^{-i\vec{p}\cdot\vec{x}} e^{-i\vec{q}\cdot\vec{y}}$$

Complete set of states

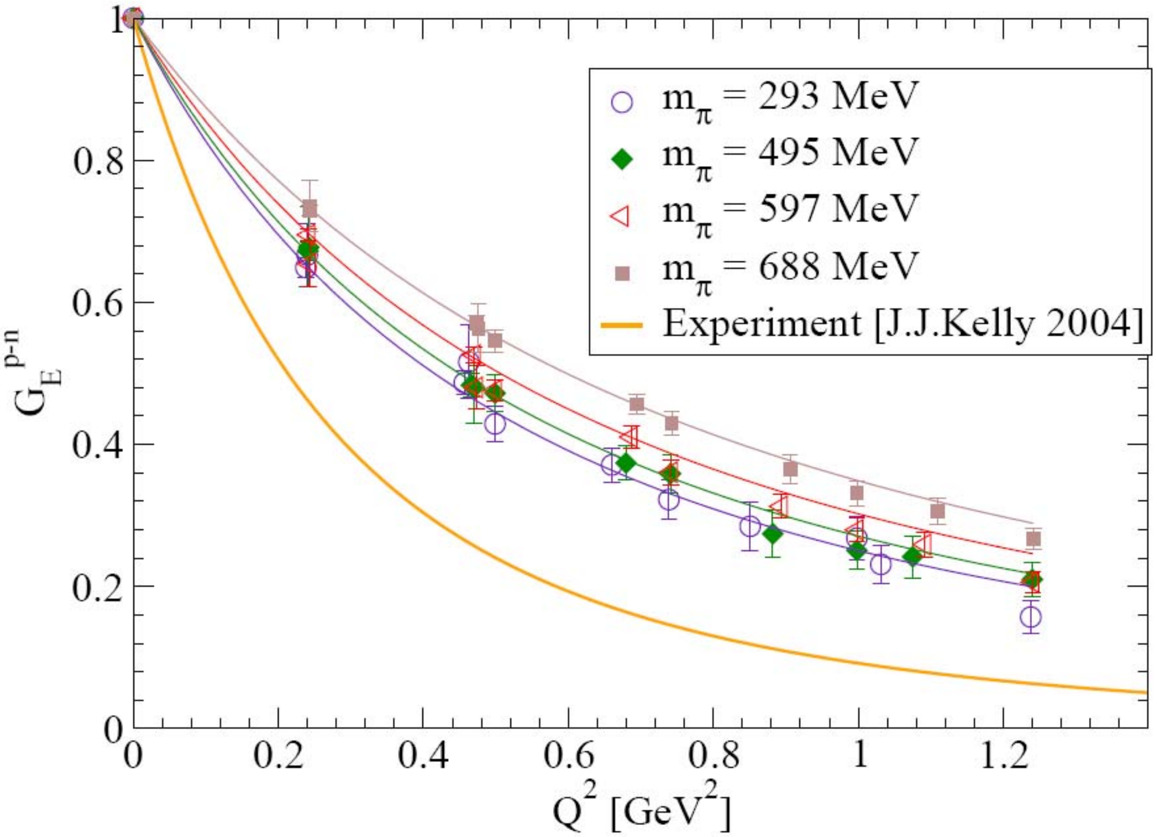
At large $t_f - t$, and t, correlator is dominated by *lowest lying state*

$$\langle 0 \mid N_1(0) \mid N_1,\vec{p}\rangle\langle N_1,\vec{p} \mid V_\mu(0) \mid N_2,\vec{p}+\vec{q}\rangle\langle N_2,\vec{p}+\vec{q} \mid \phi^\dagger \mid 0\rangle e^{-E(\vec{p}+\vec{q})(t-t_i)} e^{-E(\vec{p})(t_f-t)}$$

Lattice calculations of electromagnetic properties of some lowest-lying states well established, eg:

- EM form factors of nucleon and pion
- Moments of GPDs in DVCS for nucleon
- N-Delta transition Form Factor

Isovector Form Factor



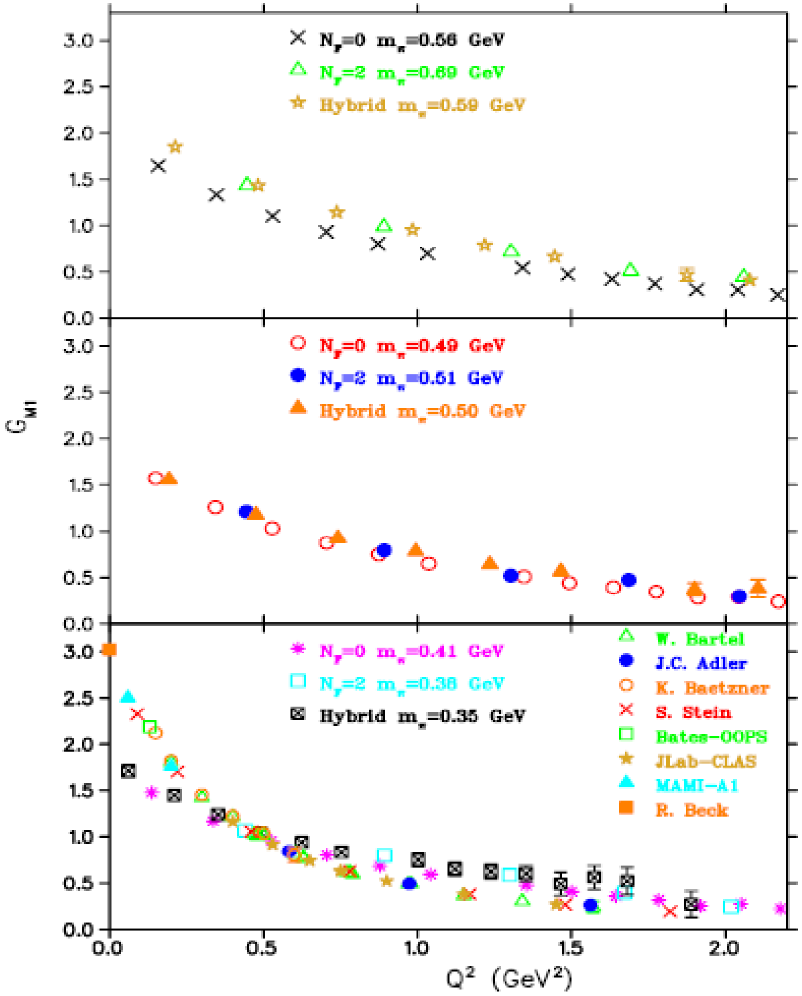
J.D.Bratt et al (LHPC), arXiv:0810.1933

Euclidean lattice: *form factors in space-like region*

Extension to higher Q^2

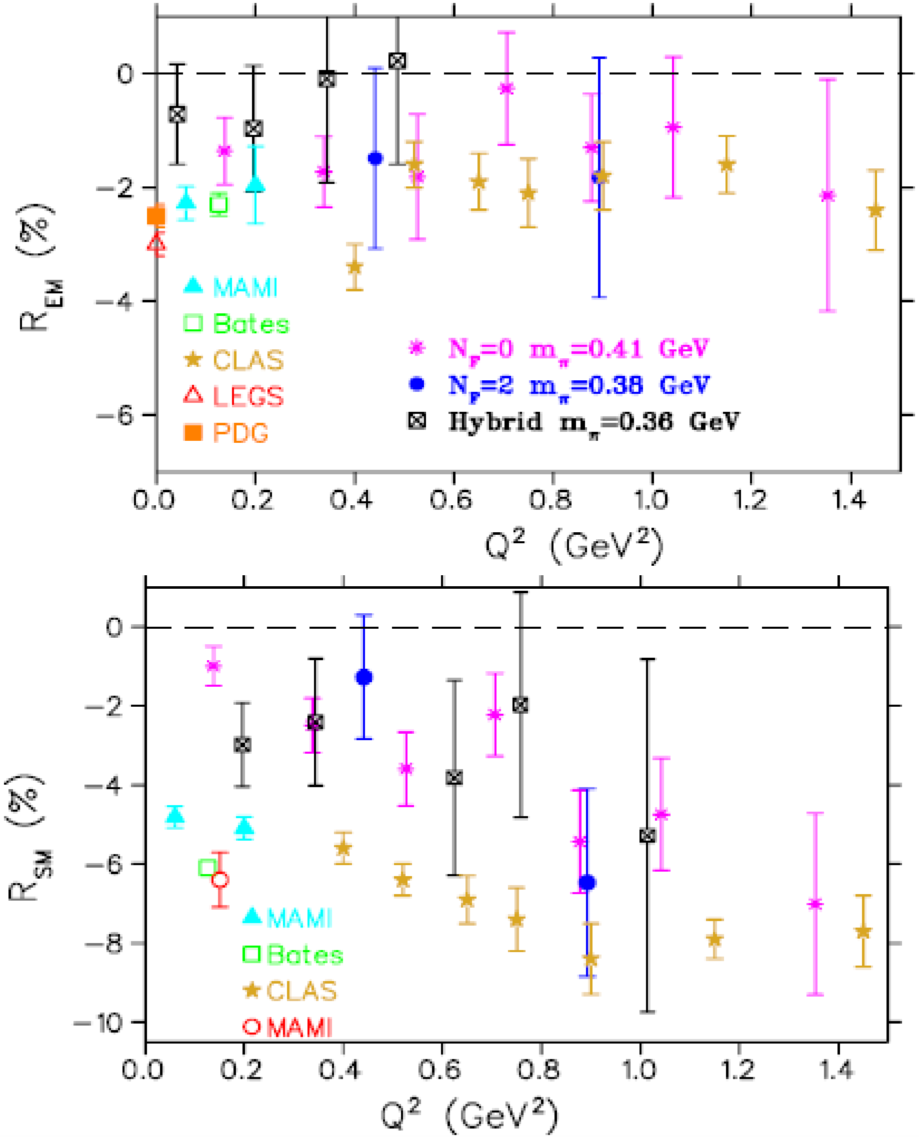
N-Δ Transition Form Factor - I

- **Transition between lowest lying I=3/2, J=3/2 (Δ), and I=1/2, J=1/2 (N)**
- **Comparison between different lattice calculations and expt.**
  - **Milder Q^2 dependence than experiment but**
  - ***Quark masses corresponding to pion masses around 350 MeV***
  - ***Q^2 range up to around 2 GeV^2***



Alexandrou et al, arXiv:0710.4621

N-Δ Transition Form Factor - II



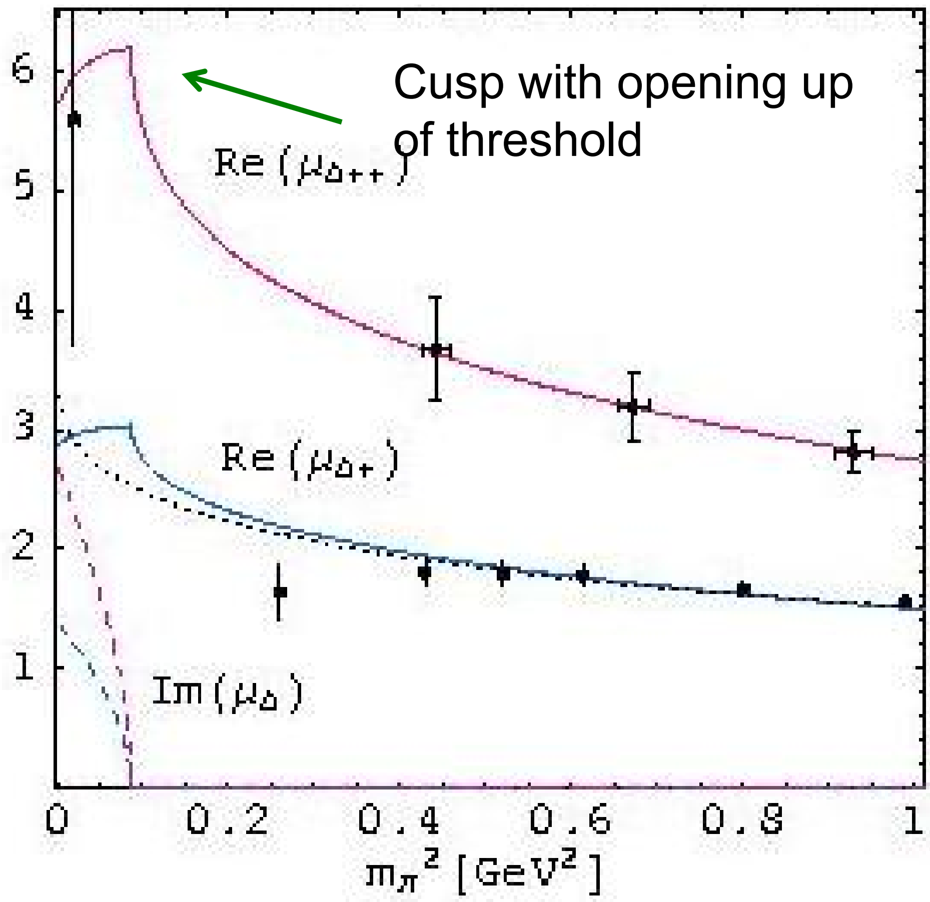
$R_{EM} \rightarrow +1$

Alexandrou et al, arXiv:0710.4621

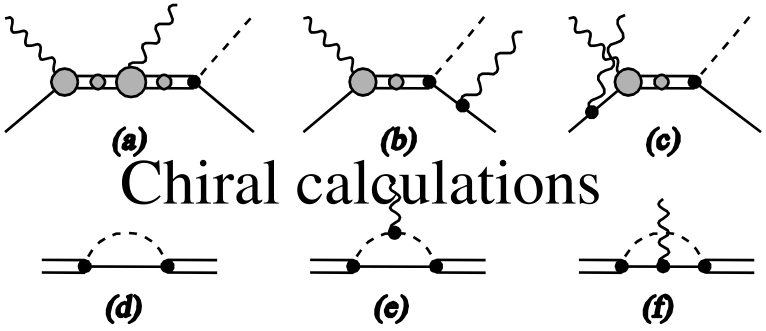
Deformation in nucleon or delta

Entering resonance region

Current calculations: Δ *stable* – P-wave decay suppressed at p = 0



Lattice points from
Leinweber (1992)
Cloet,Leinweber,Thomas (2003)
Lee *et.al*. (2004)

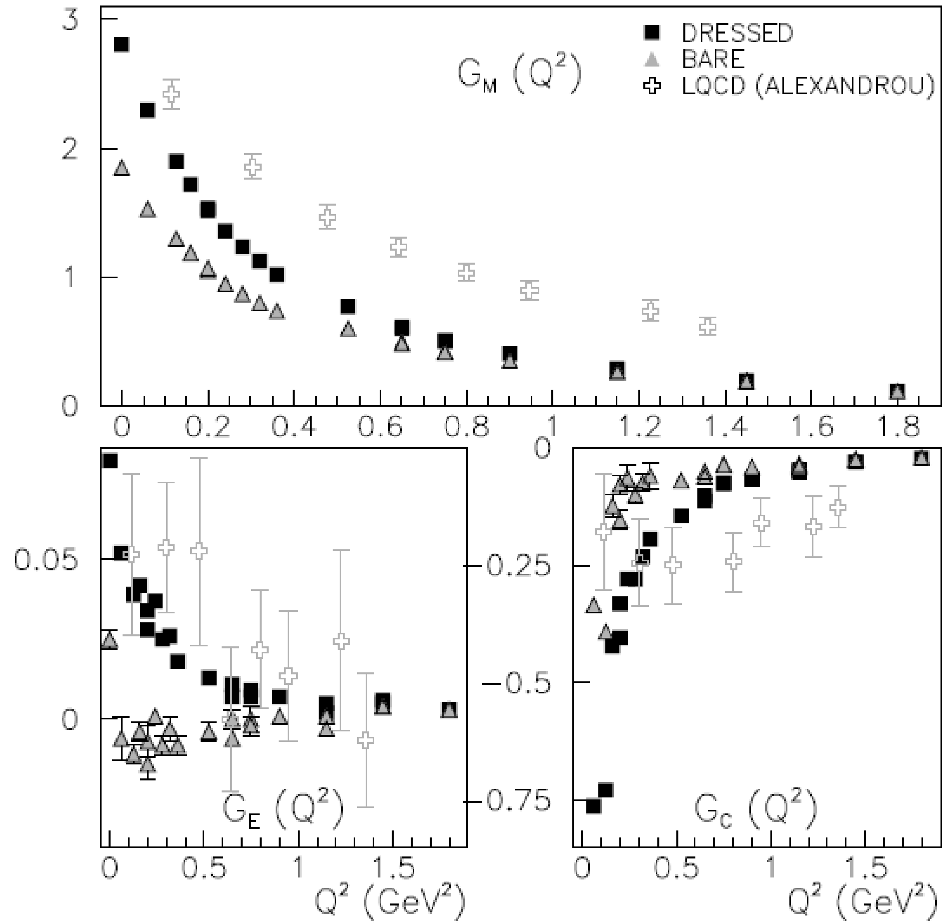


Pascalutsa, Vanderhaeghen (2004)Thomas, Young (…)

Interpretation of Parameters

Julia-Diaz et al., Phys.Rev. C75 (2007) 015205

Comparison of LQCD, EFT + expt: lattice QCD can vary quark masses

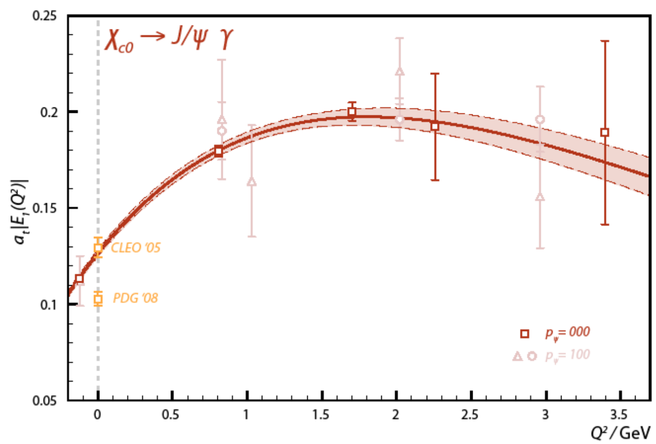


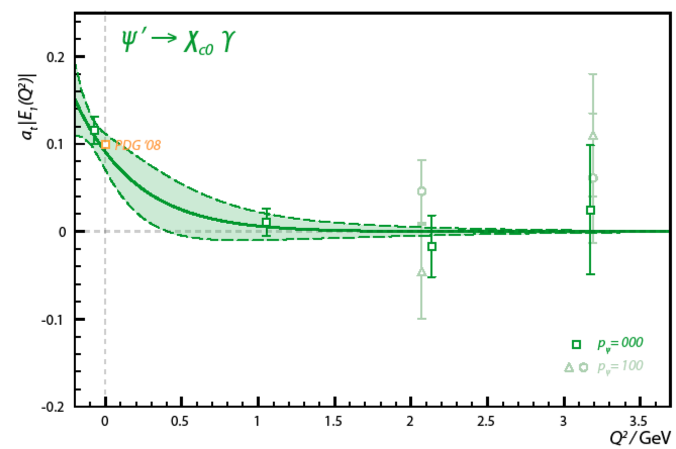
Transitions to “Radial” excitations

- At currently accessible masses, the delta is Stable – lowest energy level in its respective channel
- Transitions to resonances: use variational method to obtain interpolating operator to n[th] excited state

$$\tilde{N}^n = \sum_j A_j^n N_j$$

Radiative Transitions in Charmonium
(preliminary, thanks to Jo Dudek)

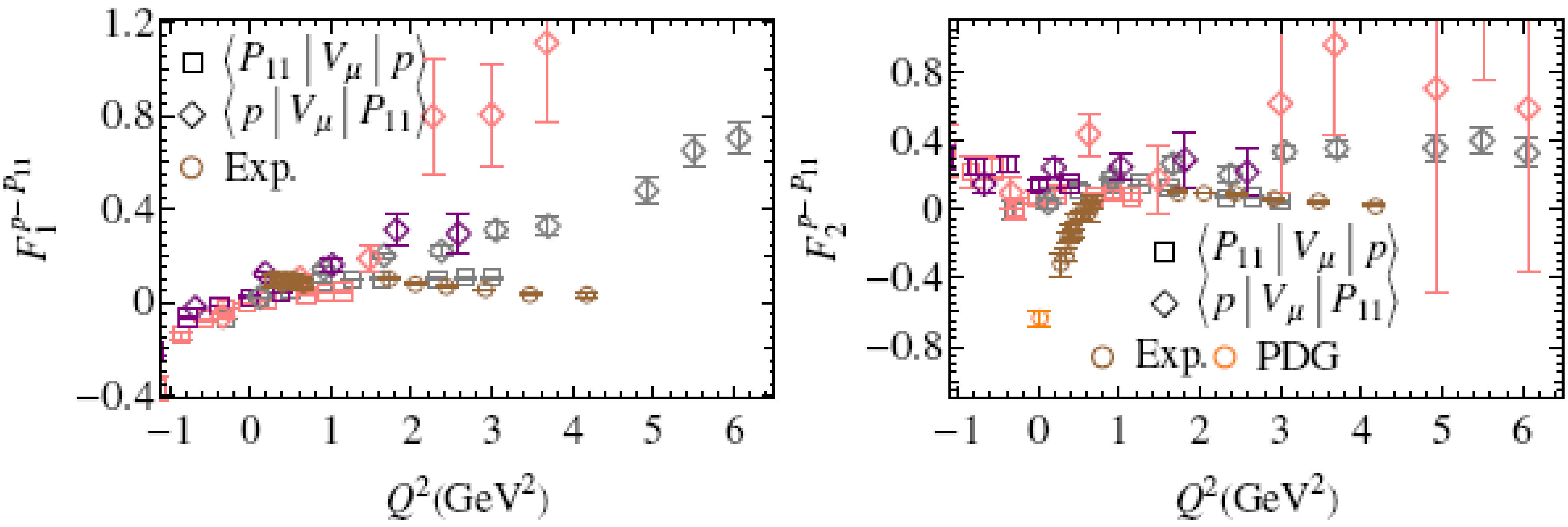




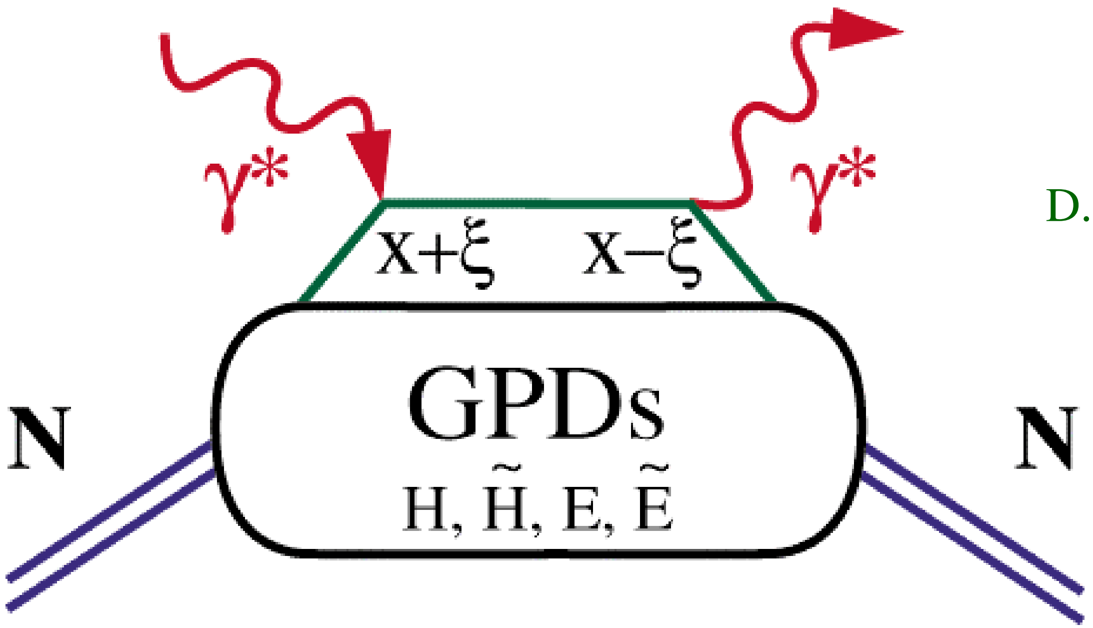
Nucleon-P11 Transition

See talk of H-W Lin

- **First measurement of nucleon-P11 transition form factor**
- **Pion masses of 480, 720, 1100 MeV**



Generalized Parton Distributions (GPDs)



HP 2008

D. Muller *et al* (1994), X. Ji & A. Radyushkin (1996)

Extensive study of Nucleon GPDs: *formulism should extend to N-N* GPDs*

Flavor off-diagonal GPDs: Huey-Wen Lin, Kostas Orginos

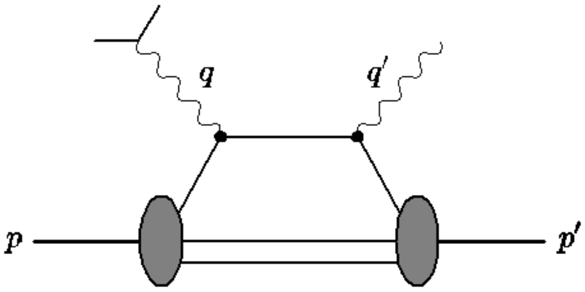
Moments of Structure Functions and GPD's

- **Matrix elements of light-cone correlation functions**

$$\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}\left(-\frac{\lambda}{2}n\right) n P e^{-ig\int_{\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(\alpha n)} \psi\left(\frac{\lambda}{2}n\right)$$

- **Expand *O(x)* around light-cone**

$$O_q^{\{\mu_1\mu_2\ldots\mu_n\}} = \bar{\psi}_q \gamma^{\{\mu_1} iD^{\mu_2} \ldots D^{\mu_n\}} \psi_q$$



- ***Off-forward* matrix element**

$$\langle P'|O_q^{\{\mu_1\ldots\mu_n\}}|P\rangle \simeq \int dx\, x^{n-1}[H(x,\xi,t), E(x,\xi,t)]$$
$$\longrightarrow A_{ni}(t), B_{ni}(t), C_n(t), \tilde{A}_{ni}(t), \tilde{B}_{ni}(t), \tilde{C}_n(t)$$

↑

Co-efficient of ξ^i

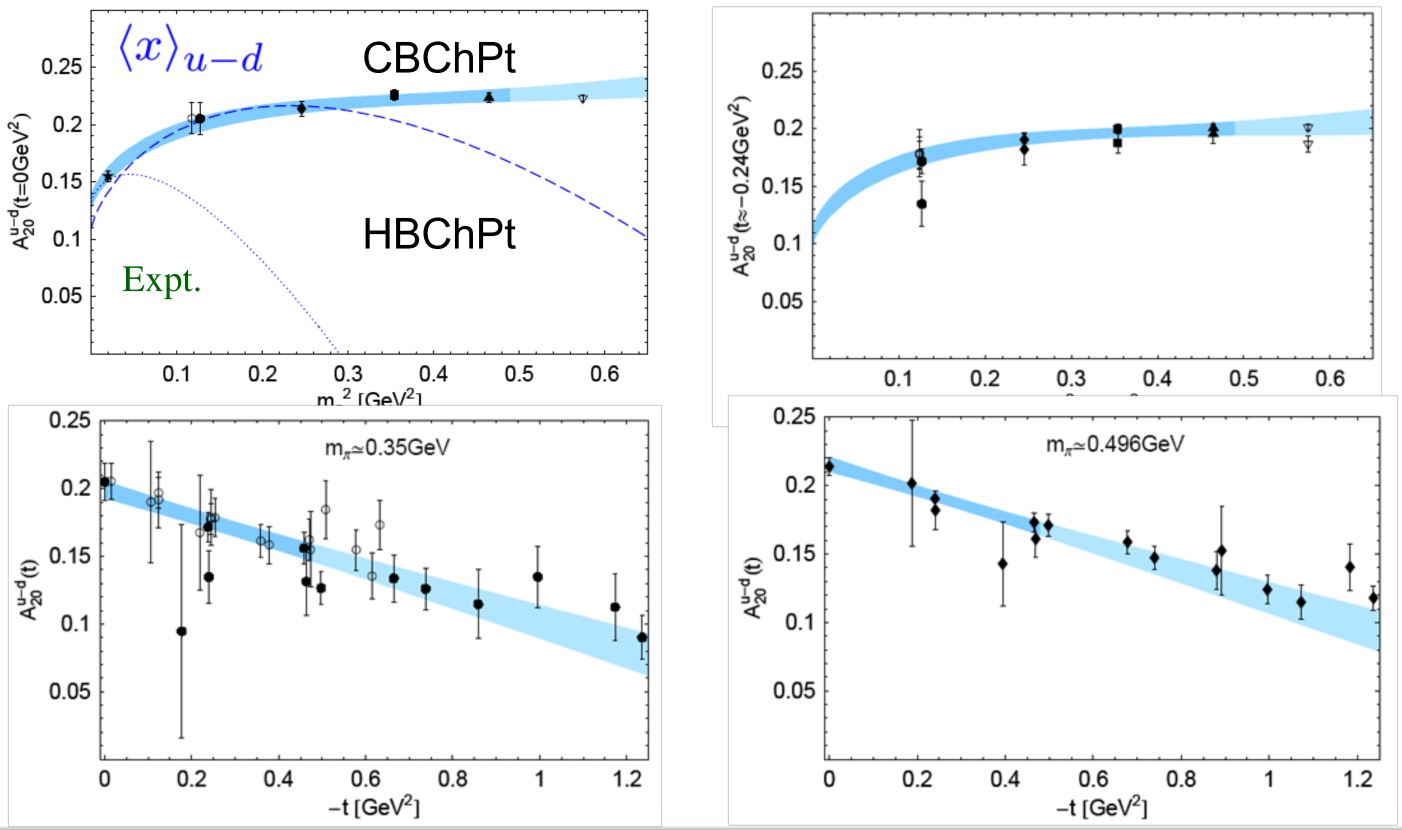
Chiral Extrap. for GPDs – $A_{20}(t,m_\pi^2)$

Joint chiral extrapolation $O(p^4)$ CBChPT (Dorati, Gail, Hemmert)

$$A_{20}^{u-d}(t,m_\pi) = A_{20}^{0,u-d}\left(f_A(m_\pi)+h_A(t,m_\pi)\right)+\tilde{A}_{20}^{0,u-d}j_A(m_\pi)+A_{20}^{m_\pi,u-d}m_\pi^2+A_{20}^{t,u-d}t$$

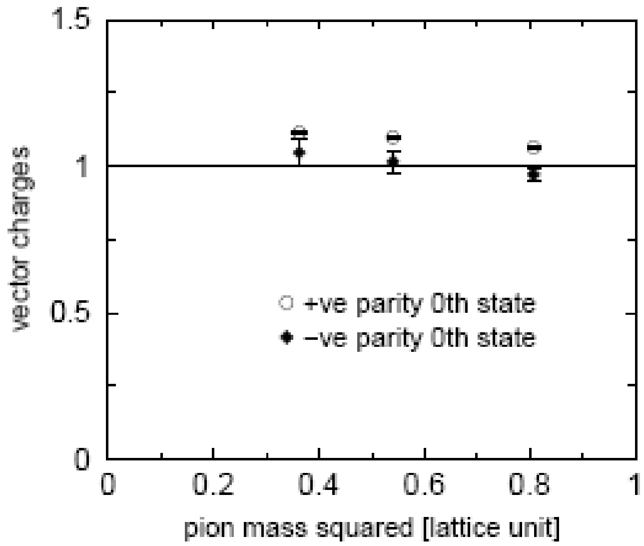
$$\langle x\rangle_{u-d} = a\left(1-\frac{(3g_A+1)}{4\pi f_\pi^2}m_\pi^2\ln m_\pi^2\right)+bm_\pi^2\ldots$$

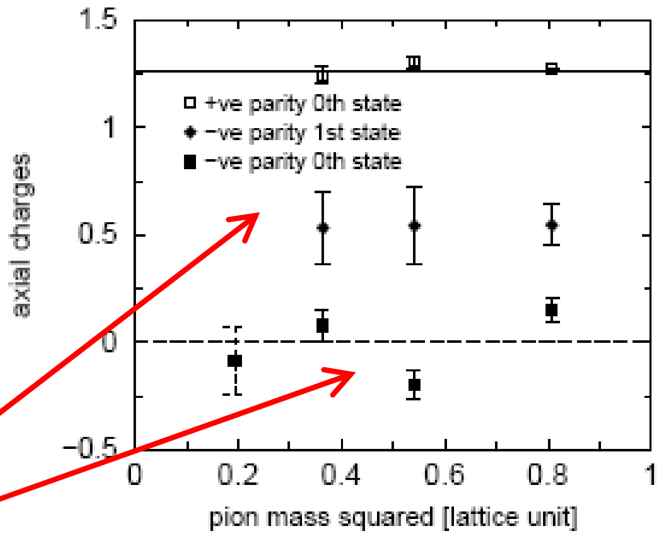
- Joint chiral extrapolation in m_π and "t"
- CBChPT describes data over wider range



Axial-vector Charges

- The axial-vector charges $g_A^{N1\,N2}$ can provide additional insight into hadron structure
- Recent calculation of axial-vector charges of two lowest-lying ½- states, associated with N(1535) and N(1650).





Takahashi, Kunihiro, arXiv:0801.4707

Consistent with NR quark model

Hadron Spectrum Collaboration

- University of Pacific
  - J Juge
- JLAB
  - S Cohen
  - J Dudek
  - R Edwards
  - B Joo
  - H-W Lin
  - D Richards
  - C Thomas
- CMU
  - J Bulava
  - J Foley
  - C Morningstar
- UMD
  - E Engelson
  - S Wallace
- Tata (India)
  - N Mathur
- Trinity College (Dublin)
  - Mike Peardon
  - Sinead Ryan

- Resonance spectrum for both mesons and baryons composed of uds quarks
- Radiative Transitions and properties of resonances
- Charmonium physics
- *Hadronic Interactions (NPLQCD)*

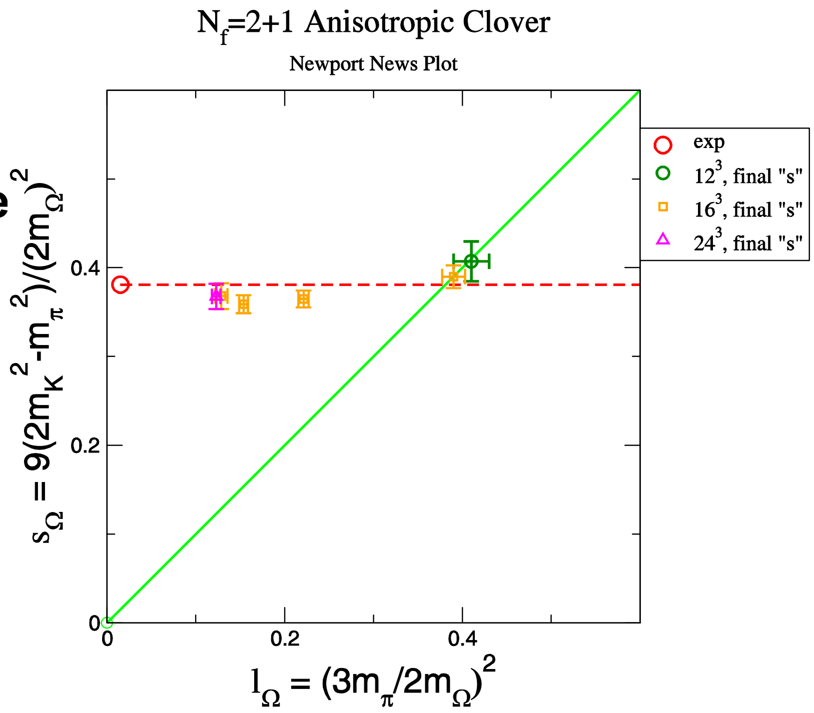
Anisotropic lattice Generation

- **Anisotropic Lattices designed for investigations of resonances**
- **Non-perturbative determination of parameters of three-flavor anisotropic-clove action completed.**

R.G. Edwards, B. Joo, H-W Lin,
arXiv:0803.3960

Prescription for setting of strange mass and lattice spacing

Robert Edwards, USQCD All-Hands



m_π(MeV)	L_s(fm) 1.92fm $16^3 \times 128$	2.4fm $20^3 \times 128$	2.9fm $24^3 \times 128$	3.8fm $32^3 \times 128$
875	10k, JLab[0.20M](8.4)			
580	20k, JLab[0.48M](5.6)			
400		20k, JLab[1.35M](4.8)		
315		30k, JLab[2.35M](3.8)	30k, ORNL[4.66M](4.5)	30k, X[13.7M](6.0)
250			30k, TACC[5.44M](3.6)	30k, ?[16.0M](4.8)

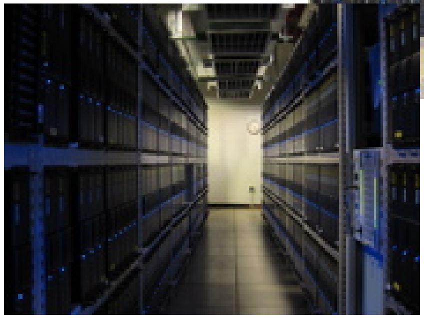
USQCD: Computing Resources

www.usqcd.org
Chroma, MILC,QDP

Leadership-class (ORNL, ANL) – petaflop at ORNL

QCDOC at BNL

Clusters at FNAL and JLAB



LQCD2: Proposal to DOE for computational facility 2010-2014

Conclusions

- Lattice calculations evolving from studies of properties of ground-state hadrons to those of resonances
- Variational method enabling us to isolate not only ground state, but first few excited states
  - Good interpolating operators → electromagnetic properties
  - Major progress: P11 transition, charmonium
- Major effort supported by USQCD:
  - Generation of Lattices
  - Development of new methods for computing correlators (“distillation”, “dilution”)
- Challenges:
  - Identification of spins
  - Delineating the single- and multiparticle states
  - Calculations at higher Q^2
  - Mapping to Chiral Perturbation Theory