The potential for "*complete*" experiments in pseudoscalar meson production

A.M. Sandorfi
(Jefferson Lab)

- *missing (?) N* states of the quark model*
- *spin-observables in $J^{\pi}=0^{-}$ photo-production*
- γ *N* $\rightarrow$ K Λ *JLab "complete" experiments*
- *Q: can such data provide total amplitudes, unique to a phase ?*
- *the mechanics of fitting out the amplitudes*
  - potentials and limitations
- *tests with mock data – work in progress* $\Leftrightarrow$ *with S. Hoblit, UVa*

All the excited states of the proton and neutron that we know about were determined from πN scattering

qqq ⇔ *conventional quark model*	q (qq) ⇔ *di-quark models*
• predict many more resonances than have been observed (in πN) • g(πN) couplings predicted to decrease rapidly with mass in each oscillator band • "*missing resonances*" may have much larger couplings to lower cross section channels: $\pi\pi$N, ρN, ηN, $K\Lambda$ or $K\Sigma$	• two quarks are quasi-bound • fewer degrees of freedom • there are no "*missing*" states

γp and γn KY reactions

Mart & Bennhold:

—————— *including $D_{13}(1895)$*

············ *without $D_{13}(1895)$*

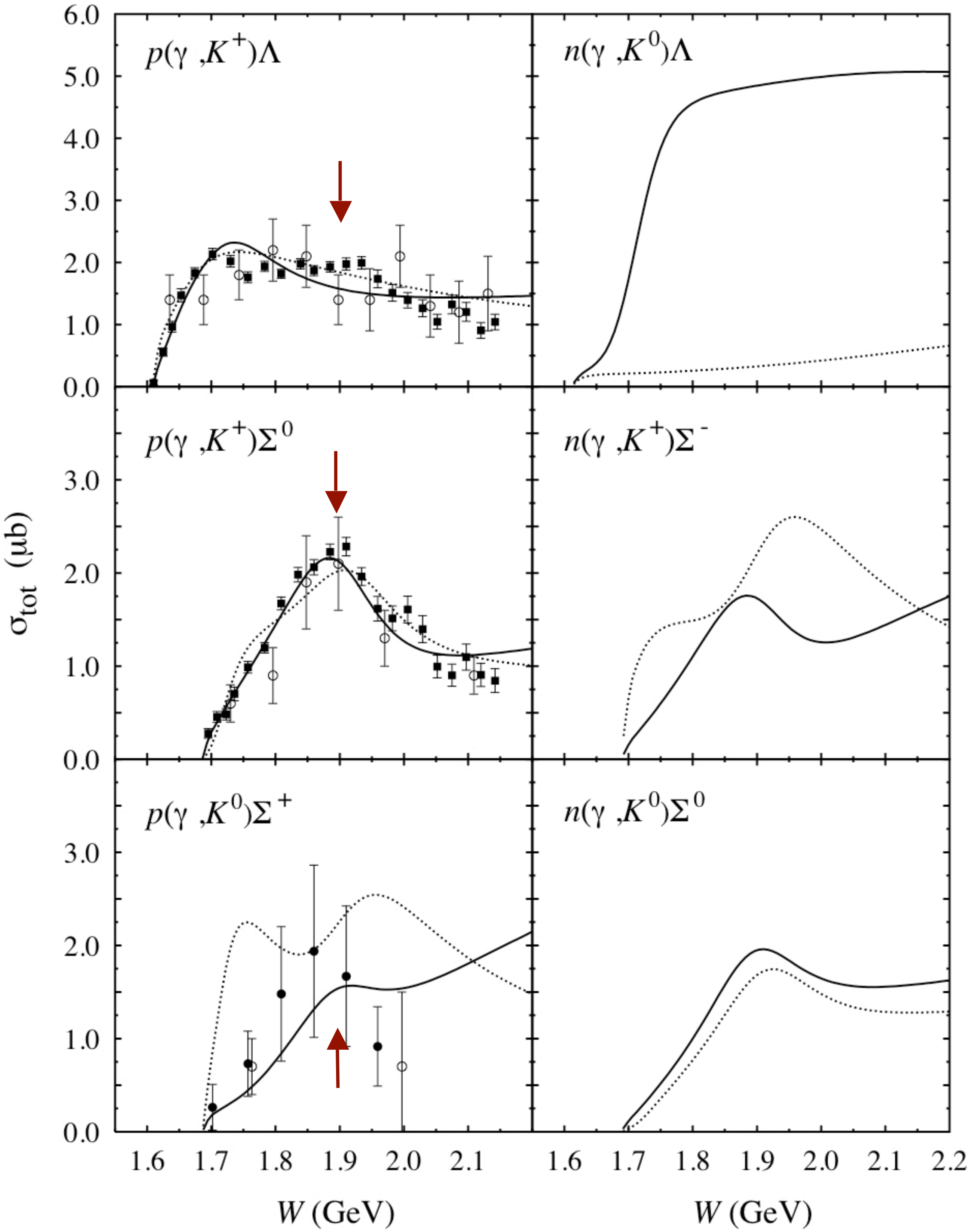
⇔ **subtle effects in σ**

History

$D_{13}(1895)$	-2000
⇒ **$D_{13}(1895)$+$P_{13}(1720)$**	-2000
⇒ ~~**$D_{13}(1900)$**~~	-2002
⇒ **$D_{13}(1740)$**	-2003
⇒ **$D_{13}(1870)$**=[$D_{13}(1520)$]*	-2005
⇒ **$D_{13}(1912)$**	-2006

Problem: 20% Eγ -dependent normalization between SAPHIR and CLAS

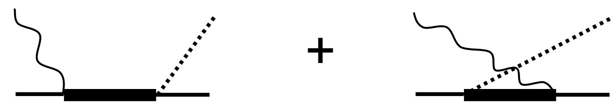
⇔ **channel separation ?**



p and n KY-Mart&Bennhold-hst06

How do you find something that "missing" ?

Issues: (1) Res + bkg divisions ~ artificial



u-ch

s-ch res *all partial waves*

(2) Electromagnetic interaction is not isospin invariant

$A(\gamma N \rightarrow \pi/\eta/K)^{I=3/2} \Leftrightarrow \Delta^*$; $A(\gamma N \rightarrow \pi/\eta/K)^{I=1/2} \Leftrightarrow N^*$ *requires both n & p*

(3) CQM: "$g_{\pi N}$ small for all but lowest in each oscillator band"

$\Rightarrow$ *need to look in ηN, KY , … channels*

(4) large number of measurements needed to define amplitude

Suppose nothing is “missing” ?

Proving “absense” is much more difficult !

best hope is a compelling case

- *need to determine the full amplitude (never done)*

Requirements from experiments ?

Pseudoscalar-meson production: $\gamma + N \rightarrow (J^{\pi} = 0^{-}) + N$

- 2×2 *spin-states* $\rightarrow$ *4 complex amplitudes*

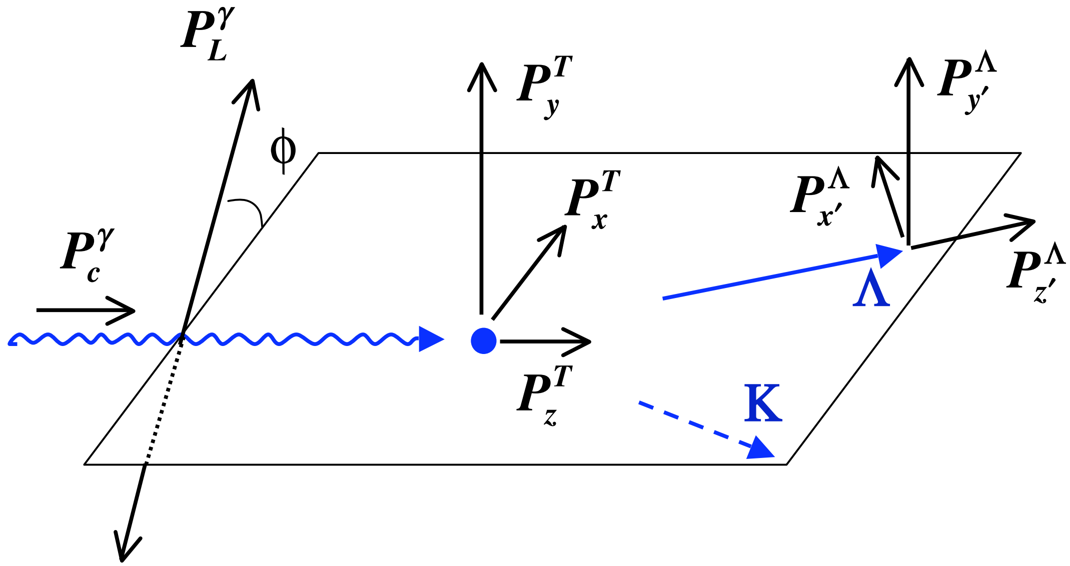
Large number of spin observables needed to define amplitude:

- *8 for* $A(\gamma N)$
- *12 for* $A(\gamma^{*} N)$

(both) *for <u>both</u>* $N = n, p$

Polarized Pseudoscalar meson photo-production:

$$\vec{\gamma} + \vec{N} \rightarrow K + \vec{\Lambda}$$



Pseudoscalar meson photo-production

$$\frac{1}{\sigma_0} d\sigma(\theta,\phi) = 1$$

Leading Pol dependence

$$+\Sigma \cdot \left[P_L^\gamma \cos(2\phi) \;\; - P_y^T \cdot P_{y'}^\Lambda \right]$$

$$+T \cdot \left[P_y^T \;\; - P_L^\gamma \cdot P_{y'}^\Lambda \cos(2\phi) \right]$$

$$+P \cdot \left[P_{y'}^\Lambda \;\; - P_L^\gamma \cdot P_y^T \cos(2\phi) \right]$$

Single Pol

$$+E \cdot \left[-P_c^\gamma \cdot P_z^T \;\; - P_L^\gamma \cdot P_x^T \cdot P_{y'}^\Lambda \sin(2\phi) \right]$$

$$+G \cdot \left[P_L^\gamma \cdot P_z^T \sin(2\phi) \;\; + P_c^\gamma \cdot P_x^T \cdot P_{y'}^\Lambda \right]$$

$$+F \cdot \left[P_c^\gamma \cdot P_x^T \;\; + P_L^\gamma \cdot P_z^T \cdot P_{y'}^\Lambda \sin(2\phi) \right]$$

$$+H \cdot \left[-P_L^\gamma \cdot P_x^T \sin(2\phi) \;\; - P_c^\gamma \cdot P_z^T \cdot P_{y'}^\Lambda \right]$$

beam+target

16 possible observables

$$+C_{x'} \cdot \left[P_c^\gamma \cdot P_{x'}^\Lambda \;\; - P_L^\gamma \cdot P_y^T \cdot P_{z'}^\Lambda \sin(2\phi) \right]$$

$$+C_{z'} \cdot \left[P_c^\gamma \cdot P_{z'}^\Lambda \;\; + P_L^\gamma \cdot P_y^T \cdot P_{x'}^\Lambda \sin(2\phi) \right]$$

$$+O_{x'} \cdot \left[P_L^\gamma \cdot P_{x'}^\Lambda \sin(2\phi) \;\; + P_c^\gamma \cdot P_y^T \cdot P_{z'}^\Lambda \right]$$

$$+O_{z'} \cdot \left[P_L^\gamma \cdot P_{z'}^\Lambda \sin(2\phi) \;\; - P_c^\gamma \cdot P_y^T \cdot P_{x'}^\Lambda \right]$$

beam+recoil

$$+L_{x'} \cdot \left[P_z^T \cdot P_{x'}^\Lambda \;\; + P_L^\gamma \cdot P_x^T \cdot P_{z'}^\Lambda \cos(2\phi) \right]$$

$$+L_{z'} \cdot \left[P_z^T \cdot P_{z'}^\Lambda \;\; - P_L^\gamma \cdot P_x^T \cdot P_{x'}^\Lambda \cos(2\phi) \right]$$

$$+T_{x'} \cdot \left[P_x^T \cdot P_{x'}^\Lambda \;\; - P_L^\gamma \cdot P_z^T \cdot P_{z'}^\Lambda \cos(2\phi) \right]$$

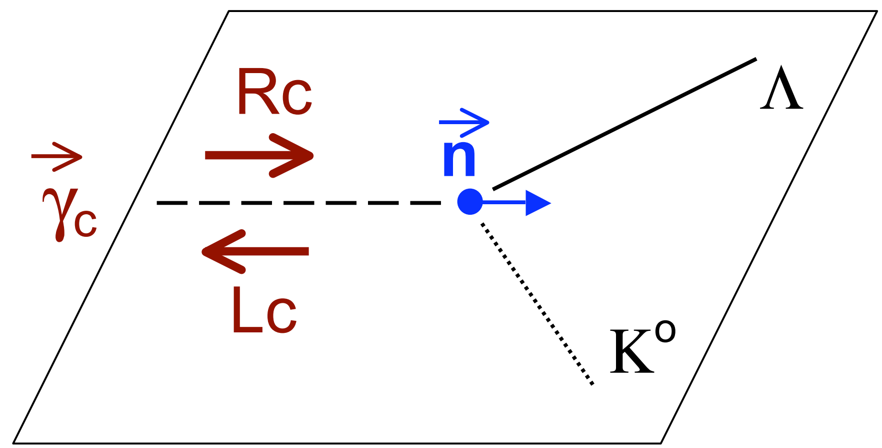
$$+T_{z'} \cdot \left[P_x^T \cdot P_{z'}^\Lambda \;\; + P_L^\gamma \cdot P_z^T \cdot P_{x'}^\Lambda \cos(2\phi) \right]$$

target+recoil

E asymmetry

leading Pol dependence

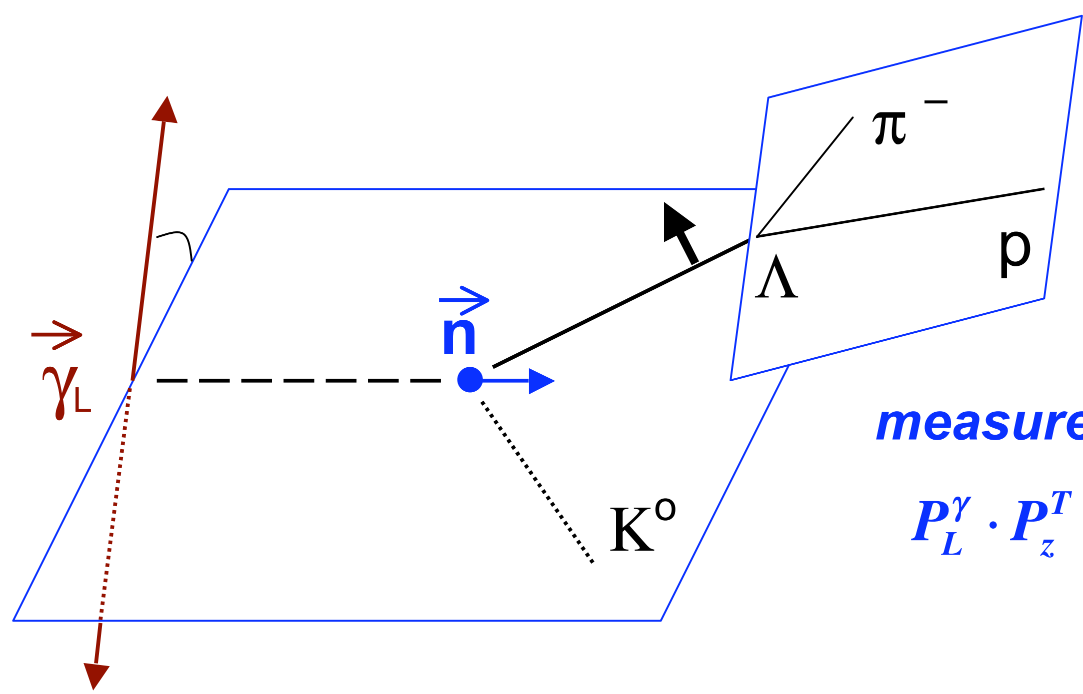
$P_c^\gamma \cdot P_z^T$



T$_{z'}$ asymmetry

leading Pol dependence

$P_x^T \cdot P_{z'}^\Lambda$



measured via

$P_L^\gamma \cdot P_z^T \cdot P_{x'}^\Lambda$

E & Tz asy planes

Polarization observables in $J^{\pi} = 0^{-}$ meson photo-production :

- ***single-pol observables measured from double-pol asy***
- ***double-pol observables measured from triple-pol asy***

Photon beam		***Target***			***Recoil***			***Target - Recoil***								
					x'	y'	z'	x'	x'	x'	y'	y'	y'	z'	z'	z'
		x	y	z				x	y	z	x	y	z	x	y	z
unpolarized	σ_0		T			P		$T_{x'}$		$L_{x'}$		Σ		$T_{z'}$		$L_{z'}$
linearly P_γ	Σ	H	P	G	$O_{x'}$	T	$O_{z'}$	$L_{z'}$	$C_{z'}$	$T_{z'}$	E		F	$L_{x'}$	$C_{x'}$	$T_{x'}$
circular P_γ		F		E	$C_{x'}$		$C_{z'}$		$O_{z'}$		G		H		$O_{x'}$	

- ***not all are independent:***

$$E^2 + F^2 + G^2 + H^2 = 1 + P^2 - \Sigma^2 - T^2$$

$$FG - EH = P - \Sigma T$$

$$C_{x'}^2 + C_{z'}^2 + O_{x'}^2 + O_{z'}^2 = 1 - P^2 - \Sigma^2 + T^2$$

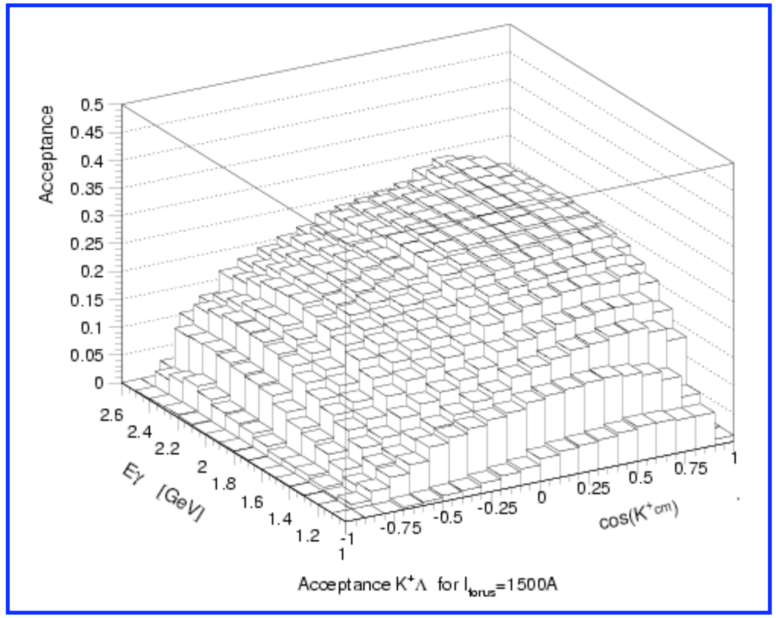
$$C_{z'} O_{x'} - C_{x'} O_{z'} = T - \Sigma P$$

$$T_{x'}^2 + T_{z'}^2 + L_{x'}^2 + L_{z'}^2 = 1 - P^2 + \Sigma^2 - T^2$$

$$L_{z'} T_{x'} - L_{x'} T_{z'} = -PT + \Sigma$$

$\gamma + p \to K^+\Lambda$ series of JLab experiments:

- ***hyperon weak-decays are self-analyzing***
- ***Hall-B proton experiments:***



e-brem
coherent e-brem
$\vec{e}$ -brem

⇓

$$\gamma + p \;\to\; K^+ + \Lambda \;\to\; K^+\, p\, \pi^-$$

⇑ ***H_2*** ***C_4H_9OH***

⇑ ***detect 2-3 charged particles in CLAS***

$\gamma + p \rightarrow K^{+}\Lambda$ *series of JLab experiments:*

Photon beam		*Target*			*Recoil*			*Target - Recoil*								
					x'	y'	z'	x'	x'	x'	y'	y'	y'	z'	z'	z'
		x	y	z				x	y	z	x	y	z	x	y	z
unpolarized	σ_0		T			P		$T_{x'}$		$L_{x'}$		Σ		$T_{z'}$		$L_{z'}$
linearly P_γ	Σ	H	P	G	$O_{x'}$	T	$O_{z'}$	$L_{z'}$	$C_{z'}$	$T_{z'}$	E		F	$L_{x'}$	$C_{x'}$	$T_{x'}$
circular P_γ		F		E	$C_{x'}$		$C_{z'}$		$O_{z'}$		G		H		$O_{x'}$	

Full set of 16

status	*CLAS run period*	*beam*	*target*	
complete	*g1*	γ , $\vec{\gamma}_c$	LH_2	*Miskimen/Schumacher*
complete	*g8*	$\vec{\gamma}_L$	LH_2	*Cole*
complete	*g9a - P^T_z*	$\vec{\gamma}_L$, $\vec{\gamma}_c$	*FROST* - $C_4\vec{H}_9O\vec{H}$	*Klein, Pasyuk*
2010	*g9b - P^T_x*	$\vec{\gamma}_L$, $\vec{\gamma}_c$	*FROST* - $C_4\vec{H}_9O\vec{H}$	*Klein, Pasyuk*

$\gamma + n(p) \rightarrow K^0 \Lambda(p)$ *measurements with* $\vec{H} \cdot \vec{D}$

coherent e-brem
$\vec{e}$ *-brem*

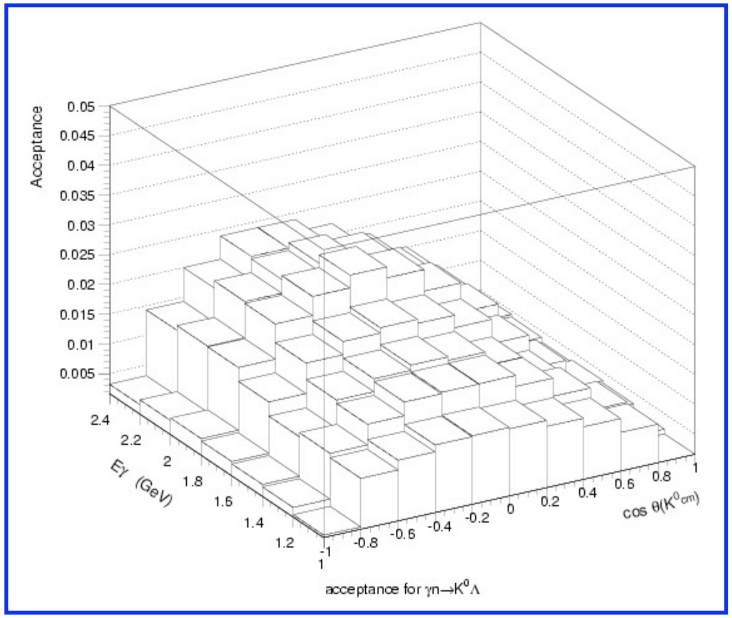
$\Downarrow$

$$\gamma + n(p) \rightarrow K^0 + \Lambda\ (p)$$

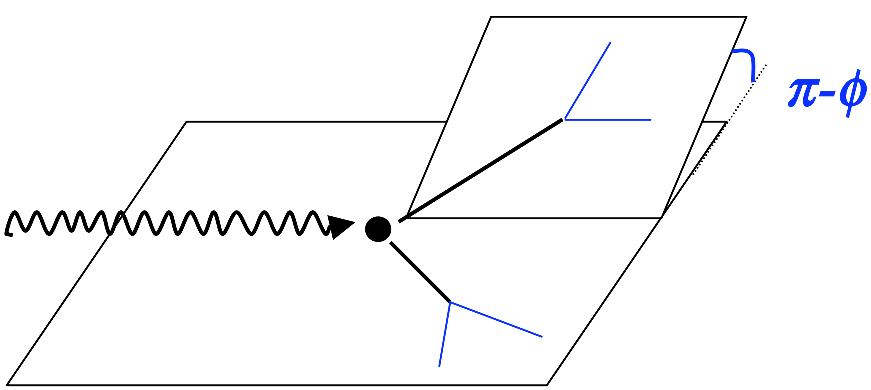
$$\rightarrow [\pi^+ \pi^-][p\ \pi^-](p)$$

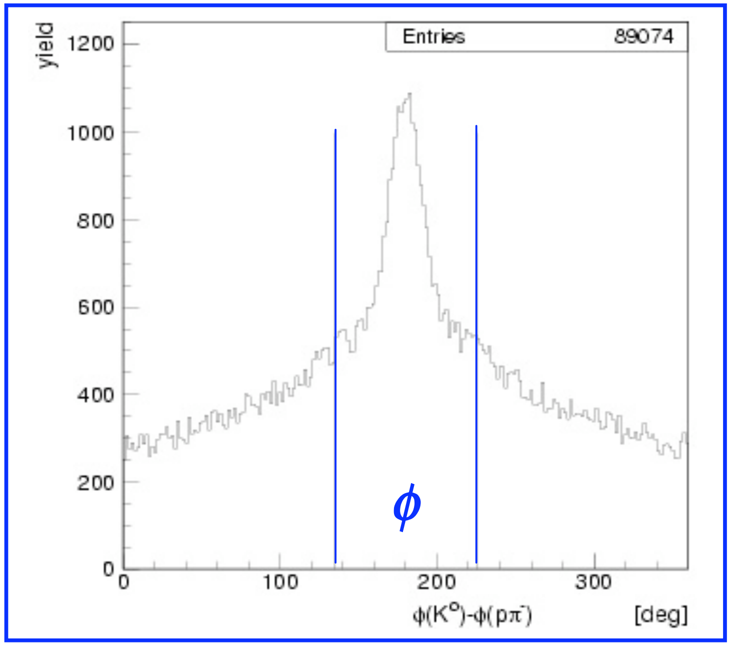
$\Uparrow$ *D·H*

$\Uparrow$ *detect 4 charged particles in CLAS*



- *coplanarity* $\Rightarrow$ *free neutron*





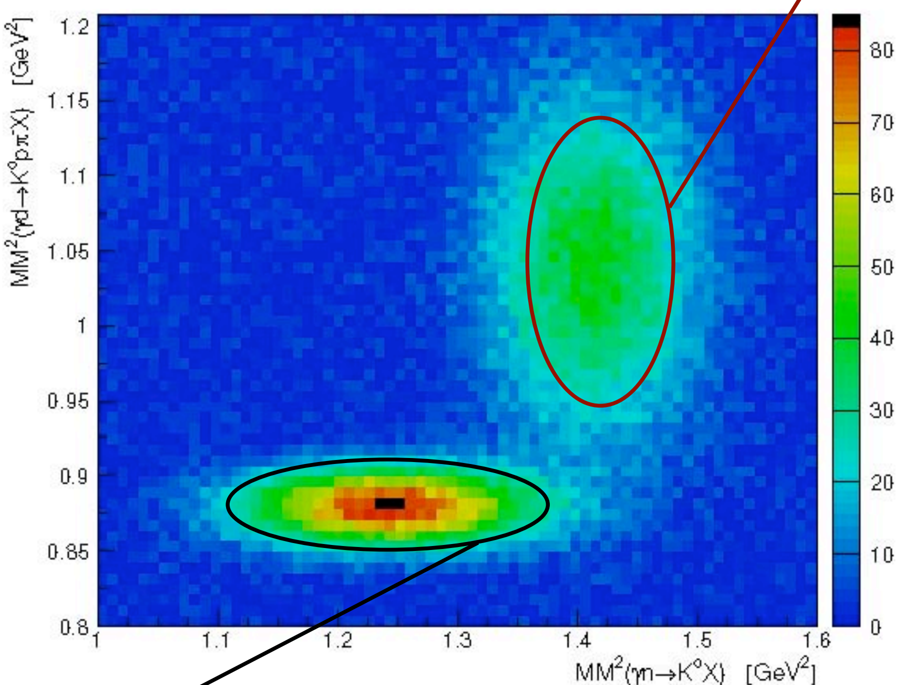
(Missing-Mass)^2 distributions:

After cuts:

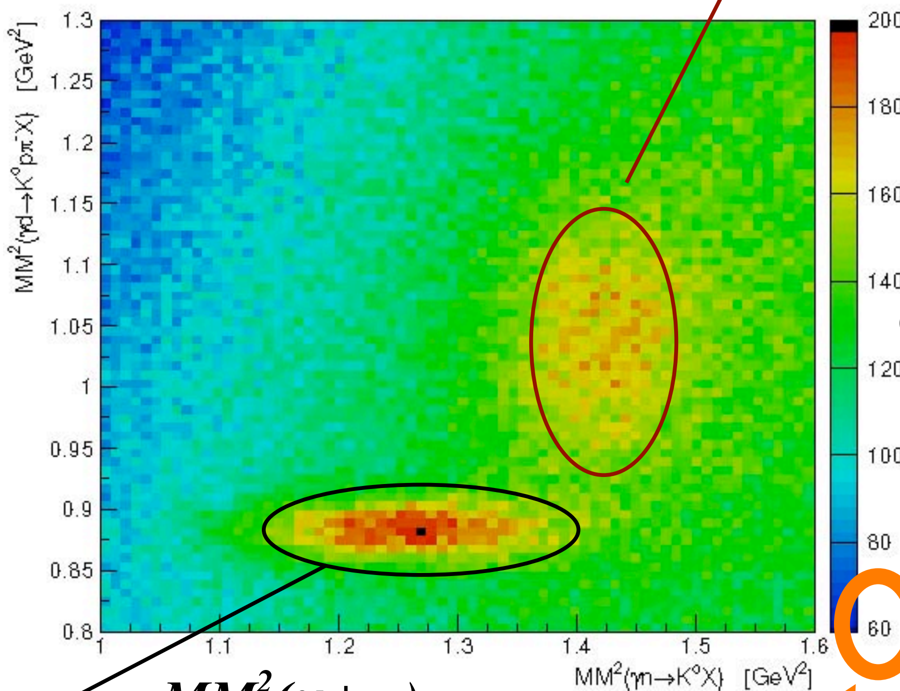
- *inv* mass($K^0 = \pi^+\pi^-$)
- $< K^0 - [p\pi^-] >$ *coplanarity*

$$\gamma n(p) \rightarrow K^0\Sigma^0(p) \rightarrow K^0 + \Lambda\gamma + (p)$$
$$\rightarrow [\pi^+\pi^-][p\pi^-](\gamma p)$$

$$\vec{\gamma} + H \cdot \vec{D}$$



$$\vec{\gamma} + C_4 \cdot \vec{D}_9 \cdot O \cdot \vec{D}$$



$$MM^2(\gamma + D)$$

$$MM^2(\gamma + n)$$

suppressed zero

$$\gamma n(p) \rightarrow K^0 + \Lambda + (p)$$
$$\rightarrow [\pi^+\pi^-][p\pi^-](p)$$

MM2(K0L) HD,FROST

$\gamma + n(p) \rightarrow K^0\Lambda$ *experiments at JLab with HDice:*

- *single-pol observables measured from double-pol asy*
- *double-pol observables measured from triple-pol asy*

Photon beam		Target			Recoil			Target - Recoil								
					x'	y'	z'	x'	x'	x'	y'	y'	y'	z'	z'	z'
		x	y	z				x	y	z	x	y	z	x	y	z
unpolarized	σ_0		T			P		$T_{x'}$		$L_{x'}$		Σ		$T_{z'}$		$L_{z'}$
linearly P_γ	Σ	H	P	G	$O_{x'}$	T	$O_{z'}$	$L_{z'}$	$C_{z'}$	$T_{z'}$	E		F	$L_{x'}$	$C_{x'}$	$T_{x'}$
circular P_γ		F		E	$C_{x'}$		$C_{z'}$		$O_{z'}$		G		H		$O_{x'}$	

⇑

simultaneous B-R with HD ⇔ ***Full set of 16***

schedule	CLAS run period	beam	target	spokesmen
2010-2011	g14	$\vec{\gamma}_L$, $\vec{\gamma}_c$	$H \cdot \vec{D}$ ice	Sandorfi/Klein

gp-K0L HDice tbl

Requirements for complete determination of the amplitude, free of ambiguities:

⇔ ***8 observables*** — *- Chiang & Tabakin, PRC* ***55****,2054(97)*

(I) cross section and the three single-polarization observables: {σ, Σ, P, T}

(II) four double-polarization asymmetries:

	Beam-Target	***Beam-Recoil***	***Target-Recoil***
• **2 + 2 cases:**			
	(E, G)		(L_x, T_x) or (L_z, T_z)
	(E, G)	(C_x, O_x) or (C_z, O_z)	
		(C_x, C_z) or (O_x, O_z)	(L_x, L_z) or (T_x, T_z)
• **2 + 1 + 1 cases:**			
	(E, G)	1 of $\{C_x, C_z, O_x, O_z\}$	1 of $\{L_x, L_z, T_x, T_z\}$
	E	(C_z, O_x)	1 of $\{L_x, T_z\}$
	G	(C_z, O_x)	1 of $\{L_z, T_x\}$
	1 of $\{E, G\}$	1 of $\{C_x, O_z\}$	(L_z, T_x)
	G	1 of $\{C_x, O_z\}$	(L_x, T_z)

⇒ ***huge redundancy in determining the*** $\gamma n \rightarrow K^0 \Lambda$ ***amplitude !***

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- *γ N $\rightarrow K\ \Lambda$ JLab "complete" experiments*
- ***Q: can such data provide total amplitudes, unique to a phase ?***
- *the mechanics of fitting out the amplitudes*
  - potentials and limitations
- *tests with mock data - work in progress $\Leftrightarrow$ with S. Hoblit, UVa*

The mechanics of inferring amplitudes from data

(I) *theory ⇒ experiment:* *multipoles ⇒ observables*

$$\frac{d\sigma}{d\Omega} = \frac{\left|P^{CM}_{\pi,\eta,K}\right|}{E^{cm}_{\gamma}} \cdot \left| \langle N(\pi,\eta,K)| \sum F_i \; |\gamma N\rangle \right|^2 ,$$ *CGLN, PR106(1957)*

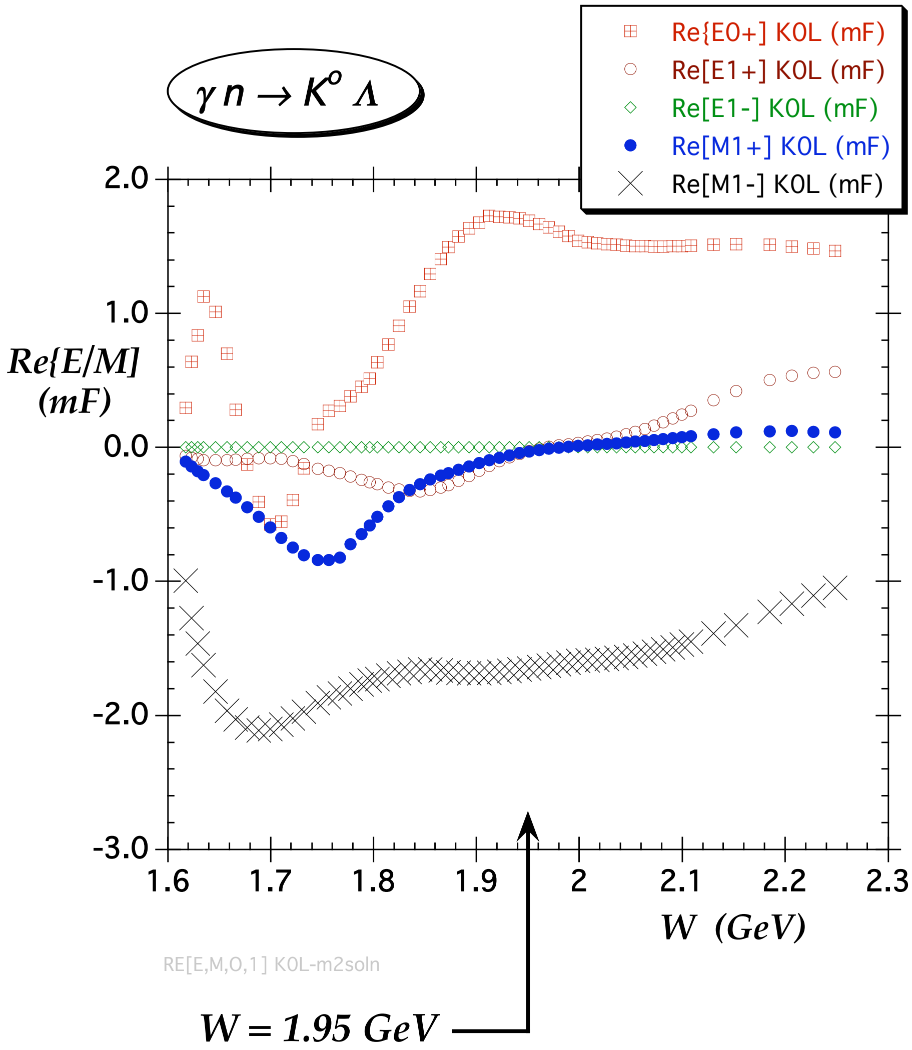
↑ *(Cartesian) ⇔ (Spherical) H_i helicity amplitudes*

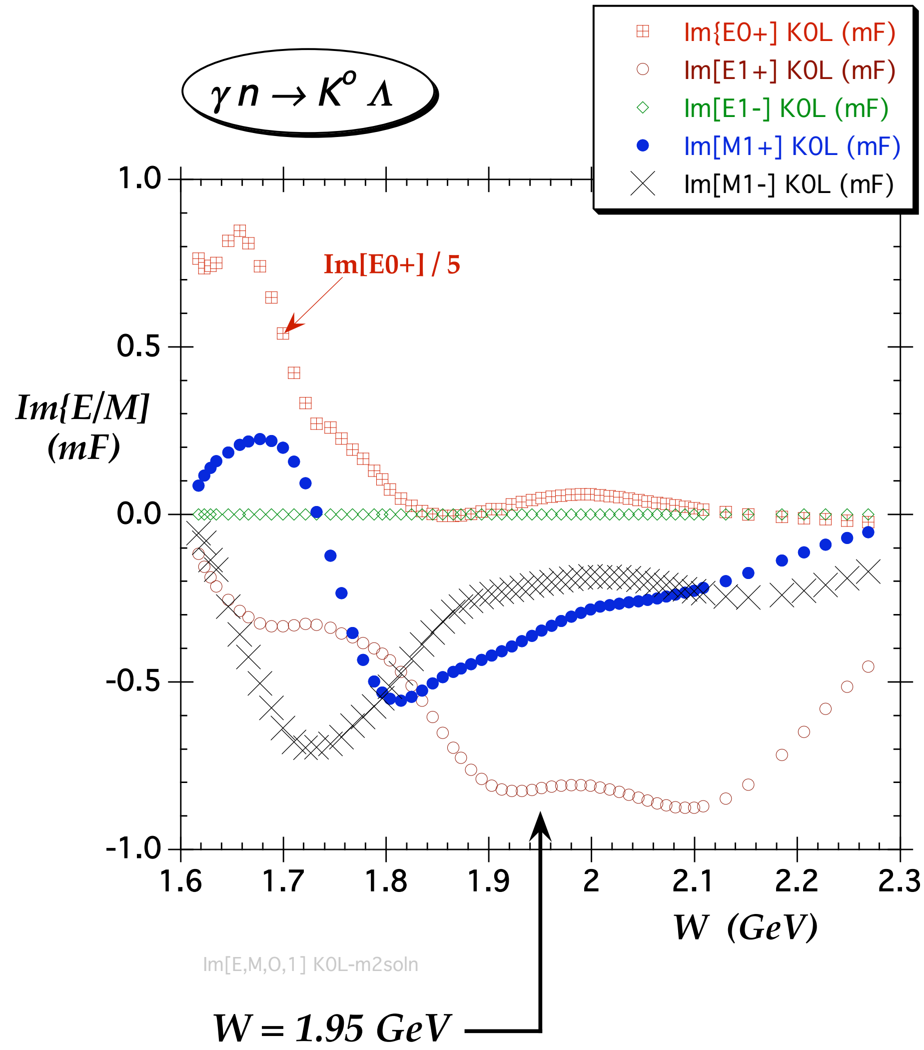
- $\{E_{\ell\pm}\ ,M_{\ell\pm}\}\otimes\{P_\ell\ ,P'_\ell\ ,P''_\ell\}\ \Leftrightarrow\ \{functions\ of\ W\,/\,E_\gamma\}\otimes\{functions\ of\ \theta_{CM}\}$

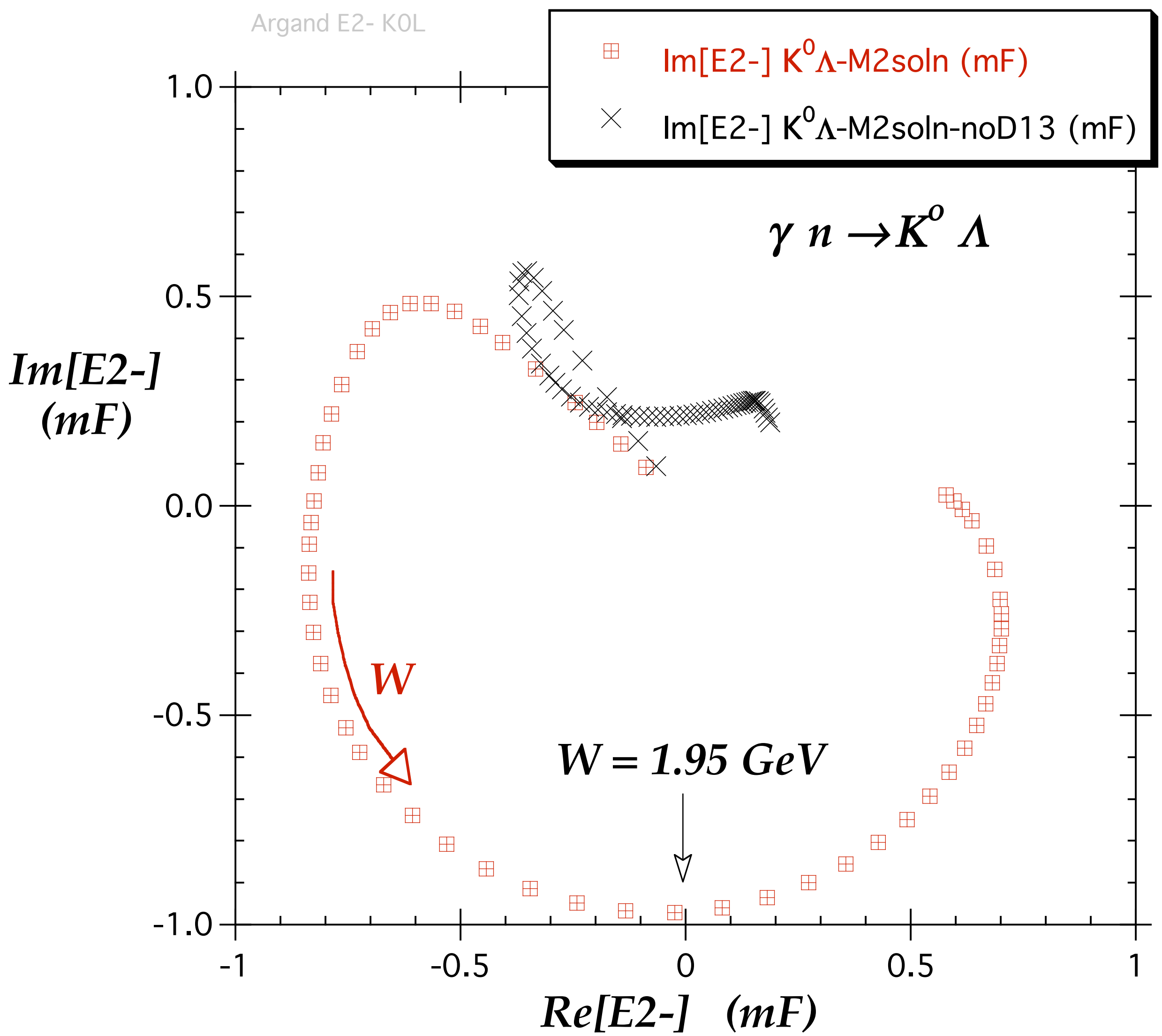
$\Rightarrow \{F_1\ ,F_2\ ,F_3\ ,F_4\}_{CGLN} \quad \Leftrightarrow\ \{functions\ of\ (W\,/\,E_\gamma\ ,\theta_{CM})\}$

$\Rightarrow \{\sigma_0\ ,\Sigma\ ,T\ ,P\ ,E\ ,\ldots L_{z'}\}\Leftrightarrow\ \{functions\ of\ (W\,/\,E_\gamma\ ,\theta_{CM}\ ,\phi)\}$

eg. ***multipoles from: B. Julia-Diaz, T-S. H. Lee***
- M2 (full) solution -







Argand E2- K0L
Im[E2-] K$^0\Lambda$-M2soln (mF)
Im[E2-] K$^0\Lambda$-M2soln-noD13 (mF)
$\gamma\ n \rightarrow K^o\ \Lambda$
Im[E2-] (mF)
W
W = 1.95 GeV
1.0
0.5
0.0
-0.5
-1.0
-1
-0.5
0
0.5
1
Re[E2-] (mF)

Constructing the CGLN F_i amplitudes:

$$F_1(x) = E_{0+} + \left(M_{1+} + E_{1+} \right) \cdot P_2'(x) + \left(2M_{2+} + E_{2+}\right) \cdot P_3'(x) + \left(3M_{2-} + E_{2-}\right) + \left(3M_{3+} + E_{3+}\right) \cdot P_4'(x) + \left(4M_{3-} + E_{3-}\right) \cdot P_2'(x) + \left(4M_{4+} + E_{4+}\right) \cdot P_5'(x) + \left(5M_{4-} + E_{4-}\right) \cdot P_3'(x)$$

$$F_3(x) = + \left(E_{1+} - M_{1+} \right) \cdot P_2''(x) + \left(E_{2+} - M_{2+}\right) \cdot P_3''(x) + \left(E_{3+} - M_{3+}\right) \cdot P_4''(x) + \left(E_{3-} + M_{3-}\right) \cdot P_2''(x) + \left(E_{4+} - M_{4+}\right) \cdot P_5''(x) + \left(E_{4-} + M_{4-}\right) \cdot P_3''(x)$$

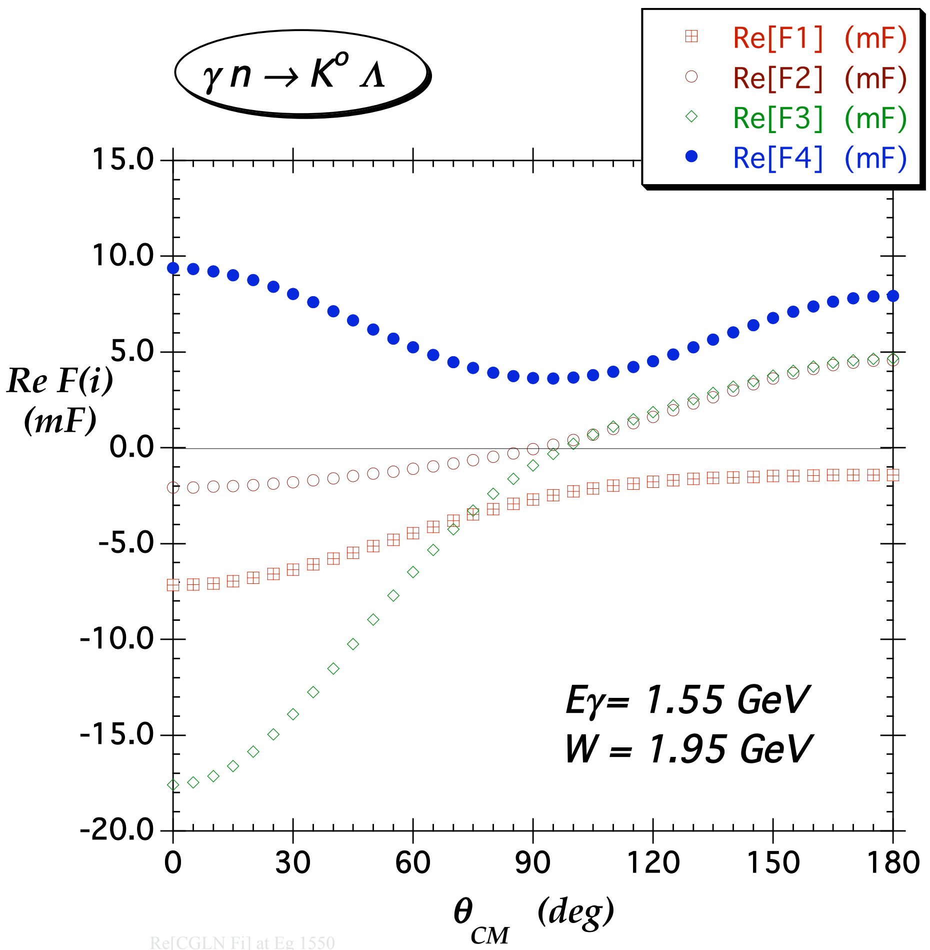
$$F_2(x) = \left(2M_{1+} + M_{1-}\right) + \left(3M_{2+} + 2M_{2-}\right) \cdot P_2'(x) + \left(4M_{3+} + 3M_{3-}\right) \cdot P_3'(x) + \left(5M_{4+} + 4M_{4-}\right) \cdot P_4'(x)$$

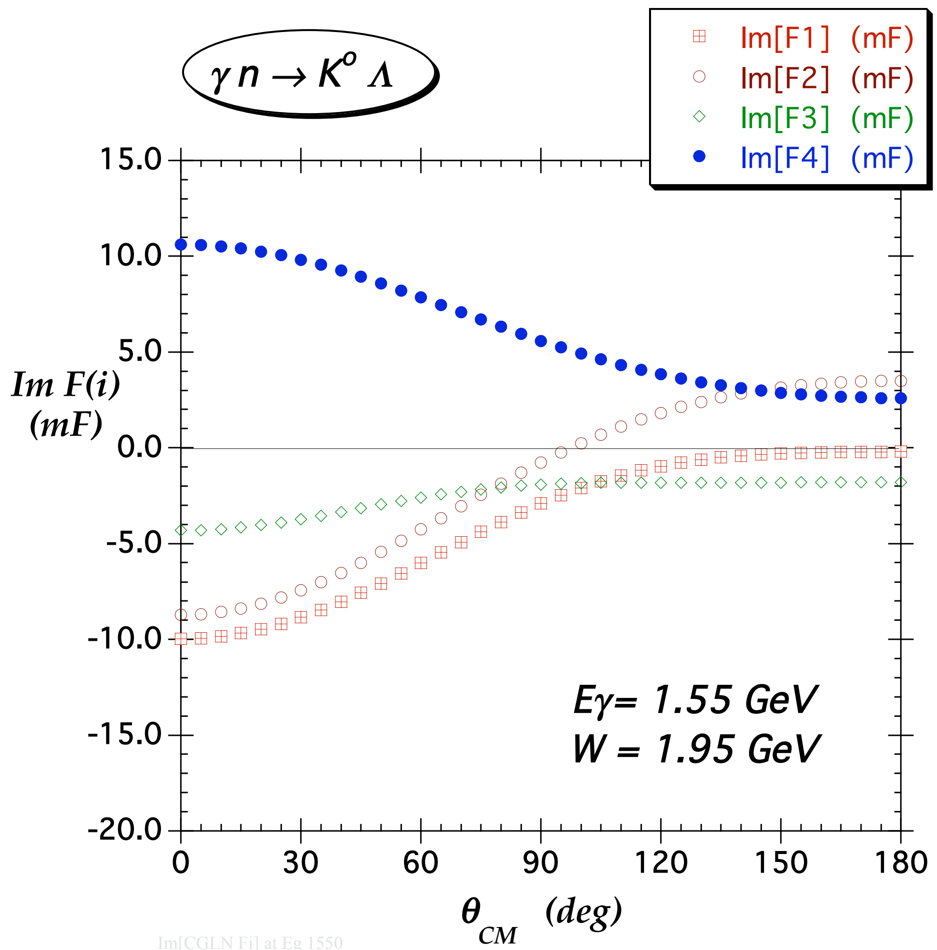
$$F_4(x) = \left(M_{2+} - E_{2+} - E_{2-} - M_{2-} \right) \cdot P_2''(x) + \left(M_{3+} - E_{3+} - E_{3-} - M_{3-} \right) \cdot P_3''(x) + \left(M_{4+} - E_{4+} - E_{4-} - M_{4-} \right) \cdot P_4''(x)$$

P_ℓ , P_ℓ' , P_ℓ'' $\equiv$ ***Legendre functions***

$$x = cos(\theta_{CM})$$

eg. ***CGLN amplitudes at W = 1.95 GeV***
- calculated from the multipoles of B. Julia-Diaz, T-S. H. Lee

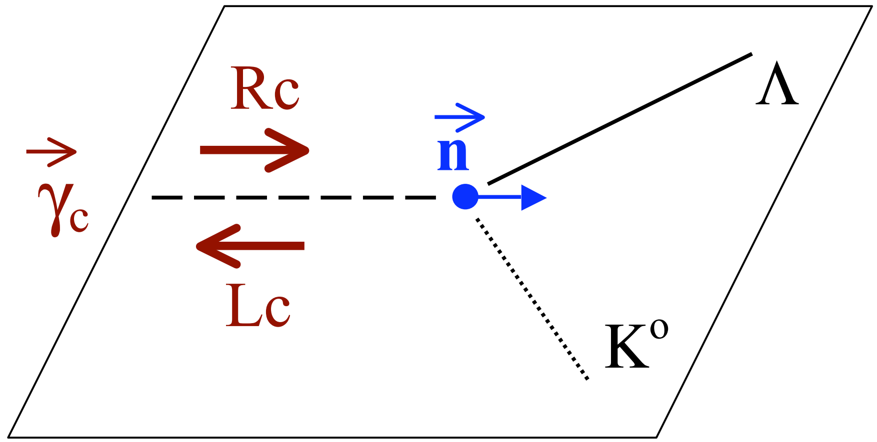




notation: $\hat{A}_{asy} = \sigma_o \cdot A_{asy}$

eg:

$$\left.\begin{aligned} E &= \frac{\sigma_{Anti} - \sigma_{Par}}{\sigma_{Anti} + \sigma_{Par}} \\ \sigma_o &= \tfrac{1}{2}\left(\sigma_{Anti} + \sigma_{Par}\right) \\ \Rightarrow \hat{E} &= \tfrac{1}{2}\left(\sigma_{Anti} - \sigma_{Par}\right) \end{aligned}\right\}$$



$$\{F_1\ ,\ F_2\ ,\ F_3\ ,\ F_4\}_{CGLN} \Leftrightarrow \{\textit{functions of}\ (W / E_\gamma\ ,\ \theta_{CM})\}$$

$$\Rightarrow \{\sigma_0\ ,\ \hat{\Sigma}\ ,\ \hat{T}\ ,\ \hat{P}\ ,\ \hat{E}\ ,\ \ldots \hat{L}_{z'}\} \Leftrightarrow \{\textit{functions of}\ (W / E_\gamma\ ,\ \theta_{CM}\ ,\ \phi)\}$$

Observables in terms of the CGLN F_i amplitudes:

$$\rho = \left|P_{\pi,\eta,K}^{CM}\right| / E_\gamma^{cm}$$

$$\hat{T} = \Im\mathrm{m}\left\{\sin\theta\left[F_1^*F_3 - F_2^*F_4 + \cos\theta\cdot\left(F_1^*F_4 - F_2^*F_3\right) - \sin^2\theta\cdot F_3^*F_4\right]\right\}\cdot\rho$$

$$\hat{P} = \Im\mathrm{m}\left\{\sin\theta\left[-2F_1^*F_2 - F_1^*F_3 + F_2^*F_4 + \cos\theta\cdot\left(F_2^*F_3 - F_1^*F_4\right) + \sin^2\theta\cdot F_3^*F_4\right]\right\}\cdot\rho$$

$$\hat{T}_{x'} = \Re e\left\{\sin^2\theta\left[-F_1^*F_3 - F_2^*F_4 - F_3^*F_4 - \tfrac{1}{2}\cos\theta\cdot\left(\left|F_3\right|^2 + \left|F_4\right|^2\right)\right]\right\}\cdot\rho$$

$$\hat{L}_{x'} = -\Re e\left\{\sin\theta\left[\left|F_1\right|^2 - \left|F_2\right|^2 + \tfrac{1}{2}\sin^2\theta\cdot\left(\left|F_4\right|^2 - \left|F_3\right|^2\right) - F_2^*F_3 + F_1^*F_4 + \cos\theta\left(F_1^*F_3 - F_2^*F_4\right)\right]\right\}\cdot\rho$$

$$\hat{T}_{z'} = \Re e\left\{\sin\theta\left[-F_2^*F_3 + F_1^*F_4 + \cos\theta\left(F_1^*F_3 - F_2^*F_4\right) + \tfrac{1}{2}\sin^2\theta\cdot\left(\left|F_4\right|^2 - \left|F_3\right|^2\right)\right]\right\}\cdot\rho$$

$$\hat{L}_{z'} = \Re e\left\{\begin{array}{l}2F_1^*F_2 - \cos\theta\left(\left|F_1\right|^2 + \left|F_2\right|^2\right) + \sin^2\theta\cdot\left(F_1^*F_3 + F_2^*F_4 + F_3^*F_4\right)\\ +\tfrac{1}{2}\cos\theta\sin^2\theta\cdot\left(\left|F_3\right|^2 + \left|F_4\right|^2\right)\end{array}\right\}\cdot\rho$$

$$\hat{\Sigma} = -\sin^2\theta\cdot\left[\tfrac{1}{2}\left\{\left|F_3\right|^2 + \left|F_4\right|^2\right\} + \Re e\left\{F_2^*F_3 + F_1^*F_4 + \cos\theta\cdot\left(F_3^*F_4\right)\right\}\right]\cdot\rho$$

$$\hat{G} = +\sin^2\theta\cdot\Im m\left\{F_2^*F_3 + F_1^*F_4\right\}\cdot\rho$$

$$\hat{O}_{x'} = \sin\theta\cdot\Im\mathrm{m}\left[F_2^*F_3 - F_1^*F_4 + \cos\theta\cdot\left(F_2^*F_4 - F_1^*F_3\right)\right]\cdot\rho$$

$$\hat{O}_{z'} = -\sin^2\theta\cdot\Im\mathrm{m}\left[F_1^*F_3 + F_2^*F_4\right]\cdot\rho$$

$$\hat{E} = \left[\left|F_1\right|^2 + \left|F_2\right|^2 + \Re e\left\{\sin^2\theta\cdot\left(F_2^*F_3 + F_1^*F_4\right) - 2\cos\theta\cdot\left(F_1^*F_2\right)\right\}\right]\cdot\rho$$

$$\hat{C}_{x'} = -\sin\theta\cdot\Re e\left\{-\left|F_1\right|^2 + \left|F_2\right|^2 + F_2^*F_3 - F_1^*F_4 + \cos\theta\cdot\left(F_2^*F_4 - F_1^*F_3\right)\right\}\cdot\rho$$

$$\hat{C}_{z'} = -\Re e\left\{-2F_1^*F_2 + \cos\theta\left(\left|F_1\right|^2 + \left|F_2\right|^2\right) - \sin^2\theta\cdot\left(F_1^*F_3 + F_2^*F_4\right)\right\}\cdot\rho$$

$$\sigma_0 = \left\{\begin{array}{l}\left|F_1\right|^2 + \left|F_2\right|^2 + \tfrac{1}{2}\sin^2\theta\cdot\left(\left|F_3\right|^2 + \left|F_4\right|^2\right)\\ +\Re e\left[\sin^2\theta\cdot\left(F_2^*F_3 + F_1^*F_4 + \cos\theta\cdot F_3^*F_4\right) - 2\cos\theta\cdot F_1^*F_2\right]\end{array}\right\}\cdot\rho$$

$\gamma n \rightarrow K^0 \Lambda$

$E_\gamma = 1.55$ GeV
$W = 1.95$ GeV

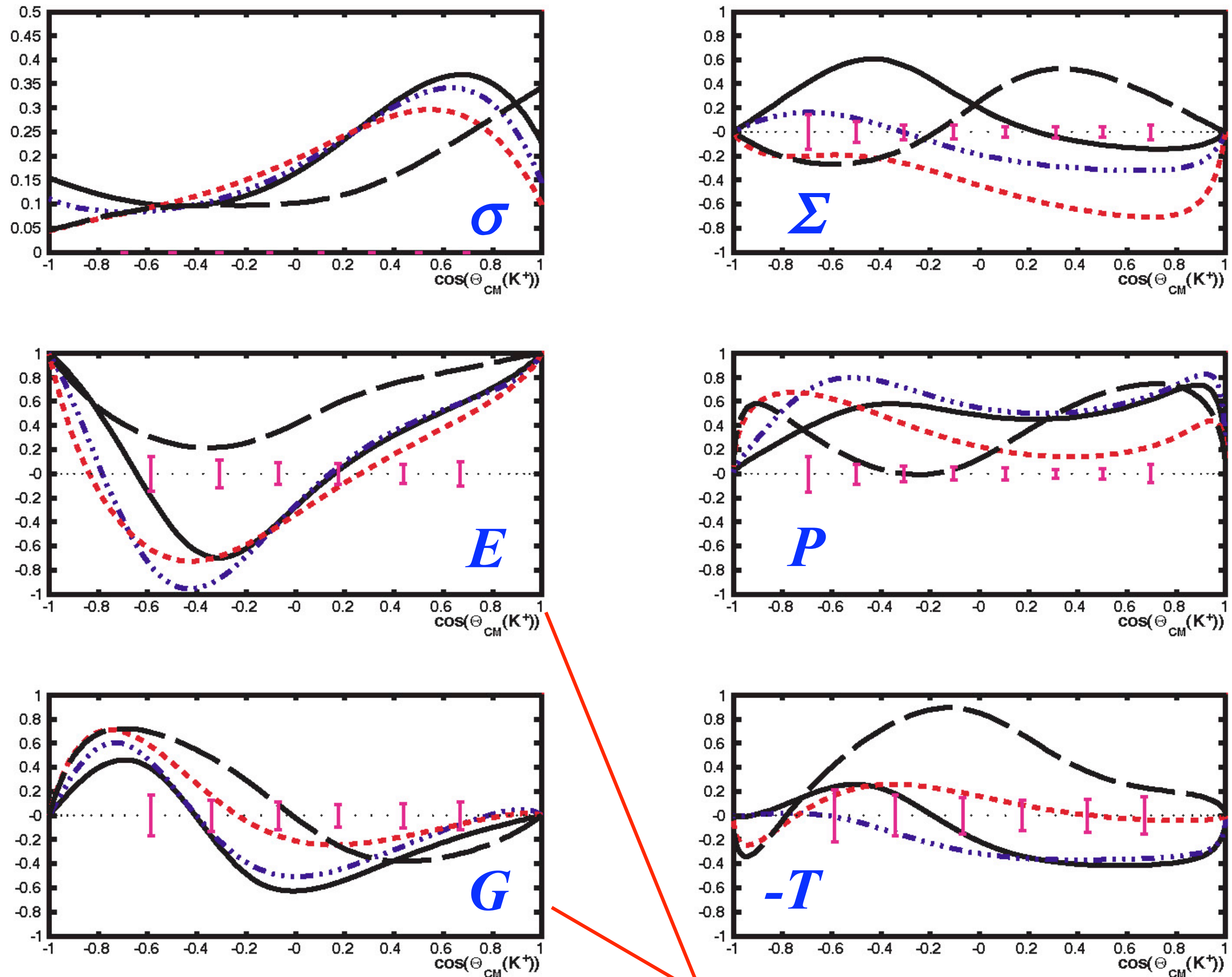
expected data

M2 solution

no π-cloud

no coupled-ch

no $D_{13}(1910)$



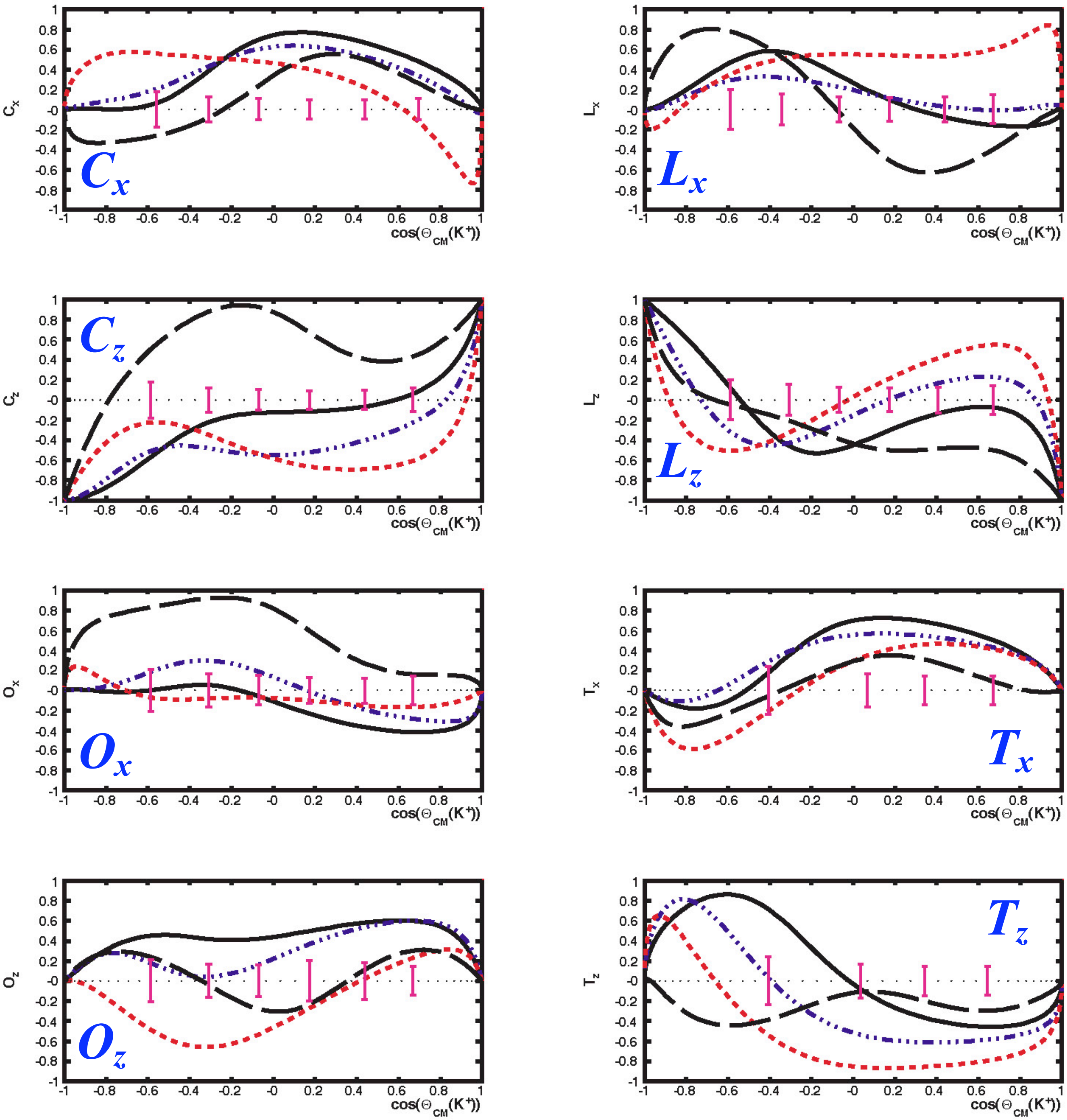
Beam-Target

-B. Juliá-Díaz, T-S. H. Lee (preliminary)

EBAC gn->K0L [‘S’,BT]W1.95

$\gamma n \to K^0 \Lambda$ $E_\gamma = 1.55$ GeV (W = 1.95 GeV)

Beam-Recoil **Target-Recoil**



— M2 solution

– – – no D_{13}(1910)

······ no π-cloud

– · · – no coupled-ch

-B. Juliá-Díaz, T-S. H. Lee (preliminary)

EBAC gn->K0L [BR,TR]W1.95

Mechanics (II): experiment $\Rightarrow$ multipoles

(a) <u>reverse the theory→observable path</u>: (& jump the F_i)

measurements for sets of $(E_\gamma\ ,\ \theta_{CM}\ ,\ \phi)$ *with different* $(P^{\gamma}_{c,\,L}\ ,\ P^{T}_{x,y,z}\ ,\ P^{\Lambda}_{x',y',z'})$

$\Downarrow$

- $\left\{\sigma_0\ ,\ \hat{\Sigma}\ ,\ \hat{T}\ ,\ \hat{P}\ ,\ \hat{E}\ ,\ \ldots\ \hat{L}_{z'}\right\} \quad \Leftrightarrow \quad \left\{\textit{tabluation vs } (W\ /\ E_\gamma\ ,\ \theta_{CM})\right\}$

- $\left.\begin{array}{c}\left\{\sigma_0(F_i)\ ,\ \hat{\Sigma}(F_i)\ ,\ \hat{T}(F_i)\ ,\ \ldots\ \hat{L}_{z'}(F_i)\right\} \\ \left\{\begin{array}{l} F_1(E_{\ell\pm}, M_{\ell\pm})\ ,\ F_2(E_{\ell\pm}, M_{\ell\pm}) \\ F_3(E_{\ell\pm}, M_{\ell\pm})\ ,\ F_4(E_{\ell\pm}, M_{\ell\pm}) \end{array}\right\}\end{array}\right\rangle \Leftrightarrow$ *fit* $\left\{\sigma_0\ ,\ \hat{\Sigma}\ ,\ \hat{T}\ ,\ \hat{P}\ ,\ \hat{E}\ ,\ \ldots\ \hat{L}_{z'}\right\}$ *directly, varying* $E_{\ell\pm}, M_{\ell\pm}$

$\Rightarrow$ *introduces an* ℓ_{max} *dependence*

<u>note</u>: $\hat{A}($ *bilinear combinations of* $E_{\ell\pm}, M_{\ell\pm}) \Rightarrow$ ***derivatives are <u>linear</u>***

Fitting observables to multipoles - tests of uniqueness:

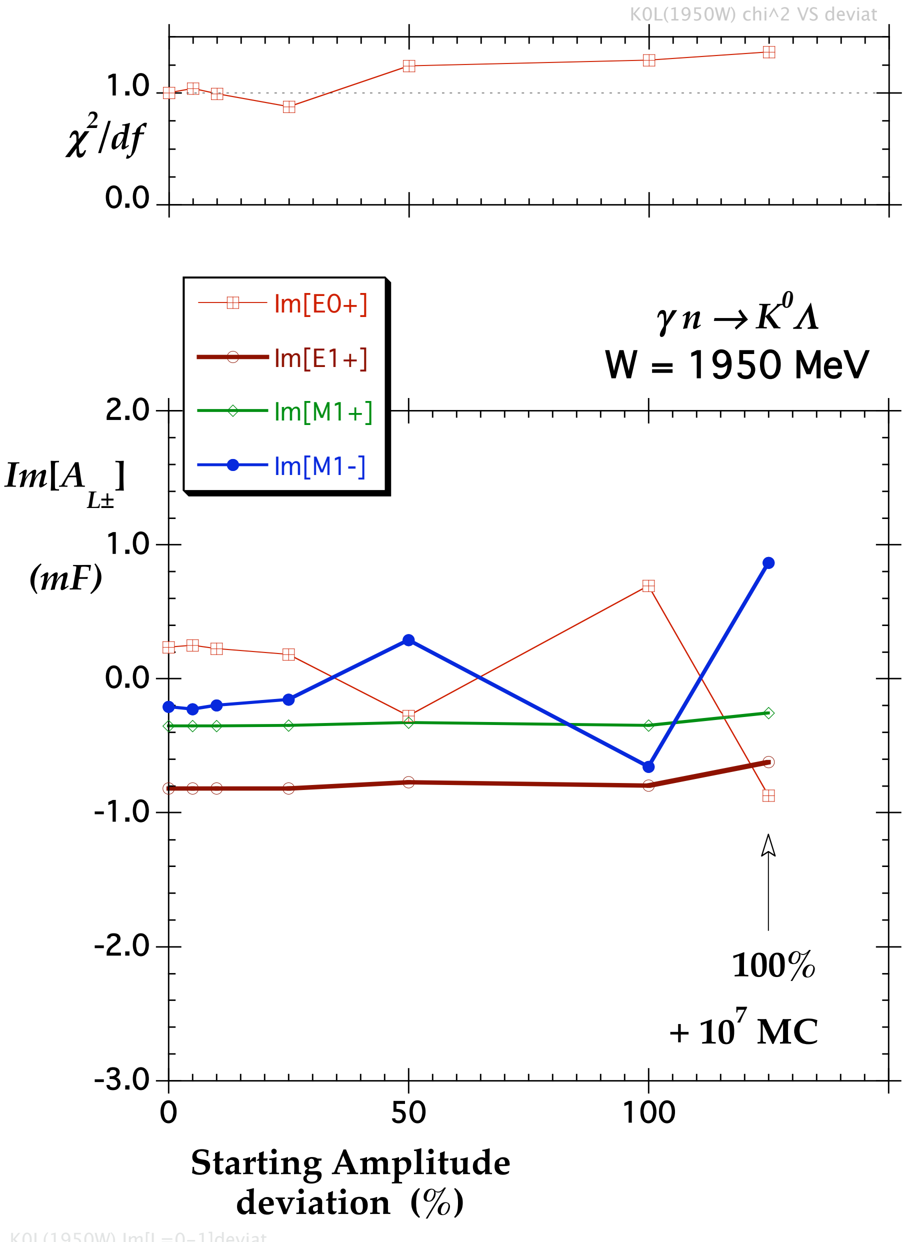
(1) create *mock data*:

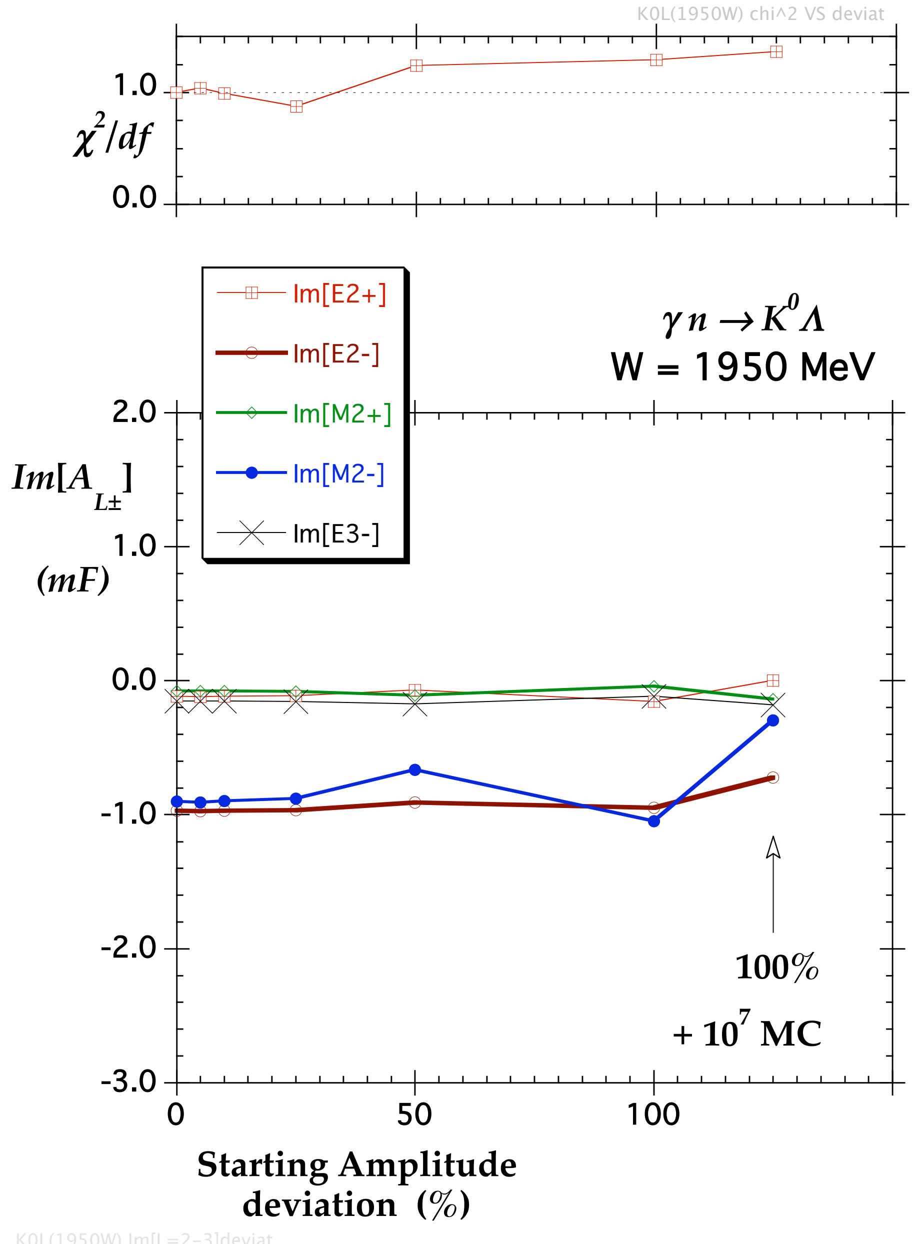
- predict observables from Juliá-Días/Lee/Sato "M2" amplitudes in 10 deg steps from 10^0 to 170^0 CM
- assign each an error of 0.1% and Gaussian smear the predictions

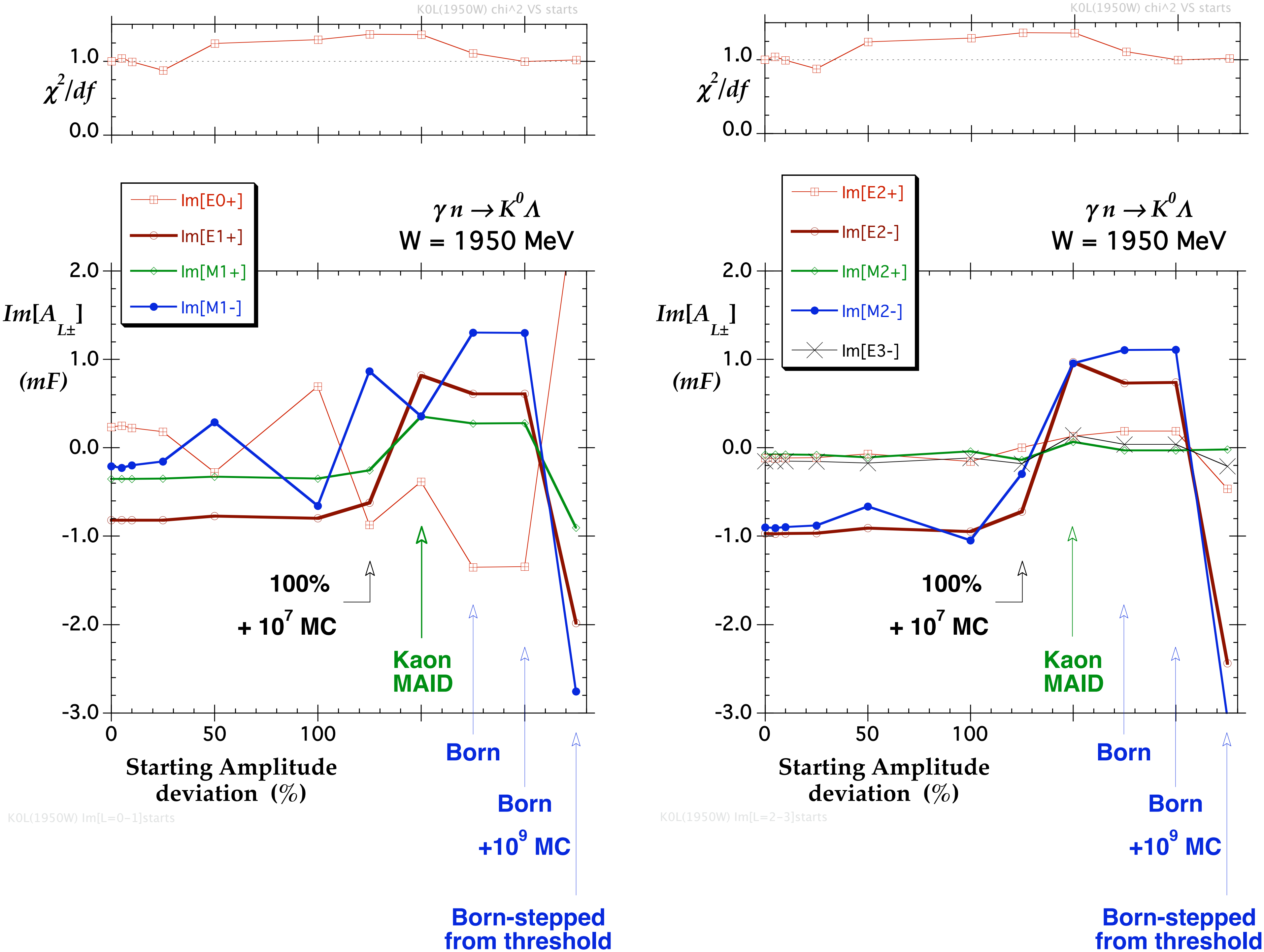
  (Note: no data set will ever be that good !)

(2) fit the *mock data,* varying multipoles, with different starting pts

- as a starting point for a search, use M2 values randomly smeared by 0, 5%, 10%, 25%, 50%, 100%, …
- start search with Kaon-MAID, Born, …
- repeat with more realistic data errors (~5 %)


K0L(1950W) chi^2 VS deviat
1.0
χ^2/df
0.0
Im[E0+]
Im[E1+]
Im[M1+]
Im[M1-]
$\gamma n \to K^0 \Lambda$
W = 1950 MeV
2.0
$Im[A_{L\pm}]$
1.0
(mF)
0.0
-1.0
-2.0
-3.0
100%
+ 10^7 MC
0
50
100
Starting Amplitude
deviation (%)
K0L(1950W) Im[L=0-1]deviat

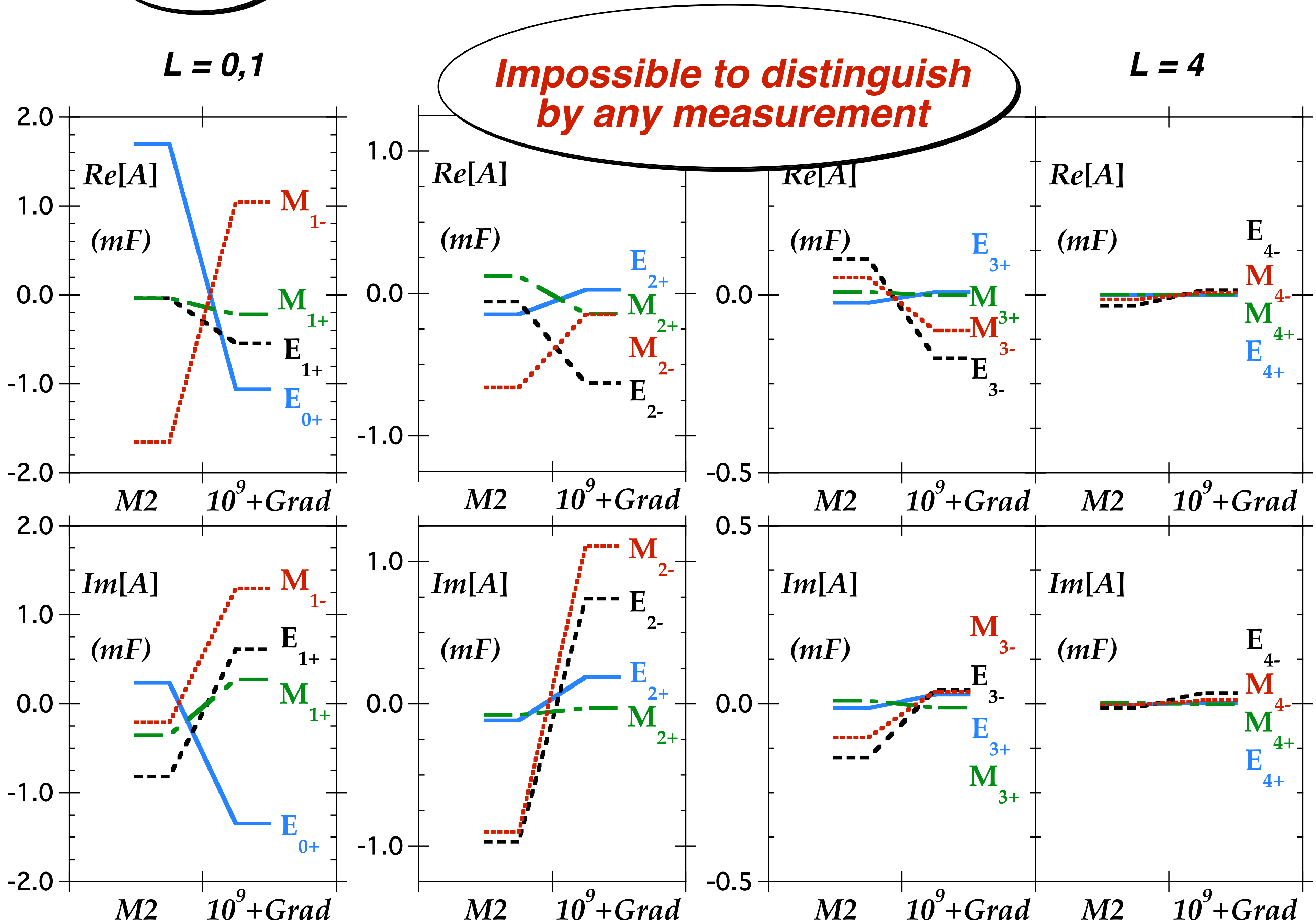

K0L(1950W) chi^2 VS deviat
1.0
χ^2/df
0.0
Im[E2+]
Im[E2-]
Im[M2+]
Im[M2-]
Im[E3-]
$\gamma n \to K^0 \Lambda$
W = 1950 MeV
2.0
$Im[A_{L\pm}]$
1.0
(mF)
0.0
-1.0
-2.0
-3.0
100%
+ 10^7 MC
0
50
100
Starting Amplitude
deviation (%)
K0L(1950W) Im[L=2-3]deviat



K0L(1950W) chi^2 VS starts
1.0
χ²/df
0.0
Im[E0+]
Im[E1+]
Im[M1+]
Im[M1-]
γ n → K⁰Λ
W = 1950 MeV
2.0
Im[A_L±]
1.0
(mF)
0.0
-1.0
-2.0
-3.0
100%
+ 10⁷ MC
Kaon
MAID
0
50
100
Starting Amplitude
deviation (%)
Born
Born
+10⁹ MC
Born-stepped
from threshold
K0L(1950W) Im[L=0-1]starts
K0L(1950W) chi^2 VS starts
Im[E2+]
Im[E2-]
Im[M2+]
Im[M2-]
Im[E3-]
K0L(1950W) Im[L=2-3]starts

$\vec{n}(\vec{\gamma}, K^o\vec{\Lambda})$

EBAC M2 vs Born + 10^9 MC + gradient search



Mechanics (II): *experiment* $\Rightarrow$ *multipoles*

(b) <u>Stepwise path</u>: $\Leftrightarrow$ ***fit first the*** F_i

measurements for sets of $(E_\gamma\ ,\ \theta_{CM}\ ,\ \phi)$ *with different* $(P^{\gamma}_{c,L}\ ,\ P^{T}_{x,y,z}\ ,\ P^{\Lambda}_{x',y',z'})$

$\Downarrow$

- $\left\{\sigma_0\ ,\ \hat{\Sigma}\ ,\ \hat{T}\ ,\ \hat{P}\ ,\ \hat{E}\ ,\ \ldots\ \hat{L}_{z'}\right\}$ $\Leftrightarrow$ $\left\{\textit{tabluation vs}\ (W\ /\ E_\gamma\ ,\ \theta_{CM})\right\}$

- *fit* $\left\{\sigma_0(F_i)\ ,\ \hat{\Sigma}(F_i)\ ,\ \hat{T}(F_i)\ ,\ \ldots\ \hat{L}_{z'}(F_i)\right\}$ $\Rightarrow$ $\left\{F_1\ ,\ F_2\ ,\ F_3\ ,\ F_4\right\}_{CGLN}$ *at each* θ

<u>note</u>: $\hat{A}$(***bilinear combinations of*** F_i) $\Rightarrow$ ***derivatives*** $\partial A / \partial F_i$ ***are <u>linear</u>***

$\Rightarrow$ ***fits well behaved; rapid convergence***

$\Downarrow$

CGLN F_i $\left\{\textit{functions of}\ (W\ /\ E_\gamma\ ,\ \theta_{CM})\right\}$

- *with $\{F_1, F_2, F_3, F_4\}$, vs W and $x = \cos(\theta)$ in hand,*

  project out any desired multipole:

$$E_{\ell\pm} = \frac{1}{2\binom{\ell+1}{\ell+0}} \int_{-1}^{+1} dx \left\{ \begin{array}{l} P_\ell F_1 \quad - P_{\ell\pm1} F_2 \\ + \frac{\binom{\ell+0}{\ell+1}}{(2\ell+1)} \left[P_{\ell\mp1} - P_{\ell\pm1} \right] F_3 \quad + \frac{\binom{\ell+1}{\ell+0}}{\binom{2\ell+3}{2\ell-1}} \left[P_\ell - P_{\ell\pm2} \right] F_4 \end{array} \right\}$$

$$M_{\ell\pm} = \frac{1}{2\binom{\ell+1}{\ell+0}} \int_{-1}^{+1} dx \left\{ \pm P_\ell F_1 \quad \mp P_{\ell\pm1} F_2 \quad + \frac{1}{(2\ell+1)} \left[P_{\ell\pm1} - P_{\ell\mp1} \right] F_3 \right\}$$

potential

- ***independent of ambiguities associated with truncation of multipole expansions***
- ***smaller dimensional space for χ^2 search***

limitation

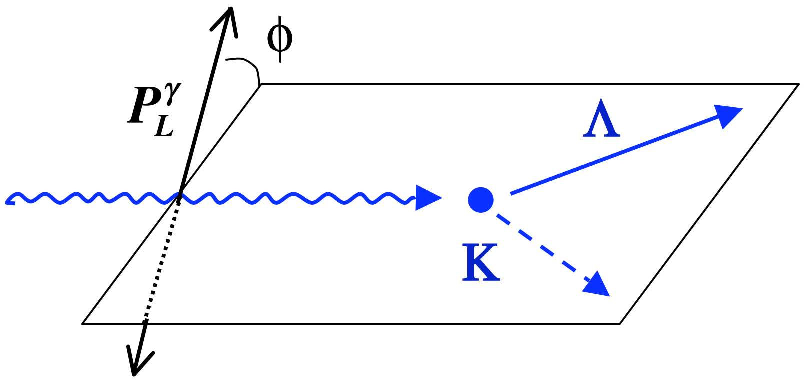
- ***F_i cannot be determined near extreme angles $(0,\pi)$***

  $\Rightarrow$ some limitations on the accuracy of the integrals

Projection of multipoles from CGLN F_i amplitudes requires integration over all θ

- but symmetries send all $\hat{A}$ observables $\rightarrow$ 0 or σ at extreme angles

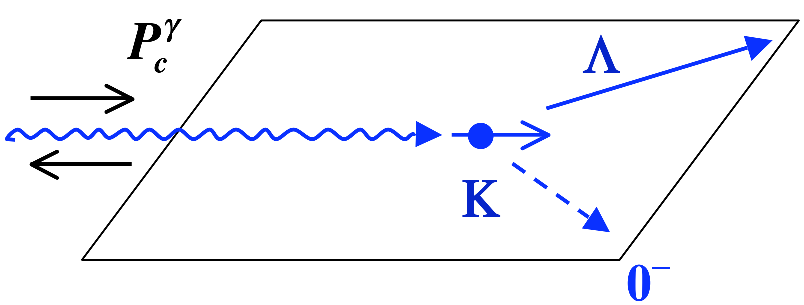
eg. 1 $$\hat{\Sigma} = -\sin^2\theta \cdot \left[\tfrac{1}{2}\left\{ |F_3|^2 + |F_4|^2 \right\} + \Re e\left\{ F_2^* F_3 + F_1^* F_4 + \cos\theta \cdot \left(F_3^* F_4 \right) \right\} \right] \cdot \rho$$



$\rightarrow$ *0 as $\theta \rightarrow 0^o$ or 180^o*

$\Rightarrow$ **1 eqn in 8 unknowns**

eg. 2 $$\hat{E} = \left[|F_1|^2 + |F_2|^2 + \Re e\left\{ \sin^2\theta \cdot \left(F_2^* F_3 + F_1^* F_4 \right) - 2\cos\theta \cdot \left(F_1^* F_2 \right) \right\} \right] \cdot \rho$$



$$\rightarrow \left\{ \begin{matrix} |F_1|^2 + |F_2|^2 \\ \mp \Re e\left[2 \cdot F_1^* F_2 \right] \end{matrix} \right\} \cdot \rho = \sigma_0(0,180)$$

Partial Summary - of a work in progress

- complete sets of spin-observables will soon be available for γ+p $\rightarrow$ $K^{+}\Lambda$
  (and large partial sets for many other channels)

- complete sets of spin-observables for γ+n $\rightarrow$ $K^{0}\Lambda$ scheduled for 2010-2011
  (and large partial sets for many other channels)

  - using kinematic cuts on γ+D $\Rightarrow$ γ+n observables,
  - checked by comparing γ+p(n)/γ+p and iterating with theory

- prospects for directly fitting out the full (Res+Bgk+cc+…) multipoles:

  path (a)
  - 16 observables X N angles fitted to 32 parameters (L=0-4) at each W
  - χ^2 space has a large number of valleys (even when the Chiang-Tabakin requirements are met)
  - valleys are not that narrow $\Leftrightarrow$ spin data will force moderate variations in the amplitude to return to the "true" solution, but a quasi-reasonable approximation is needed to start the search.

- potential search strategy: start at threshold where L=0 (at most 1) and use solutions at E(i) to start the search at E(i+1), …
  - difficult for KY channels; S_{11} at threshold ⇒ starting multipoles are very different from Born
  - may be able to start with a reasonable model at threshold, …

The challenge: how to guide fits up the physically meaningful $\chi^2(W)$ valley and still keep the exercise relatively model-independent !

- Alternate strategy - fit out the CGLN F_i amplitudes:

- 16 observables (with 6 inter-relations) to determine 4 complex F_i
- must be done at each angle
- conventional multipoles can be projected out of these F_i with integrals over angle, but 0 and 180^0 will not be constrained

- stay tuned !