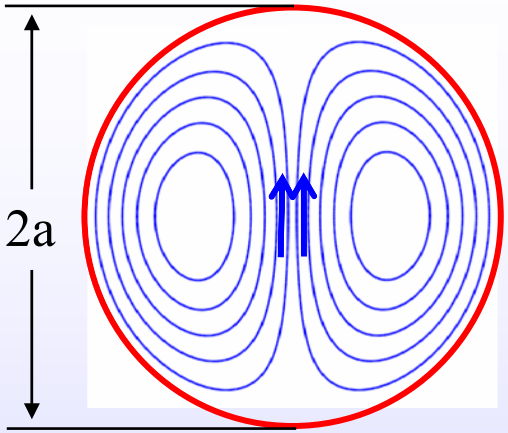
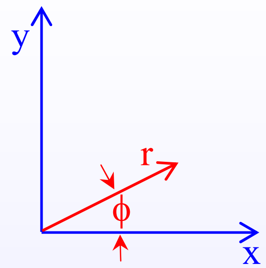
Optics to Minimize BBU

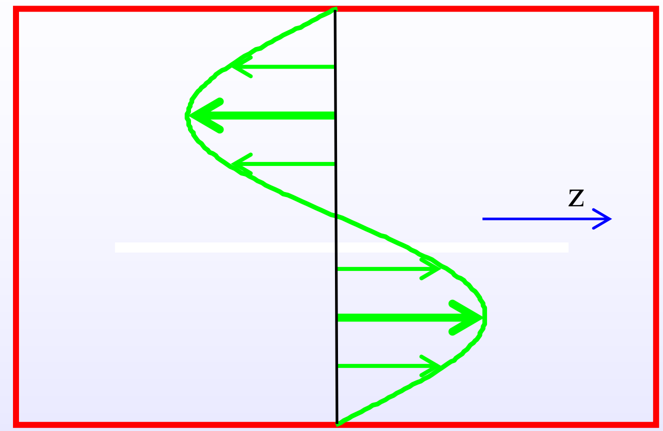
Todd I. Smith
(Todd.Smith@Stanford.edu)
W.W Hansen Experimental Physics Laboratories
Stanford, CA 94305-4085

R.E.Rand and T.I. Smith, *Beam Optical Control of Beam Breakup in a Recirculating Electron Accelerator*, *Particle Accelerators*, **11** (1980), 1-13.

TM_{011} Fields







$$B_r = -B_0 \frac{J_1(kr)}{kr}\cos(\phi)\sin(\omega t) \qquad E_z = B_0 c J_1(kr)\sin(\phi)\cos(\omega t)$$

$$B_\phi = B_0 J_1^{'}(kr)\sin(\phi)\sin(\omega t) \qquad k = \frac{\omega}{c} = \frac{2\pi}{\lambda} = \frac{3.833}{a}$$

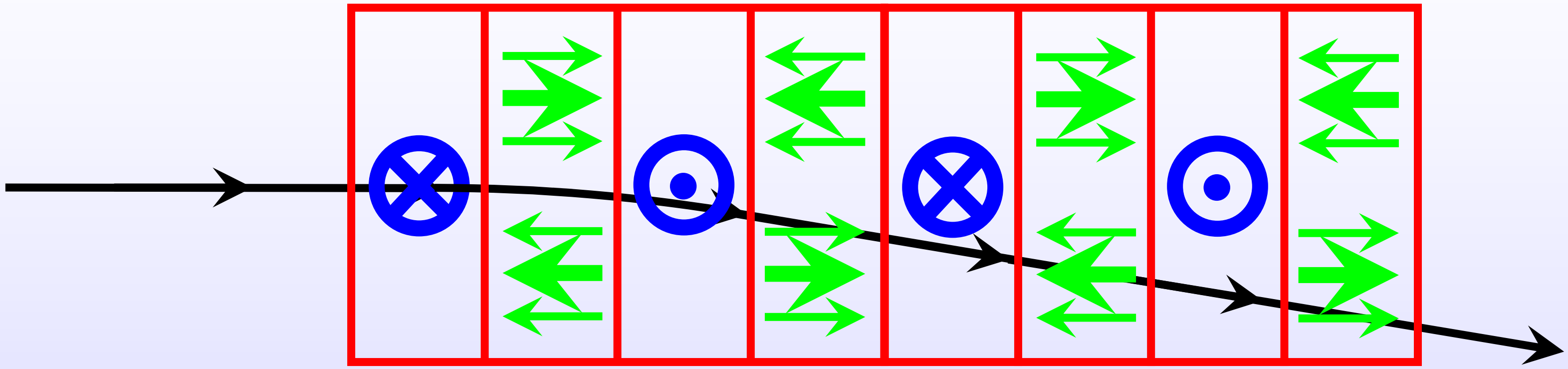
Near the axis the fields can be approximated as

$$B_y = (B_0/2)\sin(\omega t) \qquad E_z = (B_0/2)\,\omega x \cos(\omega t)$$

$$\omega_{dipole} \sim 1.5\ \omega_{accelerator}$$

Remember, there is another TM_{011} mode rotated 90 degrees to this one. Assume that Δf is several bandwidths.

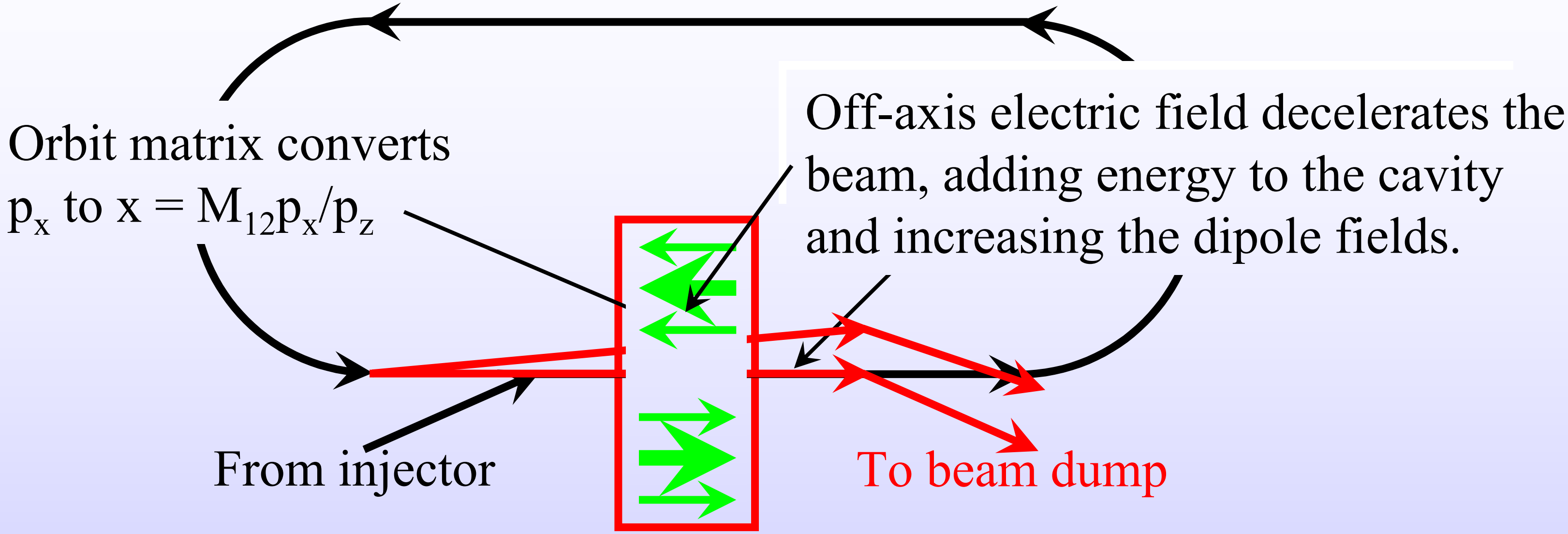
Single Pass Regenerative BBU



$$I_{threshold} \approx \frac{(\pi/2L)^2 \lambda_d V_{beam}}{(r_d/Q)Q_L} \qquad r_d = \frac{1}{k^2}\frac{(dE_z/dr)^2}{(P/L)_{cavity}} \approx \frac{100\Omega}{\lambda_{dipole}} Q$$

If L = 1 m, E_{beam}= 10 MeV, $Q_L = 10^5$, and $f_{dipole} \sim 1.5\ f_{acc} = 1.5$ GHz then $I_{threshold} \sim 100$ ma.

Multipass Regenerative BBU



$$I_{threshold} \approx \frac{-\lambda_d V_{beam}}{\pi (R_d/Q) Q_L M_{12} \sin(\omega_d T_r)} \qquad R_d = r_d L_{cavity}$$

recirculation time

With the same cavity as before, and assuming M_{12} = 1 meter, the BBU threshold is now only 10 ma, a factor of 10 lower.

A simple model of regenerative BBU in a two pass machine

Assumptions:

- Single dipole HOM
- Cavity-beam interaction is impulsive
- Mode splitting is large enough that only one polarization is relevant (for the moment, let the magnetic field be vertical)
- Mode is excited on resonance
- Electron beam is CW
- HOM fields change slowly w.r.t. recirculation time
- Transverse coordinates (x and y, x' and y') are not coupled
- ???

On the first pass through the HOM the electron beam (energy γ) gets a transverse deflection due to the magnetic field:

$$x'(t) \propto \frac{B_0(t)L_c}{\gamma}\sin(\omega t) \propto \frac{B_0(t)\lambda}{\gamma}\sin(\omega t)$$

When the beam returns to the cavity on the second pass after a time T_r, the deflection has become a displacement given by:

$$x(t) \propto M_{12}x'(t+T_r) \propto M_{12}\frac{B_0(t+T_r)\lambda}{\gamma}\sin(\omega(t+T_r))$$

The off-axis beam can now interact with the cavity electric field and exchange energy. The power delivered to the cavity is:

$$P_b(t) = I_b E_c(x) L_c \cos(\omega t) \propto I_b \frac{x(t)}{\lambda}\lambda B_0(t)\cos(\omega t)$$

$$P_b \propto I_b M_{12}\frac{B_0\lambda}{\gamma}\langle \sin(\omega(t+T_r))B_0\cos(\omega t)\rangle = I_b M_{12}\frac{B_0^2\lambda}{\gamma}\frac{\sin(\omega T_r)}{2}$$

But, $B_0^2 \propto \dfrac{U}{\lambda^3}$, therefore $$P_b \propto I_b M_{12}\frac{U}{\gamma\lambda^2}\sin(\omega T_r)$$

Define the proportionality constant to be 2πK. Then we have

$$P_b = 2\pi K I_b M_{12} \frac{\sin(\omega T_r)}{\gamma\lambda^2} U$$

By the definition of Q, the cavity loses energy at the rate

$$P_c = \frac{\omega U}{Q} = \frac{2\pi}{\lambda}\frac{U}{Q}$$

The total rate of energy change in the cavity is then given by

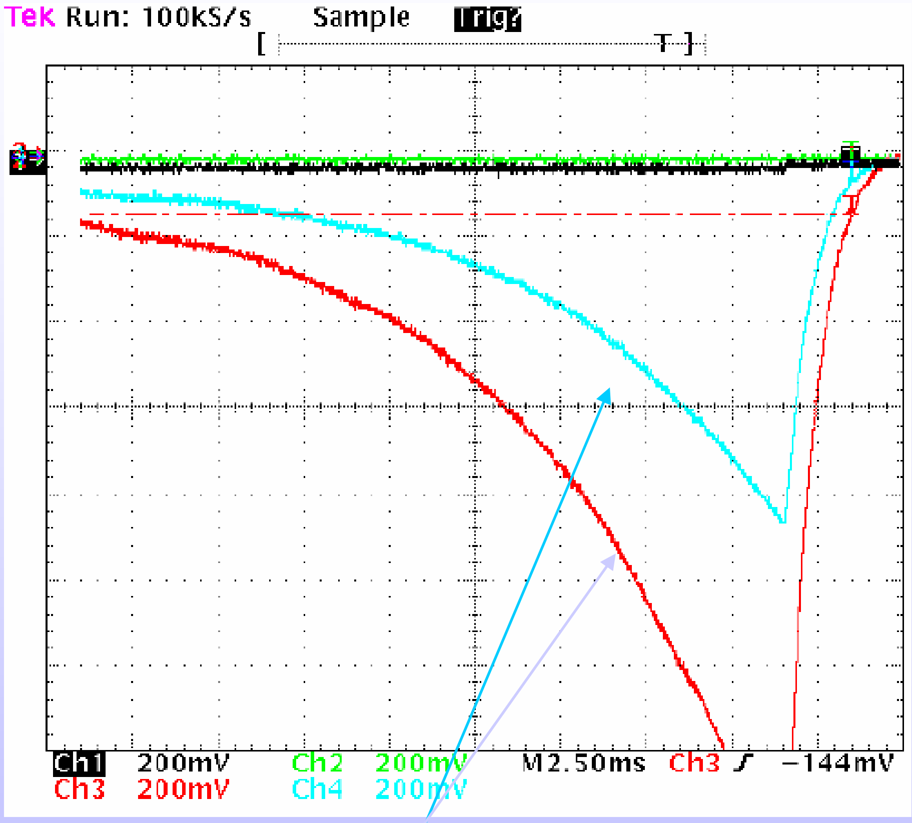
$$\frac{dU}{dt} = P_b - P_c = \omega\left(K I_b M_{12} \frac{\sin(\omega T_r)}{\gamma\lambda} - \frac{1}{Q}\right) U$$

The time dependence of the cavity energy is

$$U\left(t\right) = U_0 e^{-\frac{\omega t}{Q}\left(1 - K I_b M_{12} Q \frac{\sin(\omega T_r)}{\gamma\lambda}\right)}$$

If $I_{th} \equiv \dfrac{\gamma\lambda}{K I_b M_{12} Q \sin(\omega T_r)}$, then $U\left(t\right) = U_0 e^{-\frac{\omega t}{Q}\left(1 - \frac{I_b}{I_{th}}\right)}$

First Observation of BBU in the 10kW FEL
(May 27,2004)



The blue and red traces show the exponential growth of the HOM power from the two HOM coupler ports in cavity 4 due to BBU. (The average current was ~ 3 mA and the energy 88 MeV)

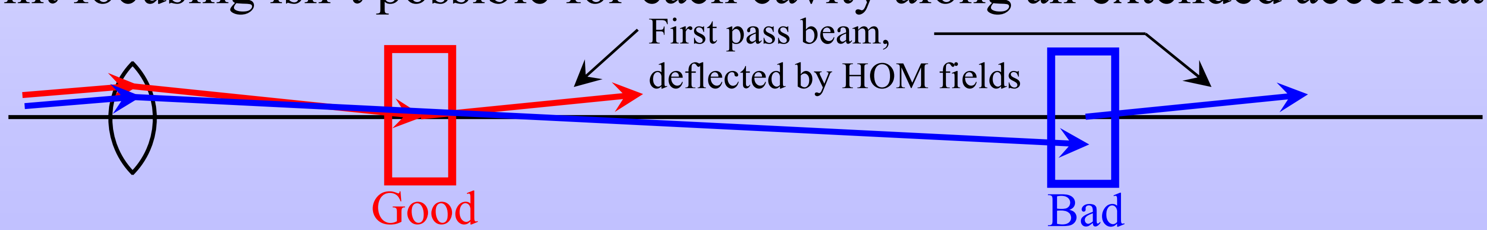
Conventional Methods of Controlling Multipass Regenerative BBU

$$I_{threshold} \approx \frac{-\lambda_d V_{beam}}{\pi (R_d / Q) Q_L M_{12} \sin(\omega_d T_r)} \qquad R_d = r_d L_{cavity}$$

recirculation time

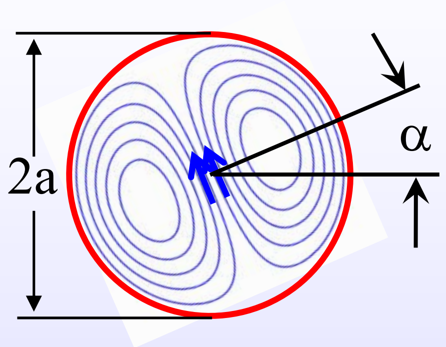
The only parameters that are readily adjustable are Q_L, ω_d, and M_{12}.

- Reducing Q_L enough for all modes is difficult, particularly given the existence of trapped modes in multi-cell cavities.
- Adjusting the frequency of each possible BBU mode while maintaining a fixed accelerating frequency isn't realistic.
- Point to point focusing from a cavity back to itself would set M_{12} to zero and can be effective when dealing with a single bad cavity. Unfortunately, point to point focusing isn't possible for each cavity along an extended accelerator.



- Active mode damping might be effective if only a few modes are a problem.

BBU in a TM_{011} like mode polarized at an angle α with respect to the vertical (or horizontal) axis



M_{12} in the earlier formula must be replaced by M^*. M_{12} came from $x'_1 x_2 = M_{12}\, x'^2_1$. Now the dot product of r'_1 and r_2 must be used.

$$\vec{r}_1' \cdot \vec{r}_2 = r_1'\left(\vec{i}\cos\alpha + \vec{j}\sin\alpha\right) \cdot r_1'\left(\vec{i}\left(M_{12}\cos\alpha + M_{14}\sin\alpha\right) + \vec{j}\left(M_{32}\cos\alpha + M_{34}\sin\alpha\right)\right)$$

Thus, $M^* \equiv M_{12}\cos^2\alpha + (M_{14} + M_{32})\sin\alpha\cos\alpha + M_{34}\sin^2\alpha$, and

$$I_{threshold} \approx \frac{\lambda_d V_{beam}}{\pi (R_d/Q) Q_L M^* \sin(\omega_d T_r)}$$

Three interesting cases:

- No x-y coupling ($M_{14} = 0 = M_{34}$)
- Reflection about 45° (x→y, y→x)
- Rotation by 90° (x→y, y→-x)

Beam Optical Suppression Techniques

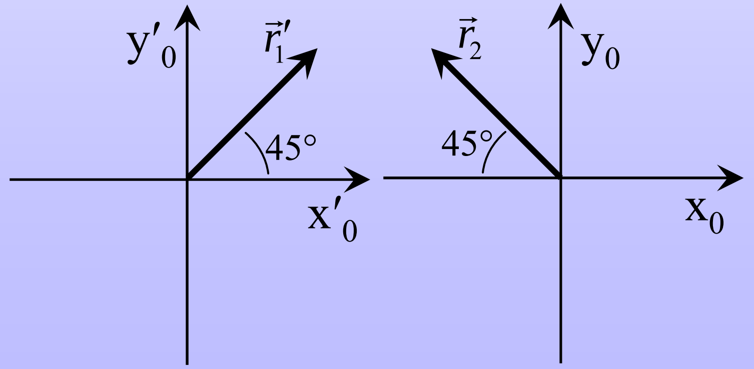
(no x-y coupling)

$$I_{threshold} \propto \frac{1}{M_{12}\cos^2\alpha + M_{34}\sin^2\alpha}$$

If $\mathbf{M_x}$ and $\mathbf{M_y}$ are 2 x 2 matrices, the transport matrix is of the form $M = \begin{vmatrix} M_x & 0 \\ 0 & M_y \end{vmatrix}$ $M_{14} = 0 = M_{32}$ and the BBU situation appears similar to before.

However, the physics of setting M_{12} or M_{34} to zero is quite different from setting $M_{12}\cos^2(\alpha) + M_{34}\sin^2(\alpha) = 0$. In the first case, the transport matrix is required to prevent an HOM deflection from resulting in any displacement. In the second , much more flexible case, the deflection is orthogonal to the displacement so the displacement magnitude is irrelevant.

Generally speaking, M_{12} and M_{34} being of opposite sign constitutes a reflection about the x or y axis. Notice in particular that if $\alpha = 45°$, then $M_{12} = -M_{34}$, and BBU is suppressed for both polarizations of an HOM–as long as the frequency degeneracy is adequately lifted.



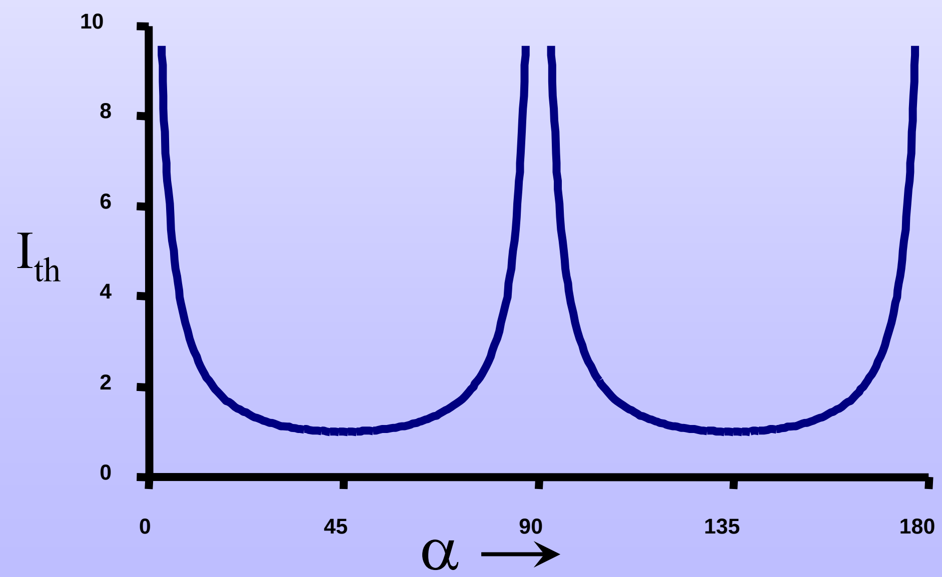
Beam Optical Suppression Techniques
Reflection about 45° (x→y, y→x)

$$I_{threshold} \propto -\frac{1}{(M_{14}+M_{32})\sin\alpha\cos\alpha}$$

In a reflection about 45°, the transport matrix takes the form $M_{12} = 0 = M_{34}$ by definition, and now BBU is suppressed completely if the HOM's are oriented at $\alpha = 0°$ or $90°$.

$$M = \begin{vmatrix} 0 & M_x \\ M_y & 0 \end{vmatrix}$$

However, if α is different from 0° or 90°, the effectiveness of reflecting optics in the suppression of BBU diminishes rapidly.



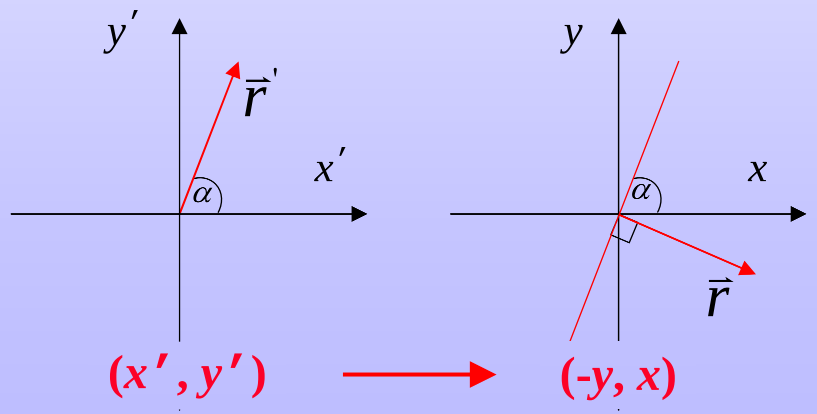
Beam Optical Suppression Techniques

Rotation by 90° (x→y, y→-x)

$$I_{threshold} \propto \frac{1}{(M_{14}+M_{32})\sin\alpha\cos\alpha}$$

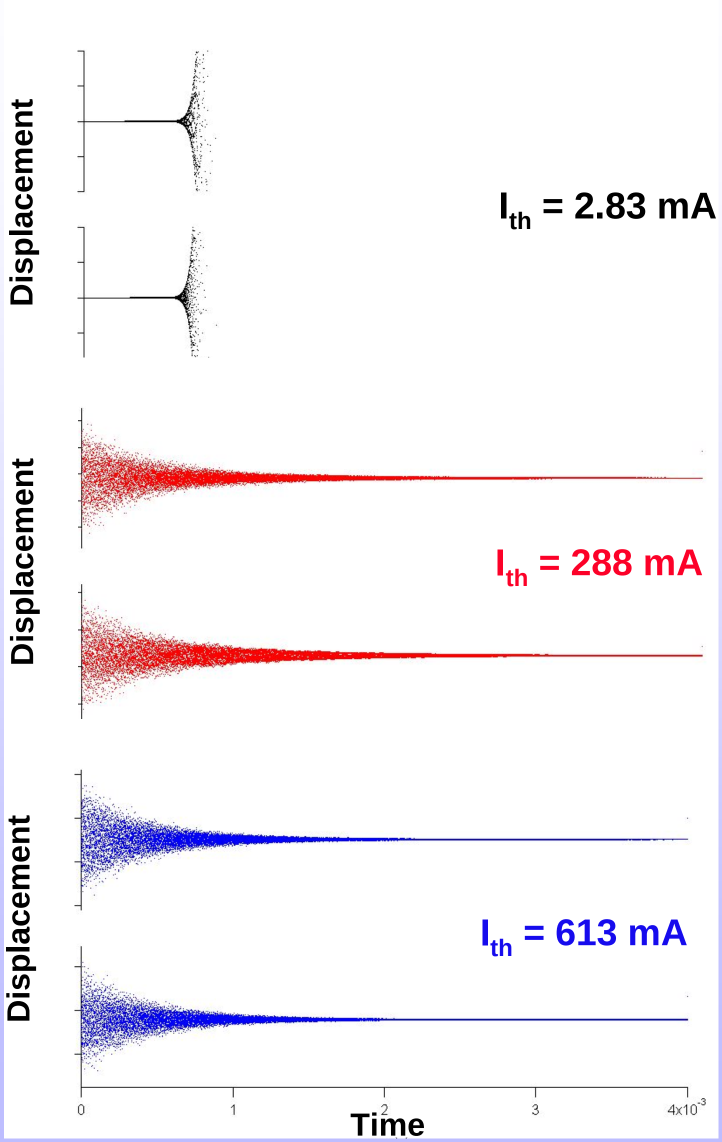
In a rotation by 90°, the transport matrix takes the form $M_{12} = 0 = M_{34}$ as in the reflection about 45°. But the requirement is added that $M_{14} = -M_{32}$ is added, and now BBU is suppressed for all HOM orientations.

$$M = \begin{vmatrix} 0 & M_x \\ M_y & 0 \end{vmatrix}$$



Furthermore, if $\mathbf{M_x} = \mathbf{M_y}$, and if transport along the linac axis is cylindrically symmetric (i.e. no quadrupoles), the suppression holds for all cavities.

Simulation Results: "*Aligned Modes*"



All simulation results are for 100 mA of average beam current. The top (bottom) plot shows the vertical (horizontal) beam offset versus time. The simulation run-time was 4ms.

Nominal Optics

With no suppression techniques applied, the threshold current is just under 3 mA.

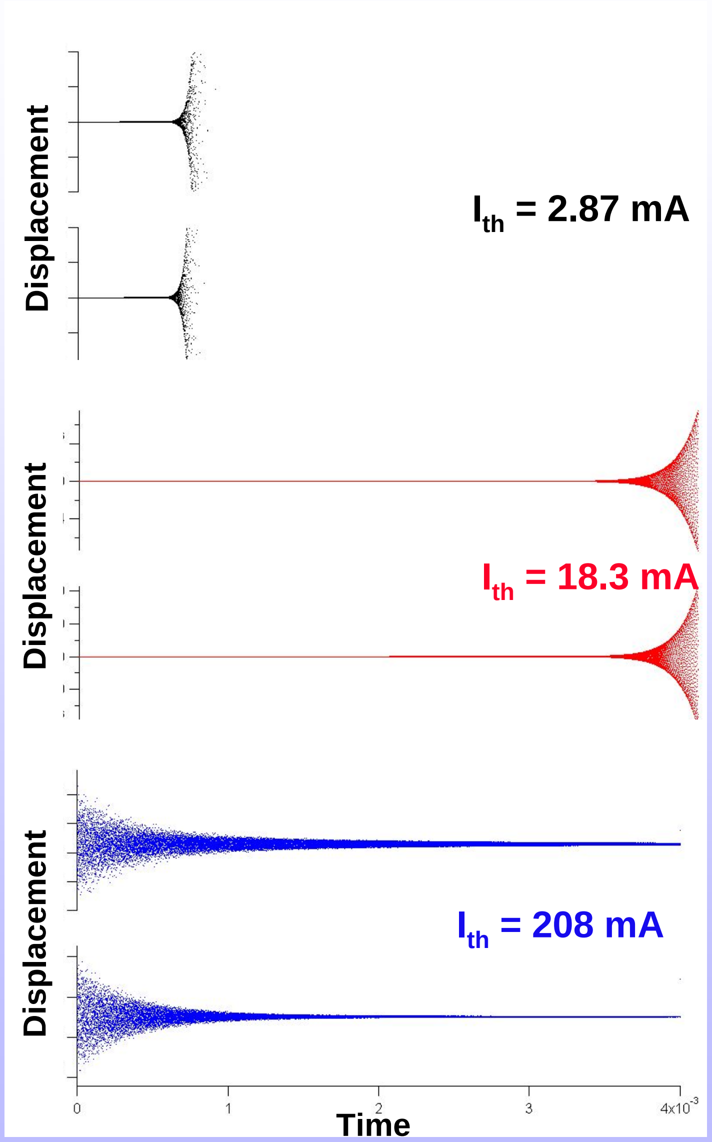
Reflecting Optics

The threshold current is increased substantially (by a factor of ~100). The threshold is not infinity (as one might expect), possibly because of "cross" excitation of orthogonal HOMs. (Insufficient frequency spacing)

Rotating Optics

The threshold current is increased by a factor of ~200. In theory, BBU should be eliminated altogether with a pure 90 degree rotation of betatron modes. However, the re-circulation matrix used for the unstable region back to itself is not a perfect rotation (i.e. the 2x2 off-diagonal matrices are of opposite sign, but are not exactly equal). Hence BBU occurs at a finite current.

Simulation Results: “*Skewed Modes*”



All simulation results are for 100 mA of average beam current. The top (bottom) plot shows the vertical (horizontal) beam offset versus time. The simulation run-time was 4ms.

Nominal Optics

The threshold current remains virtually unchanged at 3 mA.

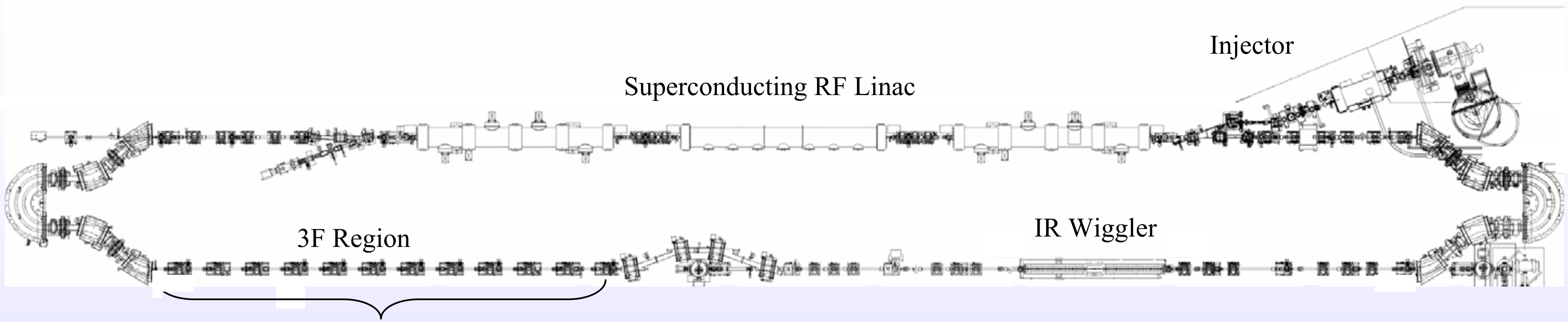
Reflecting Optics

Due to mode orientations being “skewed”, a reflection alone does not effectively suppress BBU. The threshold current is increased but is significantly less effective than the previous case (288 mA versus 18.3 mA).

Rotating Optics

A rotation is still very effective in raising the threshold current. This is expected since this technique is effective *regardless of the mode orientations*.

But does it actually work ?



A reflector using skew quadrupoles was implemented in the 3F backleg region of the JLab FEL Upgrade Driver.

Operational Experience

•Without using the reflector the BBU driven current limit could be varied from 1 ma to 5 ma.

•With the reflector operation was possible at 8 ma with no BBU observed. (The current was limited by other factors.)

How well will the suppression really work ?
or
What could possibly go wrong ?

- How accurately can the beam be rotated by 90º?
- How precisely can the HOM's be forced to lie at 0º and 90º?
- Non cylindrically symmetric focusing fields along the linac axis will prevent the suppression from being ideal for all cavities regardless of the answer to the above. (Quadrupole components of the accelerating fields?)
- Orthogonal modes overlapping in frequency clearly weaken the suppression. How bad is this effect likely to be?
- What if the HOM's have a helical component? Then there is no straight off axis path through the cavity where there is no electric field.
- When will single pass regenerative BBU appear?
- When will cumulative BBU appear?
- What else???