Optics effects of RF Cavities

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- **Ponderomotive focusing by non-synchronous RF waves**

accelerating fields in multi-cell cavities have counter-propagating wave components (standing waves) and/or higher spatial harmonics (standing and traveling waves) which are non-synchronous with particles → net ponderomotive focusing arises from coupling between 1st order transv. particle oscillation and gradient of e.m. energy density → 2nd order focusing in field gradient (relevant at low energy and high field!)

- **Envelope description and Transport Matrix of RF focusing**

Cell-to-cell averaging allows analytical calculation of focusing gradient, while change of variable (z --> $\gamma(z)$) gives analytical solution for transp. matrix of a generic RF cavity (field expansion in spatial harmonics)

Optics effects of RF Cavities

- **Transport Matrix directly measured by RF kicks on misaligned beam**

  impact on beam transport at low energy, strong focusing may lead to beam steering difficult to correct (cavities are not localized)

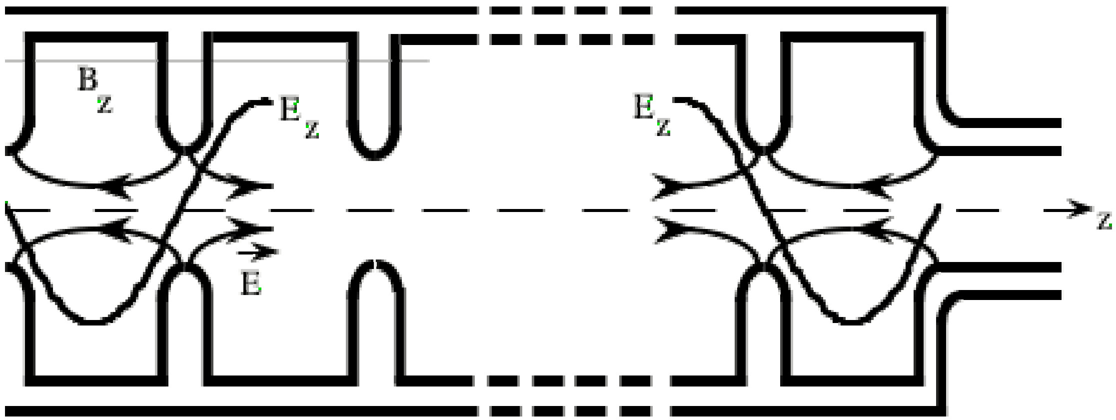
  decelerating beams: interference between ponderomotive focusing (slightly phase dependent) and adiabatic anti-damping

- **Beam focusing by counter-propagating e.m. waves**

  (TEM_{01} laser pulses)

Following Hartman and Rosenzweig

Phys. Rev. E **47** (1993) 2031



On-axis expansion of the $TM_{010\text{-}\pi}$ standing mode

E_0 = peak field
$k \equiv 2\pi/\lambda = \omega/c$
a_n = spatial harmonic coefficients functions of cavity geometry

$$\begin{cases} E_z = \mathcal{E}_z(r,z)\cdot\sin(\omega t+\varphi_0) \quad ; \quad \mathcal{E}_z(r,z) = E_0 \sum_{n=1,odd}^{\infty} a_n \cos(nkz) \\ E_r = \mathcal{E}_r(r,z)\cdot\sin(\omega t+\varphi_0) \quad ; \quad \mathcal{E}_r(r,z) = \frac{kr}{2} E_0 \sum_{n=1,odd}^{\infty} n\cdot a_n \sin(nkz) \quad ; \quad a_1 = 1 \\ B_\theta = B_\theta(r,z)\cdot\cos(\omega t+\varphi_0) \quad ; \quad B_\theta(r,z) = c\,\frac{kr}{2}\mathcal{E}_z(r,z) \end{cases}$$

Following Hartman and Rosenzweig

- **The first order transverse motion is oscillatory under the action of a radial force (electric + magnetic component of backward wave)**

$$F_r = -eE_0kr\left[\sin(kz+\Delta\phi)\sum_{n=1}^{\infty} a_n\cos(nkz) + n\cos(kz+\Delta\phi)\sum_{n=1}^{\infty} a_n\sin(nkz)\right] *$$

- **Being the reference (synchronous) particle defined by**

$$\sin(\omega t+\phi_0)=\sin\left[kz+\frac{\pi}{2}+\Delta\phi\right]=\cos(kz+\Delta\phi)$$

* $\sin(kz+\Delta\varphi)\cos(kz)+\cos(kz+\Delta\varphi)\sin(kz)=\sin(2kz+\Delta\varphi)=\sin(kz+\omega t+\Delta\varphi)$

Backward wave of fundamental

Following Hartman and Rosenzweig

- **In general, if the first order oscillatory motion is perturbative**

$$\ddot{x} = \frac{F_x}{\gamma m} = \frac{x_0}{\gamma m} \sum_{n=-\infty}^{\infty} A_n \exp(in\omega t)$$

$$x = x_0 \left[1 - \frac{1}{\gamma m \omega^2} \sum_{n=-\infty}^{\infty} \left[\frac{A_n}{n^2} \exp(in\omega t) \right] \right]$$

$$\frac{A_1}{\gamma m \omega^2} << 1$$

- **We can average the momentum imparted by the force over one oscillation period (substit. x_0 with x)**

$$\Delta p_x = \int_0^T F_x dt$$

$$= x_0 \int_0^T \left[1 - \frac{1}{\gamma m_e \omega^2} \sum_{n=-\infty}^{\infty} \frac{A_n}{n^2} \exp(in\omega t) \right]$$

$$\times \sum_{m=-\infty}^{\infty} A_m \exp(im\omega t) dt \; .$$

Following Hartman and Rosenzweig

- **Obtaining the average force, which comes out to be always inward (net focusing) !**

$$\overline{F_x} = \frac{\Delta p_x}{T}$$

$$= -\frac{x_0}{\gamma m \omega^2} \sum_{n=-\infty}^{\infty} \frac{|A_n|^2}{n^2}$$

- **Applying same procedure to our linear RF transverse force**

$$\overline{F_r} = -r\frac{(eE_0)^2}{8\gamma mc^2} \sum_{n=1}^{\infty} \{a_{n-1}^2 + a_{n+1}^2 + 2a_{n-1}a_{n+1}[\cos(2\Delta\phi)]\}, \quad a_0 = 0,$$

Following Hartman and Rosenzweig

- **The associated focusing gradient is of second order in the field amplitude, typical of ponderomotive focusing**

$$K_r = -\frac{\overline{F_r}}{r\gamma\beta^2 mc^2} = \frac{1}{8}\left[\frac{eE_0}{\beta\gamma mc^2}\right]^2 \sum_{n=1}^{\infty}\{a_{n-1}^2 + a_{n+1}^2 + 2a_{n-1}a_{n+1}[\cos(2\Delta\phi)]\}$$

$\eta(\phi)$

η=1 for pure TM_{010} π mode

- **Assumptions**

$$\frac{\Delta\gamma_{cell}}{\gamma} << 1$$

"averaging"

$$2k >> k_\beta = \sqrt{K_r} \simeq \frac{eE_0}{\sqrt{8}p_z c}$$

"average force perturbative w.r.t. first order transverse motion"

- **Both conditions are easily fulfilled if T > few Mev**

Envelope Equation with adiabatic damping and second order focusing

$$\sigma'' + \sigma'\frac{\gamma'}{\gamma} + \sigma\frac{\Omega^2\gamma'^2}{\gamma^2} - \xcancel{\frac{I(\zeta)}{2I_A\sigma\gamma^3}} = \frac{\varepsilon_n^2}{\sigma^3\gamma^2}$$

$$\gamma = \gamma_0 + \gamma' z \qquad \gamma' \equiv \frac{eE_0\cos\varphi}{mc^2} \qquad \sigma' \equiv \frac{d\sigma}{dz} \qquad \sigma \equiv \langle x^2 \rangle$$

Normalized focusing gradient

$$\Omega^2 = \left(\frac{eB_{sol}}{mc\gamma'}\right)^2 + \left\{ \begin{array}{ll} \approx \eta/8 & SW \\ \approx 0 & TW \end{array} \right\}$$

Solenoid magnetic field

RF ponderomotive focusing

Envelope Equation with adiabatic damping and second order focusing

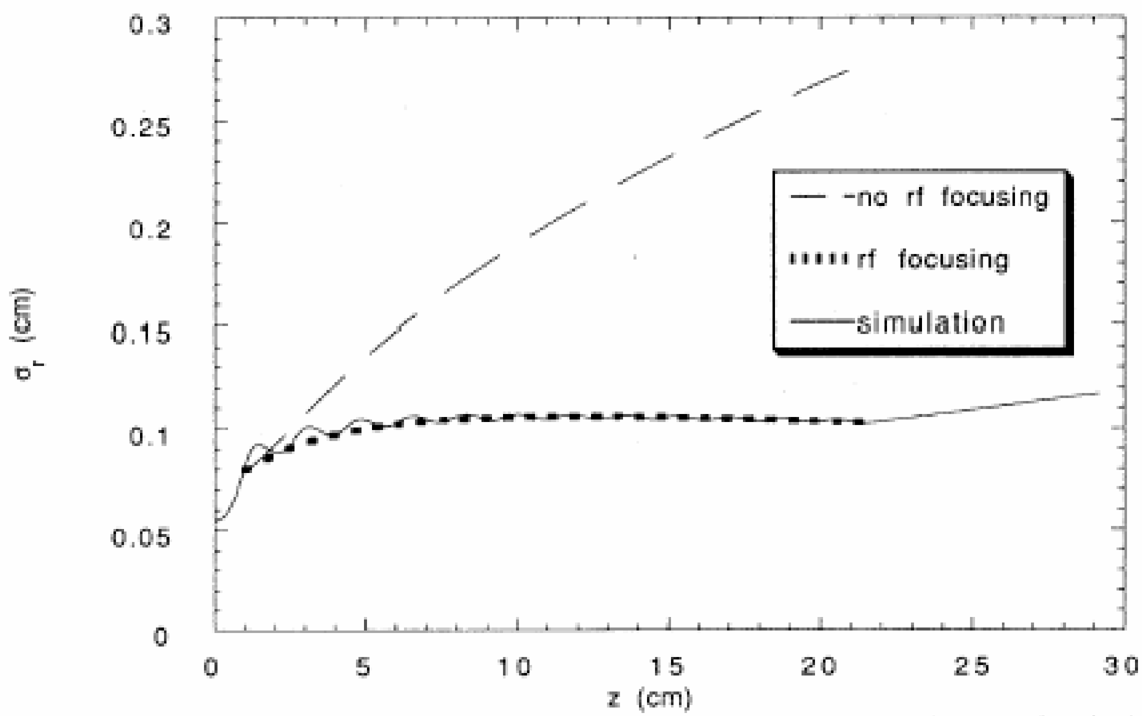


Figure 1. Comparison of PARMELA simulation to envelope equation results, including and omitting rf focusing. Parameters of gun: σ_z=.15 mm,1 nC of charge, E_0=225 MV/m.

Envelope Equation (laminar beams) becomes simple harmonic oscillator (freq. Ω) in Cauchy space

Rosenzweig, Serafini, *Phys. Rev. E* **49** (1994) 1599

Serafini, Rosenzweig, *Phys. Rev. E* **55** (1997) 7565

$$\sigma'' + \sigma' \frac{\gamma'}{\gamma} + \sigma \frac{\Omega^2 \gamma'^2}{\gamma^2} = 0$$

$$y \equiv \ln(\gamma/\gamma_2).$$

$$\Rightarrow \quad \frac{d^2\sigma}{dy^2} + \Omega^2 \sigma = 0$$

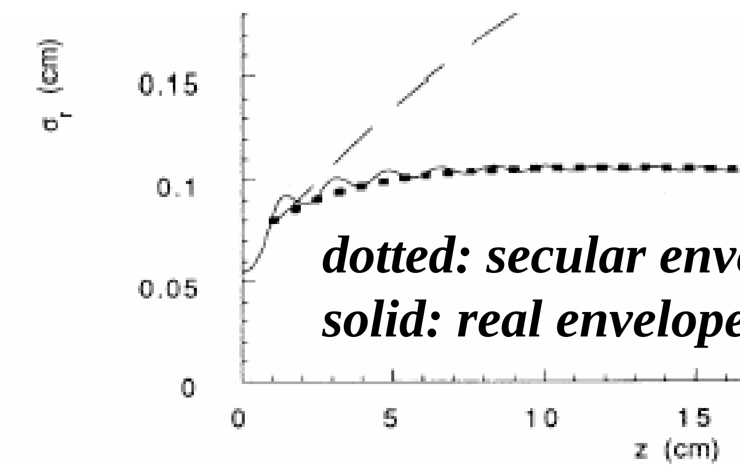
$$\Omega^2 = \frac{\eta}{8} + \left(\frac{eB_{sol}}{mc\gamma'}\right)^2$$

same transformation for single particle trajectory ($B_{sol}=0$)

$$x'' + \left(\frac{\gamma'}{\gamma}\right) x' + \frac{\eta(\Delta\phi)}{8\cos^2(\Delta\phi)} \left(\frac{\gamma'}{\gamma}\right)^2 x = 0$$

Transport matrix for secular envelope

$$\begin{bmatrix} x \\ x' \end{bmatrix}_f = \begin{bmatrix} \cos(\alpha) & \left(\frac{8}{\eta(\Delta\phi)}\right)^{1/2}\frac{\gamma_i}{\gamma'}\cos(\Delta\phi)\sin(\alpha) \\ -\left(\frac{\eta(\Delta\phi)}{8}\right)^{1/2}\frac{\gamma'}{\gamma_f\cos(\Delta\phi)}\sin(\alpha) & \frac{\gamma_i}{\gamma_f}\cos(\alpha) \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix}_i$$

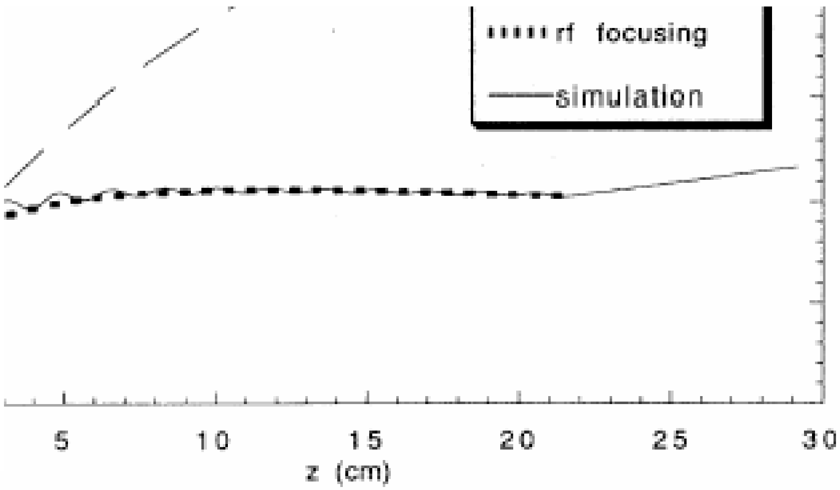


dotted: secular envelope
solid: real envelope

$$\alpha = \left(\frac{\sqrt{\eta(\Delta\phi)/8}}{\cos(\Delta\phi)}\right)\ln\left(\frac{\gamma_f}{\gamma_i}\right)$$

phase advance
(log. damping from Cauchy space to trace space)

Transport matrix for secular envelope matched to injection/extraction out of the cavity (entrance/exit RF kicks)



RF kick matrix

$$\begin{bmatrix} 1 & 0 \\ \mp \dfrac{\gamma'}{2\gamma_{i(f)}} & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\alpha) - \left(\dfrac{2}{\eta(\Delta\phi)}\right)^{1/2} \cos(\Delta\phi)\sin(\alpha) & \left(\dfrac{8}{\eta(\Delta\phi)}\right)^{1/2} \dfrac{\gamma_i}{\gamma'} \cos(\Delta\phi)\sin(\alpha) \\ -\dfrac{\gamma'}{\gamma_i}\left[\dfrac{\cos(\Delta\phi)}{\sqrt{2\eta(\Delta\phi)}} + \left(\dfrac{\eta(\Delta\phi)}{8}\right)^{1/2} \dfrac{1}{\cos(\Delta\phi)}\right]\sin(\alpha) & \dfrac{\gamma_i}{\gamma_f}\left[\cos(\alpha) + \left(\dfrac{2}{\eta(\Delta\phi)}\right)^{1/2} \cos(\Delta\phi)\sin(\alpha)\right] \end{bmatrix}$$

Transport matrix for a pure π mode (no spatial harmonics)

$$\begin{bmatrix} \cos(\alpha)-\sqrt{2}\sin(\alpha) & \sqrt{8}\dfrac{\gamma_i}{\gamma'}\sin(\alpha) \\ -\dfrac{3\gamma'}{\sqrt{8}\gamma_f}\sin(\alpha) & \dfrac{\gamma_i}{\gamma_f}[\cos(\alpha)+\sqrt{2}\sin(\alpha)] \end{bmatrix}$$

$$\det M = \frac{\gamma_i}{\gamma_f}$$

→ *adiabatic damping + ponderomotive RF focusing*

Same as in Chambers, SLAC-rep 1965, unpublished

Optics effects of RF Cavities

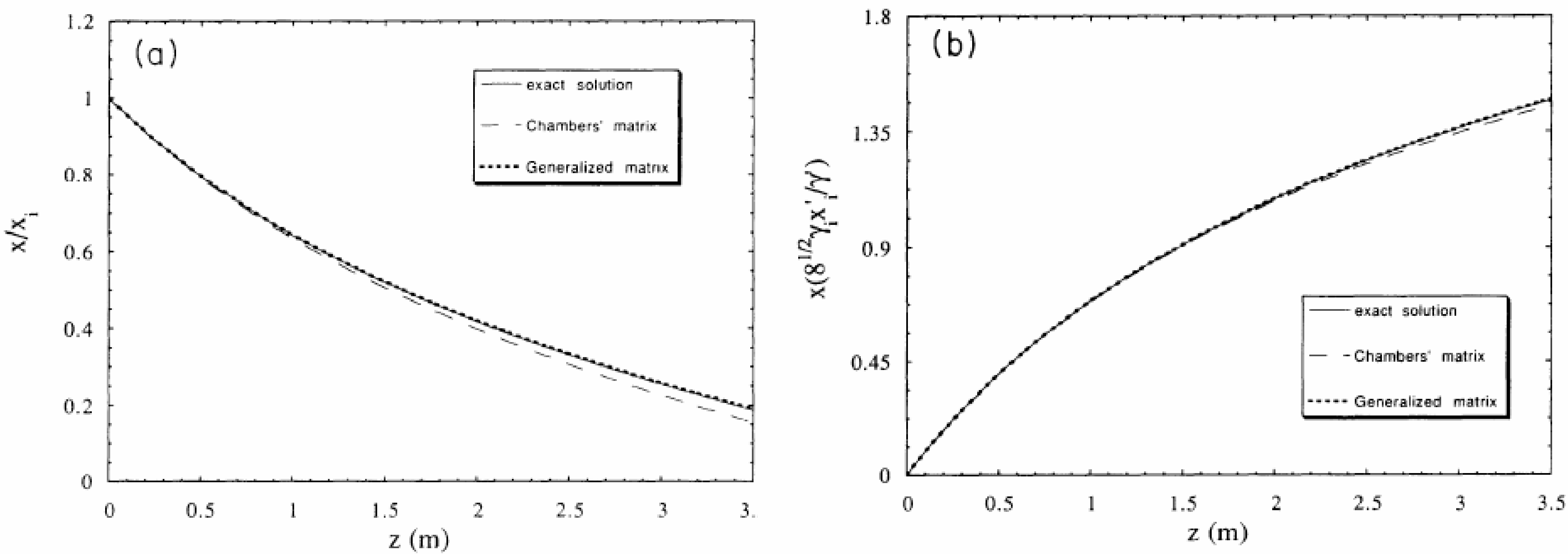


FIG. 1. Comparison of the numerical solution to the exact equations of motion in a π-mode standing-wave cavity [$\gamma_i = 100$, $E_0 = 50$ MV/m, $k_0 = 59.8$ m^{-1} ($f = \omega/2\pi = 2856$ MHz)] containing a higher spatial harmonic ($b_1 = b_{-2} = -0.2$), with the predictions of Chambers' matrix and our generalized matrix. Initial conditions are (a) $(x_i, x_i') = (1,0)$ and (b) $(x_i, x_i') = (0,1)$.

Optics effects of RF Cavities

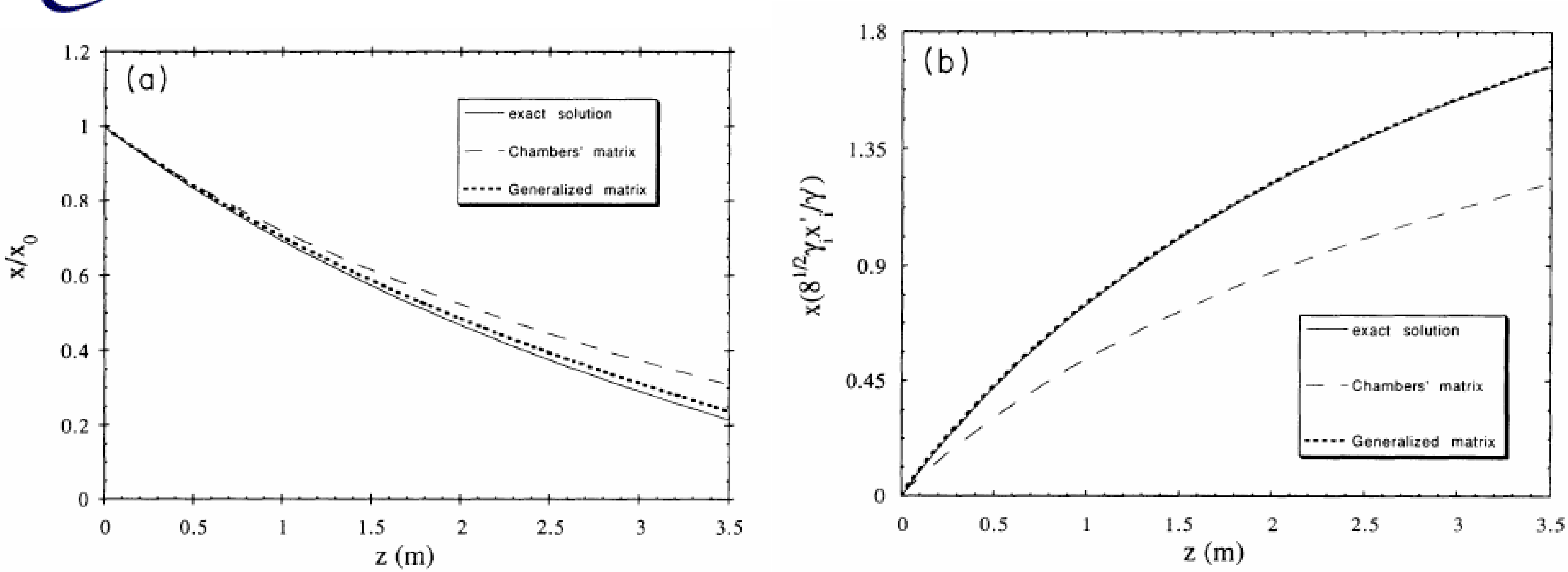


FIG. 2. Comparison of the numerical solution to the exact equations of motion in a pure π-mode standing-wave cavity [$\gamma_i = 100$, $E_0 = 50$ MV/m, $k_0 = 59.8$ m^{-1} ($f = \omega/2\pi = 2856$ MHz)] of a particle injected at $\Delta\phi = \pi/4$, with the predictions of Chambers' matrix and our generalized matrix. Initial conditions are (a) $(x_i, x_i') = (1,0)$ and (b) $(x_i, x_i') = (0,1)$.

Envelope Equation with space charge

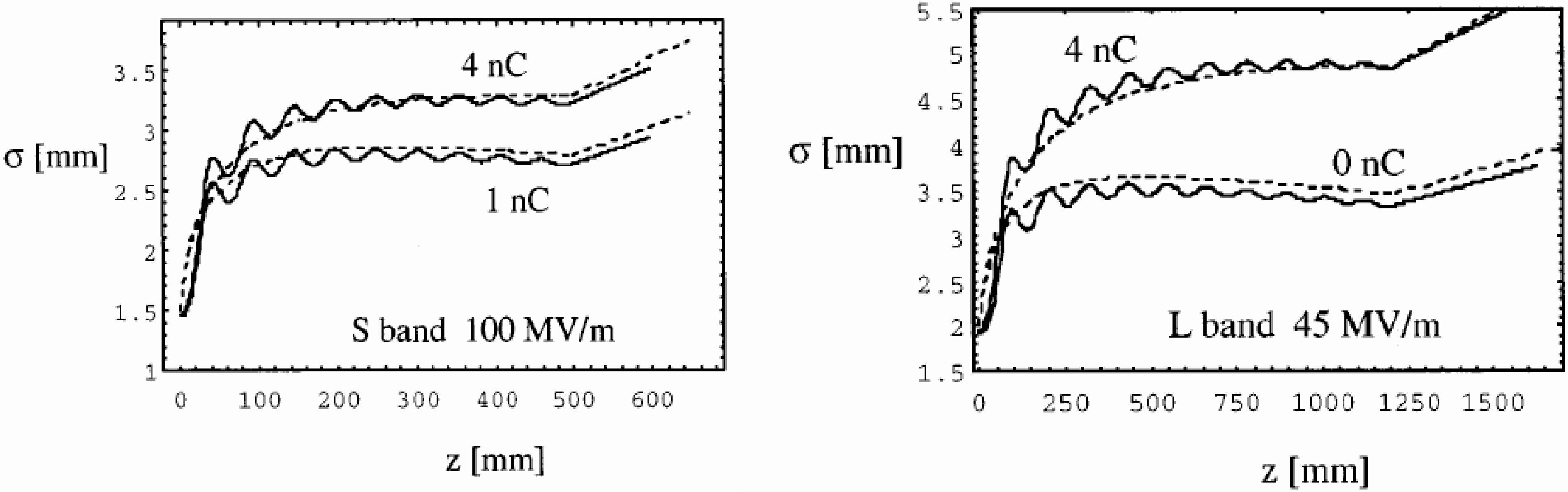


FIG. 4. Beam envelopes through two different 10+1/2 cell rf guns operated without external solenoid focusing, i.e., $B_0=0$ ($\nu_{rf}=2.856$ GHz $E_0=100$ MV/m upper diagram, $\nu_{rf}=1.3$ GHz $E_0=45$ MV/m lower diagram). Dashed lines give the secular orbits analytically predicted, while solid lines are numerical simulation results. Various bunch charges have been used, as indicated.

PHYSICAL REVIEW E VOLUME 56, NUMBER 3 SEPTEMBER 1997

Experimental confirmation of transverse focusing and adiabatic damping in a standing wave linear accelerator

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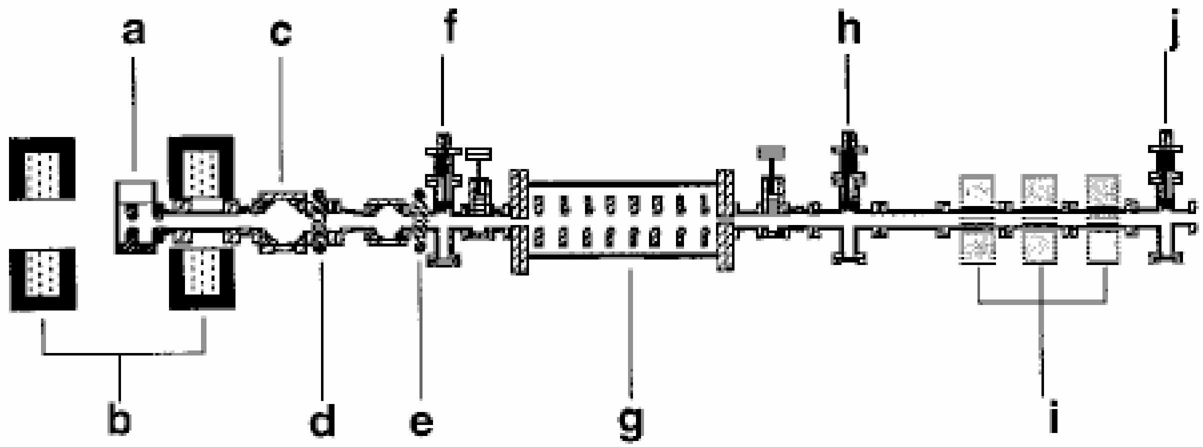
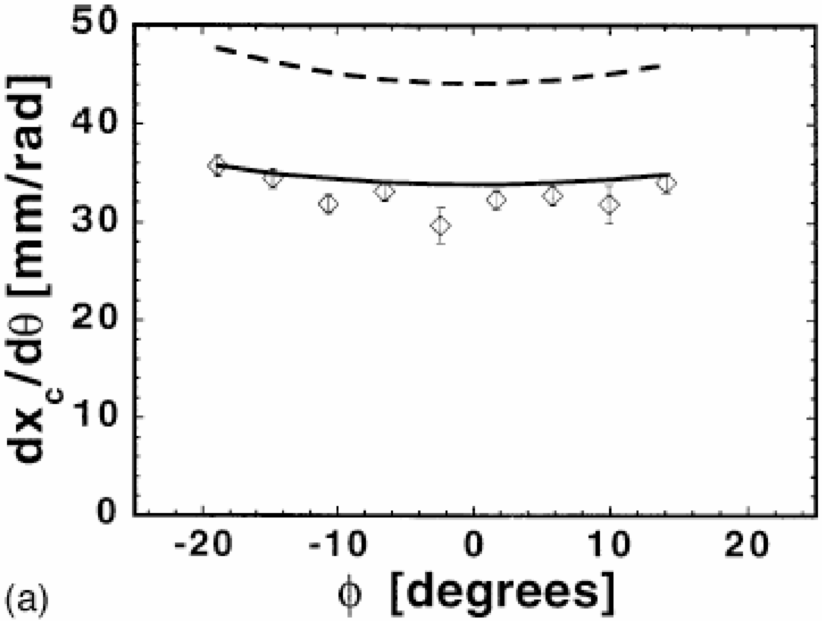
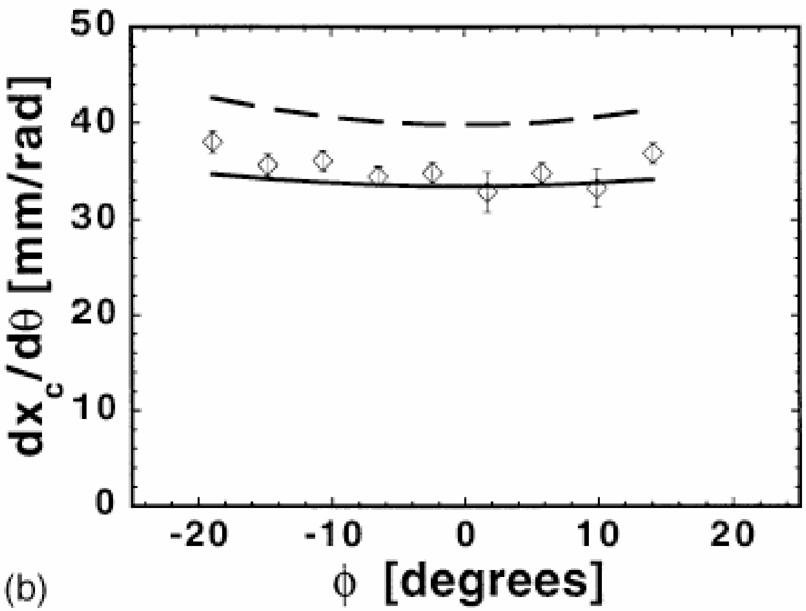


FIG. 1. Layout of the UCLA photoinjector with a (a) 1.5 cell rf gun, (b) focusing and bucking solenoid, (c) mirror box, steering magnets (d) $K1$ and (e) $K2$, (f) phosphor screen $P1$, (g) PWT linac, (h) phosphor screen $P2$, (i) quadrupole triplet, and (j) phosphor screen $P3$.



(a)



(b)

FIG. 2. (a) Centroid motion $dx_c/d\theta$ due to steering magnet $K1$ measured on phosphor screen $P2$, (b) centroid motion $dx_c/d\theta$ due to steering magnet $K1$ measured on phosphor screen $P3$, (c) centroid motion $dx_c/d\theta$ due to steering magnet $K2$ measured on phosphor screen $P2$, and (d) centroid motion $dx_c/d\theta$ due to steering magnet $K2$ measured on phosphor screen $P3$, for different acceleration phases ϕ. Data obtained are marked by diamonds, with predictions from the matrix shown in Eq. (4) given by the solid line and the dashed line showing predictions if focusing effects are ignored (by the limit $\eta \rightarrow 0$) in the matrix.

Ponderomotive Focusing with TEM01 laser pulses

$$E_{xj}(x,y,z,t) = E_0 \frac{e^{i\varphi} e^{i\varphi_j^s}}{\sqrt{1+z^2/z_0^2}} \cdot \tau_j \qquad j = 0,1$$

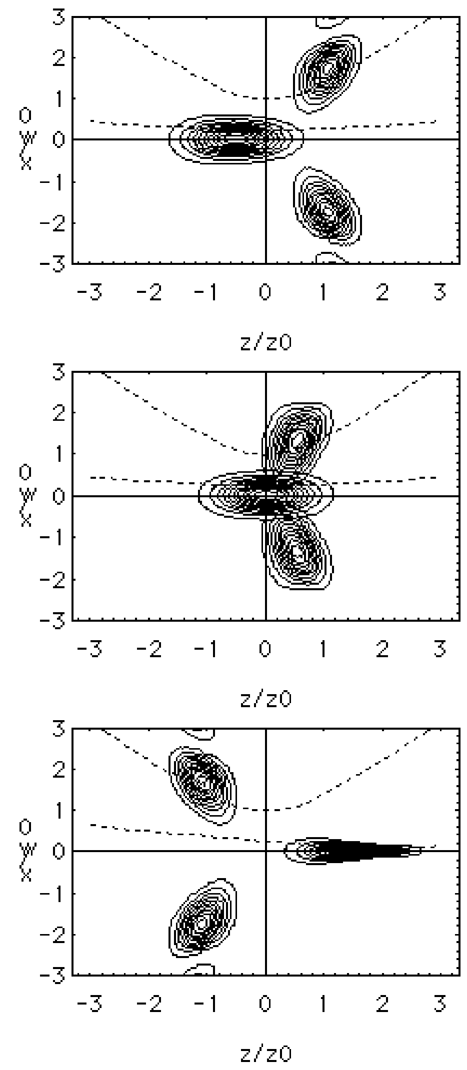
$$\varphi_j^s = \frac{\left(x^2+y^2\right)/w_0^2}{(z/z_0 + z_0/z)} - (j+1)\tan^{-1}\left(\frac{z}{z_0}\right)$$

$$w = w_0\sqrt{1+z^2/z_0^2} \qquad \tau_0 = e^{-\frac{x^2+y^2}{w^2}} \qquad \tau_1 = 2\sqrt{2}\,\frac{x}{w}\,e^{-\frac{x^2+y^2}{w^2}}$$

Ponderomotive Focusing with TEM01 laser pulses*

$$K_x = \left(\frac{4\alpha}{\gamma w_0}\right)^2 \qquad \alpha = 1.22\,\frac{\lambda}{w_0}\sqrt{P[TW]}$$

$$K_{RF} = \alpha^2 k^2/(8\gamma^2)$$



* L. Serafini, *Ponderomotive Focusing using Laser Beams*
AIP CP **335** (1995) 666