BBU Codes Overview

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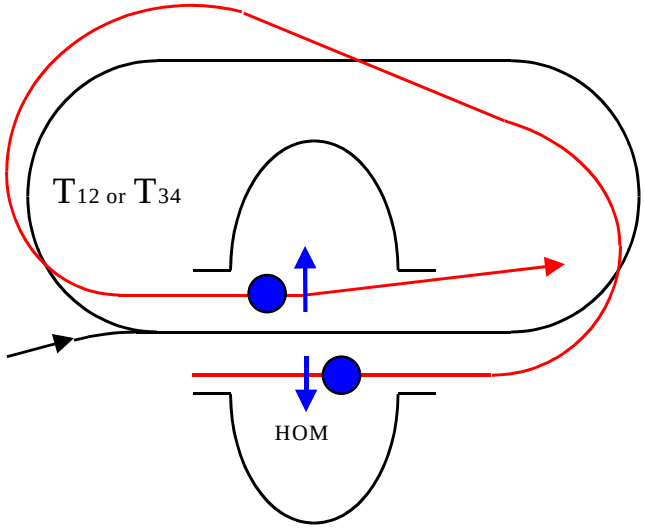
Outline

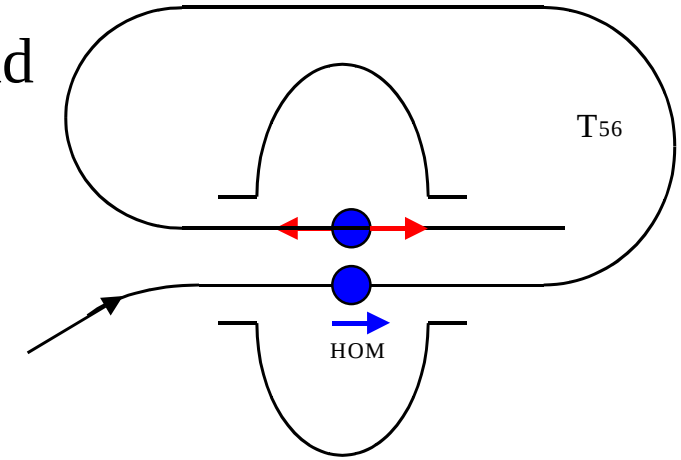
- Introduction
- Beam transport Equation
- How to solve (BBU-R, TDBBUU, bi, MATBBU etc.)
- Comparison of BBU codes

Introduction

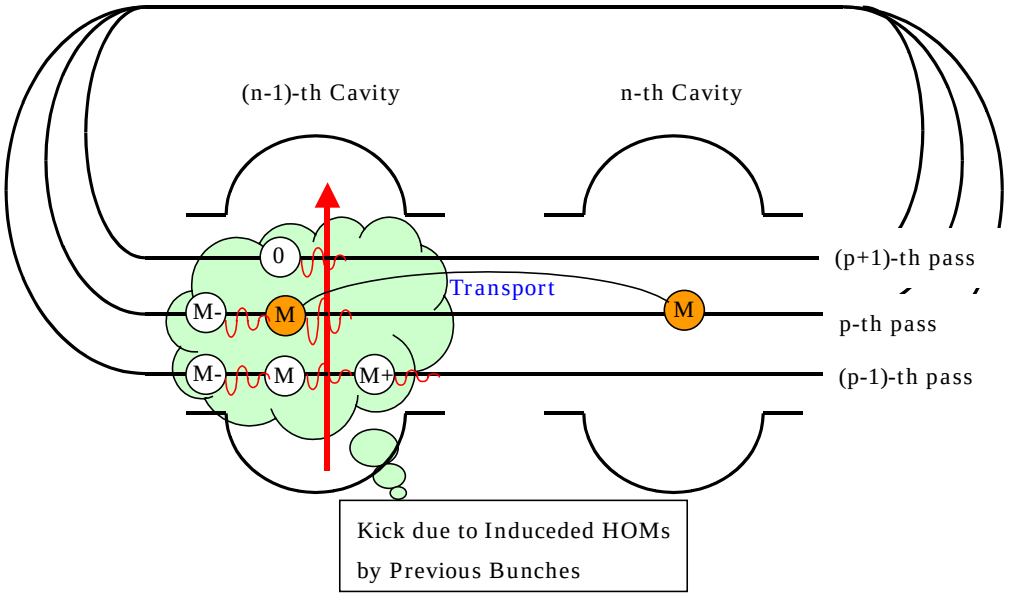
ERL current limited by

- Beam Breakup
  - Transverse
    deflect beam
  - Longitudinal
    arrival time difference
    HOM-induced energy spread





Beam Transport Equation



$T_{n,m}^{pq}$: *transfer matrix*

$G = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

Z_n : *transverse impedance*

M_0 : *number of bunches in one recirculation*

l : *cavity length*

$s_k(\omega_n\tau) = e^{-\frac{k\omega_n\tau}{2Q}} \sin(k\omega_n\tau)$

n_p : *passes*

n_0 : *number of cavity sites*

I : *average current*

•transport

•kick by HOM

$$U_p(n,M) = T_{n,n-1}^{pp} U_p(n-1,M) + T_{n,n-1}^{pp} G \; IZ_{n-1} \sum_{r=1}^{n_p} \sum_{k=1}^{M+(p-r)M_0-1} U_r(n-1, M+(p-r)M_0+k) s_k(\omega_n\tau)$$

Induced HOM

Decay & Phase shift

How to Solve

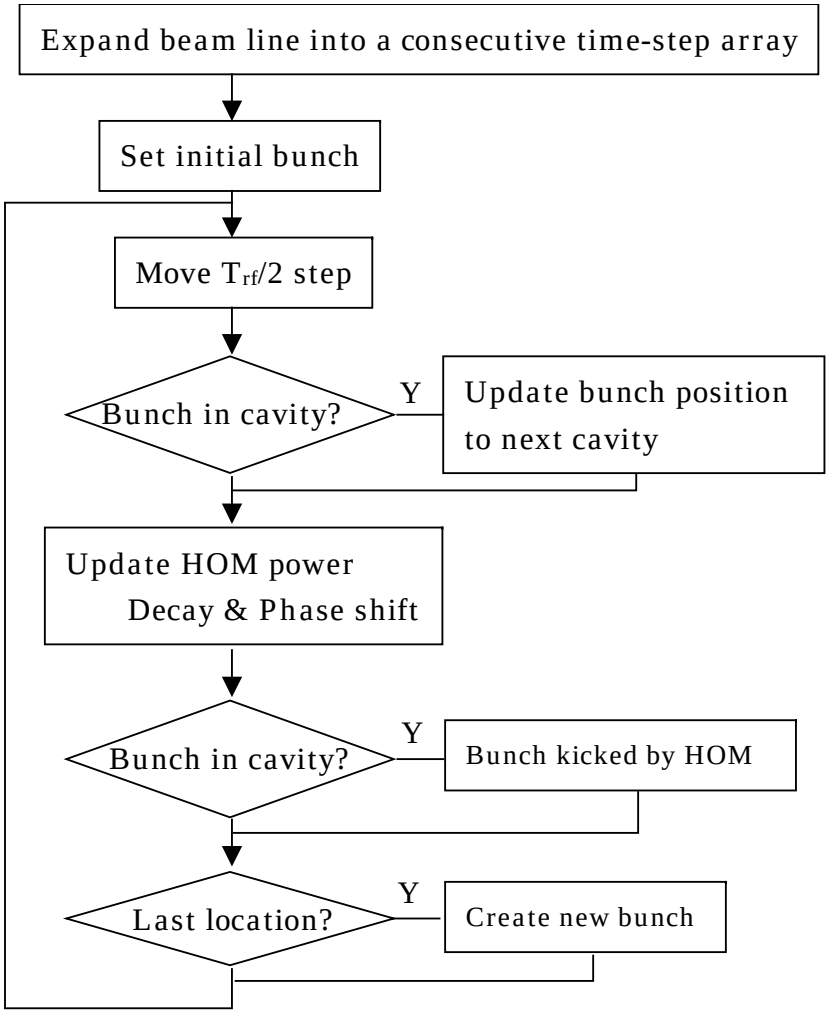
- Beam Tracking (Beam position vs. Time)
  - BBU-R (JAERI)
  - TDBBU (JLab)
  - bi (Cornell Univ.)
  - new code (JLab)
- Eigenvalue Solution (Current vs. Frequency)
  - MATBBU (JLab)

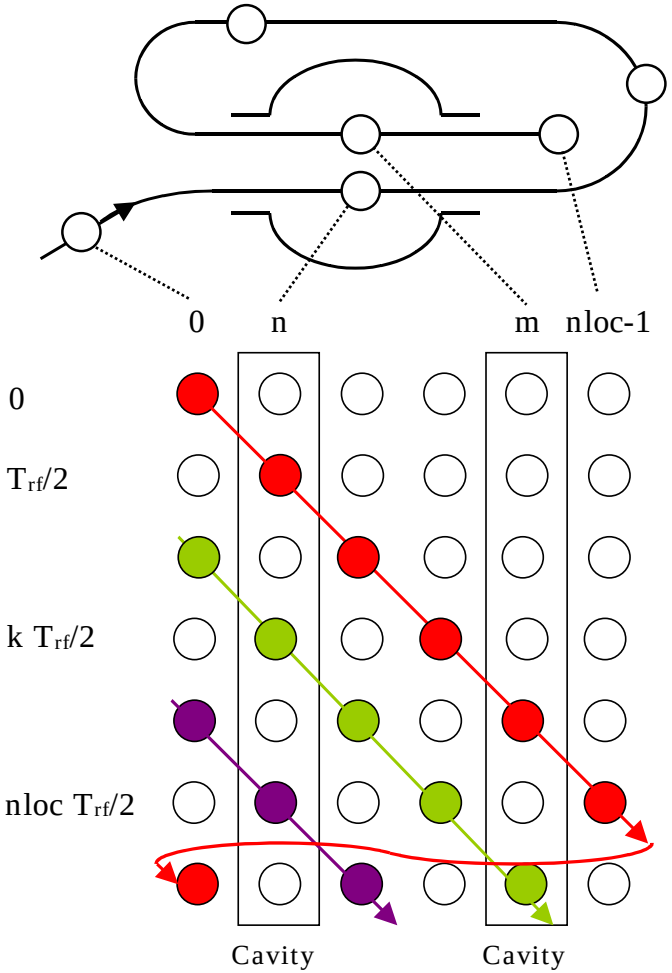
BBU-R

feature

- Transverse BBU
- Beam position vs. time (time increment $T_{rf}/2$)
- Point-like bunch
- X,Y HOM orientation
- X- & Y-axis independently
- Impulse of HOM kick
- Using 2X2 Transfer Matrix
- Two-pass recirculation

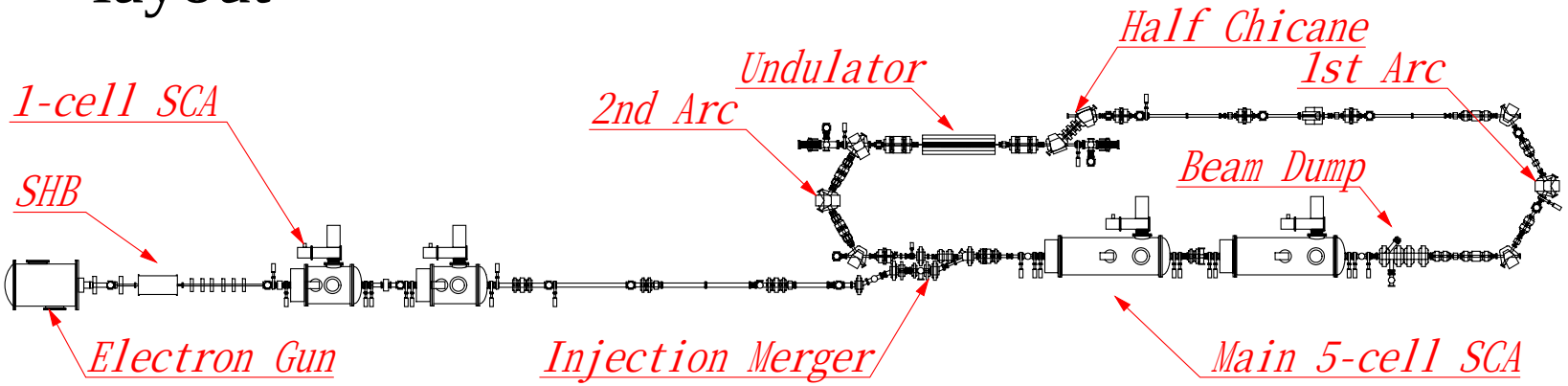
BBU-R Algorism

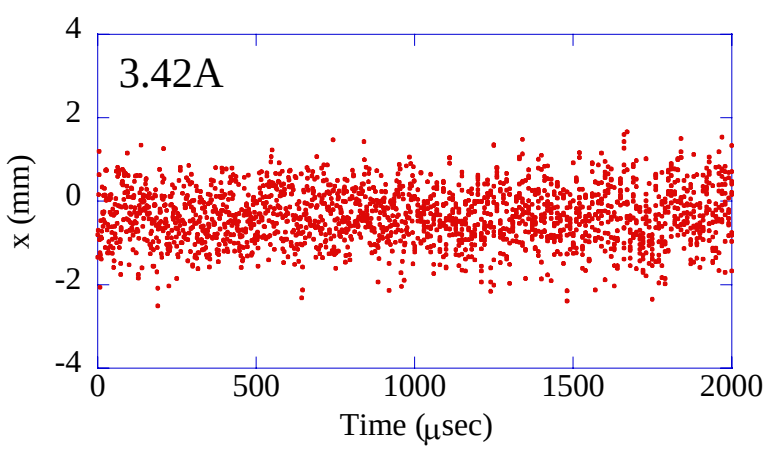


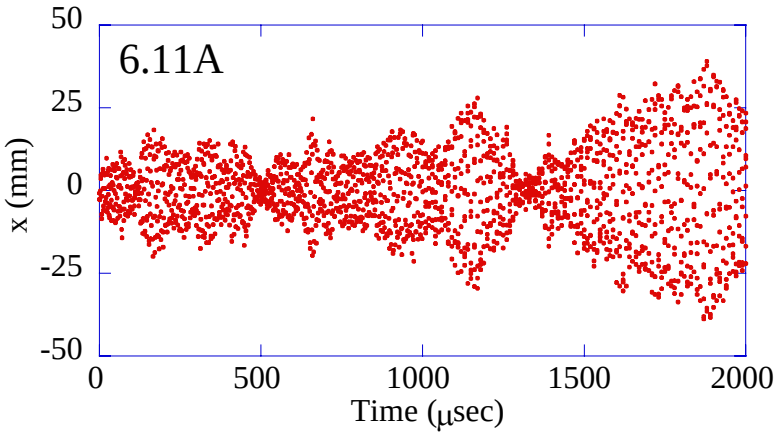


Threshold current of JAERI ERL-FEL

- layout



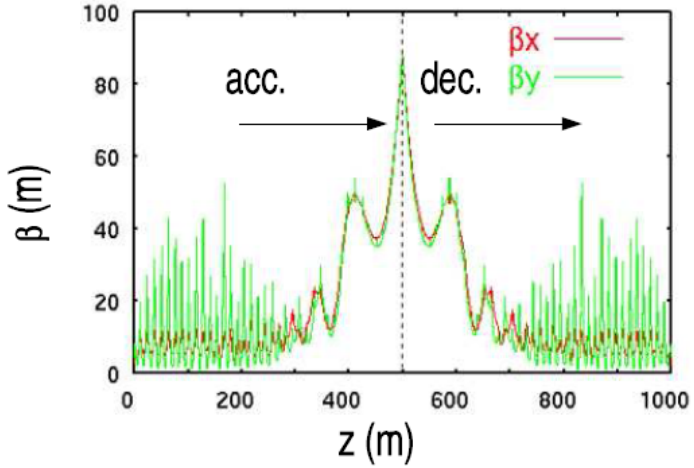




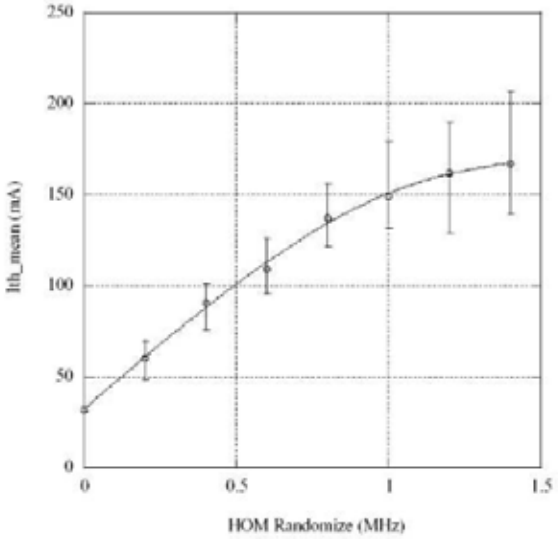
Example for future ERL design

6GeV 1.3GHz 15MV/m

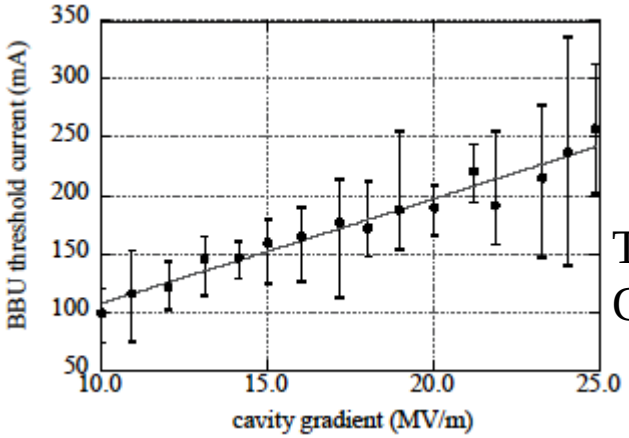
48 cryomodules 47 external QT



β-function in the linac



Threshold current vs.
HOM randomization

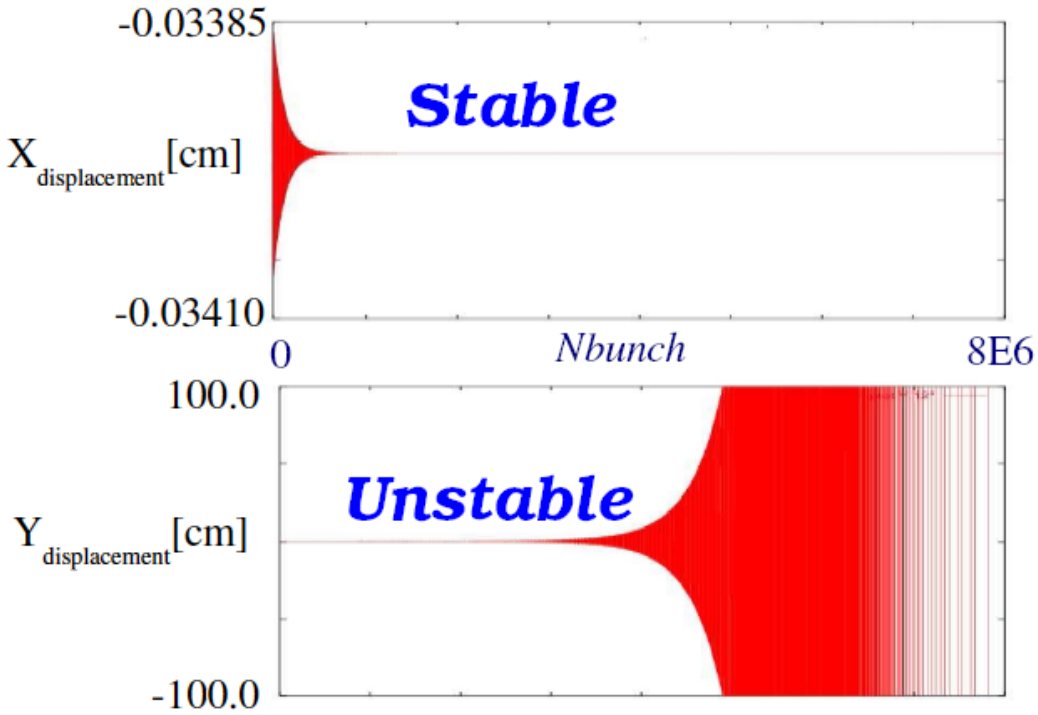


Threshold current vs.
Cavity gradient

TDBBU

- Transverse BBU
- (a) Move bunch
  (b) calculate position of bunch entering the cavity
  (c) update all HOM excitation level
- Turn the current up/down to find instability
- X- &Y-axis entirely independent
- Suitable for large HOM numbers or
  short characteristic time (2Q/ω)

Example of TDBBU results



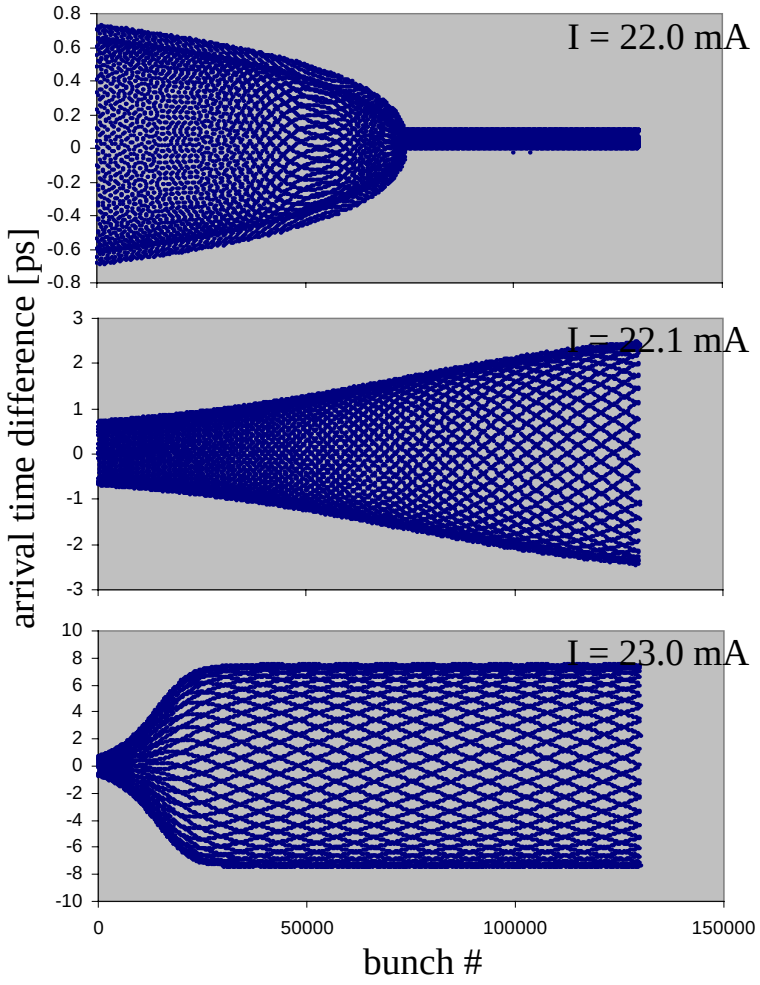
[K.Beard et al., PAC2003 Proc. 332 (2003)]

bi

- Transverse or longitudinal BBU
- Allow any ERL topology
- transient effect for arbitrary bunch pattern
- Arbitrary HOM orientation

Example of bi results

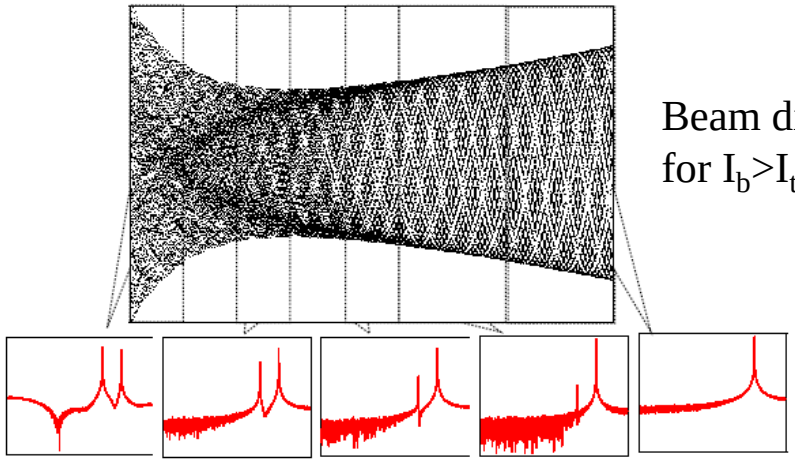
Longitudinal instability



New code at JLab

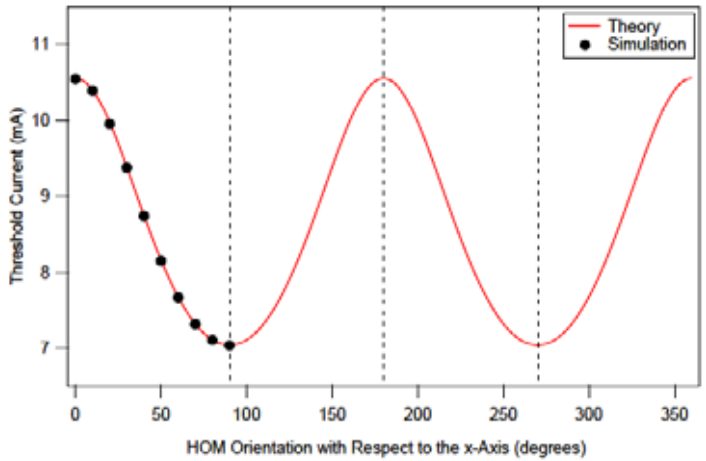
- Transverse BBU
- Two-path machine
- Full 2D particle tracking (4X4) or 1D (2X2)
- Arbitrary HOM angle
- Decoupled transverse motion and coupled motion
  effect of rotated HOM
  effects of rotated HOMs and of rotated optics
- Include FB for BBU suppression

Example of new code results



Beam displacement as a function of time for $I_b > I_{th}$

FFTs of the indicated"slices" of the beam displacement



Threshold current versus HOM orientation for a single cavity with one HOM and a decoupled recirculation matrix

[C.Tennant and E.Pozdeyev, JLAB-TN-02-020 (2004)]

MATBBU

- Solve eigenvalue of matrix representing system
- Transverse BBU
- X,Y axis treated sequentially, entirely independently

Algorism of MATBBU

$$U_p(n,M) = T^{pp}_{n,n-1} U_p(n-1,M) + T^{pp}_{n,n-1} G\ IZ_{n-1} \sum_{r=1}^{n_p} \sum_{k=1}^{M+(p-r)M_0-1} U_r(n-1, M+(p-r)M_0+k) s_k(\omega_n r)$$

↓ assume a steady state solution $U_p(n,M) = e^{i\Omega M\tau} V_p(n)$ & $M >> 0$

Ω : coherent frequency

$$V_p(n) = T^{pp}_{n,n-1} V_p(n-1) + IZ_{n-1} T^{pp}_{n,n-1} G \sum_{r=1}^{n_p} e^{i(p-r)M_0\Omega\tau} h_n(\Omega) V_r(n-1)$$

$$h_n(\Omega) = \frac{H_n(\Omega)\sin(\omega_n\tau)}{1+H_n(\Omega)^2 - 2H_n(\Omega)\cos(\omega_n\tau)}$$

↓ sum over all passes $\bar{V}(n) = \sum_{k=0}^{n_p} e^{ikM_0\tau} V_k(n)$

$$H_n(\Omega) = e^{-\frac{\omega_n\tau}{2Q}} e^{-i\Omega\tau}$$

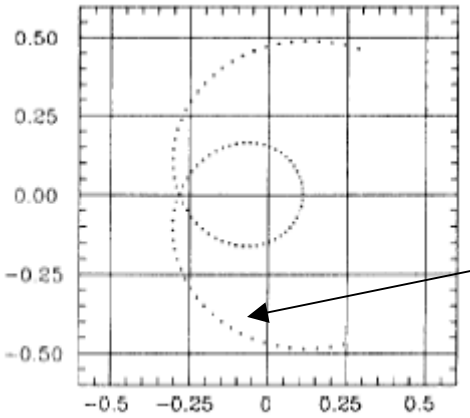
$$D_i = I \sum_{p=2}^{n_p} \sum_{r<p} \sum_{l=1}^{n_0} (T^{pr}_{i,l})_{12} e^{M_0\Omega\tau(p-r)} Z_l h_l(\Omega) D_l + I \sum_{p=1}^{n_p} \sum_{l=1}^{i-1} (T^{pp}_{i,l})_{12} Z_l h_l(\Omega) D_l$$

D_i $x-component\ of\ \bar{V}(i)$

Eigenvalue problem $M(\Omega)D = \frac{1}{I} D$

Ω is also unknown → Scan Ω and find *Im(I)=0* & *I>0*

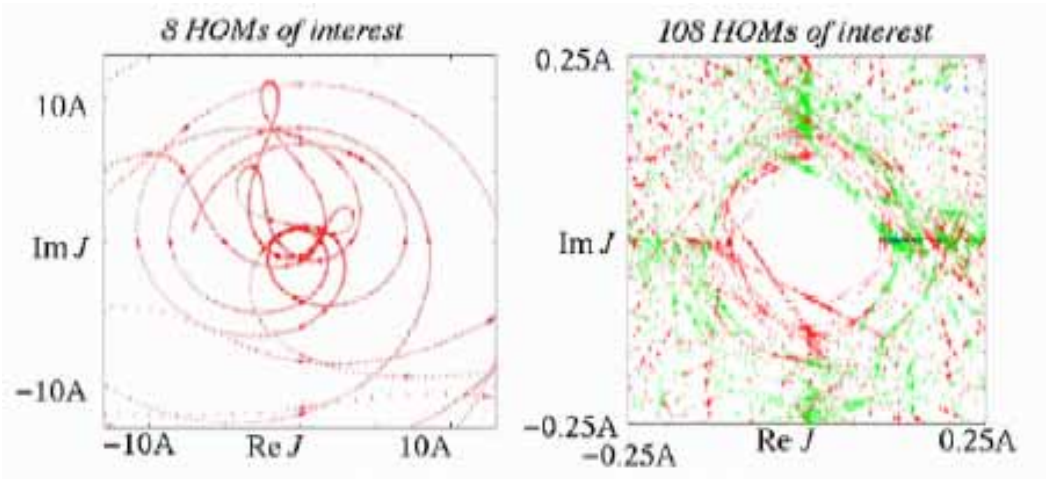
Example of MATBBU results



Stability plot of complex threshold current (amperes) for one site, two-pass configuration

Sweep coherent frequency and search Positive real current

[J.J.Bisognano et al., 1987 PAC Proc. 1078 (1987)]



[K.Beard et al., PAC2003 Proc. 332 (2003)]

Comparison of BBU codes

Name	BBU-R	TDBBU	bi	New code @JLab	MATBBU
Developer	JAERI	JLab	Cornell Univ.	JLab	JLab
Transverse /Longitudinal	T	T	T/L	T	T
Solve	Tracking	Tracking	Tracking	Tracking	Eigenvalue
HOM direction	X,Y	X,Y	Arbitrary	Arbitrary	X,Y
Dimension	1D	1D	2D	1D/2D	1D
No. of Recirculation	2	Arbitrary	Arbitrary	2	Arbitrary
Programming Language	C	Fortran/C	C++	C++	Fortran/C

Other problems

- Thin cavity
  Impulse kick
  Where does a kick work -entrance, middle or exit?
  Single- or multi- kick for low energy?

- Faster methods to determine whether stable or instable beam displace or HOM voltage?