

The diagram illustrates a circular particle accelerator ring with three parallel paths in blue, yellow, and green. Four square markers are placed at the top, bottom, left, and right of the ring. A green rectangular section on the left is labeled "Place for doubling energy linac". At the bottom, a large coil of magnets is shown with a right-pointing arrow below it. A smaller coil of magnets is positioned above it, with a purple arrow pointing from the larger coil to the smaller one. On the right side, there are two smaller coils of magnets, each with a blue starburst symbol next to it. Large, curved arrows in purple and pink are also present on the right side of the ring.

(Optimum???)
Merger for an ERL
- dispersion aspects

Dmitry Kayran and Vladimir Litvinenko

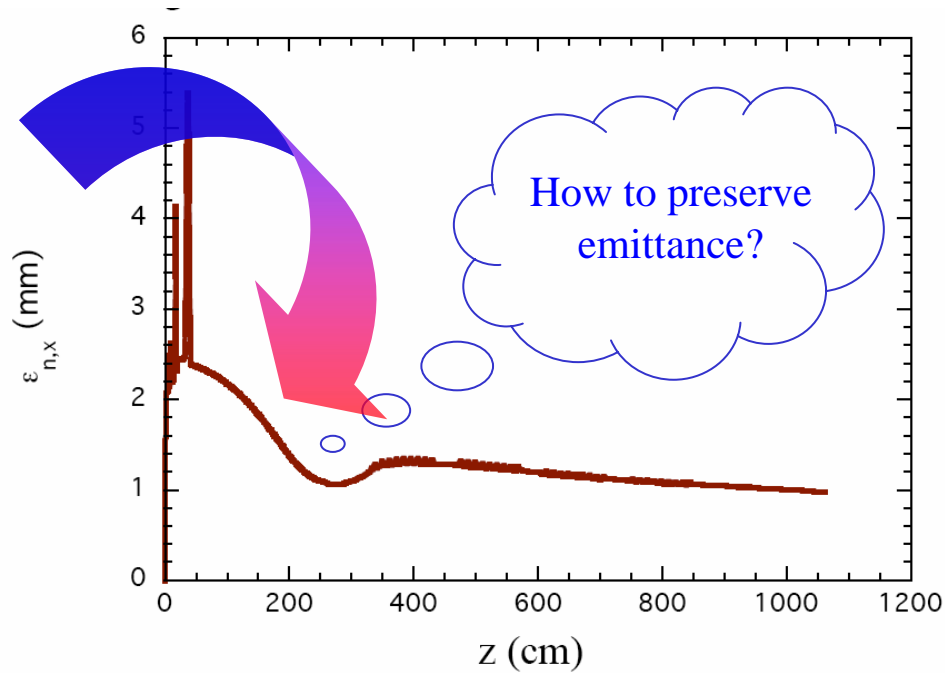
Collider Accelerator Department, Brookhaven National Laboratory, Upton, NY, USA

[Work is supported by US Department of Energy grant DE-AC02-98XH10886](#)

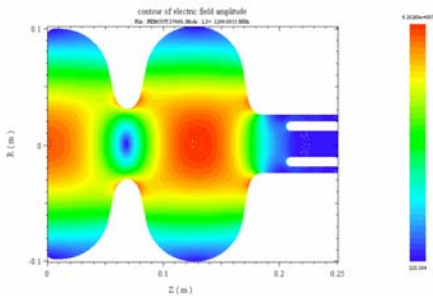
Motivation

- Each ERL has at least one merging system, which includes dipoles **↑ Potential for the mixing of longitudinal and transverse motions**
- Low energy injection into high current ERL is strongly desirable: (a) no residual radiation; (b) less MWs in RF power **↑ Strong space charge effects in a merger**
- Emittance compensation schemes do not allow using a strong focusing in a merger **↑ Necessity to use of a smooth optics**

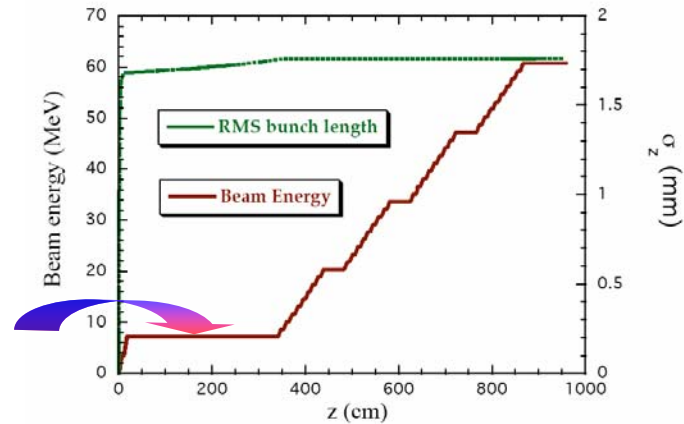
Merger is going here

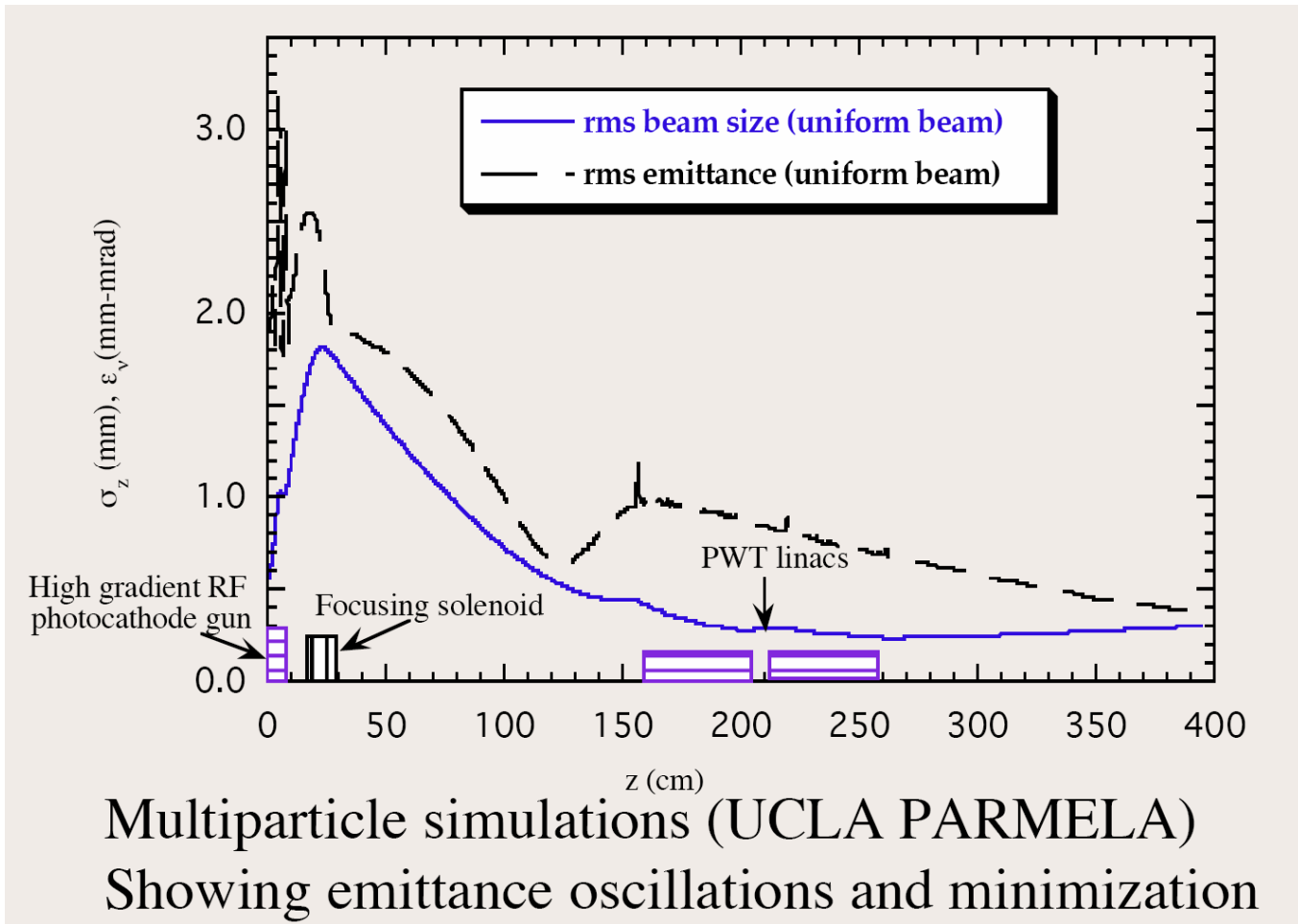


1.6 cell gun

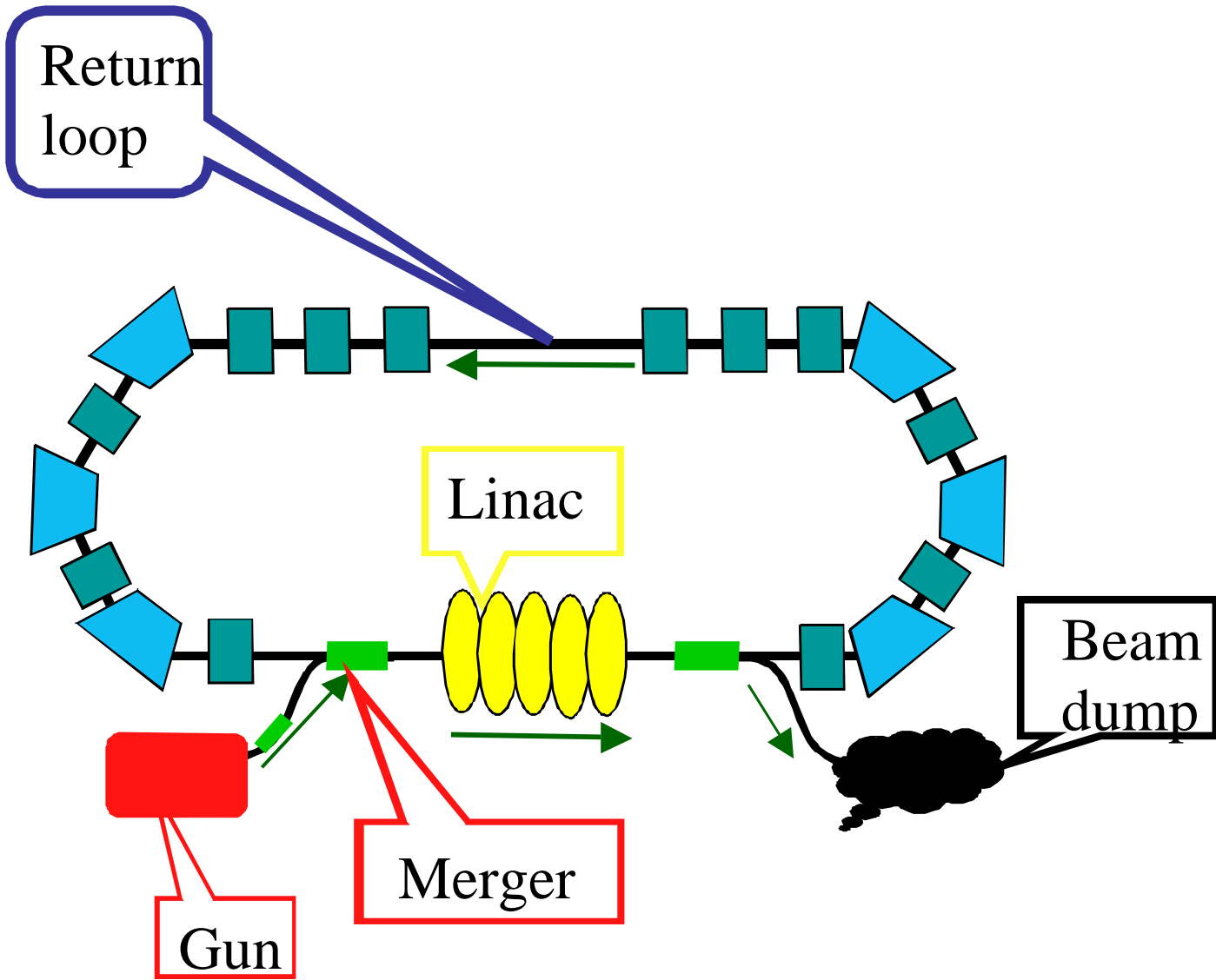


Merger is going here





An ERL



Emittance compensation

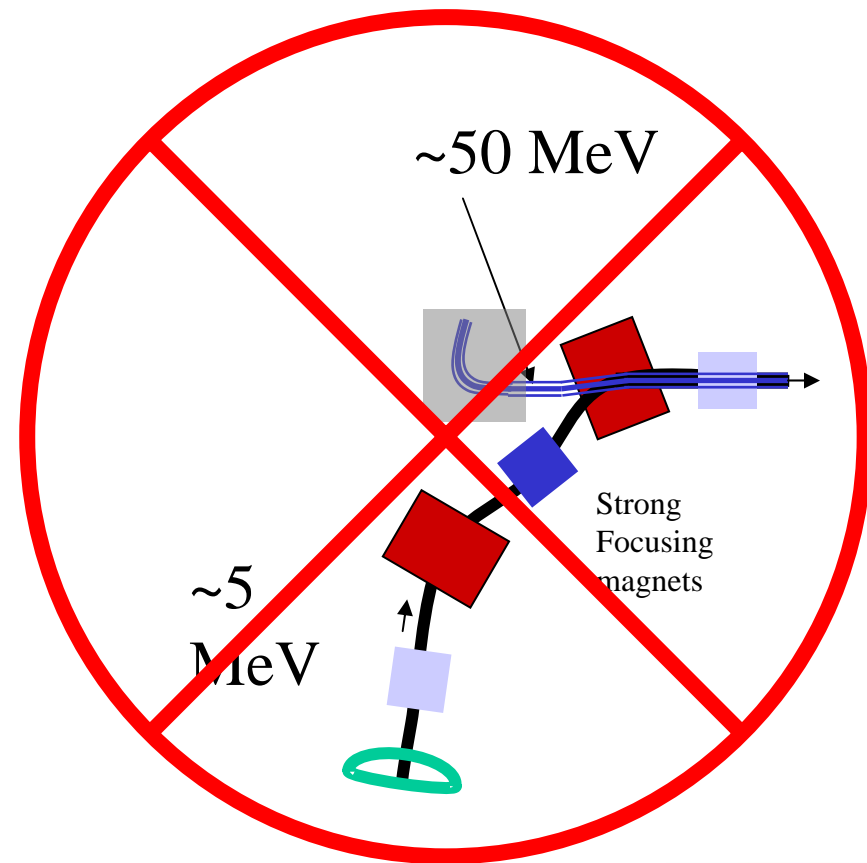
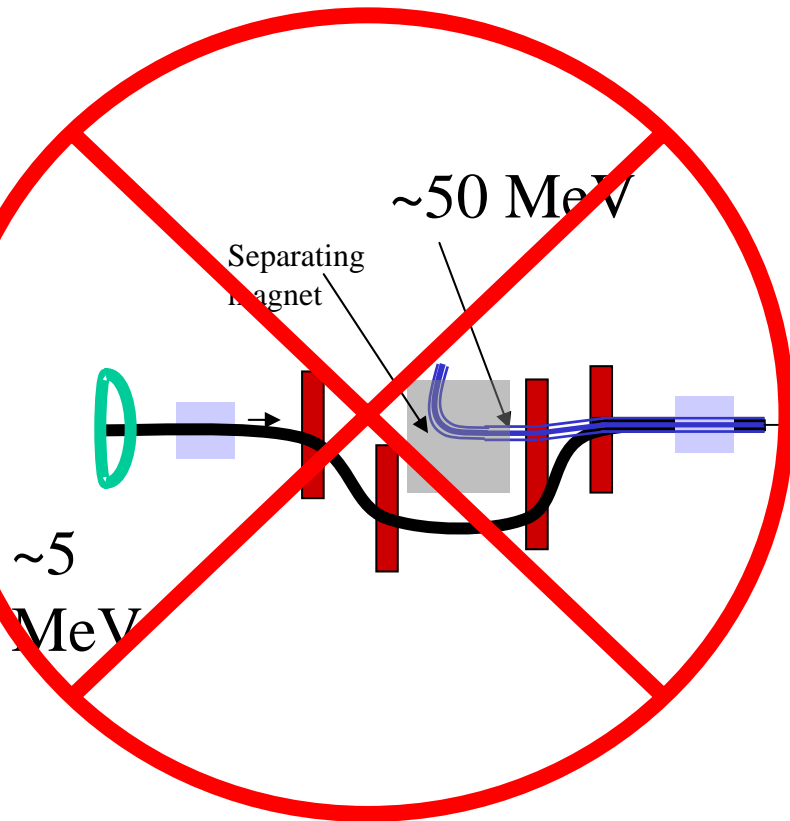
- After initial acceleration, space-charge field is mainly transverse (beam is long in rest frame).
- Both radial and longitudinal forces scale as γ^{-2}
- Transverse force dependent almost exclusively on local value of current density I / σ^2

$$\sigma_x''(\zeta, s) + K_\beta^2 \cdot \sigma_x(\zeta, s) = \frac{r_e \lambda(\zeta)}{2\gamma^3 \sigma_x(\zeta, s)} + \frac{\mathcal{E}_{n,x}^2}{2\gamma \sigma_x^3(\zeta, s)}$$

$$\zeta = s - v_b t$$

$$I(\zeta) = \lambda(\zeta) \cdot v_b$$

Simple-minded merger for ERL - an achromatic system simply does not work



New Emittance spoilers - nonlinear coupling between longitudinal motion and transverse motion in the bending plane

$$Z = \begin{pmatrix} x \\ x' \\ \xi \\ \delta \end{pmatrix}$$

$$Z(s_1) = M(s_0|s_1) : Z(s_0)$$

$$Z = Z_o + \delta Z; \quad \delta Z(s_1) = M(s_0|s_1, Z_o) \cdot \delta Z(s_0);$$

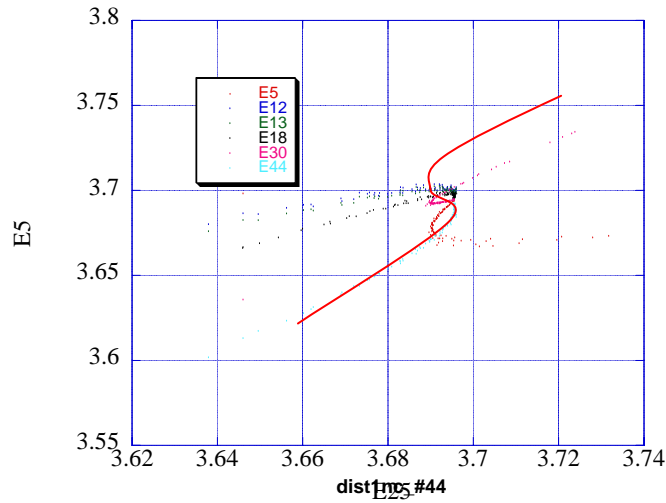
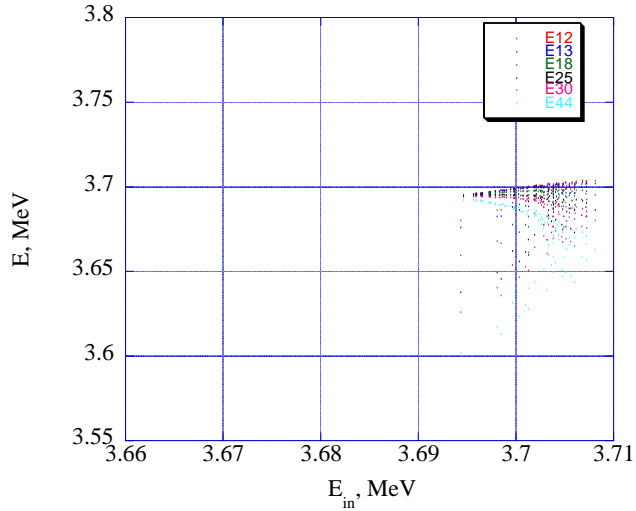
$$M^T \cdot S \cdot M = S; \quad M = \begin{bmatrix} M & P \\ Q & N \end{bmatrix}; \quad S = \begin{bmatrix} \sigma & 0 \\ 0 & \sigma \end{bmatrix}; \quad \sigma = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Decoupling separates the bending from the emittance compensation:

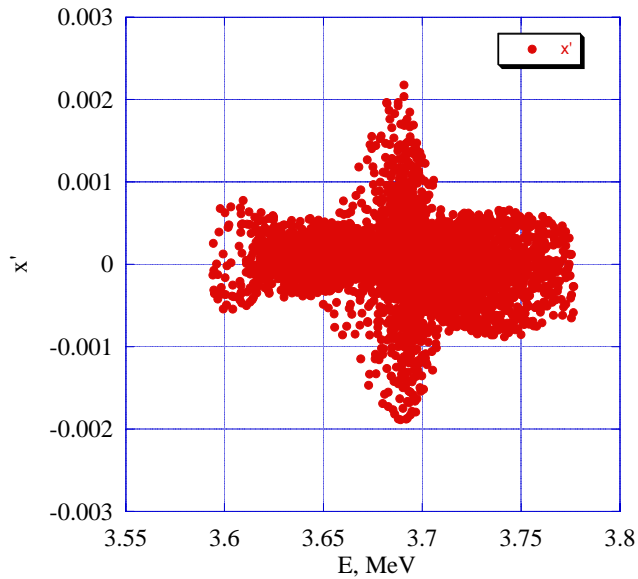
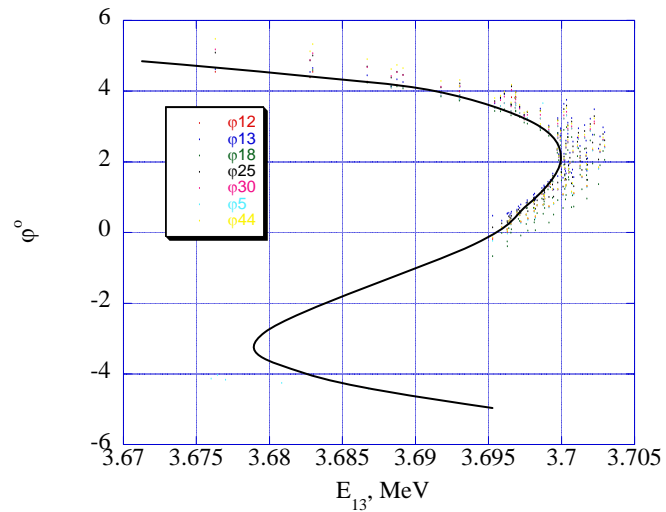
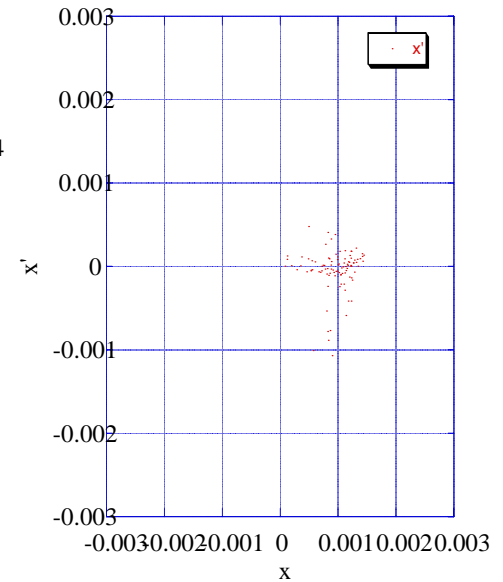
$$M^T \sigma P + Q^T \sigma N = 0 \quad \Rightarrow \quad P = \sigma M^{-1T} Q^T \sigma N \quad \Rightarrow \quad Q \equiv 0!!!!$$

Mess or what?

1.5 cell SRF gun

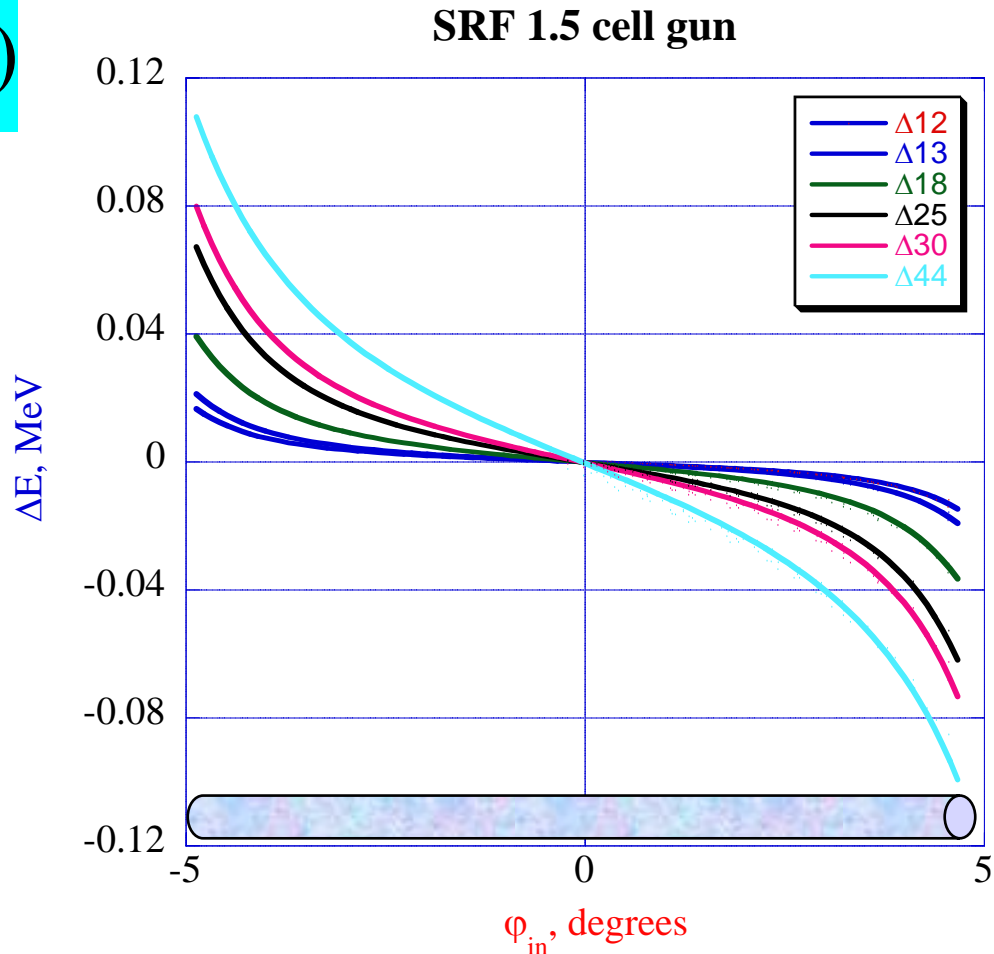


dist1nc_#44



Something is predictable

$$\frac{dE}{ds} \cong f(\zeta_i)$$



Almost ideal fit to the field of evenly charged cylinder

$$\frac{dE}{ds} \cong eE(\zeta); \quad E(\zeta) = \frac{2Q}{r^2 \cdot 2l} \left(2\zeta - \sqrt{r^2 + (\zeta + l)^2} + \sqrt{r^2 + (\zeta - l)^2} \right)$$

There is a lot of well ordered correlations

$$\Delta E \cong \Delta E_i + f(\zeta_i) \cdot (s + \alpha \cdot s^2)$$

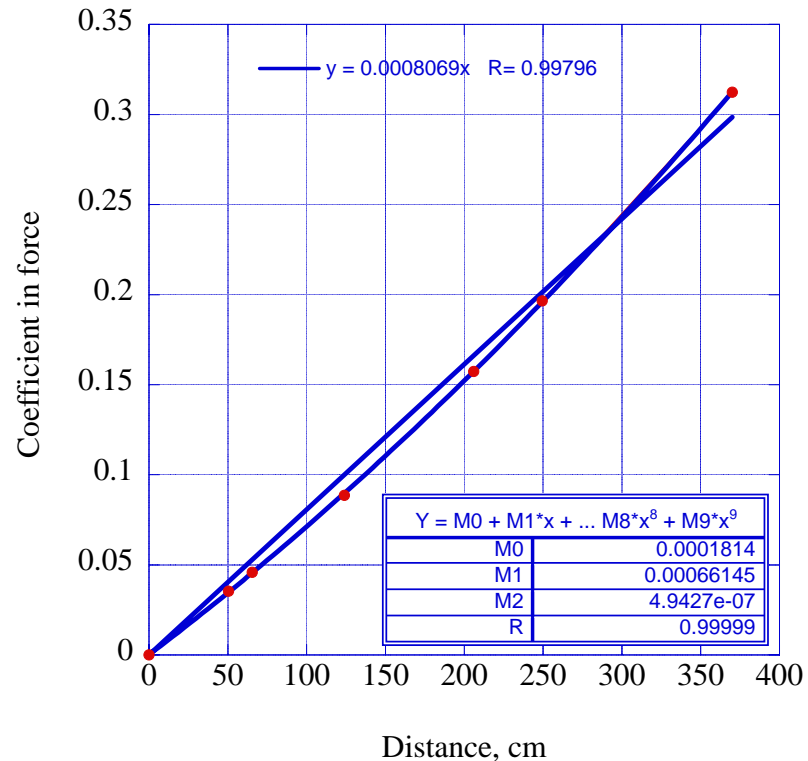
Thus, energy dependence vs s for any electron depends on two parameters - initial energy and initial phase

In general, we seek a general 2-parameter parametrization

$$E_i(s) = a_i \cdot g_1(s) + b_i \cdot g_2(s)$$

—●— D

1.5 cell SRF gun



Concept

$$X = \begin{bmatrix} x \\ x' \end{bmatrix}; \quad \frac{d}{ds} X \equiv X' = D(s) \cdot X$$

free oscillations $X(s) = M(s) \cdot X(0)$

$$M' = D(s) \cdot M; \quad \det M = 1; \quad M(0) = \hat{1}$$

$$\delta = \frac{E - E_o}{E_o}$$

$$\frac{d}{ds} \Psi \equiv \Psi' = D(s) \cdot \Psi + K_o(s) \cdot \delta(s) \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \quad \Psi(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow$$

$$\underline{\underline{\Psi(s) = M(s) \cdot A(s)}} \Rightarrow A' = K_o \cdot \delta \cdot M^{-1} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix};$$

$$M^{-1}(s) = \begin{bmatrix} m_{22} & -m_{12} \\ -m_{21} & m_{11} \end{bmatrix} \Rightarrow A' = K_o \cdot \delta \cdot \begin{bmatrix} -m_{12} \\ m_{11} \end{bmatrix};$$

$$A(s) = \begin{bmatrix} -\int_0^s K_o(s') \cdot \delta(s') m_{12}(s') ds' \\ 0 \\ \int_0^s K_o(s') \cdot \delta(s') m_{11}(s') ds' \\ 0 \end{bmatrix} \Rightarrow A = 0!$$

Concept - cont.

$$\delta = \frac{E - E_o}{E_o}$$

Parametrization for all electrons in the bunch

$$\delta_i(s) = a_i \cdot g_1(s) + b_i \cdot g_2(s) \Rightarrow 4 \text{ "Achromat" conditions}$$

$$\int_0^s K_o(s') \cdot g_1(s) \cdot m_{11}(s') ds' = 0; \int_0^s K_o(s') \cdot g_2(s) \cdot m_{11}(s') ds' = 0;$$
$$\int_0^s K_o(s') \cdot g_1(s) \cdot m_{12}(s') ds' = 0; \int_0^s K_o(s') \cdot g_2(s) \cdot m_{12}(s') ds' = 0;$$

Simple examples: "frozen" longitudinal motion

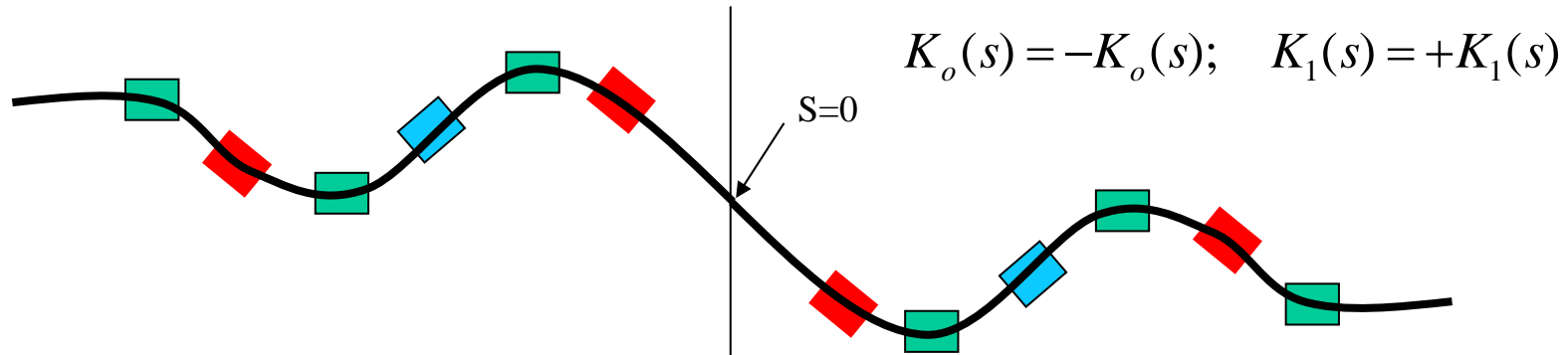
$$\delta = g(\zeta)$$

$$\delta_i(s) = \delta_{i0} + s \cdot g(\zeta_i) \Rightarrow 4 \text{ "Achromat" conditions}$$

$$\int_0^s K_o(s') \cdot m_{11}(s') ds' = 0; \int_0^s K_o(s') \cdot s \cdot m_{11}(s') ds' = 0;$$
$$\int_0^s K_o(s') \cdot m_{12}(s') ds' = 0; \int_0^s K_o(s') \cdot s \cdot m_{12}(s') ds' = 0;$$

System with bilateral symmetry (ZigZag):

Concept - cont.



$$M(-s) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} M(s) \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \Rightarrow m_{11}(-s) = m_{11}(s); m_{12}(-s) = -m_{12}(s)$$

$$K_o(-s) \cdot m_{11}(-s) = -K_o(s) \cdot m_{11}(s) \Rightarrow \int_{-L}^L K_o(s') \cdot m_{11}(s') ds' \equiv 0$$

$$K_o(-s) \cdot (-s) \cdot m_{12}(-s) = -K_o(s) \cdot (s) \cdot m_{12}(s) \Rightarrow \int_{-L}^L K_o(s') \cdot m_{12}(s') s' \cdot ds' \equiv 0$$

$$\int_0^L K_o(s') \cdot m_{12}(s') ds' = 0;$$

$$\int_0^L K_o(s') \cdot s \cdot m_{11}(s') ds' = 0;$$

2 conditions are automatically satisfied

2 conditions remain -> Two elements

Concept - cont.

No focusing

$$m_{11} = 1; \quad m_{12} = s;$$

$$\delta_i(s) = \delta_{i0} + s \cdot g(\zeta_i) \Rightarrow 3 \text{ "Achromat" conditions}$$

$$\int_0^s K_o(s') \cdot ds' = \sum_k \theta_k = 0; \quad \int_0^s K_o(s') \cdot s' \cdot ds' = \sum_k s_k \cdot \theta_k = 0;$$

$$\int_0^s K_o(s') \cdot s' \cdot ds' = \sum_k s_k \cdot \theta_k = 0; \quad \int_0^s K_o(s') \cdot s'^2 \cdot ds' = \sum_k s_k^2 \cdot \theta_k = 0;$$

In such system with bilateral symmetry (ZigZag)

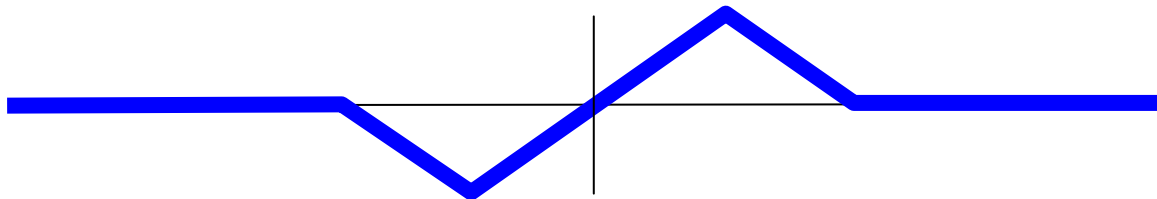
$$K_o(s) = -K_o(s); \quad K_1(s) = +K_1(s)$$

only one condition remains

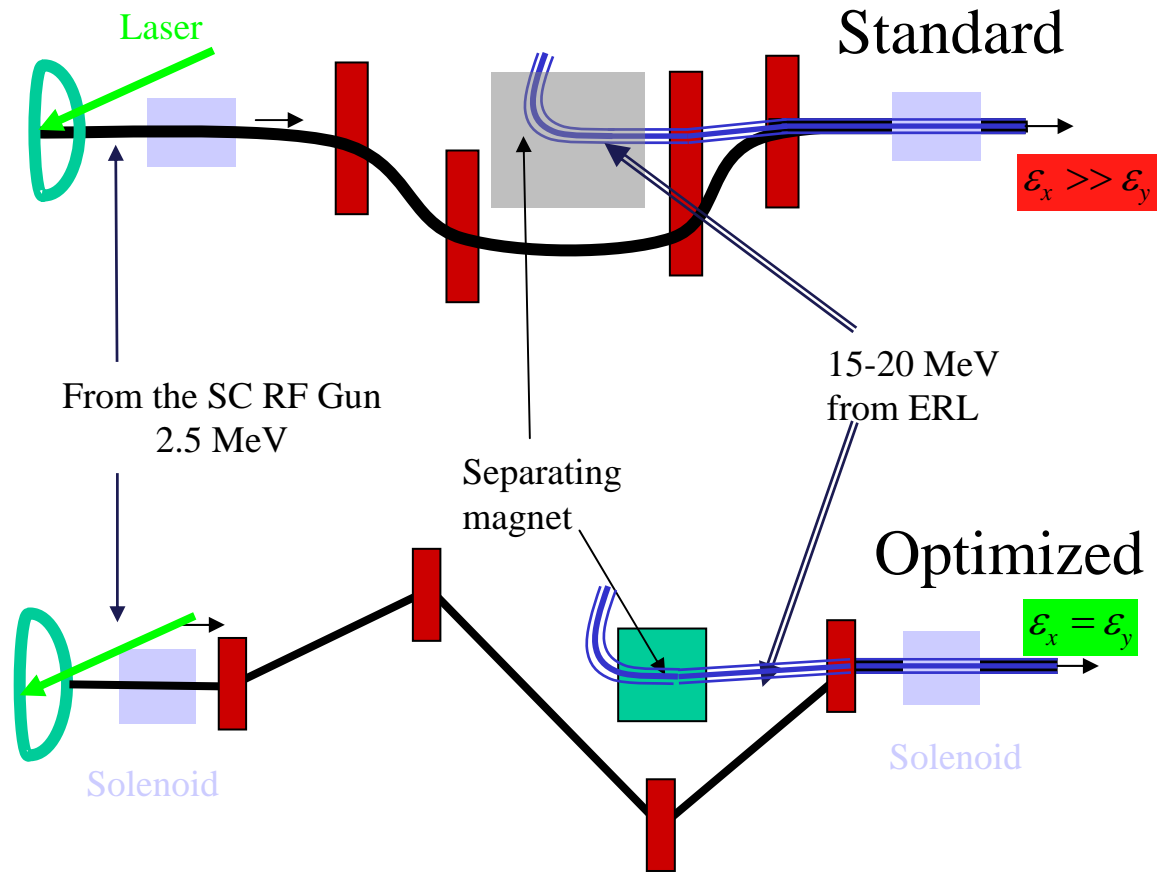
$$\sum_{k=1}^K s_k \cdot \theta_k = 0$$

and it is trivial to satisfy in many ways with $K=2$.

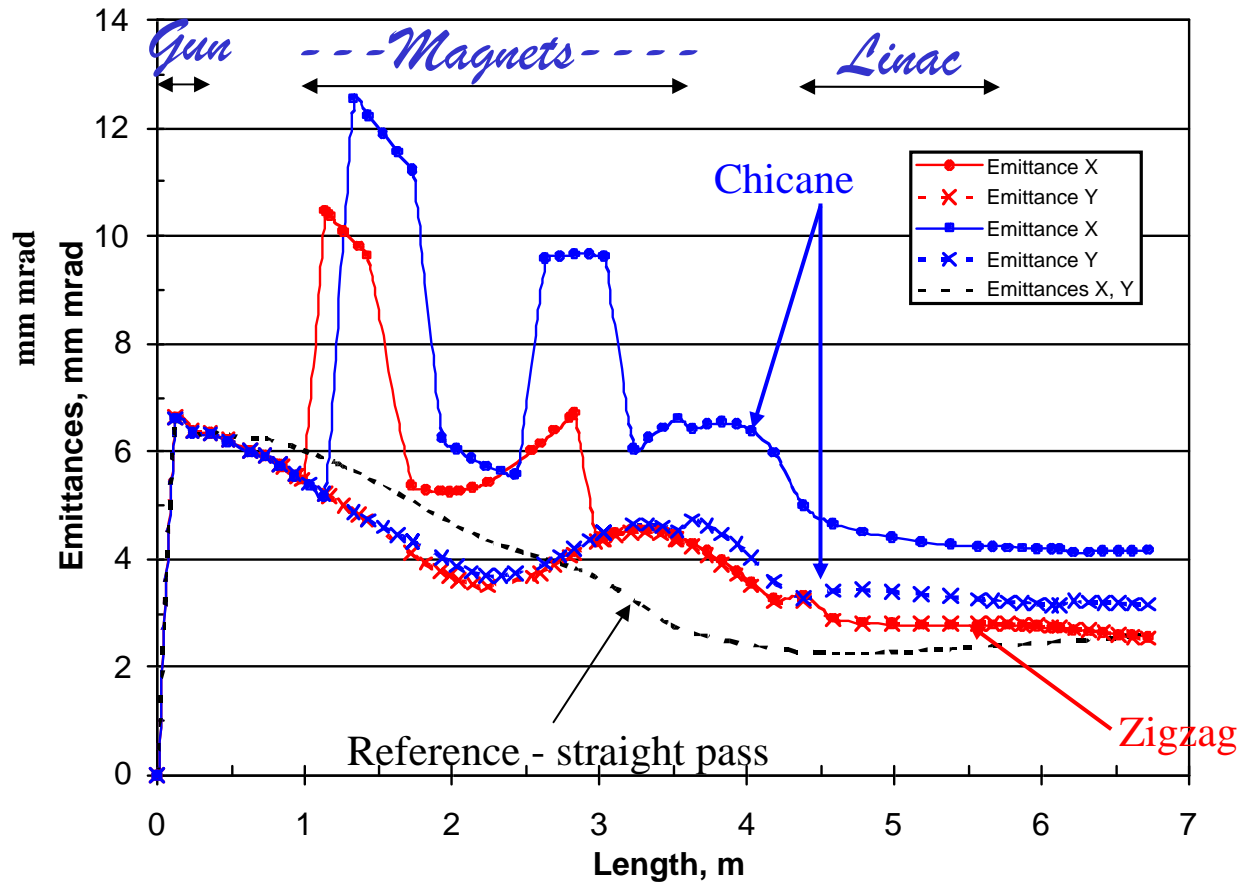
Example: simplest ZigZag $s_2 = 2s_1; \quad \theta_1 = -2\theta_2$



Standard and optimized merging systems



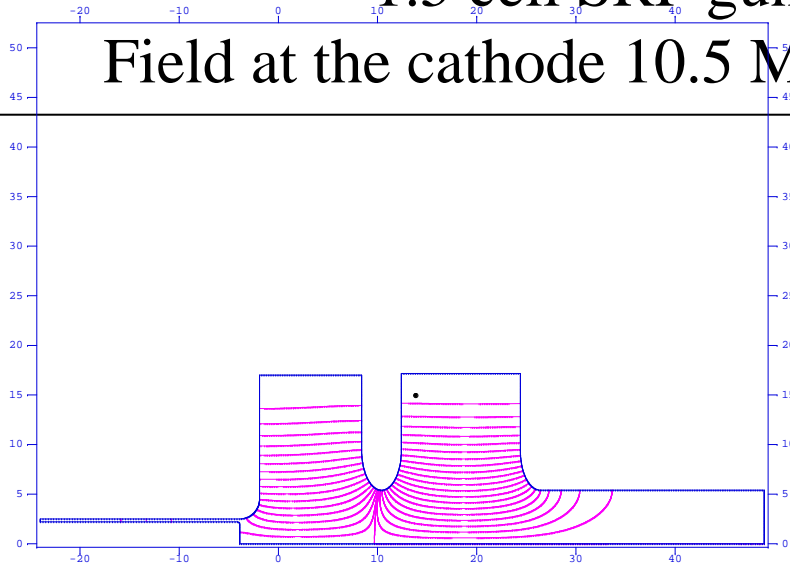
First test



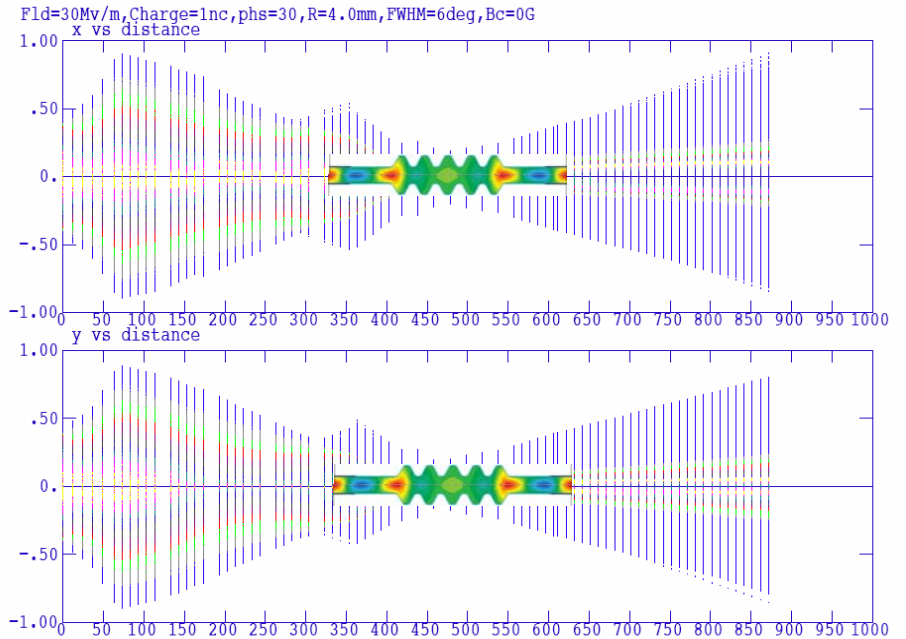
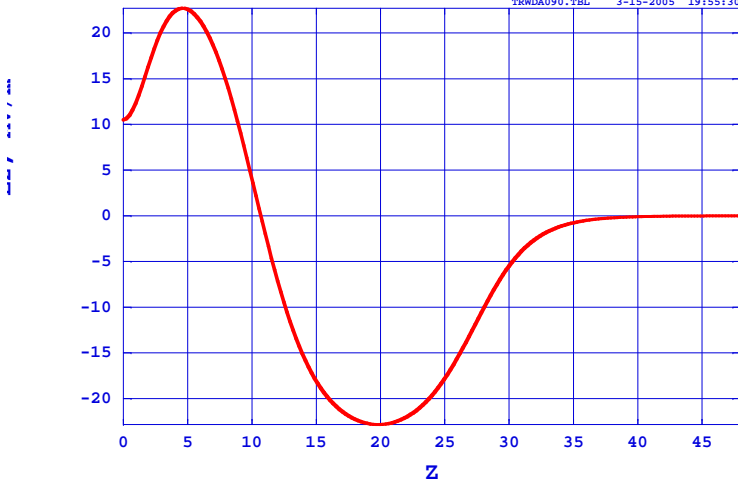
Results of Parmela simulation for 1 nC.

1.5 cell SRF gun - for simulations

Field at the cathode 10.5 MV/m, max field 22.7 MV/m



NPRINT= 200 Z from 0.000000 to 48.00000
PRINTED AT R1=0, R2= 0.000000 WT= 90.000de



Energy after the 3.7 MeV gun $\gamma mc^2=4.2$ MeV,
after the linac $E=18$ MeV.

ZigZag parameters:

all dipoles are chevron, $\rho=1/K_0 = 15$ cm

Lattice 10° bend, 40 cm drift, -20° bend, 81.6 cm,
 20° bend, 40 cm drift, 10° bend

Chicane parameters: the same radii, the same total
focusing and the length:

Lattice 12.4° bend, 447.5 cm drift, -11.36° bend,
96.6 cm, 11.36° bend, 47.5 cm drift, -12.4° bend

Both configurations are achromats for particle with
constant energy.

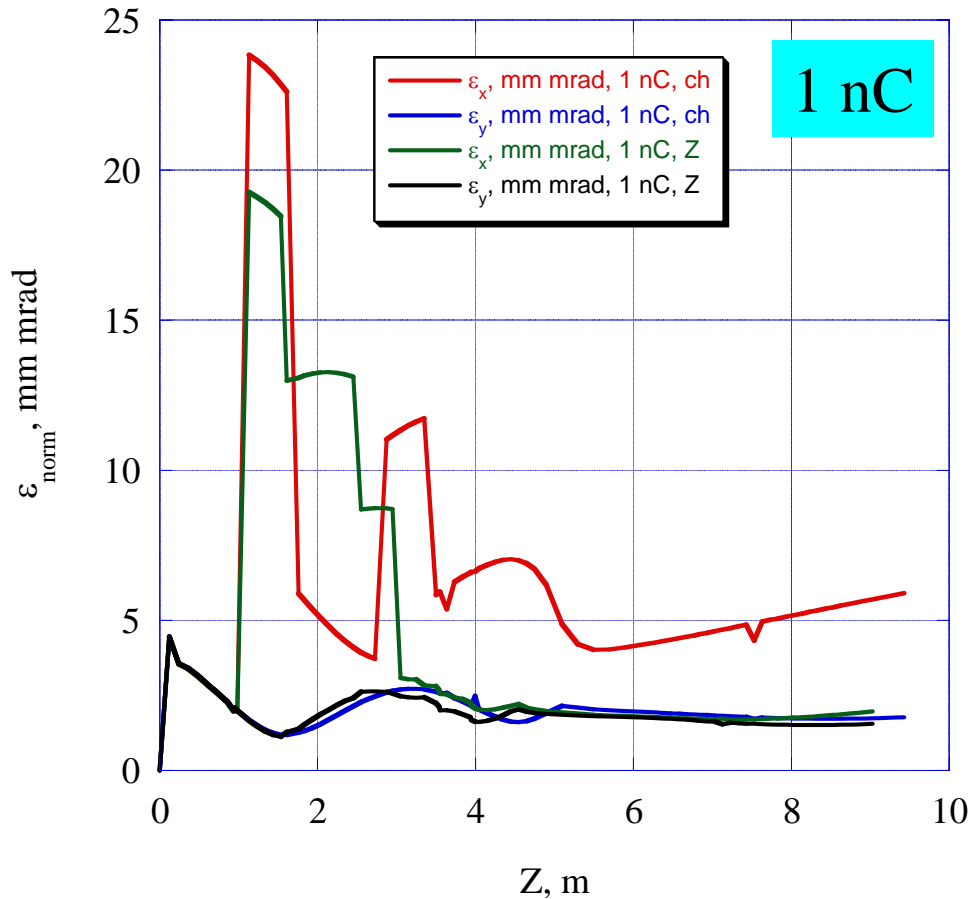
Beer-can distribution, 1.5 cell gun, Q=1, 2, 4 nC

Pulse length 12°, $r_{\text{cath}}=4, 5, 6$ mm

1.5 Cell gun

Z-bend: $\epsilon_{x \text{ norm}} = 1.7$ mm mrad, $\epsilon_{y \text{ norm}} = 1.5$ mm mrad;

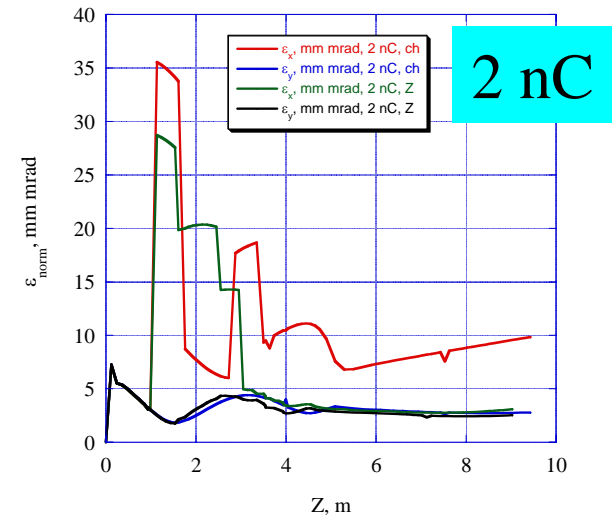
Chicane: $\epsilon_{x \text{ norm}} > 4$ mm mrad, $\epsilon_{y \text{ norm}} = 1.7$ mm mrad



1.5 Cell gun

Z-bend: $\epsilon_{x \text{ norm}} = 2.8$ mm mrad, $\epsilon_{y \text{ norm}} = 2.4$ mm mrad;

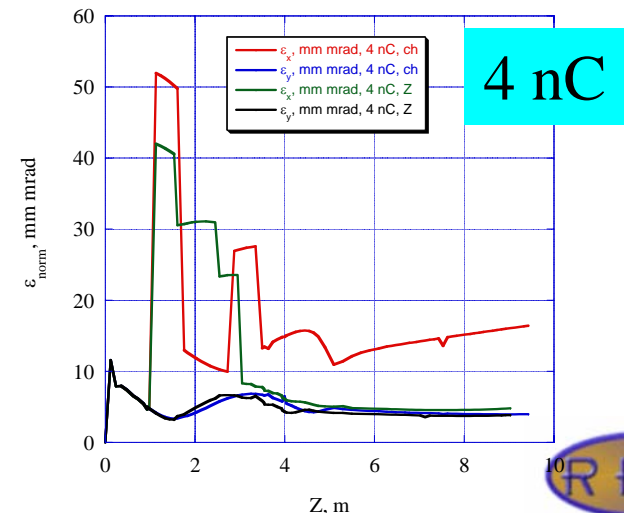
Chicane: $\epsilon_{x \text{ norm}} > 7$ mm mrad, $\epsilon_{y \text{ norm}} = 2.7$ mm mrad



1.5 Cell gun

Z-bend: $\epsilon_{x \text{ norm}} = 4.5$ mm mrad, $\epsilon_{y \text{ norm}} = 3.7$ mm mrad;

Chicane: $\epsilon_{x \text{ norm}} > 11$ mm mrad, $\epsilon_{y \text{ norm}} = 3.9$ mm mrad

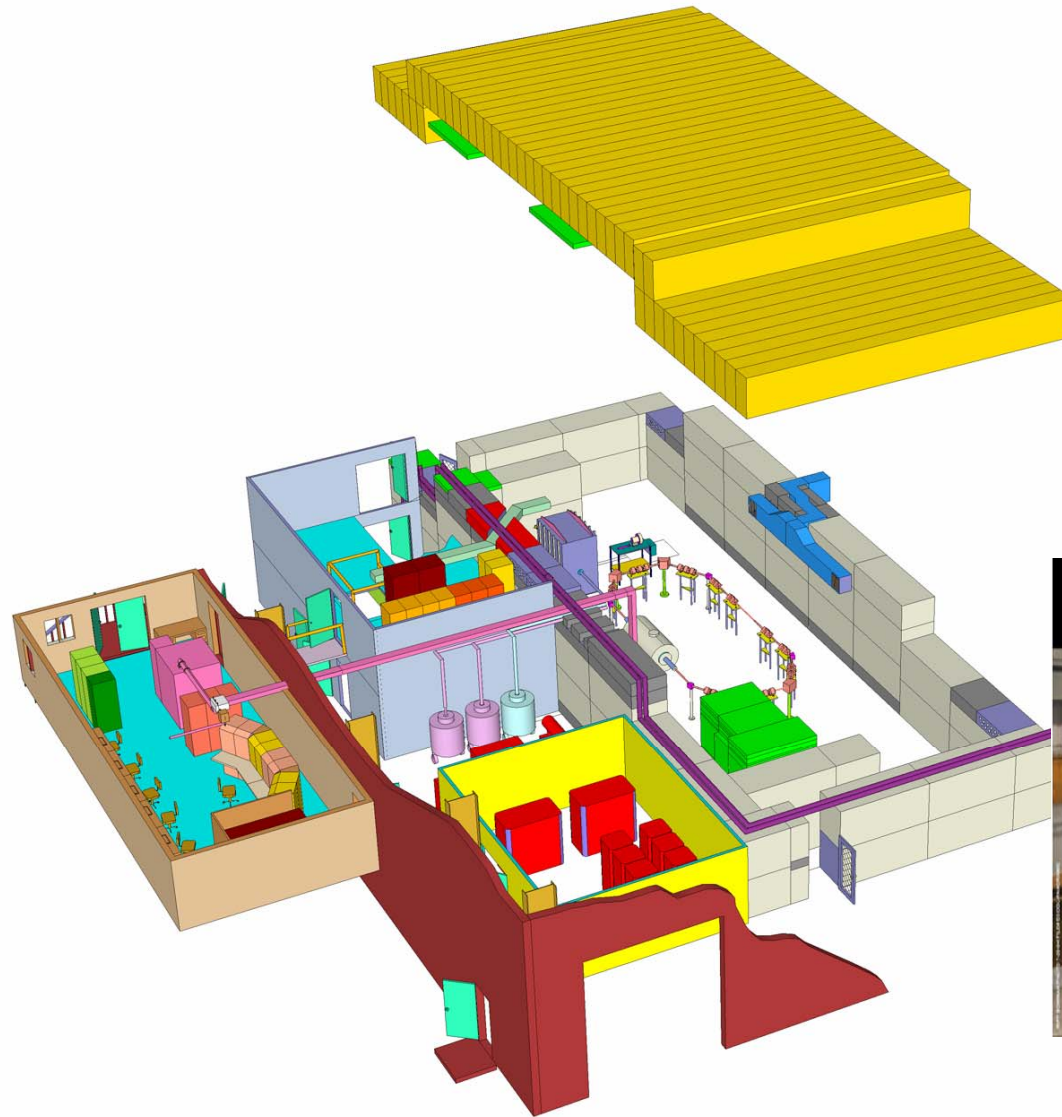


Zigzag works as a magic

Much better than can be expected from a
very-very simple concept

**Most Importantly it is Compatible
with Emittance Compensation scheme**

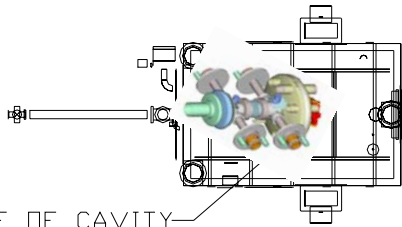
R&D ERL in bldg. 912



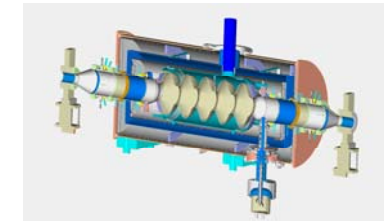
R&D ERL in Bldg.912 - Start-to-End simulations

Half cell, Max Field=28 MV/m, Field at cathode 18 MV/m,
Beer-can, Chg=1.4nc, R=2.3 mm, L=12 deg

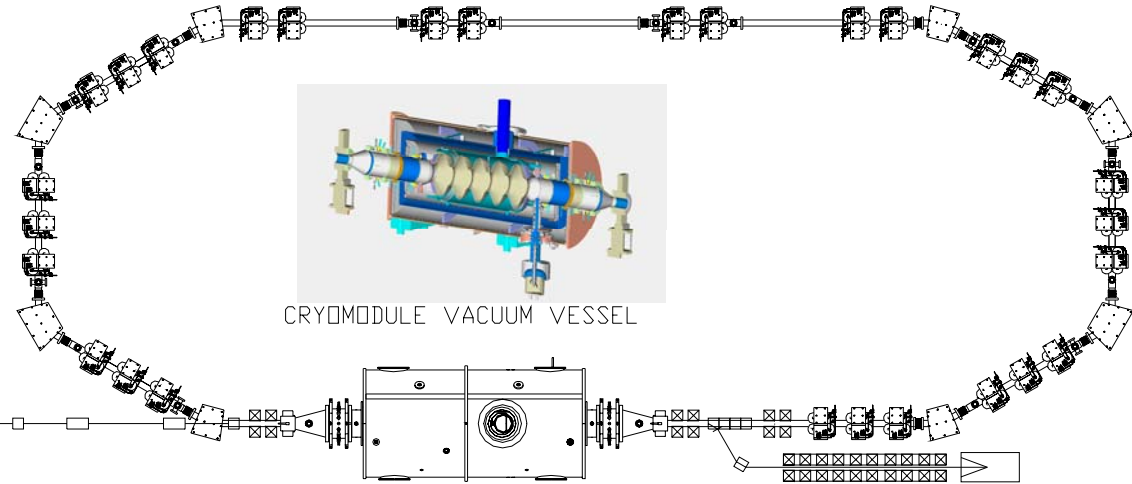
ELECTRON GUN



FACE OF CAVITY

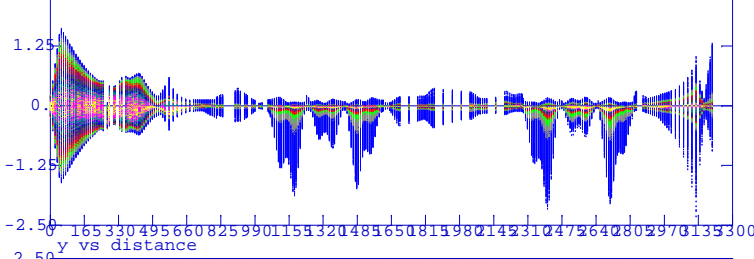


CRYOMODULE VACUUM VESSEL

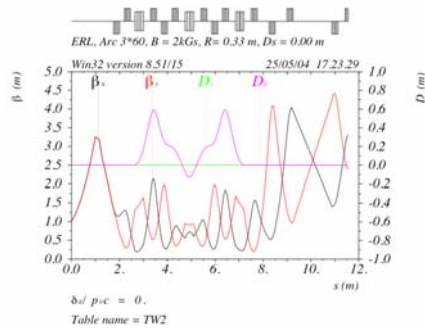
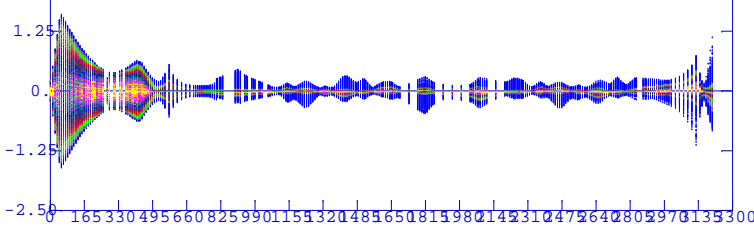


Half cell, Max Fld=28 Mv/m, Fld at cathode =18 MeV/m, Beer-can, Chg=1.4nc, R=2.3 mm, L=12 deg

2.50 x vs distance



2.50 y vs distance



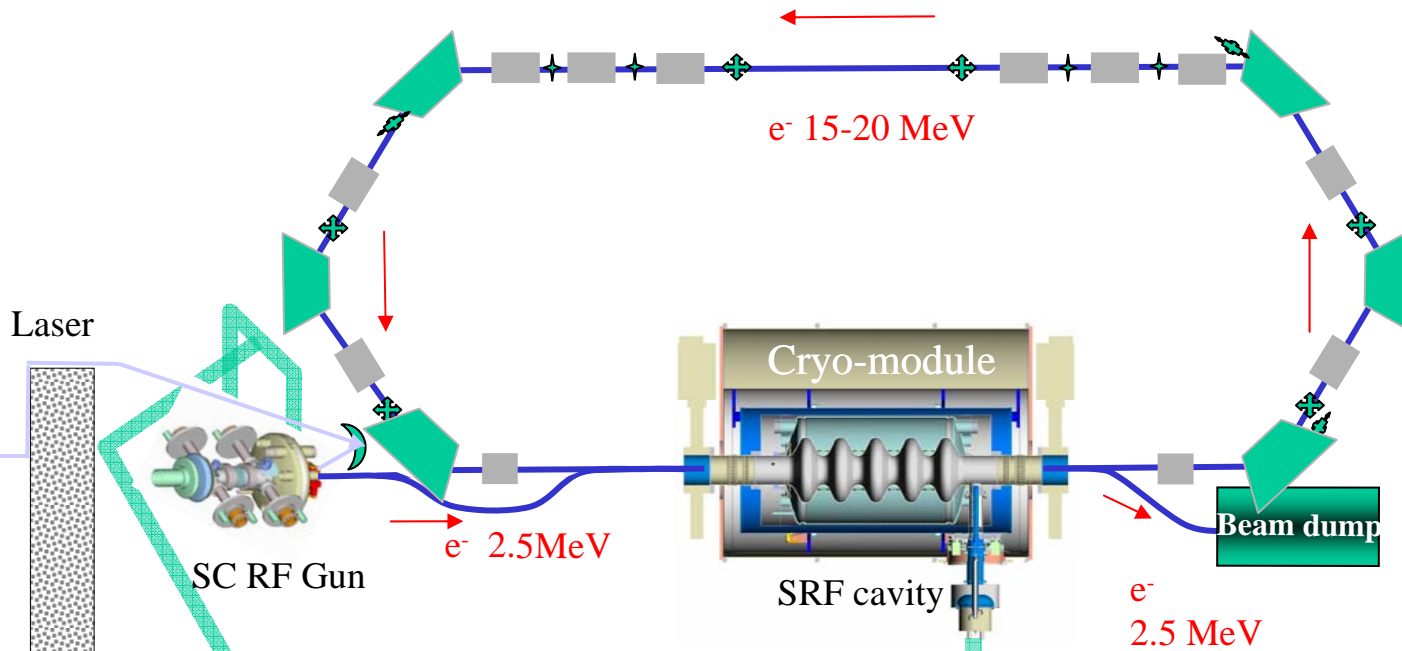
Conclusions

Zigzag works as a magic

Both for low energy and space charge dominated e-beam - works much better than can be expected from a very simple concept

Most Importantly it is Compatible with Emittance Compensation scheme

BNL's 0.5 A average current ERL test-bed in Bldg.912



1 MW
703.75 MHz
Klystron

50 kW 703.75 MHz
system

Control room