Matrix analysis for the effects of CSR and longitudinal space charge force in an ERL

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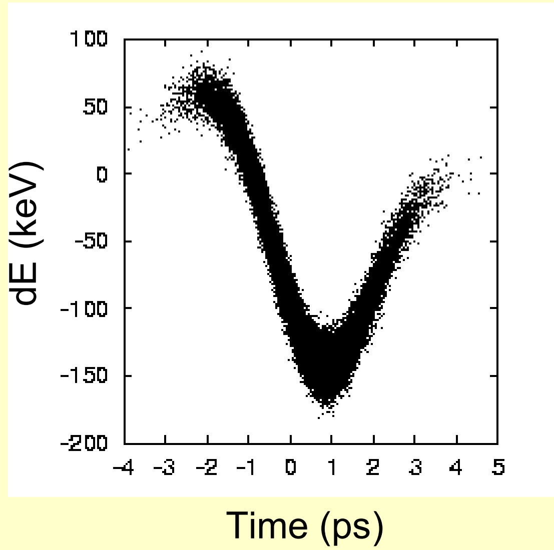
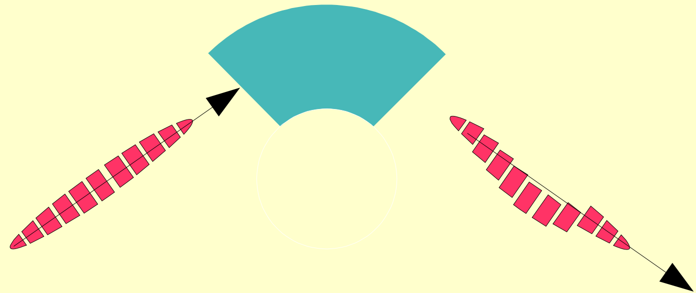
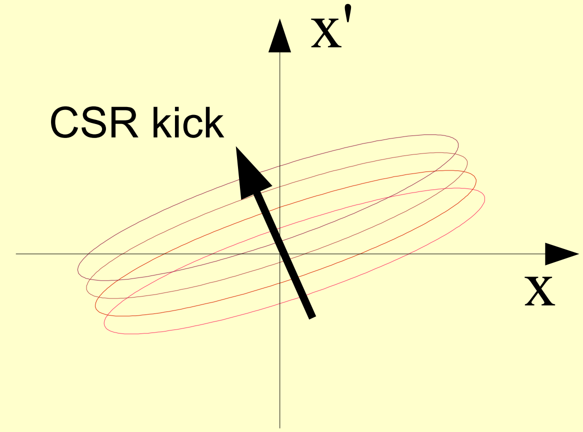
Emittance Growth due to the Coherent Synchrotron Radiation (CSR)







energy change in a bending path → errors in bending angle → emittance growth

$$\Delta E_{ave} = -0.3505 \frac{r_e Q L_b}{e \gamma (R^2 \sigma_s^4)^{1/3}}$$

--- critical for a short bunch

Many studies on bunch compressors for SASE-FELs, experiments and simulations

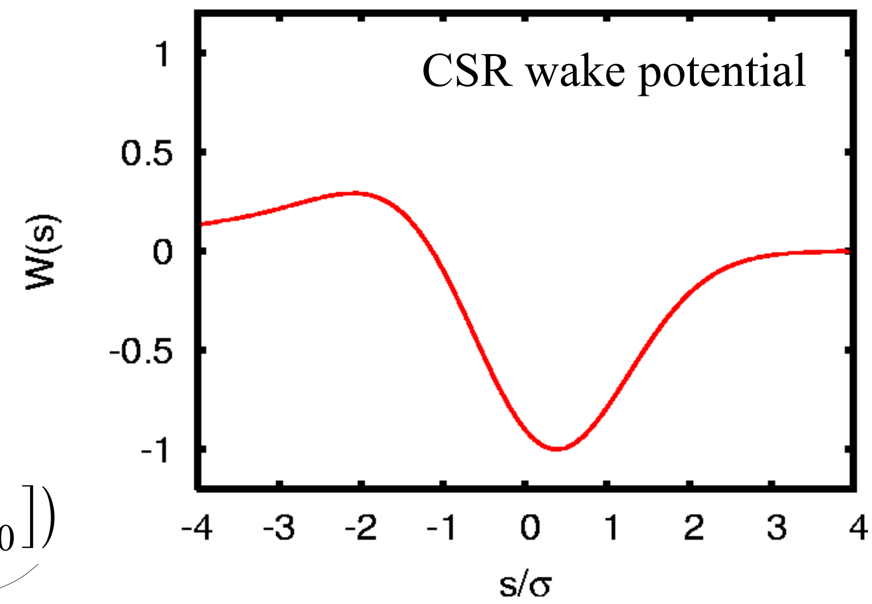
For ERL light sources (femtoseconds, $\varepsilon_n \leq 1$ mm-mrad)

CSR is one of the sources of emittance growth.

Linear Analysis of the CSR Effect on the Transverse Beam Dynamics

1st-order analysis

$$\frac{x}{\rho} \ll 1 \quad \text{and} \quad \frac{\Delta P_{CSR}}{P_0} \ll 1$$



electron's motion in a bending plane

$$x'' = -\frac{x}{\rho^2} + \frac{1}{\rho}\left(\delta_0 + \underbrace{\delta_{CSR} + \kappa\,[s - s_0]}_{\text{for CSR}}\right)$$

x deviation from the reference path

ρ curvature radius of the bending

δ_0 initial momentum error

s coordinate along the path

κ normalized CSR wake potential

δ_{CSR} momentum error by CSR in the upstream path

We assume

- all the dipoles have the same bending radius,
- the bunch does not change its longitudinal profile,
- the transient CSR effect is not large.

→ constant CSR wake regime, valid for GeV ERLs

we can attribute κ to each electron

5x5 R-Matrix for the CSR Analysis

electron's motion in the bending plane is expressed by a vector

$$\vec{x}(s)=(x, x', \delta_0, \kappa L_b, \kappa)^T \qquad \delta_{CSR}(s)=\kappa L_b(s)$$

$L_b(s)$: sum of bending path length in the upstream

before the bend

$$\vec{x}(s_0)=(x, x', \delta_0, \kappa L_b, \kappa)^T$$

after the bend

$$\vec{x}(s_1)=(x, x', \delta_0, \kappa L_b, \kappa)^T$$

transfer matrix of the bend

$$\vec{x}(s_1)=R_{bend}\vec{x}(s_0)$$

5x5 R-matrix for a sector bending magnet

$$R_{bend}=\begin{pmatrix} \cos\theta & \rho\sin\theta & \rho(1-\cos\theta) & \rho(1-\cos\theta) & \rho^2(\theta-\sin\theta) \\ -\rho^{-1}\sin\theta & \cos\theta & \sin\theta & \sin\theta & \rho(1-\cos\theta) \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & \rho\theta \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

extension of the conventional 3x3 R-matrix

R. Hajima, JJAP 42, L974 (2003).

CSR-wake dispersion function

Following the momentum dispersion function “η”,
we define the CSR wake dispersion function “ζ”

momentum dispersion function

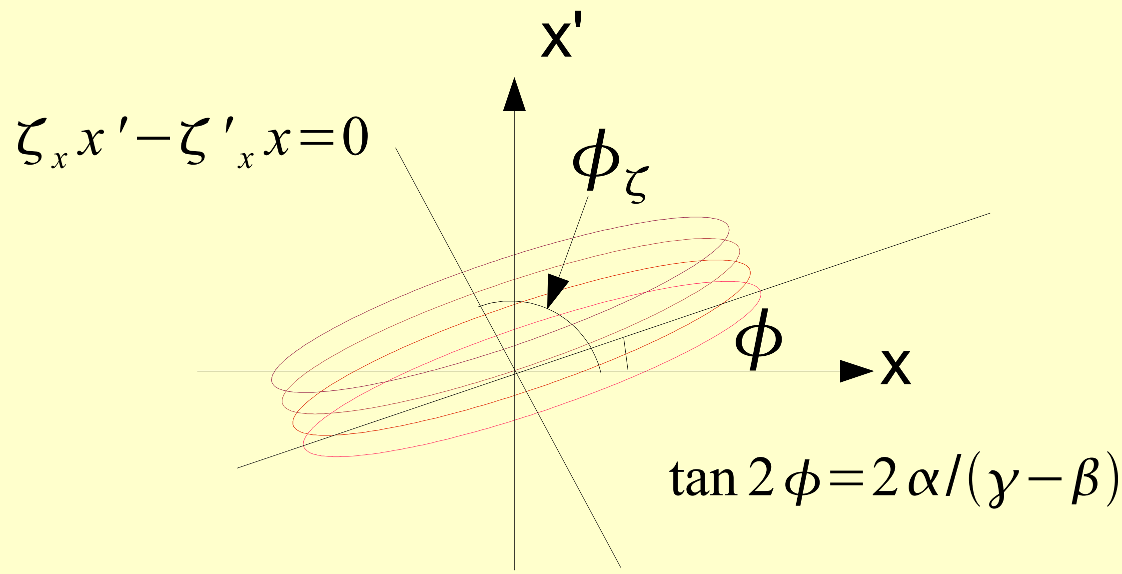
$$(\eta , \eta ')$$

$$\begin{pmatrix} \eta_x(s_1) \\ \eta'_x(s_1) \\ 1 \\ 0 \\ 0 \end{pmatrix} = R_{0\to 1} \begin{pmatrix} \eta_x(s_0) \\ \eta'_x(s_0) \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

CSR wake dispersion function

$$(\zeta , \zeta ')$$

$$\begin{pmatrix} \zeta_x(s_1) \\ \zeta'_x(s_1) \\ 0 \\ L_b(s_1) \\ 1 \end{pmatrix} = R_{0\to 1} \begin{pmatrix} \zeta_x(s_0) \\ \zeta'_x(s_0) \\ 0 \\ L_b(s_0) \\ 1 \end{pmatrix}$$



bunch slices align on a line :

$$\zeta_x x' - \zeta'_x x = 0$$

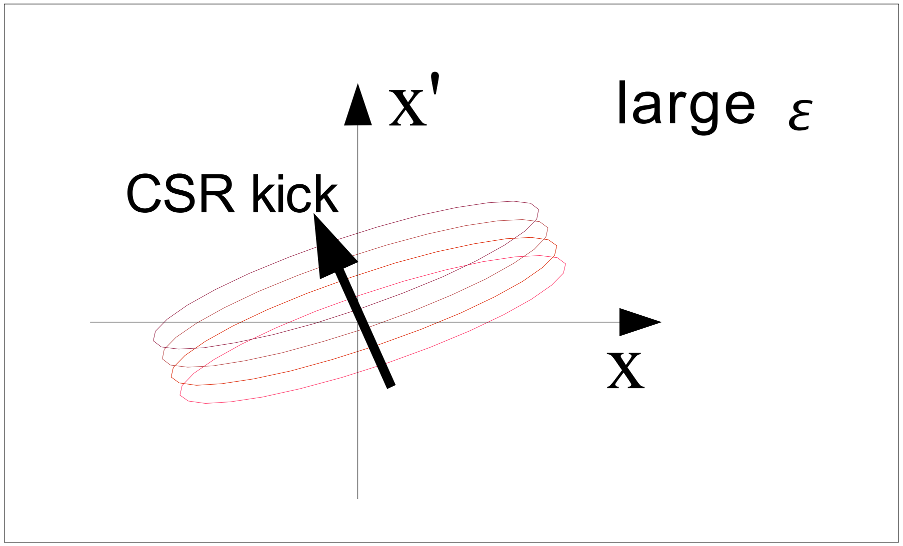
We can track the motion of bunch slices.

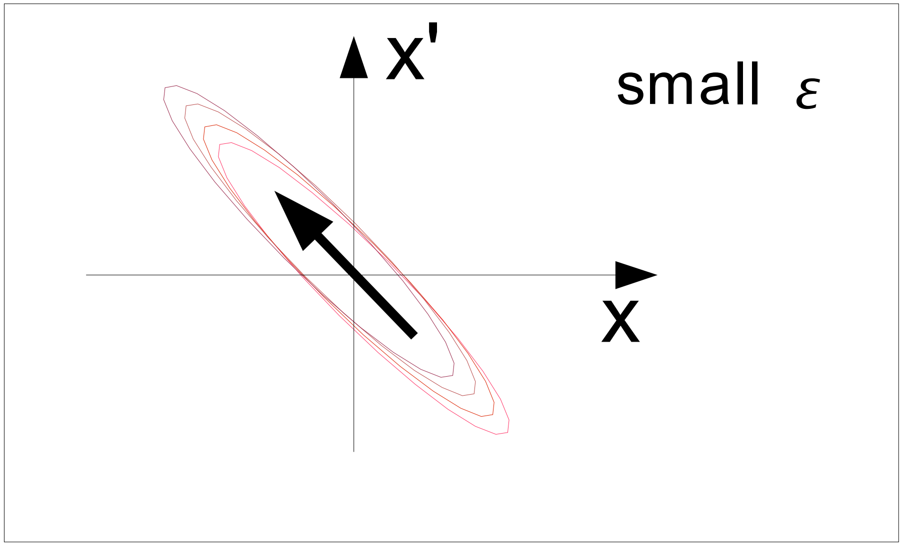
Emittance Growth by CSR, and its Compensation

coincidence between the CSR kick and the phase ellipse orientation.

↓

minimum emittance growth





projection emittance is evaluated by

$$\varepsilon^2 = (\varepsilon_0 \beta_x + D^2)(\varepsilon_0 \gamma_x + D'^2) - (-\varepsilon_0 \alpha_x + D D')^2$$

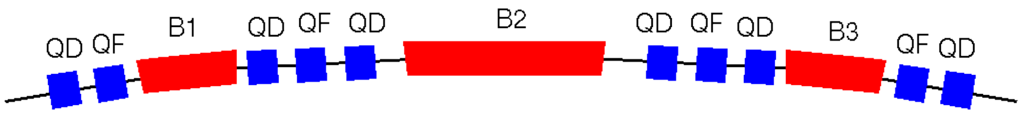
$$(D, D') = \Delta \kappa_{rms} (\zeta_x, \zeta_x')$$

$$\Delta \kappa_{rms} = \frac{\Delta E_{rms}}{E_0 L_b}$$

$$\varepsilon^2 - \varepsilon_0^2 = \varepsilon_0 \underbrace{(\Delta \kappa_{rms})^2}_{\text{bunch parameter}} \underbrace{(\beta_x \zeta'^2 + \gamma_x \zeta^2 - 2 \alpha_x \zeta \zeta')}_{\text{beam transport}}$$

bunch parameter — beam transport

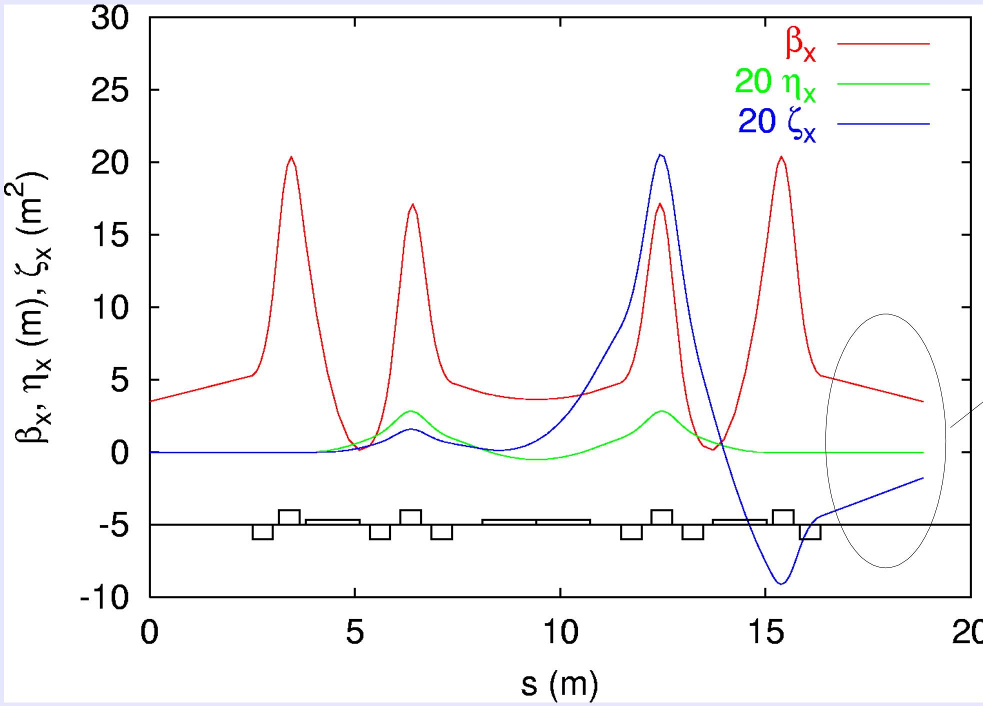
Optimization of a 3-GeV ERL



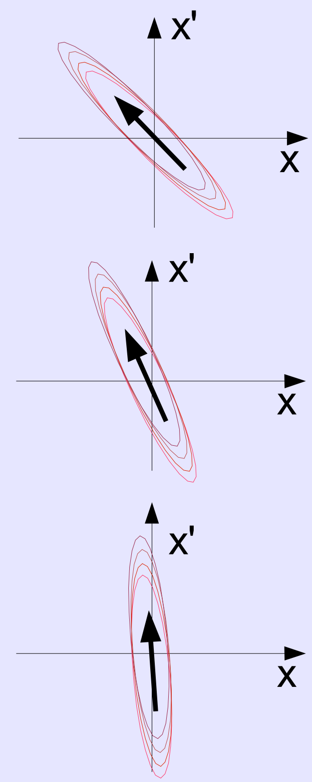
Triple Bend Achromat

$$\rho = 25\,m\,,\ \theta = 3+6+3 = 12\,\text{deg.}$$

design for minimum emittance growth



after the last bend



β_x and ζ_x have the same envelope after the last bend

-- envelope matching

- R-matrix analysis gives the optimum design.
- This analysis is independent of beam parameters, charge, bunch length, ...

Comparison with Particle Simulations

Designed TBA cell and
bunch parameters

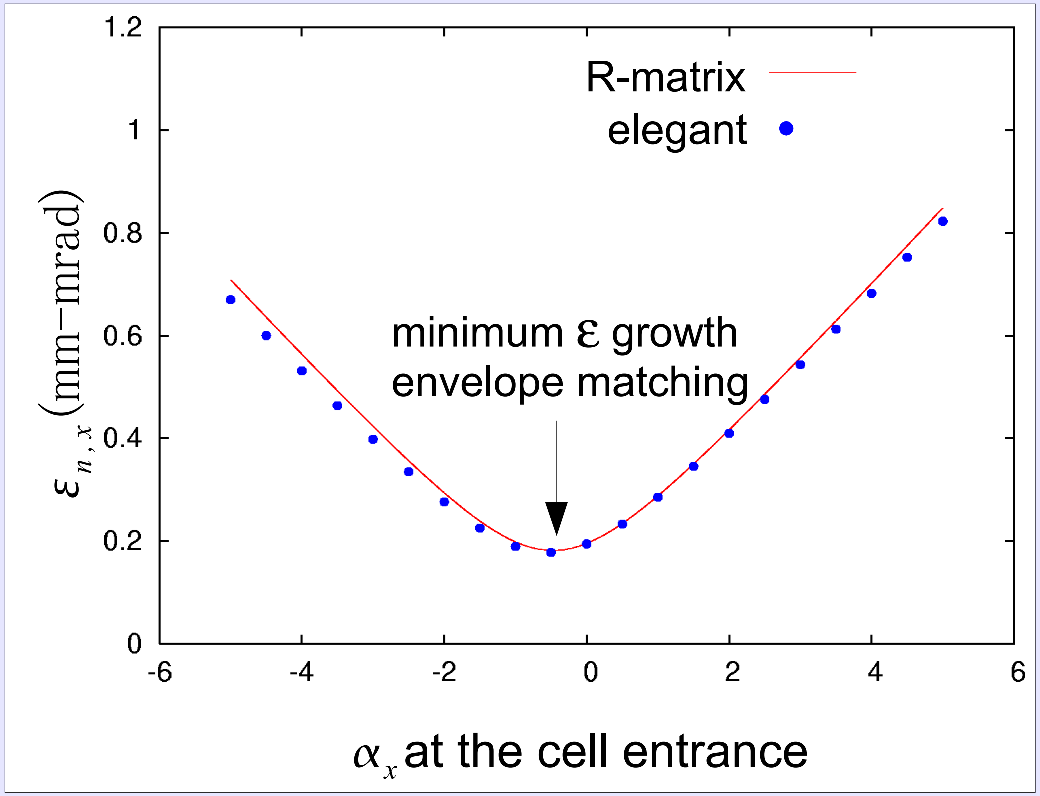
$E_0 = 3.07\,\mathrm{GeV}$ $\sigma_s = 30\,\mu m\,(100\mathrm{fs})$
$Q = 770\,\mathrm{pC}$ $\varepsilon_{n,0} = 0.1\,\text{mm-mrad}$

calculation of emittance
with scanning beam envelope

$-5 < \alpha_x < 5, \gamma_x = 0.29\,m^{-1}$
(at the cell entrance)

comparison with
a simulation code `elegant`*

* M.Borland, PR-STAB 4, 070701(2001)

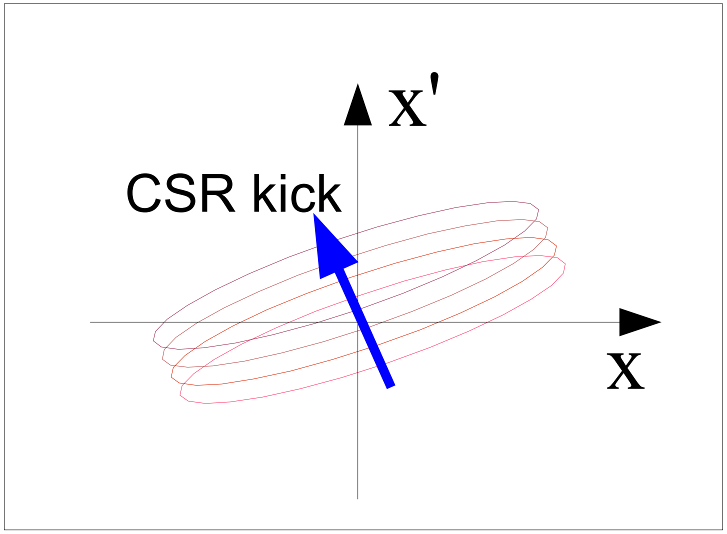


- R-Matrix method gives an optimum design and predicts the emittance growth.

Emittance Growth by Shielded CSR

projection emittance is evaluated by

$$\varepsilon^2 - \varepsilon_0^2 = \varepsilon_0 (\Delta \kappa_{rms})^2 (\beta_x \zeta'^2 + \gamma_x \zeta^2 - 2\alpha_x \zeta \zeta')$$



emittance is a function of CSR wake dispersion and "beam energy-spread" caused by CSR.

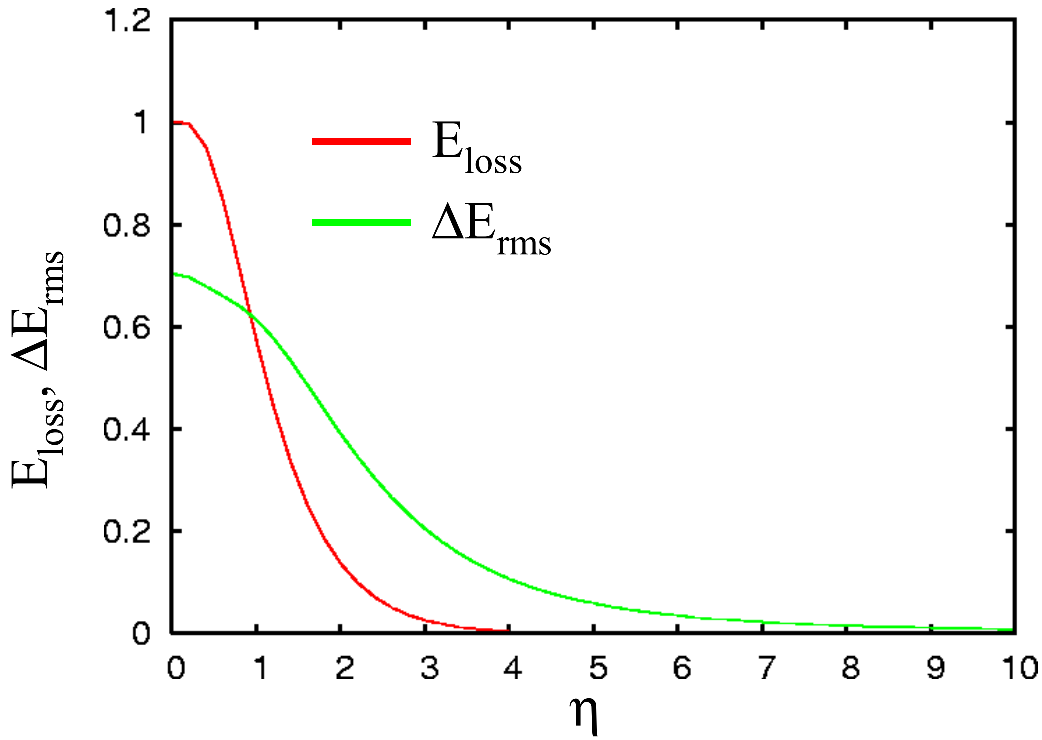
CSR wake dispersion : determined by beam transport geometry

beam energy spread: determined by bunch parameters and shielding.

shielded CSR is given by impedance analysis.

T. Agoh, K. Yokoya, PRST-AB 7, 054403 (2004).
R.L. Warnock, SLAC-PUB-5375 (1990)

Energy Loss, Energy Spread by Shielded CSR



shielding parameter

$$\eta = \sqrt{2/3}\left(\frac{\pi\rho}{h}\right)^{3/2}\left(\frac{\sigma_s}{\rho}\right)$$

ρ : bending radius
h: full gap of two parallel plates
σ_s: RMS bunch length

strong shielding

$\rho = 25\text{m},\ h = 2\text{cm},\ \sigma_s = 900\,\mu\text{m (3ps)} \rightarrow \eta = 7.2$

$$\frac{P_{CSR}(\text{shielded})}{P_{CSR}(\text{free space})} = \frac{E_{loss}(\text{shielded})}{E_{loss}(\text{free space})} = 1.2\times10^{-5}$$

$$\frac{\Delta\varepsilon(\text{shielded})}{\Delta\varepsilon(\text{free space})} = \frac{\Delta E_{rms}(\text{shielded})}{\Delta E_{rms}(\text{free space})} = 0.029$$

we define: $\Delta\varepsilon = \sqrt{(\varepsilon^2 - \varepsilon_0^2)}$

weak shielding

$\rho = 25\text{m},\ h = 2\text{cm},\ \sigma_s = 30\,\mu\text{m (100fs)} \rightarrow \eta = 0.24$

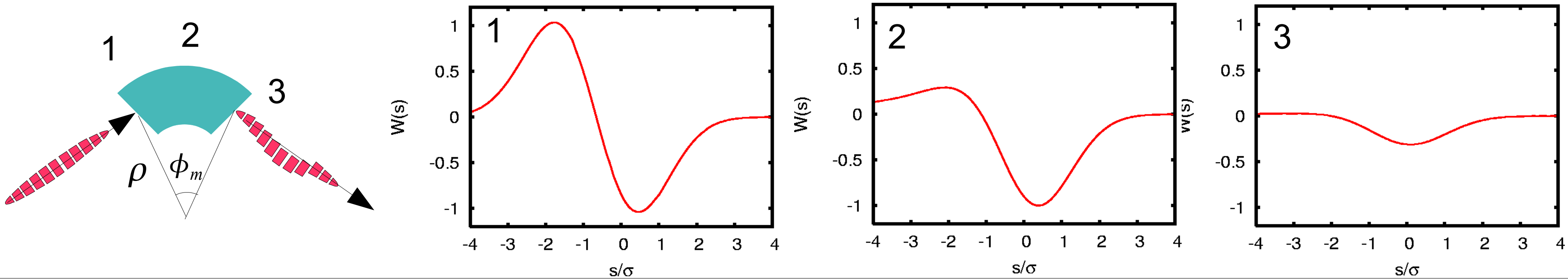
$$\frac{P_{CSR}(\text{shielded})}{P_{CSR}(\text{free space})} = \frac{E_{loss}(\text{shielded})}{E_{loss}(\text{free space})} = 0.99$$

$$\frac{\Delta\varepsilon(\text{shielded})}{\Delta\varepsilon(\text{free space})} = \frac{\Delta E_{rms}(\text{shielded})}{\Delta E_{rms}(\text{free space})} = 0.99$$

Effect of Transient CSR

transient CSR

CSR wake potential shows different profile along the path.



transient CSR is neglected for

$$\rho/\gamma^3 \ll l_b \ll \rho\phi_m^3/24$$

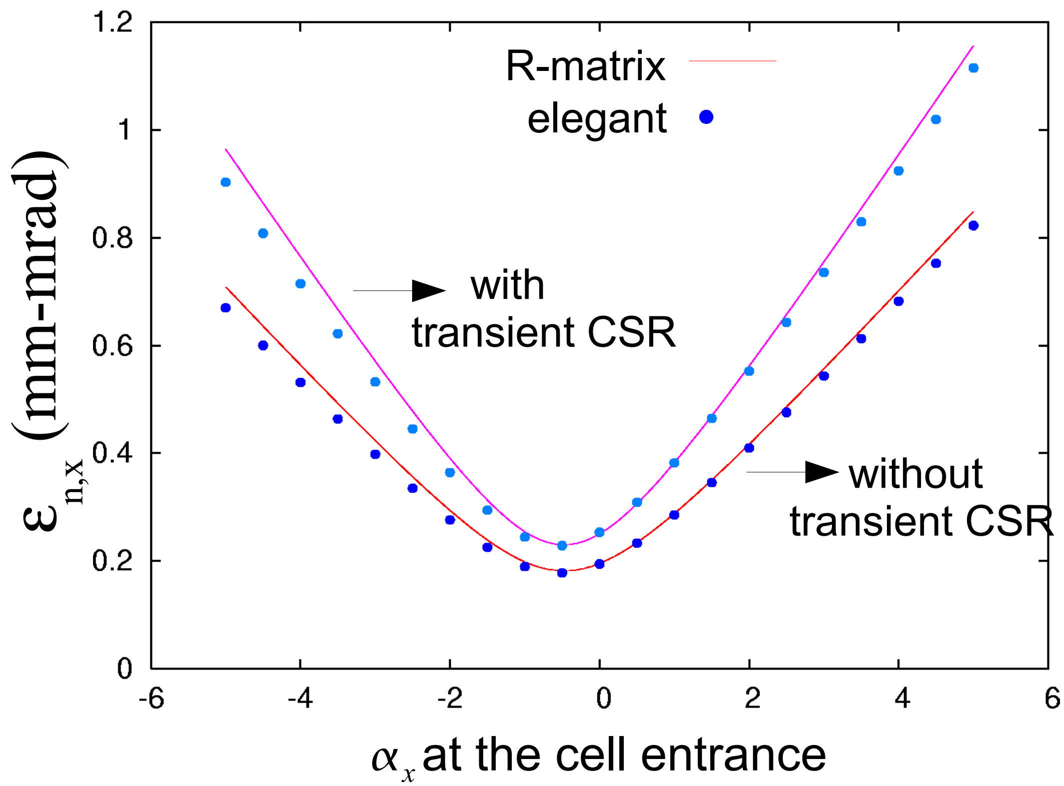
in the 3GeV ERL,

$$\rho/\gamma^3 = 1\times10^{-10},\ l_b = 3\times10^{-5},\ \rho\phi_m^3/24 = 1.5\times10^{-4}$$

correction of $\Delta\kappa_{rms}$ for the transient CSR

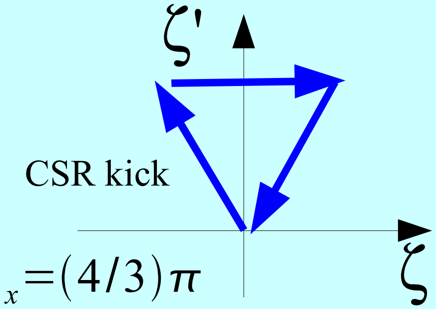
$$\Delta\kappa_{rms}^{transient} = \frac{\Delta E_{rms}^{transient}}{E_0 L_b}$$

← from elegant

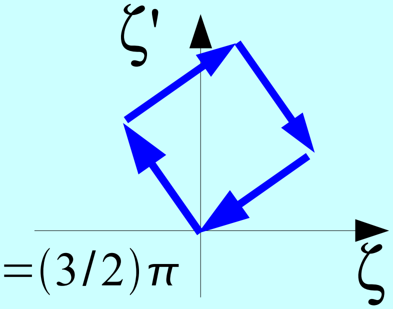


Emittance Compensation by cell-to-cell Phase Matching

cancellation of the CSR kick after a series of cells



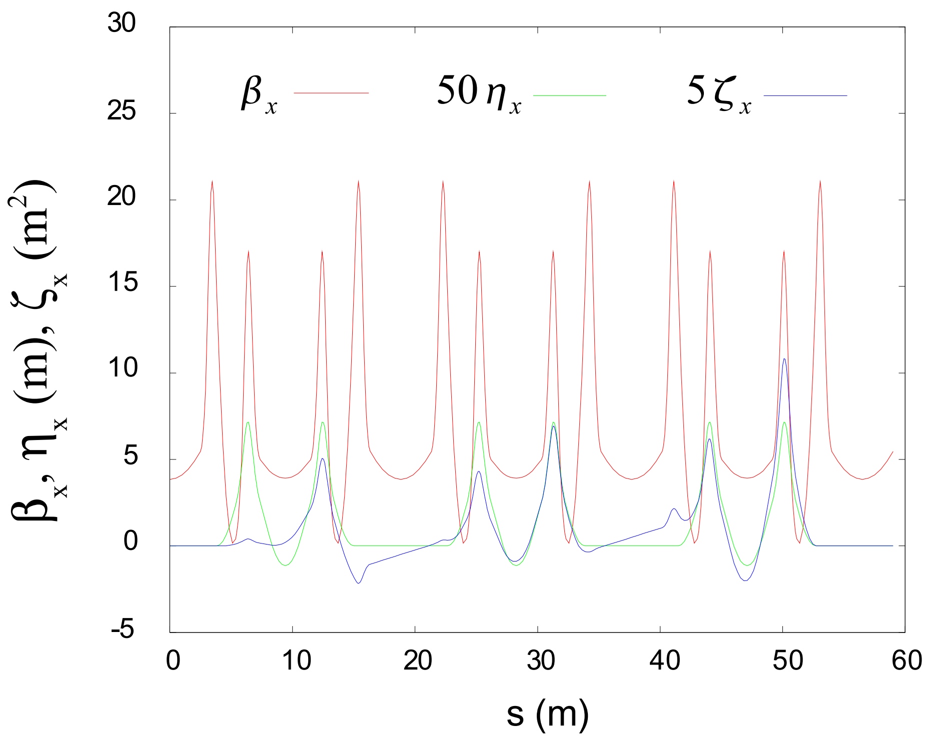
$\Delta\psi_x = (4/3)\pi$



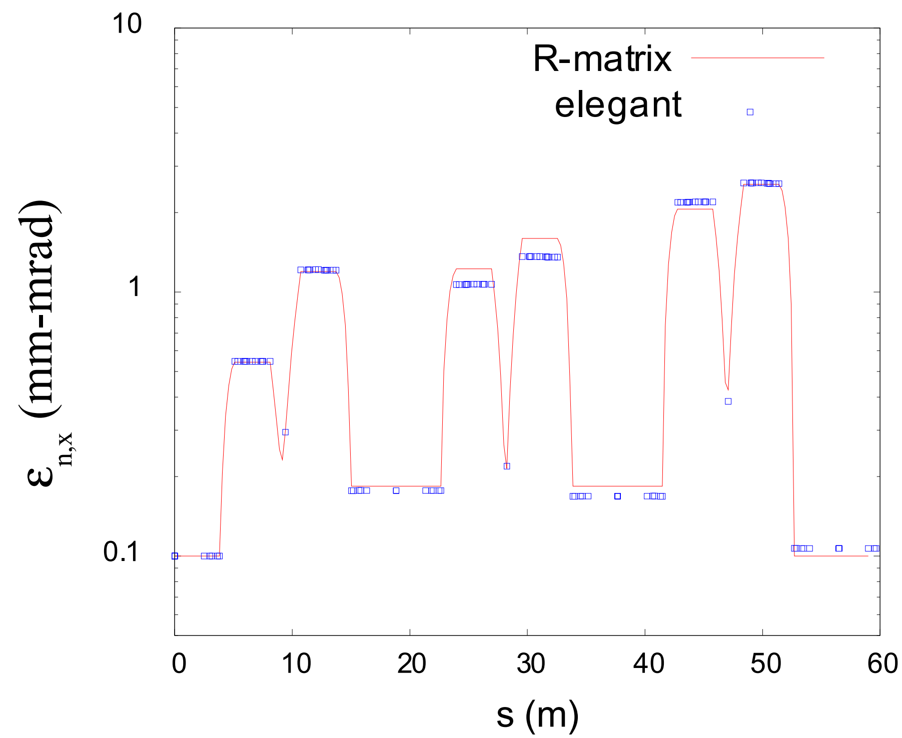
$\Delta\psi_x = (3/2)\pi$

The CSR kick is canceled, if $\Delta\psi_x(1) + \Delta\psi_x(2) + \ldots = 2N\pi$

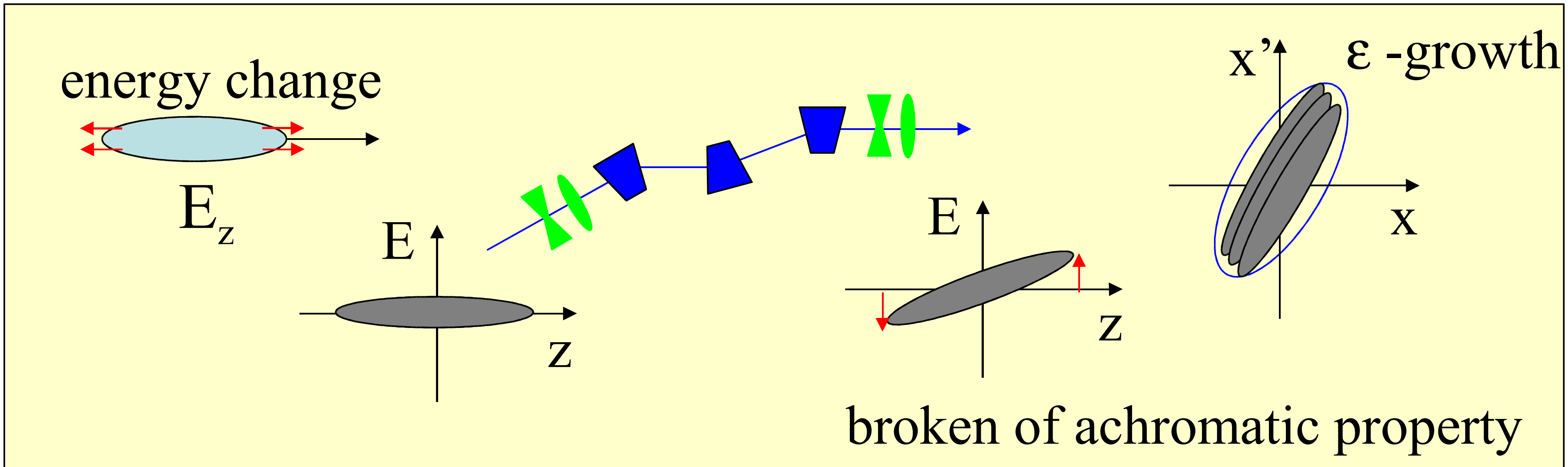
example for $\Delta\psi_x = (8/3)\pi$



emittance compensation every 3-cells



Transverse emittance growth by Ez

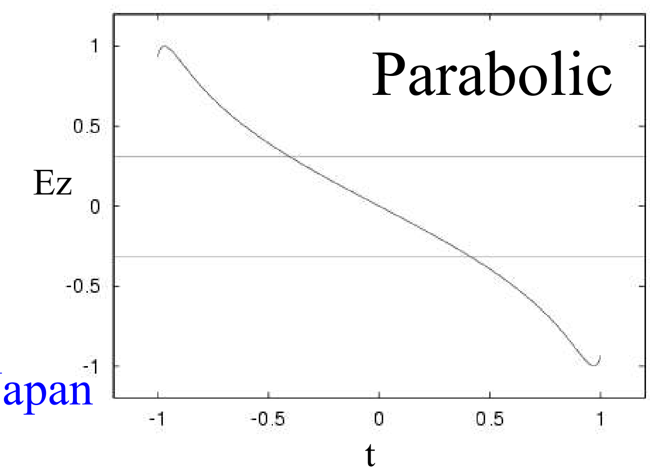


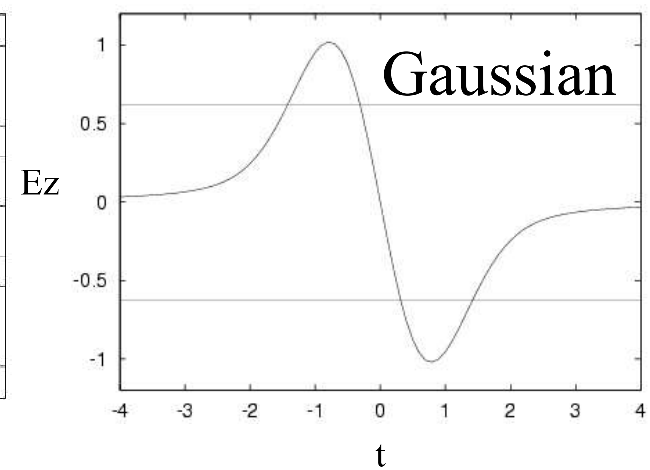
see for example, B.E. Carlsten et al., IEEE QE **27**, 2580 (1991).

assuming steady-state Ez $\Delta E/E = \delta = \delta_0 + \kappa(s - s_0)$

We can track bunch slice motion by linear matrix.

applicable to
arbitrary bunch shape



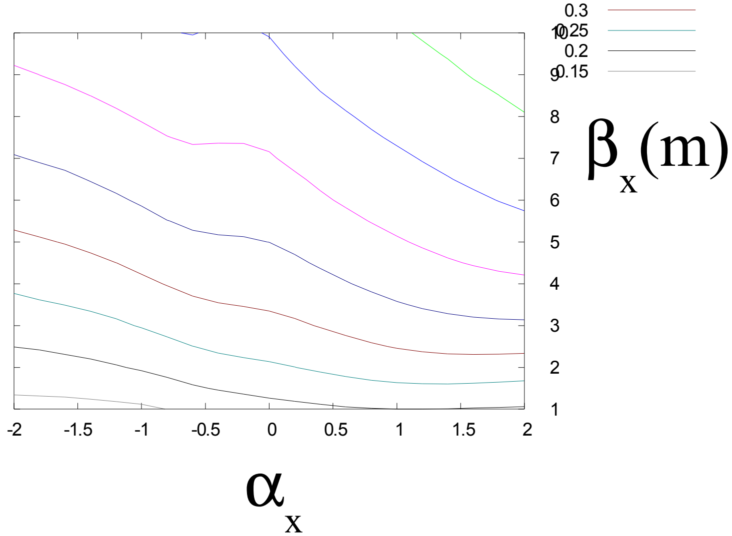


R. Hajima,
Proc. Ann. Meeting of Particle Soc. Japan
p.432 (2004).

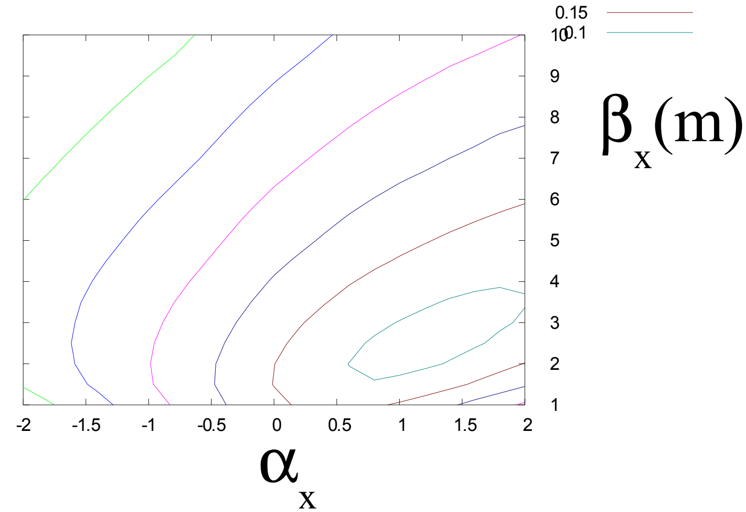
Optimum injection to a 3-dipole merger

calculate emittance growth with varying injection parameters (α_x, β_x)

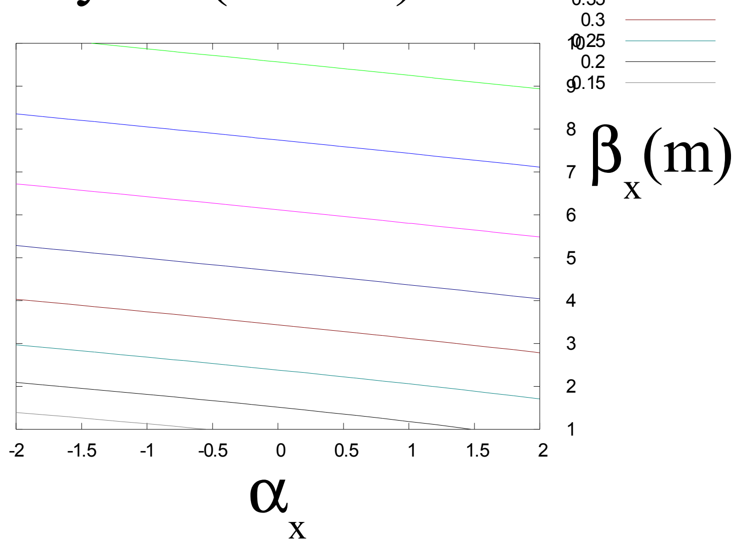
only Ez (simulation)



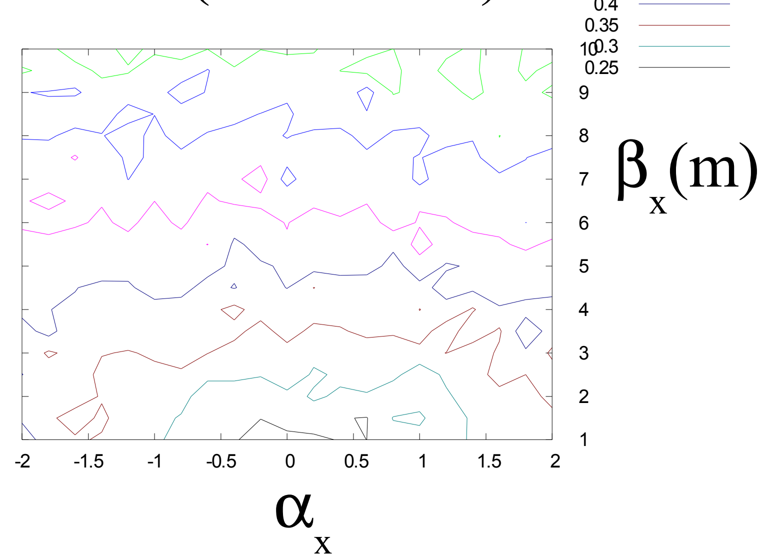
only Er (simulation)



only Ez (matrix)



Er+Ez (simulation)



Summary

- A linear analysis of beam dynamic using R-matrix is extended to the study on CSR-induced emittance growth.
- Two kind of emittance compensation techniques, "envelope matching" and "cell-to-cell phase matching" are presented.
- Emittance growth by shielded CSR and transient CSR is also discussed.
- Emittance growth by longitudinal space charge force can be calculated in a similar way.
- Results from the R-matrix analysis are compared with particle simulations. Both results show good agreement.