Modified Fragmentation Functions due to Multiple Scattering in Nuclei

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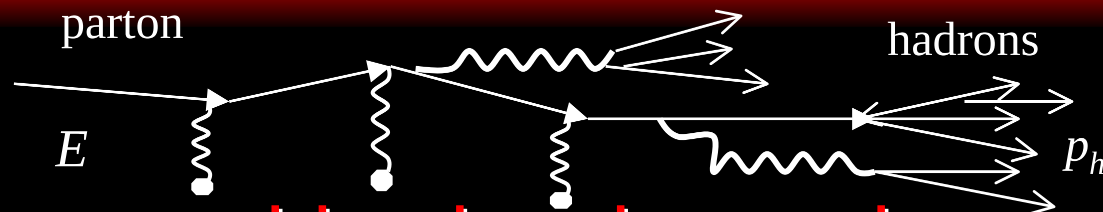
ECT, Trento --- October 4, 2005

Outline

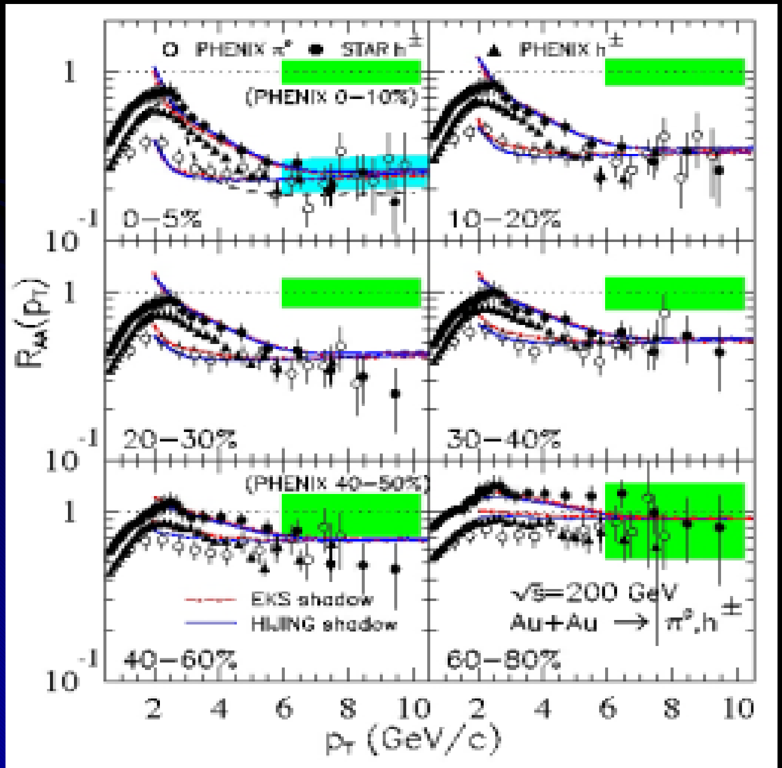
- Introduction
- Modified heavy quark fragmentation function
- Modified FF due to quark-quark double scattering in nuclei
- Summary

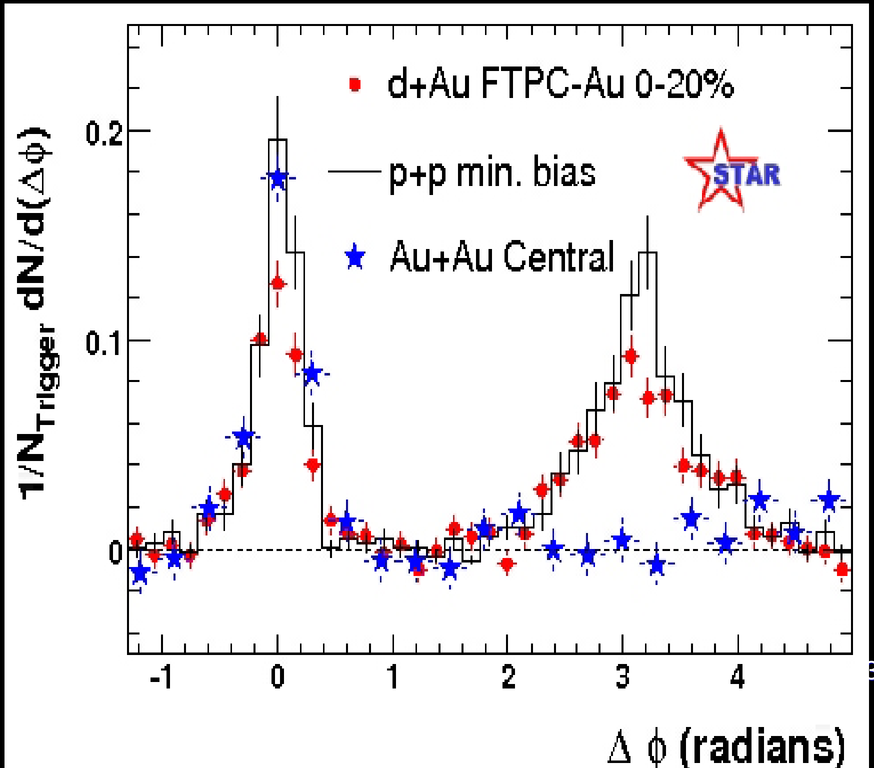
Jet Quenching



Jet quenching has been observed as a new nuclear phenomenon at RHIC

X.N. Wang, Nucl.Phys. A750 (2005) 98





Theoretical approaches

- M. Gyulassy, X.-N.Wang: GW model
- Baier, et al: BDMPS
- Gyulassy, Levai, Vitev: GLV
- Kovner, Wiedemann
- Zakharov

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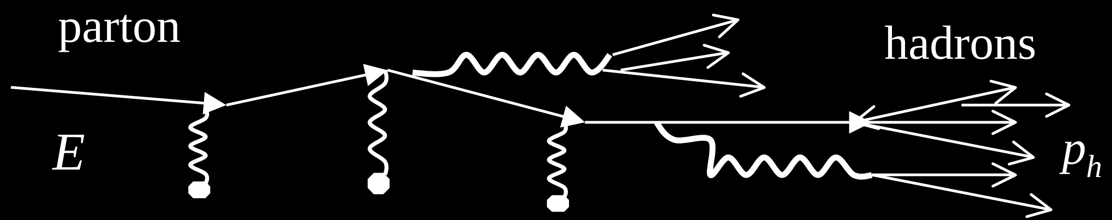
- Twist Expansion approach

X. Guo, X.-N. Wang, Enke Wang, B.W.Zhang

Jet quenching with pQCD



- How to measure the parton energy loss?
- Direct measurement is impossible
- Particle distributions within a jet
- Modification to Fragmentation Function
- Obtain the energy loss indirectly from measuring the modification of FF

$$D_{q\to h}(z_h,\mu^2) \Rightarrow \widetilde{D}_{q\to h}(z_h,\mu^2)$$

Modified FF of the Light Quark(I)

$$\frac{dW_{\mu\nu}}{dz_h} = \sum_q \int dx \tilde{f}_q^A(x, \mu_I^2) H_{\mu\nu}^{(0)}(x, p, q) \widetilde{D}_{q\to h}(z_h, \mu^2)$$

$$\begin{aligned}\widetilde{D}_{q\to h}(z_h, \mu^2) &\equiv D_{q\to h}(z_h, \mu^2) + \int_0^{\mu^2} \frac{d\ell_T^2}{\ell_T^2} \frac{\alpha_s}{2\pi} \int_{z_h}^1 \frac{dz}{z} \Big[\Delta\gamma_{q\to qg}(z, x, x_L, \ell_T^2) D_{q\to h}(z_h/z) \\ &+ \Delta\gamma_{q\to gq}(z, x, x_L, \ell_T^2) D_{g\to h}(z_h/z)\Big]\end{aligned}$$

$$\begin{aligned}\Delta\gamma_{q\to qg}(z, x, x_L, \ell_T^2) &= \left[\frac{1+z^2}{(1-z)_+} T_{qg}^{A(m)}(x, x_L) + \delta(1-z)\Delta T_{qg}^{A(m)}(x, \ell_T^2)\right] \\ &\times \frac{2\pi\alpha_s C_A}{\ell_T^2 N_c \tilde{f}_q^A(x, \mu_I^2)}, \\ \Delta\gamma_{q\to gq}(z, x, x_L, \ell_T^2) &= \Delta\gamma_{q\to qg}(1-z, x, x_L, \ell_T^2),\end{aligned}$$

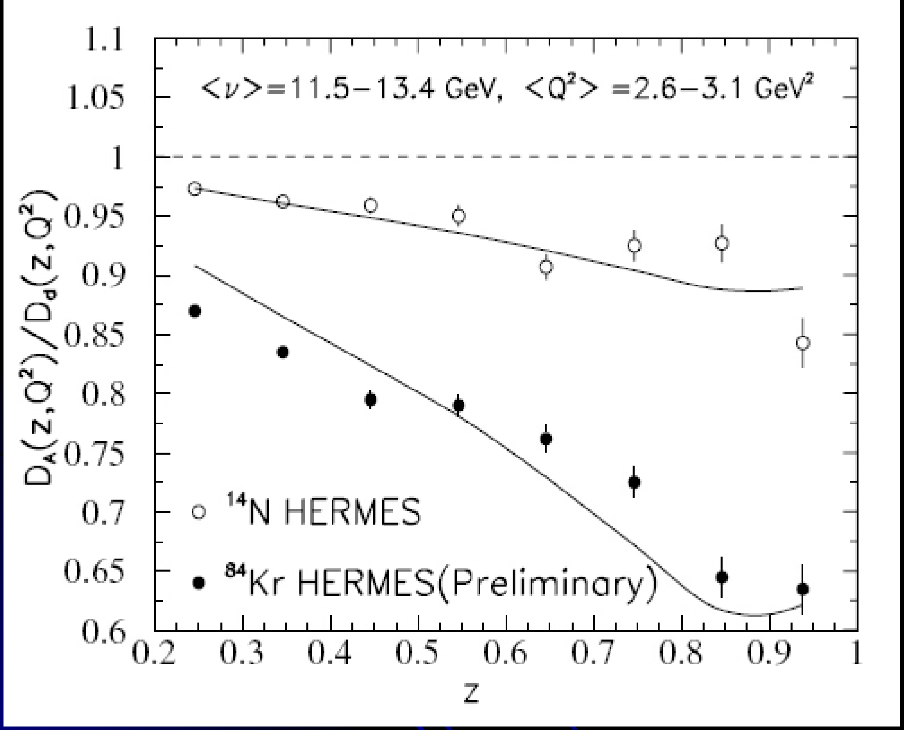
Xiao-Feng Guo, Xin-Nian Wang, PRL 85 (2000) 3591;

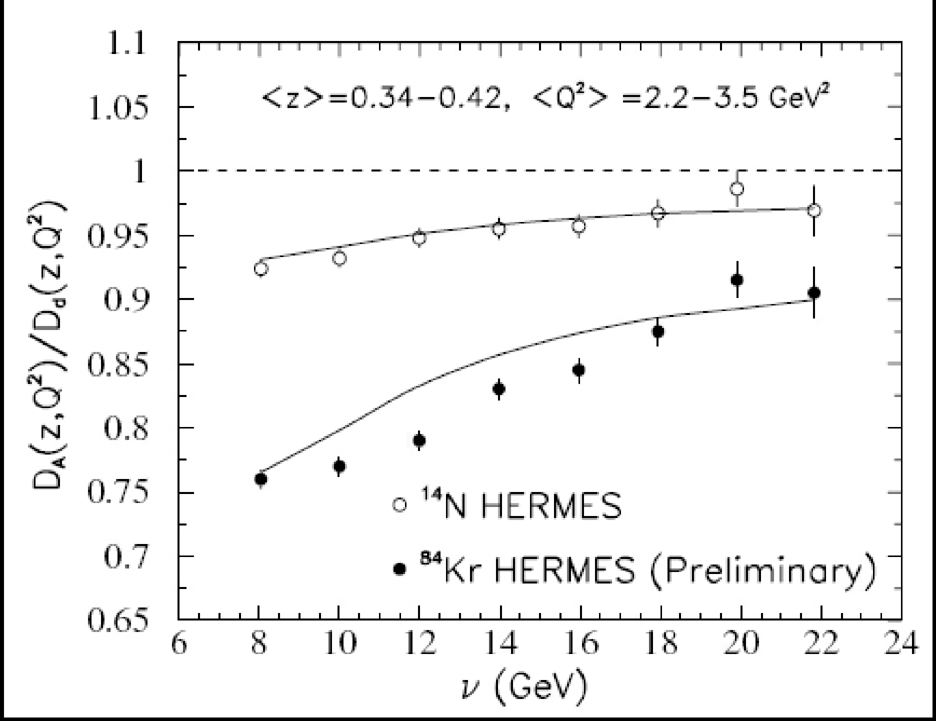
Xin-Nian Wang, Xiao-Feng Guo, NPA 696(2001) 788;

Ben-Wei Zhang, Xin-Nian Wang, NPA 720(2003) 429.

Modified FF of light quark (II)

$$\widetilde{C}(Q^2) = 0.0060 \text{ GeV}^2$$





Enke Wang, Xin-Nian Wang, PRL 89(2000)162301

From Light to Heavy

Theory:

- **Heavy quark energy loss can provide a further test of a uniform formalism of jet quenching.**
- **How does mass effect of heavy quark affect the pattern of energy loss?**

Experiment:

- **Open charm suppression, which can be measured at RHIC by comparing pt distributions of D-mesons in *D-Au* and *Au-Au* collisions, is a novel probe of QGP dynamics.**

Heavy Quark Energy Loss

- There are several theoretical calculations to obtain the heavy quark energy loss:

  Dokshitzer and Kharzeev

  Djordjevic and Gyulassy

  Armesto, Salgado and Wiedemann

  G.D. Moore and D. Teaney

  ……

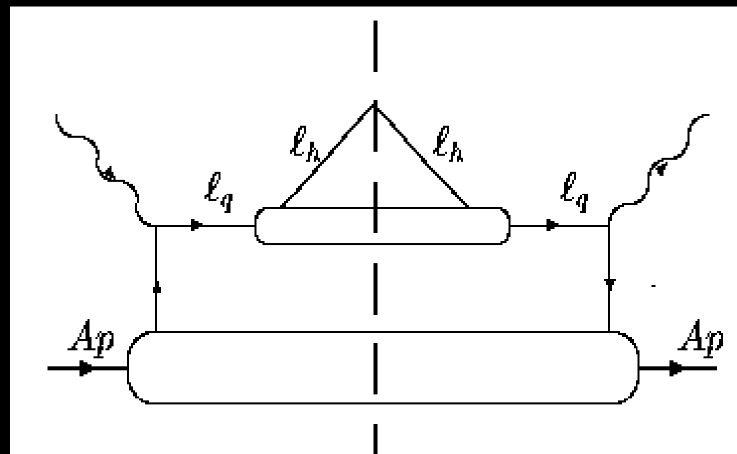
- Twist Expansion approach

B.W. Zhang, E. Wang and X.N. Wang, PRL 93(2004)072301,

B.W. Zhang, E. Wang and X.N. Wang, NPA 757(2005)493

Heavy Quark

- In parton model, for the heavy quark production we obtain :

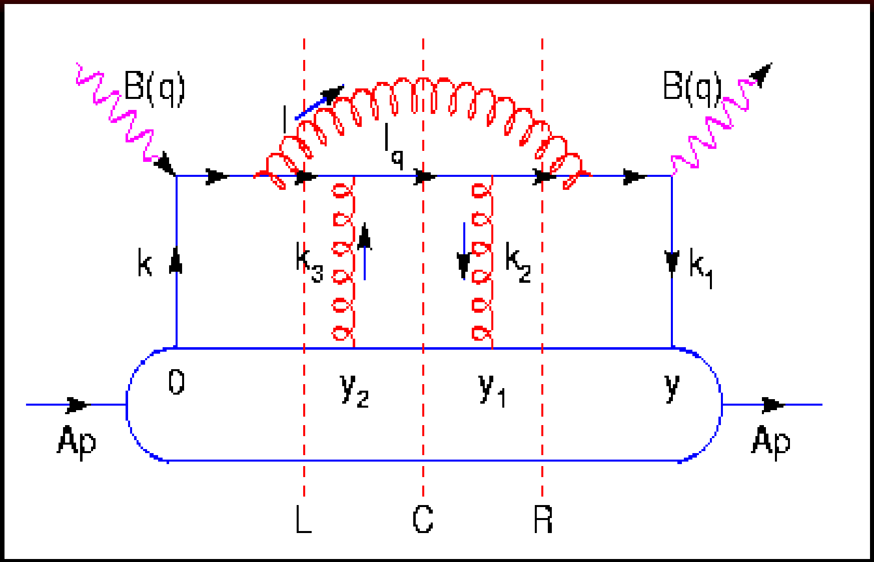


$$\frac{dW^S_{\mu\nu}}{dz_h} = \sum_q \int dx\, f^A_q(x, Q_1^2) H^{(0)}_{\mu\nu}(x, p, q, M) D_{Q\to h}(z_h, Q_2^2)$$

$$H^{(0)}_{\mu\nu}(k, q, M) = \frac{e_q^2}{2x} \mathrm{Tr}(\not{k}\gamma_\mu V(\not{q} + x\not{p})V^\dagger\gamma_\nu)\,\frac{2\pi}{2p\cdot q}\delta(x - x_B - x_M)$$

$$x_M = \frac{M^2}{2p^+q^-}, \quad x_B = \frac{Q^2}{2p^+q^-}\ .$$

Mutiple Scattering in Nuclei



In order to calculate the heavy quark energy loss induced by gluon radiation in pQCD, we should separate the 'hard' part from the 'soft' part:

factorizing.

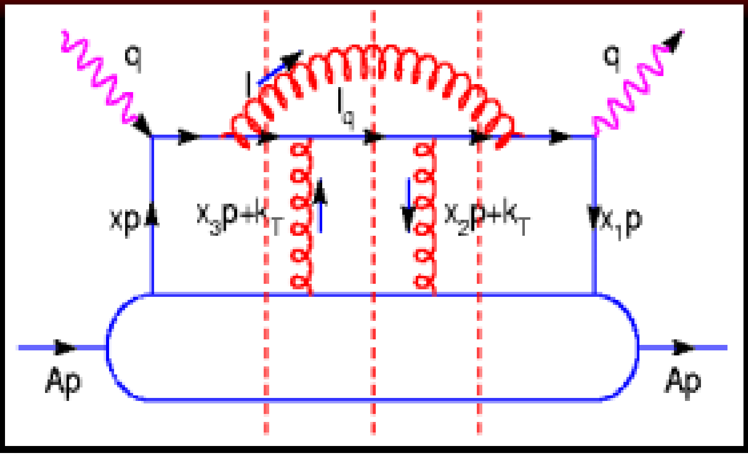
Factorization of Twist-4

$$W_{\mu\nu} = \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \int \frac{d^4k_3}{(2\pi)^4} \int d^4y \int d^4y_1 \int d^4y_2 e^{ik_1y + ik_2y_1 - ik_3y_2}$$
$$\times \mathrm{Tr}\left[\hat{H}^{\alpha\beta}_{\mu\nu}(k, k_1, k_2)\langle A|T[\bar{\psi}_q(0)\, A_\beta(y_2)\, A_\alpha(y_1)\, \psi_q(y)]|A\rangle\right].$$

$$\hat{H}^{\alpha\beta}_{\mu\nu}(xp^+, k_2, k_3) = \hat{H}^{\alpha\beta}_{\mu\nu}(xp^+, x_2p^+, x_3p^+) + \left.\frac{\partial \hat{H}^{\alpha\beta}_{\mu\nu}(xp^+, k_2, k_3)}{\partial k_{2\rho}}\right|_{\mathbf{k}_{2T}=0} (k_2 - x_2p^+)_\rho$$
$$+ \left.\frac{\partial \hat{H}^{\alpha\beta}_{\mu\nu}(xp^+, k_2, k_3)}{\partial k_{3\sigma}}\right|_{\mathbf{k}_{3T}=0} (k_3 - x_3p^+)_\sigma$$
$$+ \frac{1}{2} \left.\frac{\partial^2 \hat{H}^{\alpha\beta}_{\mu\nu}(xp^+, k_2, k_3)}{\partial k_{2\rho}\partial k_{3\sigma}}\right|_{\mathbf{k}_{2T}=\mathbf{k}_{3T}=0} (k_2 - x_2p^+)_\rho (k_3 - x_3p^+)_\sigma$$
$$+ \cdots .$$

J. Qiu, G. Sterman, NPB 353(1991)105; NPB 353(1991)137.
M. Luo,J. Qiu, G. Sterman, PLB 279(1992)377;
M. Luo,J. Qiu, G. Sterman, PRD 49(1994)4493; PRD 50(1994)1951

Generalized Factorization



$$\frac{dW^D_{\mu\nu}}{dz_H} = \sum_q \int_{z_H}^{1} \frac{dz}{z} D_{Q\to H}(z_H/z)$$

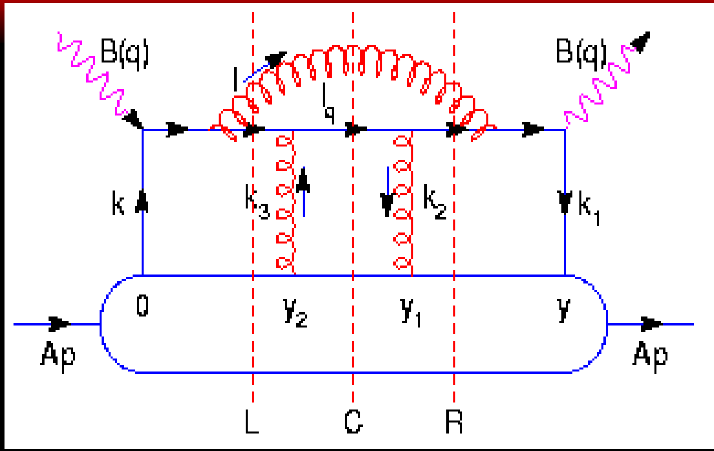
$$\times \int \frac{dy^-}{2\pi} dy_1^- dy_2^- \frac{1}{2}\langle A|\bar{\psi}_q(0)\gamma^+ F_\sigma^{\ +}(y_2^-) F^{+\sigma}(y_1^-)\psi_q(y^-)|A\rangle$$

$$\times \left(-\frac{1}{2} g^{\alpha\beta}\right)\left[\frac{1}{2}\frac{\partial^2}{\partial k_T^\alpha \partial k_T^\beta} \bar{H}^D_{\mu\nu}(y^-, y_1^-, y_2^-, k_T, p, q, M, z)\right]_{k_T=0}.$$

$$\overline{H}^D_{\mu\nu}(y^-, y_1^-, y_2^-, k_T, p, q, z) = \int dx\, dx_1\, dx_2\, e^{ix_1p^+y^- + ix_2p^+y_1^- + i(x-x_1-x_2)p^+y_2^-}$$

$$\times \mathrm{Tr}\left[\hat{H}^{\alpha\beta}_{\mu\nu}(xp^+, k_2, k_3)\not{p}\, p_\alpha p_\beta\right]\Big|_{k_T=0}$$

Quark-Gluon Double Scattering



$$\overline{H}^D_{C\,\mu\nu}(y^-,y_1^-,y_2^-,k_T,p,q,z) = \int \frac{dx}{x}\frac{dx_1}{2\pi}\frac{dx_2}{2\pi} e^{ix_1p^+y^- + ix_2p^+y_1^- + i(x-x_1-x_2)p^+y_2^-} \int \frac{d^4\ell}{(2\pi)^4}$$

$$\times \frac{p^{+2}}{2}\mathrm{Tr}\left[\not{k}\gamma_\mu V n^\sigma n^\rho \widehat{H}_{\sigma\rho} V^\dagger \gamma_\nu\right] 2\pi\delta_+(\ell^2)\,\delta(1-z-\frac{\ell^-}{q^-})\,.$$

$$\widehat{H}_{\sigma\rho} = \frac{C_F}{2N_c}g^4 \frac{\gamma\cdot(q+k_1)+M}{(q+k_1)^2-M^2-i\epsilon}\gamma_\alpha \frac{\gamma\cdot(q+k_1-\ell)+M}{(q+k_1-\ell)^2-M^2-i\epsilon}\gamma_\sigma(\gamma\cdot\ell_q+M)\gamma_\rho$$

$$\times\, \varepsilon^{\alpha\beta}(\ell)\frac{\gamma\cdot(q+k-\ell)+M}{(q+k-\ell)^2-M^2+i\epsilon}\gamma_\beta \frac{\gamma\cdot(q+k)+M}{(q+k)^2-M^2+i\epsilon}\, 2\pi\delta_+(\ell_q^2-M^2),$$

$$n = [1, 0^-, \vec{0}_\perp]$$

Collinear Approximation

- Make a collinear approximation:

$$n^\sigma \widehat{H}_{\sigma\rho} n^\rho \approx \frac{\gamma \cdot \ell_q + M}{4\ell_q^-} \mathrm{Tr}\left[\gamma^- n^\sigma \widehat{H}_{\sigma\rho} n^\rho\right]$$

$$\frac{1}{2x}\mathrm{Tr}\left[\not{k}\gamma_\mu V n^\sigma n^\rho \widehat{H}_{\sigma\rho} V^\dagger \gamma_\nu\right]$$

$$\approx \frac{1}{2x}\mathrm{Tr}\left[\not{k}\gamma_\mu V(\gamma \cdot \ell_q + M)V^\dagger \gamma_\nu\right] \frac{1}{4\ell_q^-}\mathrm{Tr}\left[\gamma^- n^\sigma \widehat{H}_{\sigma\rho} n^\rho\right]$$

- The tensor structure can be factorized:

$$\overline{H}^D_{\mu\nu}(y^-, y_1^-, y_2^-, k_T, p, q, M, z) = \int dx H^{(0)}_{\mu\nu}(k, q, M)\ \overline{H}^D(y^-, y_1^-, y_2^-, k_T, x, p, q, M, z)$$

LPM Effect

$$\overline{H}^D_{1,C}(y_i^-, k_T, x, p, q, m, z) = \int d\ell_T^2 \frac{(1+z^2)\ell_T^2 + (1-z)^4 m^2}{(1-z)(\ell_T^2 + (1-z)^2 m^2)^2} \frac{\alpha_s}{2\pi} \times C_F \frac{2\pi\alpha_s}{N_c} \overline{I}_{1,C}(y_i^-, \ell_T, k_T, x, p, q, m, z)$$

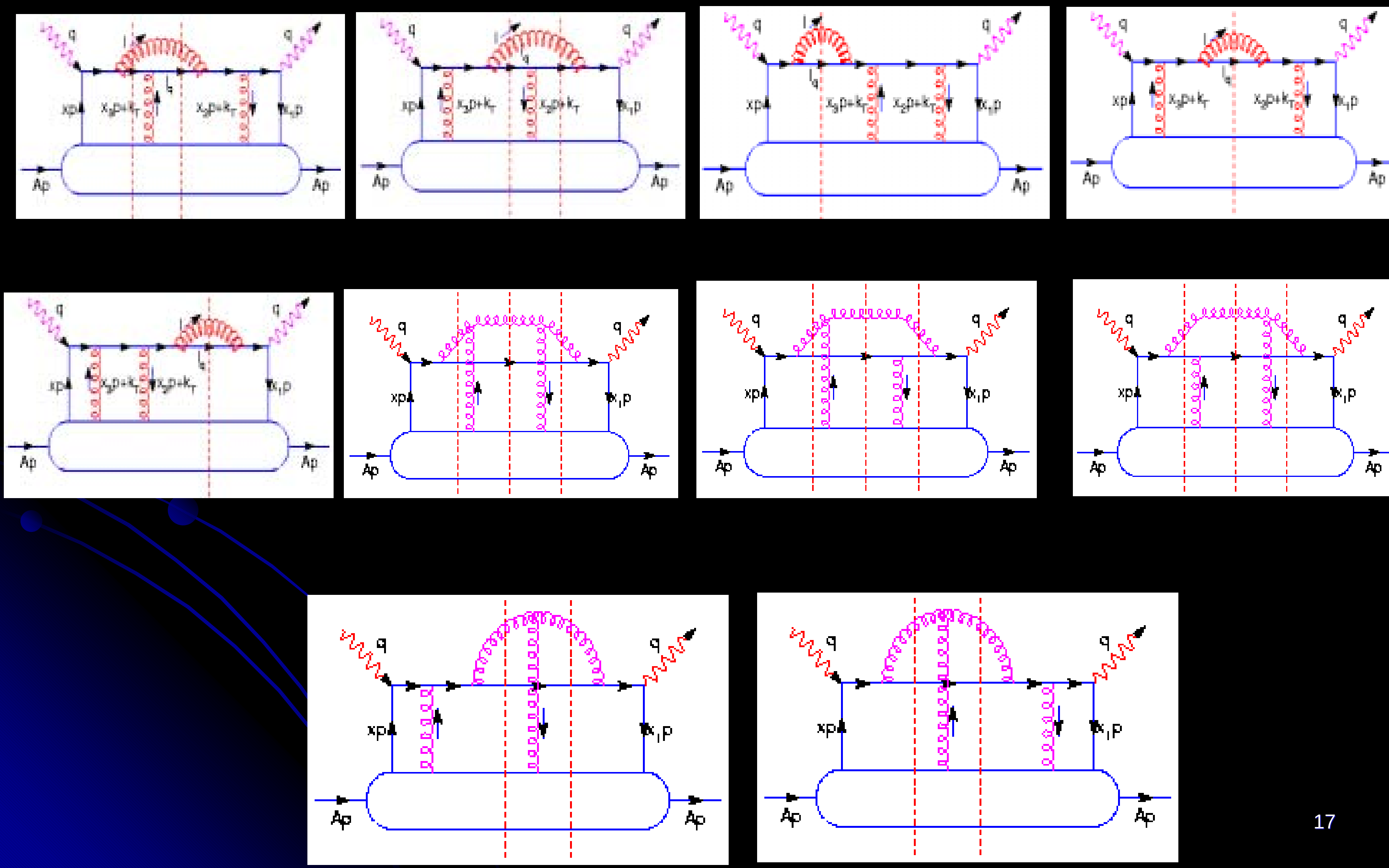
$$\overline{I}_{1,C}(y_i^-, \ell_T, k_T, x, p, q, m, z) = e^{i(x+x_L)p^+y^- + ix_D p^+(y_1^- - y_2^-)} \theta(-y_2^-)\theta(y^- - y_1^-) \times \left(1 - e^{-i(x_L + (1-z)x_M)p^+ y_2^-}\right) \times \left(1 - e^{-i(x_L+(1-z)x_M)p^+(y^- - y_1^-)}\right).$$

- LMP interference effect.
- The formation time of gluon radiation.

$$x_L = \frac{\ell_T^2}{2p^+q^- z(1-z)}$$

$$\tau_f^Q \equiv \frac{1}{(x_L + (1-z)x_M/z)p^+} < \tau_f^q \equiv \frac{1}{x_L p^+}$$

Other Processes



Hard Partonic Part

$$\nabla^2_{k_T} H^D_{C(L,R)}|_{k_T=0} = 4C_A \frac{1+z^2}{1-z} \frac{\ell_T^4}{[\ell_T^2+(1-z)^2M^2]^4} \widetilde{H^D}_{C(L,R)} + \mathcal{O}(x_B/Q^2\ell_T^2)\,,$$

$$\begin{aligned}
\widetilde{H^D_C} &= c_1(z,\ell_T^2,M^2)(1-e^{-i(x_L+(1-z)x_M/z)p^+y_2^-})(1-e^{-i(x_L+(1-z)x_M/z)p^+(y^- - y_1^-)})\\
&+ c_2(z,\ell_T^2,M^2)[e^{-i(x_L+(1-z)x_M/z)p^+y_2^-}(1-e^{-i(x_L+(1-z)x_M/z)p^+(y^- - y_1^-)})\\
&+ e^{-i(x_L+(1-z)x_M/z)p^+(y^- - y_1^-)}(1-e^{-i(x_L+(1-z)x_M/z)p^+y_2^-})]\\
&+ c_3(z,\ell_T^2,M^2)e^{-i(x_L+(1-z)x_M/z)p^+(y^- - y_1^-)}e^{-i(x_L+(1-z)x_M/z)p^+y_2^-}\\
\widetilde{H^L_C} &= c_4(z,\ell_T^2,M^2)(e^{-i(x_L+(1-z)x_M/z)p^+(y^- - y_1^-)} - e^{-i(x_L+(1-z)x_M/z)p^+(y^- - y_2^-)})\\
&+ c_5(z,\ell_T^2,M^2)(1-e^{-i(x_L+(1-z)x_M/z)p^+(y^- - y_1^-)})\\
\widetilde{H^R_C} &= c_4(z,\ell_T^2,M^2)(e^{-i(x_L+(1-z)x_M/z)p^+y_2^-} - e^{-i(x_L+(1-z)x_M/z)p^+y_1^-})\\
&+ c_5(z,\ell_T^2,M^2)(1-e^{-i(x_L+(1-z)x_M/z)p^+y_2^-}).
\end{aligned}$$

Dead-Cone effect of heavy quark propagating

$$f_{Q/q} = \left[\frac{\ell_T^2}{\ell_T^2+(1-z)^2M^2}\right]^4 = \left[1+\frac{\theta_0^2}{\theta^2}\right]^{-4}$$

$$\theta_0 = \frac{M}{q^-}, \quad \theta = \frac{\ell_T}{l^-}$$

Modified FF of Heavy Quark

- With the generalized factorization theorem we obtain the semi- inclusive hadronic tensor:

$$\frac{dW_{\mu\nu}}{dz_h} = \sum_q \int dx \widetilde{f}_q^A(x, \mu_I^2) H_{\mu\nu}^{(0)}(x, p, q, M) \widetilde{D}_{Q\to h}(z_h, \mu^2)$$

The modified effective fragmentation function is

$$\begin{aligned}\widetilde{D}_{Q\to h}(z_h, \mu^2) \equiv\ & D_Q(z_h, \mu^2) \\ & + \int_0^{\mu^2} \frac{d\ell_T^2}{\ell_T^2 + (1-z)^2 M^2} \frac{\alpha_s}{2\pi} \int_{z_h}^1 \frac{dz}{z} \Delta\gamma_{q\to qg}(z, x, x_L, \ell_T^2, M^2) D_{Q\to h}(z_h/z) \\ & + \int_0^{\mu^2} \frac{d\ell_T^2}{\ell_T^2 + z^2 M^2} \frac{\alpha_s}{2\pi} \int_{z_h}^1 \frac{dz}{z} \Delta\gamma_{q\to gq}(z, x, x_L, \ell_T^2, M^2) D_{g\to h}(z_h/z)\,,\end{aligned}$$

Twist-4 Parton Correlation

$$\Delta\gamma_{q\to qg}(z,x,x_L,\ell_T^2,M^2)=\left[\frac{1+z^2}{(1-z)_+}T_{qg}^{A,H}(x,x_L,M^2)+\delta(1-z)\Delta T_{qg}^{A,H}(x,\ell_T^2,M^2)\right]$$

$$\times\ \frac{2\pi C_A\alpha_s\ell_T^4}{[\ell_T^2+(1-z)^2M^2]^3N_c\tilde{f}_q^A(x,\mu_I^2)},$$

$$\Delta\gamma_{q\to gq}(z,x,x_L,\ell_T^2,M^2)=\Delta\gamma_{q\to qg}(1-z,x,x_L,\ell_T^2,M^2),$$

$$\Delta T_{qg}^{A,Q)}(x,\ell_T^2,M^2)\equiv\int_0^1 dz\frac{1}{1-z}\left[2T_{qg}^{A,Q}(x,x_L,M^2)|_{z=1}-(1+z^2)T_{qg}^{A,Q}(x,x_L,M^2)\right]$$

$$T_{qg}^{A,Q}(x,x_L,M^2)\equiv T_{qg}^{A,C}(x,x_L,M^2)+T_{qg}^{A,L}(x,x_L,M^2)+T_{qg}^{A,R}(x,x_L,M^2)\ ,$$

$$T_{qg}^{A,C}(x,x_L,M^2)=\int\frac{dy^-}{2\pi}\,dy_1^-dy_2^-\,\widetilde{H_C^D}\,\frac{1}{2}\langle A|\bar{\psi}_q(0)\,\gamma^+\,F_\sigma{}^+(y_2^-)\,F^{+\sigma}(y_1^-)\,\psi_q(y^-)|A\rangle$$

$$\times\ e^{i(x+x_L)p^+y^-}\theta(-y_2^-)\theta(y^--y_1^-)\ ,$$

$$T_{qg}^{A,L}(x,x_L,M^2)=\int\frac{dy^-}{2\pi}\,dy_1^-dy_2^-\,\widetilde{H_L^D}\,\frac{1}{2}\langle A|\bar{\psi}_q(0)\,\gamma^+\,F_\sigma{}^+(y_2^-)\,F^{+\sigma}(y_1^-)\,\psi_q(y^-)|A\rangle$$

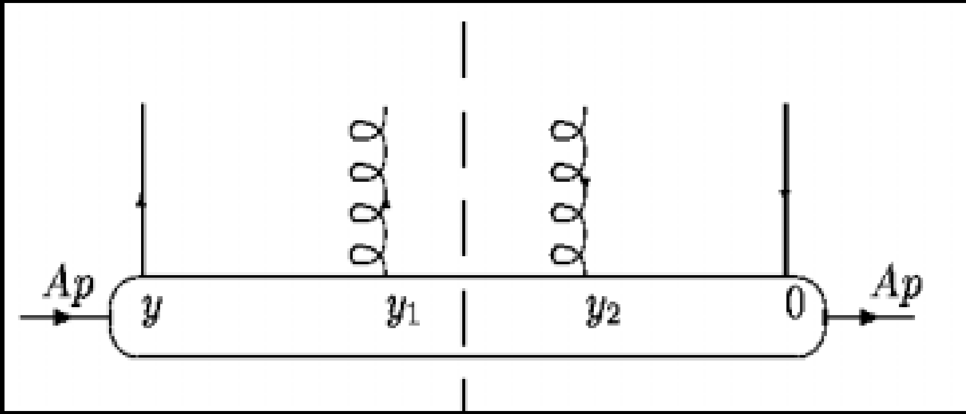
$$\times\ e^{i(x+x_L)p^+y^-}\theta(y^--y_1^-)\theta(y_1^--y_2^-)\ ,$$

$$T_{qg}^{A,R}(x,x_L,M^2)=\int\frac{dy^-}{2\pi}\,dy_1^-dy_2^-\,\widetilde{H_R^D}\,\frac{1}{2}\langle A|\bar{\psi}_q(0)\,\gamma^+\,F_\sigma{}^+(y_2^-)\,F^{+\sigma}(y_1^-)\,\psi_q(y^-)|A\rangle$$

$$\times\ e^{i(x+x_L)p^+y^-}\theta(-y_2^-)\theta(y_2^--y_1^-)$$

Two-parton Correlation Function

Twist-4 parton correlation function is in principle not calculable.



With some approximations we can estimate the twist-4 correlation functions approximated as

$$T_{qg}^{A,Q}(x, x_L, M^2)$$
$$\approx \frac{\widetilde{C}}{x_A} f_q^A(x)[(1 - e^{-(x_L+(1-z)x_M/z)^2/x_A^2})a_1(z, \ell_T^2, M^2) + a_2(z, \ell_T^2, M^2)],$$

$$\widetilde{C} \equiv 2Cx_T f_g^N(x_T) \qquad x_A = 1/m_N R_A$$

J. Osborne and Xin-Nian Wang, NPA 710(2002)281

Heavy Quark Energy Loss

Heavy quark energy loss can be derived as:

$$\langle \Delta z_g^Q \rangle (x_B, \mu^2) = \int_0^{\mu^2} d\ell_T^2 \int_0^1 dz \frac{\alpha_s}{2\pi}(1-z)\, \frac{\Delta\gamma_{q\to qg}(z, x_B, x_L, \ell_T^2)}{\ell_T^2 + (1-z)^2 M^2}$$

$$\langle \Delta z_g^Q \rangle (x_B, \mu^2) = \frac{\widetilde{C} C_A \alpha_s^2}{N_c x_A} \int_0^{Q^2} d\ell_T^2 \int_0^1 dz \frac{(1+z^2)\ell_T^4}{[\ell_T^2 + (1-z)^2 M^2]^4} \times \left[\left(1 - e^{-(x_L + (1-z)x_M/z)^2 / x_A^2}\right) a_1(z, \ell_T^2, M^2) + a_2(z, \ell_T^2, M^2) \right]$$

When M=0, we will recover the result of the light quark energy loss.

Xiao-feng Guo, Xin-Nian Wang, PRL 85(2000)3591;

Xin-Nian Wang, Xiao-feng Guo, NPA 696(2001) 788

Ben-Wei Zhang, Xin-Nian Wang, NPA 720(2003) 429

Two Limits

$$\frac{(x_L + (1-z)x_M/z)^2}{x_A^2} = \frac{L_A^-}{\tau_f} \sim \frac{x_B^2 M^4}{x_A^2 Q^4}$$

$$L_A^- = R_A m_N / p^+$$

(1) When $x_B/Q^2 \gg x_A/M^2$ → $(1 - e^{-(x_L+(1-z)x_M)^2/x_A^2}) \simeq 1$

$$\langle \Delta z_g^Q \rangle \sim C_A \frac{\widetilde{C}\alpha_s^2}{N_c} \frac{x_B}{x_A Q^2} \propto R_A$$

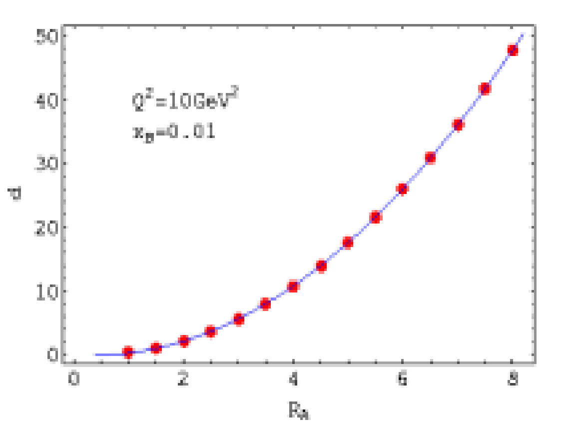
(2) For large values of Q^2 or small x_B, we have

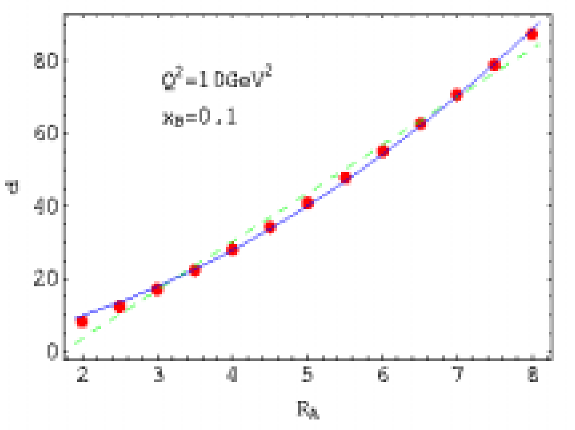
$$\langle \Delta z_g^Q \rangle \sim C_A \frac{\widetilde{C}\alpha_s^2}{N_c} \frac{x_B}{x_A^2 Q^2} \propto R_A^2$$

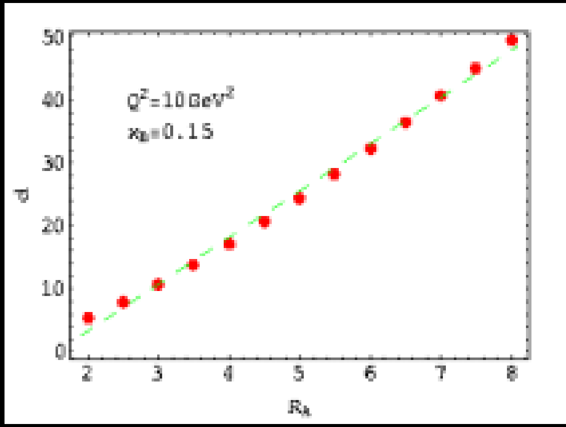
Nuclear Size Dependence

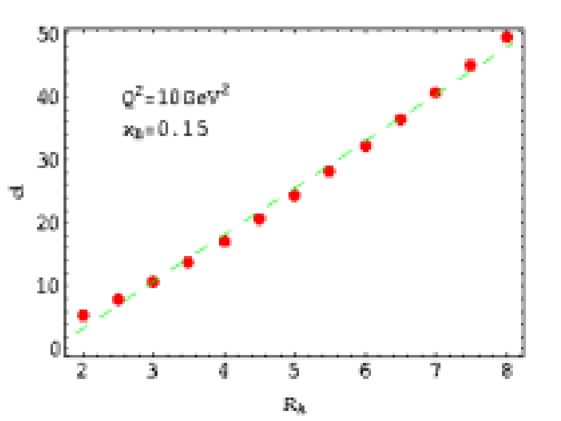
$$d = \langle \Delta z_g^Q \rangle \frac{N_C}{\widetilde{C}(Q^2) C_A \alpha_s^2(Q^2)}$$

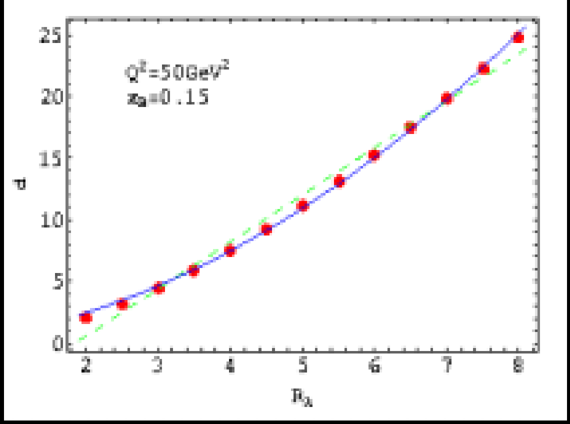
$$\widetilde{C}(Q^2) = 0.0060\ \mathrm{GeV}^2$$

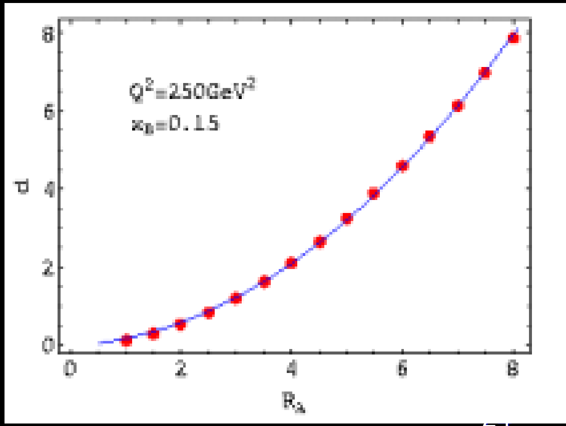






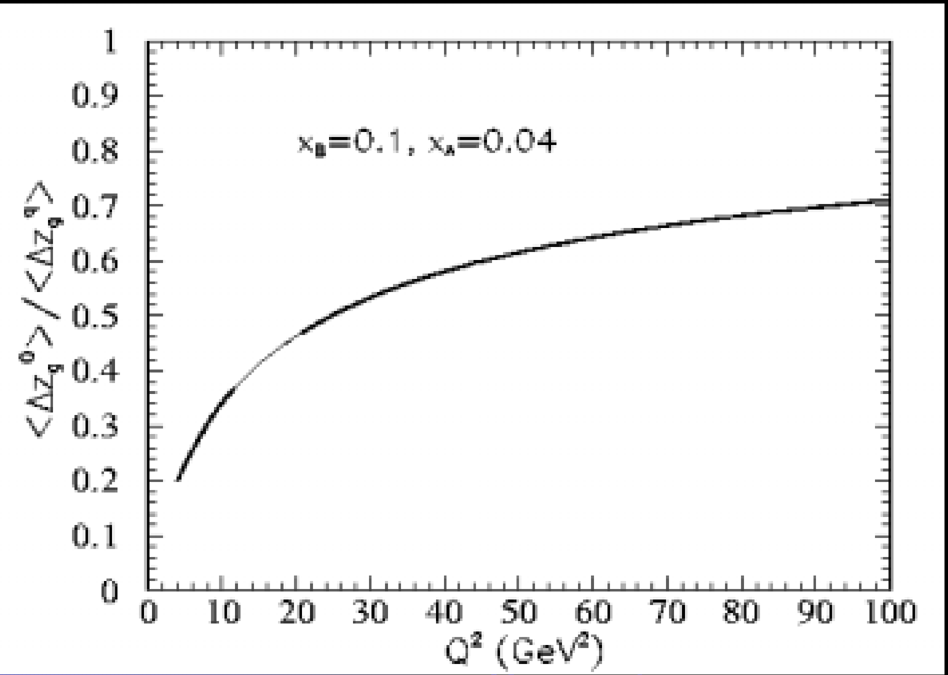


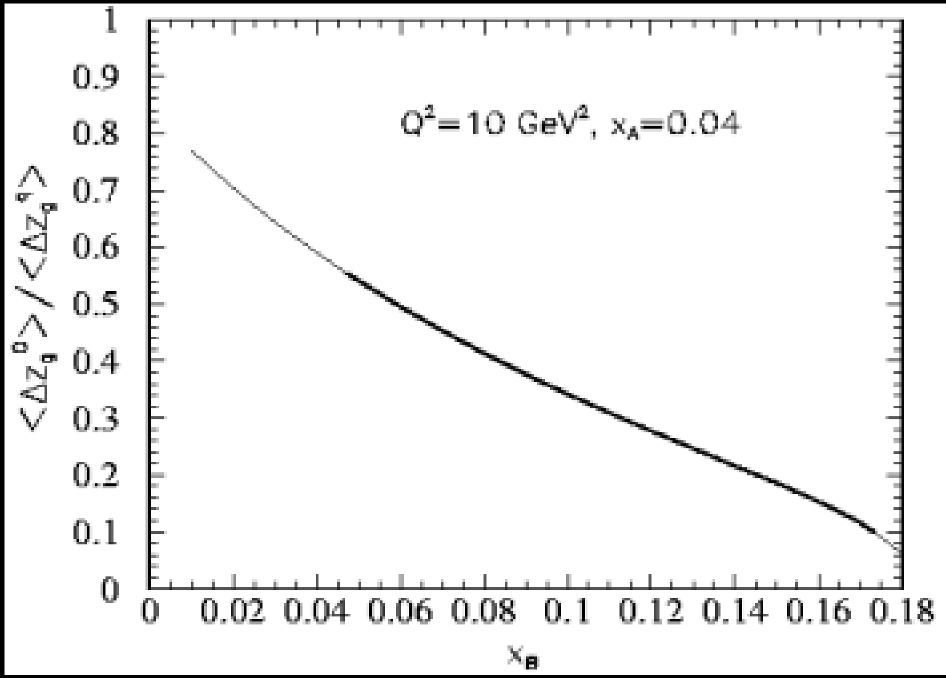




Heavy Quark VS. Light Quark

Energy loss of heavy quark is significantly suppressed due to mass effect, in particular, the dead-cone effect.



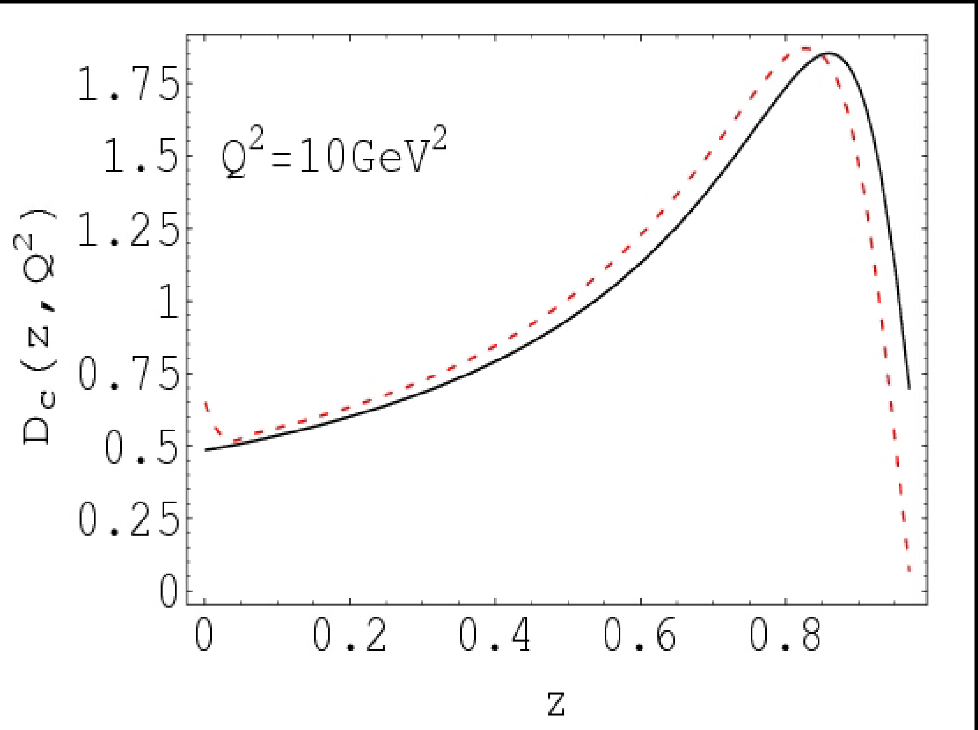


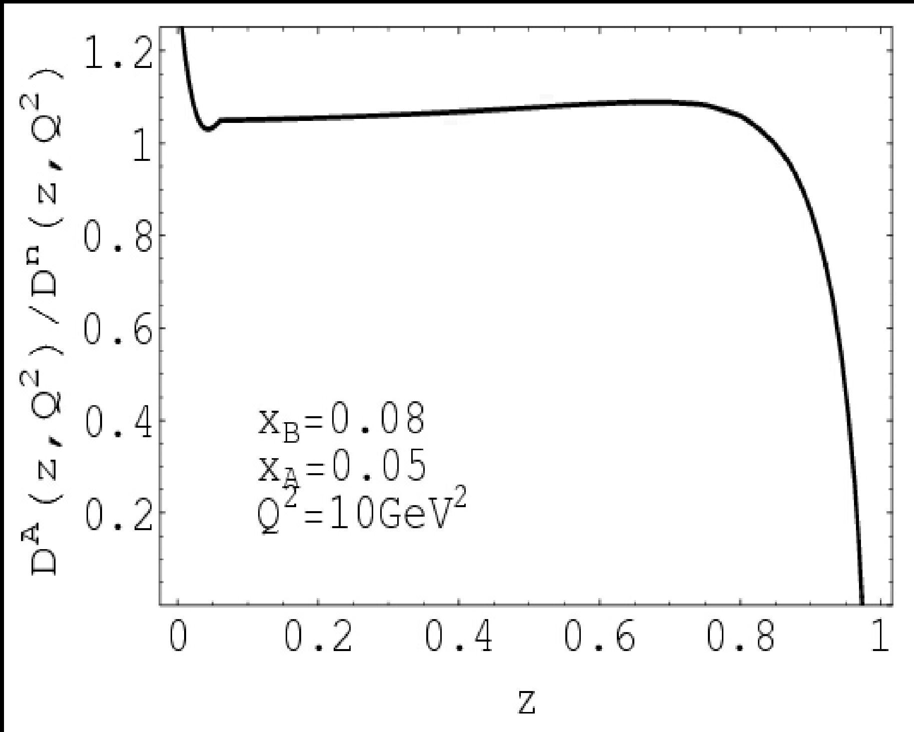
B.W. Zhang, E. Wang and X.N. Wang, PRL 93(2004)072301

Modified heavy quark FF

- Charm quark fragmentation function in vacuum:

$$D_{c\to D}(z) = \frac{N}{z[1 - z^{-1} - \varepsilon_c/(1-z)]}$$

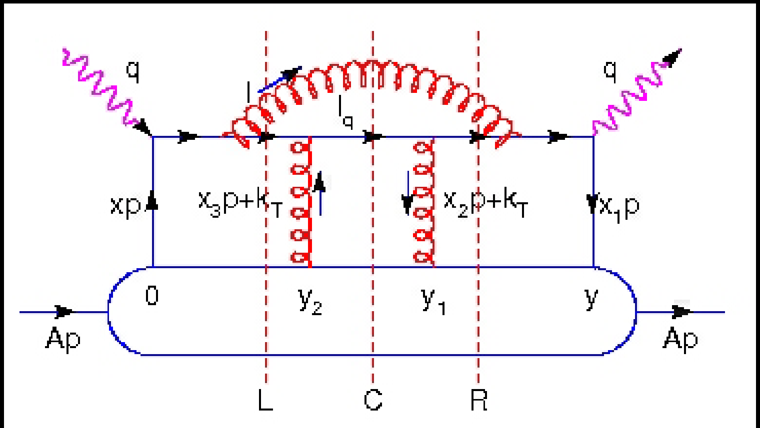




B.W. Zhang, E. Wang and X.N. Wang, NPA 757(2005)493

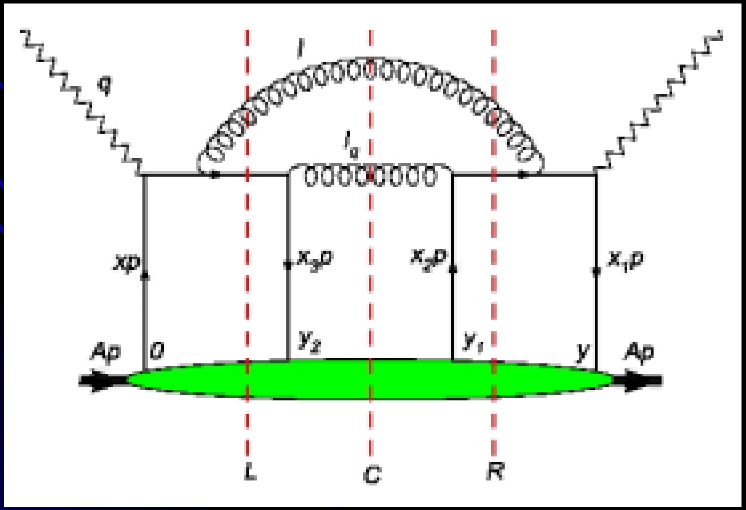
Quark-Quark Double Scattering

- Two kinds of double scattering in eA DIS



quark-gluon

double scattering

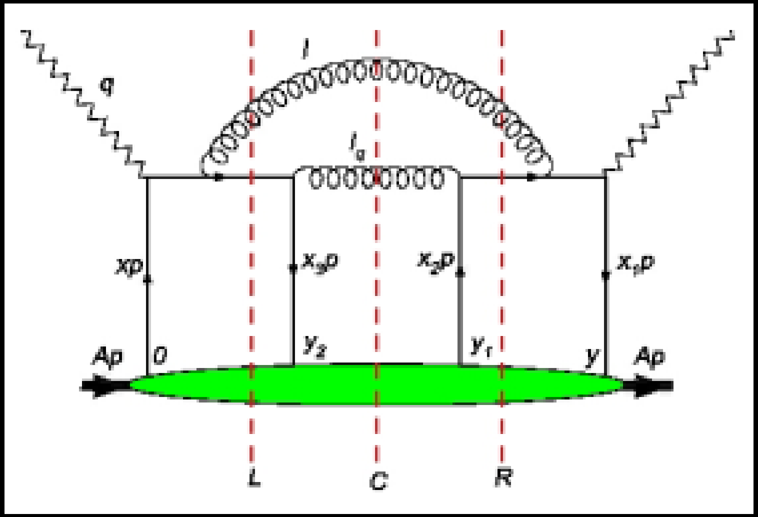


quark-quark

double scattering

Properties of q-q double scattering

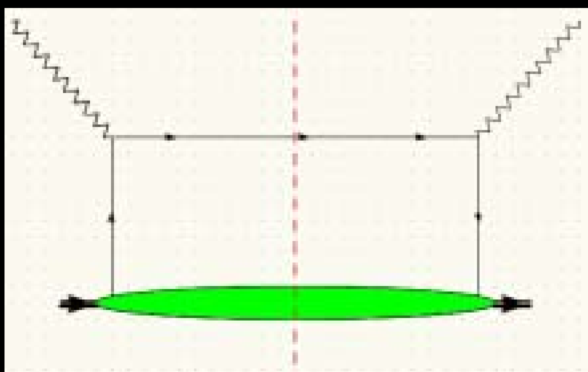
- The contributions of quark-quark double scattering are suppressed as compared to quark-gluon double scattering: quark density VS. gluon density
- Quark-quark double scattering will mix the quark and gluon fragmentation functions.
- Quark-quark double scattering may give different modifications to quark FF and anti-quark FF.



B.W.Zhang, X.N.Wang, A. Schaefer, in preparation

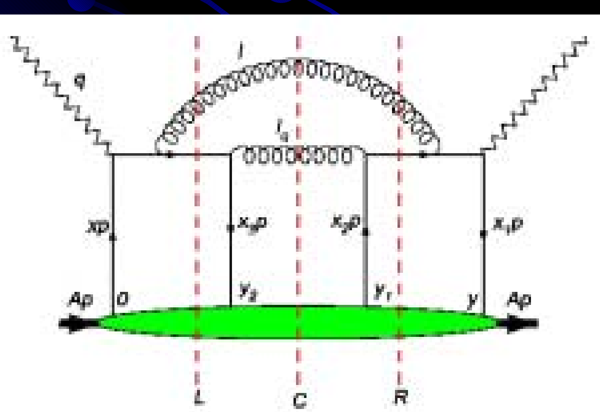
Quark-quark double scattering

- Single Scattering: leading twist contribution



$$\propto f_q^A(x,\mu_I^2) \bigotimes H_{\mu\nu} \bigotimes D_{q\to h}(z_h,\mu^2)$$

- Double Scattering: twist-4 contribution

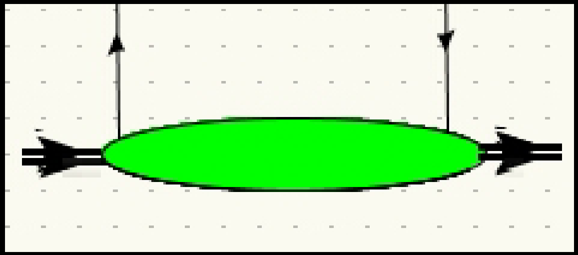


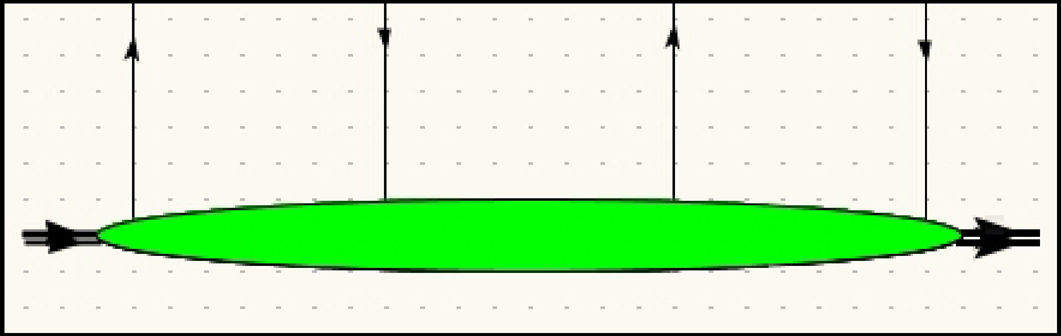
$$\propto T_{q\bar{q}}(x) \bigotimes H_{\mu\nu} \times C(z) \bigotimes D_{i\to h}(\frac{z_h}{z},\mu^2)$$

$$= f_q^A(x,\mu_I^2) \bigotimes H_{\mu\nu} \bigotimes \Delta D_{q\to h}(z_h,\mu^2)$$

$$\Delta D_{q\to h}(z_h,\mu^2) \equiv C(z) \times D_{i\to h}(\frac{z_h}{z}) \bigotimes \frac{T_{q\bar{q}}(x)}{f_q^A(x,\mu_I^2)}$$

quark-quark correlation function





$$f_q^A(x)$$

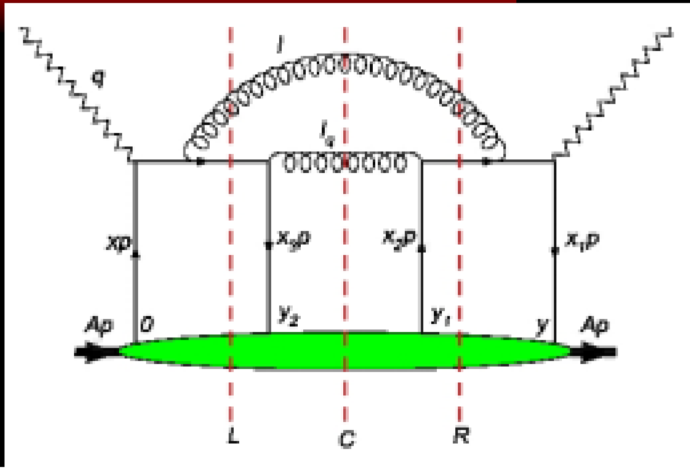
$$T_{q\bar{q}} \propto f_q^A(x_1)\, f_{\bar{q}}^N(x_2)$$

$$\Delta D_{q\to h}(z_h,\mu^2) \propto \frac{T_{q\bar{q}}(x)}{f_q^A(x,\mu_I^2)} \propto f_{\bar{q}}^N(x_2)$$

$$\Delta D_{\bar{q}\to h}(z_h,\mu^2) \propto \frac{T_{\bar{q}q}(x)}{f_{\bar{q}}^A(x,\mu_I^2)} \propto f_q^N(x_2)$$

Generalized Factorization

Consider a typical quark-quark double scattering process in semi-inclusive eA DIS:



$$\frac{dW^D_{\mu\nu}}{dz_h} = \sum_q \int_{z_h}^1 \frac{dz}{z} D_{g\to h}(z_h/z) \int \frac{p^+ dy^-}{2\pi} dy_1^- dy_2^- \overline{H}^D_{\mu\nu}(y^-, y_1^-, y_2^-, p, q, z)$$

$$\times \langle A|\bar{\psi}_q(0)\frac{\gamma^+}{2}\psi_{\bar{q}}(y^-)\bar{\psi}_q(y_1^-)\frac{\gamma^+}{2}\psi_{\bar{q}}(y_2^-)|A\rangle.$$

$$\overline{H}^D_{C\,\mu\nu}(y^-, y_1^-, y_2^-, z) = \int dx \frac{dx_1}{2\pi}\frac{dx_2}{2\pi} e^{ix_1p^+y^- + ix_2p^+y_1^- + i(x-x_1-x_2)p^+y_2^-}$$

$$\times \int \frac{d^4\ell}{(2\pi)^4} \mathrm{Tr}\left[\frac{\not{p}}{2}\gamma_\mu \widehat{H}\gamma_\nu\right] 2\pi\delta_+(\ell^2) 2\pi\delta_+(\ell_q^2)\, \delta(1 - z - \frac{\ell^-}{q^-})$$

J. Qiu, G. Sterman, NPB 353(1991)105; NPB 353(1991)137.

Hard partonic part

- Applying the collinear approximation:

$$\mathrm{Tr}\left[\frac{\not p}{2}\gamma_\mu \widehat{H}\gamma_\nu\right] \approx H_1\, \mathrm{Tr}\left[\frac{\not p}{2}\gamma_\mu(\gamma\cdot\ell_q)\gamma_\nu\right]$$

- Semi-inclusive hadronic tensor can be given:

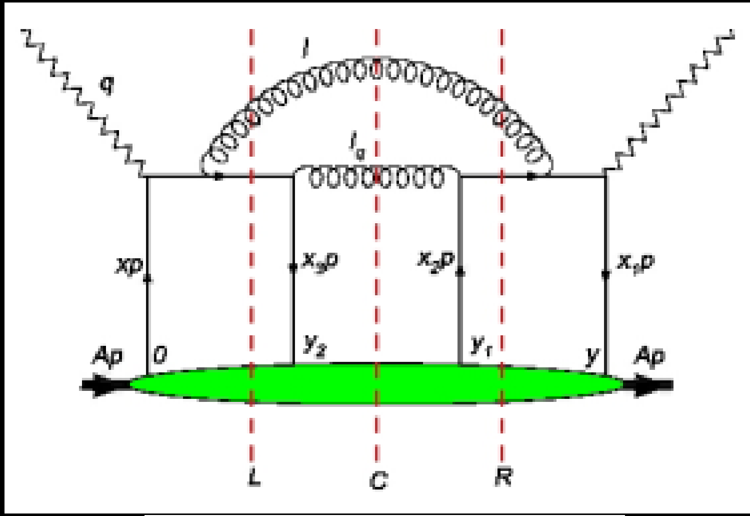
$$\frac{dW^D_{qq,\mu\nu}}{dz_h} = \sum_q \int dx H^{(0)}_{\mu\nu}(x,p,q) T^A_{q\bar q}(x) \overline{H}^D(y^-, y_1^-, y_2^-, x, p, q, z)$$

$$T^A_{q\bar q}(x) = \int \frac{p^+ dy^-}{2\pi} dy_1^- dy_2^- \langle A|\bar\psi_q(0)\frac{\gamma^+}{2}\psi_{\bar q}(y^-)\bar\psi_q(y_1^-)\frac{\gamma^+}{2}\psi_{\bar q}(y_2^-)|A\rangle$$

LPM effect

$$\overline{H}^D_{1,C} = \int_{z_h}^1 \frac{dz}{z} D_{g\to h}(z_h/z) \int \frac{d\ell_T^2}{\ell_T^2} \frac{\alpha_s^2 x_B}{Q^2} \frac{2(1+z^2)}{z(1-z)} \times \frac{C_F^2}{N_c} \overline{I}_{1,C}(y^-, y_1^-, y_2^-, \ell_T, x, p, q, z),$$

$$\overline{I}_{1,C} = e^{i(x+x_L)p^+ y^-} \theta(-y_2^-) \theta(y^- - y_1^-) \times [1 - e^{-ix_L p^+ y_2^-}][1 - e^{-ix_L p^+ (y^- - y_1^-)}].$$



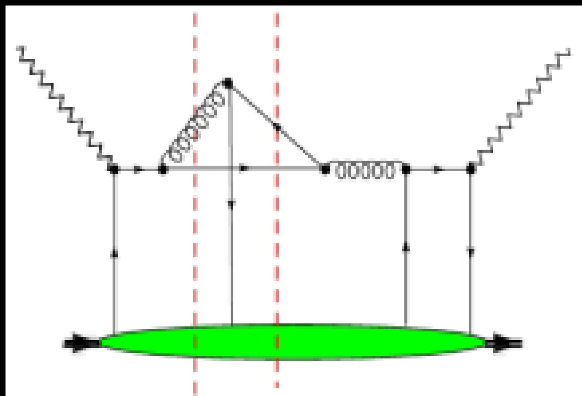
$$x_L = \frac{\ell_T^2}{2p^+ q^- z(1-z)}$$

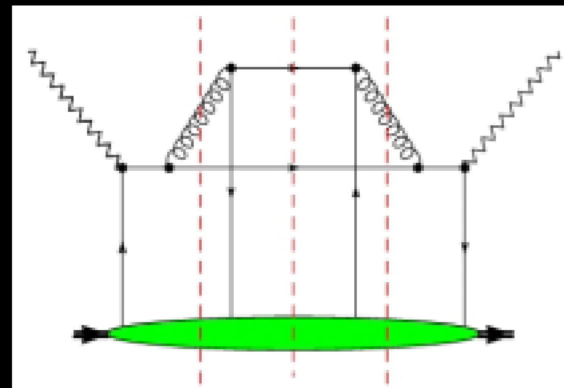
LPM effect

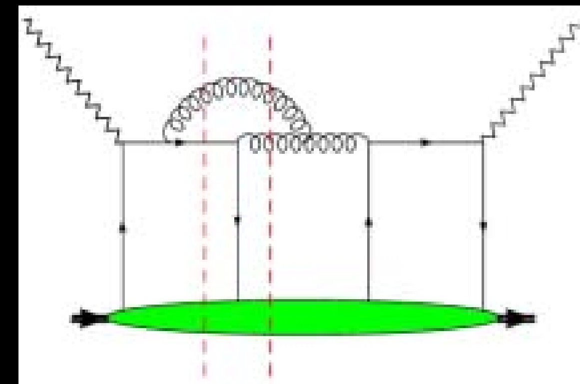
$$\overline{H}^D_{1,L(R)} = \int_{z_h}^1 \frac{dz}{z} D_{q\to h}(z_h/z) \int \frac{d\ell_T^2}{\ell_T^2} \frac{\alpha_s^2 x_B}{Q^2} \frac{2(1+z^2)}{z(1-z)} \times \frac{C_F^2}{N_c} \overline{I}_{1,L(R)}(y^-, y_1^-, y_2^-, \ell_T, x, p, q, z),$$

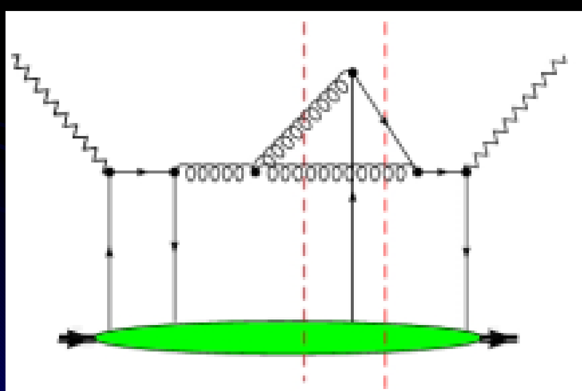
$$\overline{I}_{1,L} = -e^{i(x+x_L)p^+ y^-} \theta(y_1^- - y_2^-) \theta(y^- - y_1^-) \times (1 - e^{-ix_L p^+ (y^- - y_1^-)}),$$

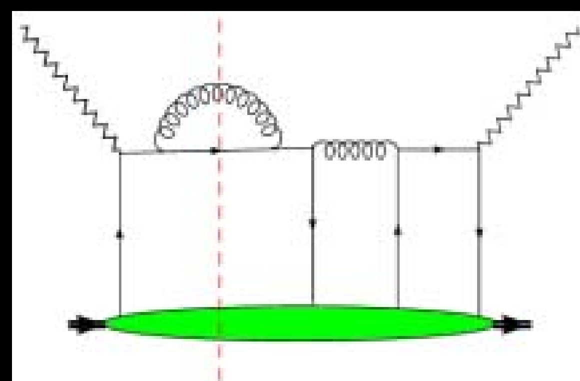
Other processes

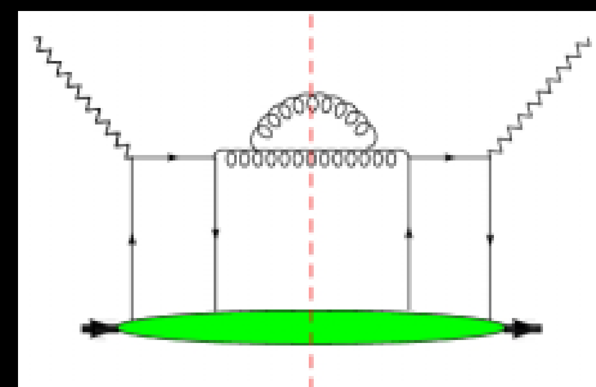






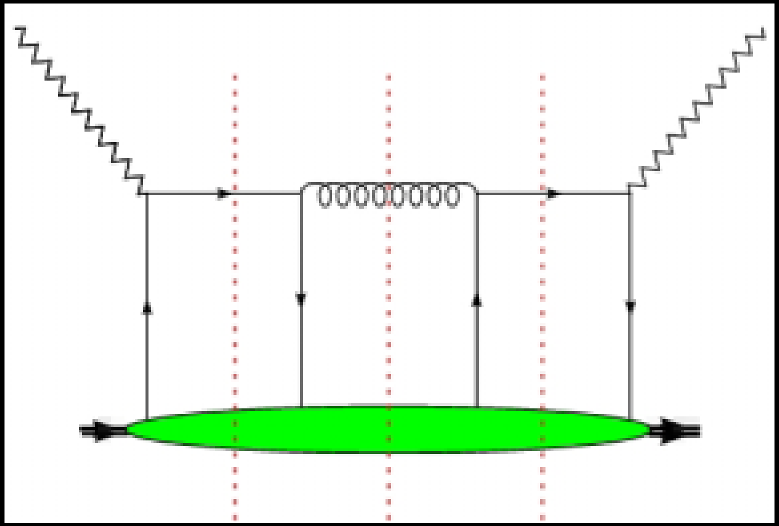






Q-Q Scattering without radiation

- Lowest order contribution:



$$\overline{H}^D_{0,C}(y^-, y_1^-, y_2^-, x, p, q, z) = D_{g\to h}(z_h)\frac{2\pi\alpha_s}{N_c}2C_F\frac{x_B}{Q^2}\theta(-y_2^-)\theta(y^- - y_1^-)\,,$$

$$\overline{H}^D_{0,L}(y^-, y_1^-, y_2^-, x, p, q, z) = D_{q\to h}(z_h)\frac{2\pi\alpha_s}{N_c}2C_F\frac{x_B}{Q^2}\theta(y_1^- - y_2^-)\theta(y^- - y_1^-)\,,$$

$$\overline{H}^D_{0,R}(y^-, y_1^-, y_2^-, x, p, q, z) = D_{q\to h}(z_h)\frac{2\pi\alpha_s}{N_c}2C_F\frac{x_B}{Q^2}\theta(-y_2^-)\theta(y_2^- - y_1^-)\,.$$

Modified Fragmenation Function

- Summing single and double scattering gives

$$\frac{dW_{\mu\nu}}{dz_h} = \sum_q \int dx\, \widetilde{f}_q^A(x,\mu_I^2)\, H_{\mu\nu}^{(0)}(x,p,q)\, \widetilde{D}_{q\to h}(z_h,\mu^2)$$

- We define the modified quark FF as:

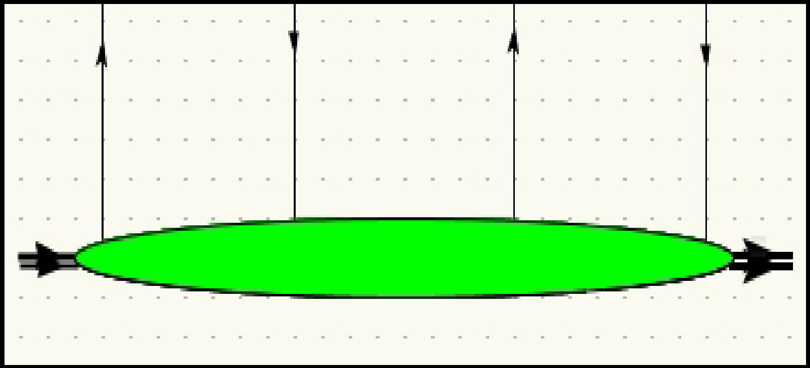
$$\widetilde{D}_{q\to h}(z_h,\mu^2) = D_{q\to h}(z_h,\mu^2) + \Delta\widetilde{D}_{q\to h}(z_h,\mu^2)\ ,$$

$$\Delta\widetilde{D}_{q\to h}(z_h,\mu^2) \equiv \frac{2\pi\alpha_s x_B}{Q^2}\frac{2C_F}{N_C}\frac{T_{q\bar{q}}^{A(I)}(x,x_L)[D_{g\to h}(z_h/z,\mu^2) - D_{q\to h}(z_h/z,\mu^2)]}{\widetilde{f}_q^A(x,\mu_I^2)}$$

$$+ \int_0^{\mu^2}\frac{d\ell_T^2}{\ell_T^2}\frac{\alpha_s^2 x_B}{Q^2}\int_{z_h}^1\frac{dz}{z}D_{q\to h}(z_h/z,\mu^2)\frac{C_F}{N_C}\frac{1}{\widetilde{f}_q^A(x,\mu_I^2)}$$

$$\times\left[\frac{1+z^2}{(1-z)_+^2}T_{q\bar{q}}^{A(II)}(x,x_L) + \delta(1-z)\Delta T_{q\bar{q}}^{A(II)}(x,\ell_T^2)\right]\ ,$$

Quark-quark correlation function



$$x_A = \frac{1}{m_N R_A}$$

$$T_{q\bar{q}} \propto f_q^A(x_1)\, f_{\bar{q}}^N(x_2)$$

M. Luo,J. Qiu and G. Sterman, PRD50(1994)1951,

X.F. Guo and X.N. Wang, PRL85(2000)3591.

Quark and anti-quark FF(I)

- We get the modification to quark FF as:

$$\Delta\widetilde{D}_{q\to h}(z_h,\mu^2) \approx \frac{2\pi\alpha_s x_B}{x_A Q^2}\frac{2C_F}{N_C}[D_{g\to h}(z_h/z,\mu^2) - D_{q\to h}(z_h/z,\mu^2)]C\, f_{\bar{q}}^N(x_T)$$
$$+\int_0^{\mu^2}\frac{d\ell_T^2}{\ell_T^2}\frac{\alpha_s^2 x_B}{x_A Q^2}\int_{z_h}^1\frac{dz}{z}D_{q\to h}(z_h/z,\mu^2)\frac{1+z^2}{(1-z)_+^2}\frac{C_F}{N_C}[1-e^{-x_L^2/x_A^2}]C\, f_{\bar{q}}^N(x_T)$$
$$-\int_0^{\mu^2}\frac{d\ell_T^2}{\ell_T^2}\frac{\alpha_s^2 x_B}{x_A Q^2}\int_0^1 dz D_{q\to h}(z_h,\mu^2)\frac{1+z^2}{(1-z)^2}\frac{C_F}{N_C}[1-e^{-x_L^2/x_A^2}]C\, f_{\bar{q}}^N(x_T)\,.$$

- Similarly the modification to anti-quark FF is:

$$\widetilde{D}_{\bar{q}\to h}(z_h,\mu^2) = D_{\bar{q}\to h}(z_h,\mu^2) + \Delta\widetilde{D}_{\bar{q}\to h}(z_h,\mu^2)\ ,$$
$$\Delta\widetilde{D}_{\bar{q}\to h}(z_h,\mu^2) \approx \frac{2\pi\alpha_s x_B}{x_A Q^2}\frac{2C_F}{N_C}[D_{g\to h}(z_h/z,\mu^2) - D_{\bar{q}\to h}(z_h/z,\mu^2)]C\, f_q^N(x_T)$$
$$+\int_0^{\mu^2}\frac{d\ell_T^2}{\ell_T^2}\frac{\alpha_s^2 x_B}{x_A Q^2}\int_{z_h}^1\frac{dz}{z}D_{\bar{q}\to h}(z_h/z,\mu^2)\frac{1+z^2}{(1-z)_+^2}\frac{C_F}{N_C}[1-e^{-x_L^2/x_A^2}]C\, f_q^N(x_T)$$
$$-\int_0^{\mu^2}\frac{d\ell_T^2}{\ell_T^2}\frac{\alpha_s^2 x_B}{x_A Q^2}\int_0^1 dz D_{\bar{q}\to h}(z_h,\mu^2)\frac{1+z^2}{(1-z)^2}\frac{C_F}{N_C}[1-e^{-x_L^2/x_A^2}]C\, f_q^N(x_T)\,.$$

Quark and anti-quark FF(II)

- The difference between modified quark FF and modified anti-quark FF:

$$\Delta D_{q\to h}(z_h,\mu^2) \propto \frac{T_{q\bar{q}}(x)}{f_q^A(x,\mu_I^2)} \propto f_{\bar{q}}^N(x_2)$$

$$\Delta D_{\bar{q}\to h}(z_h,\mu^2) \propto \frac{T_{\bar{q}q}(x)}{f_{\bar{q}}^A(x,\mu_I^2)} \propto f_q^N(x_2)$$

$$D_{q\to h}(z_h,\mu^2) = D_{\bar{q}\to h}(z_h,\mu^2)$$

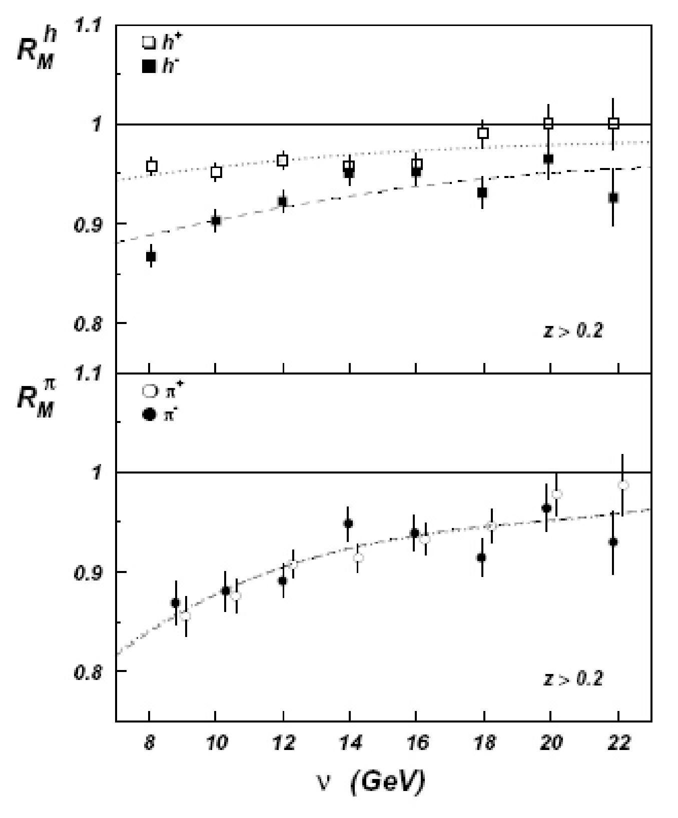
$$\frac{\Delta\widetilde{D}_{q\to h}(z_h,\mu^2)}{\Delta\widetilde{D}_{\bar{q}\to h}(z_h,\mu^2)} = \frac{f_{\bar{q}}^N(x_T)}{f_q^N(x_T)} < 1$$

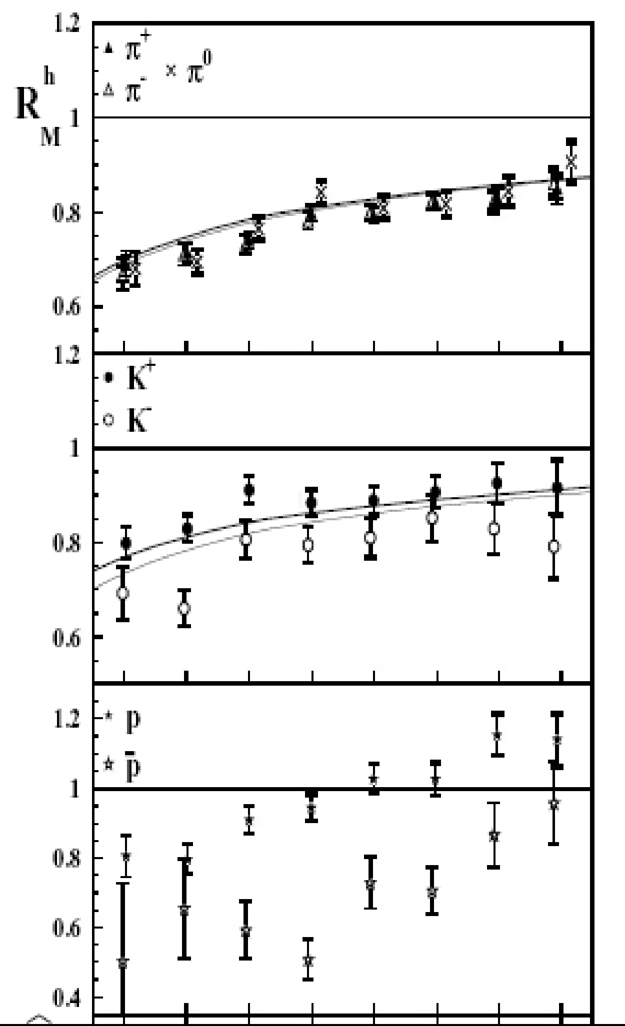
B.W.Zhang, X.N.Wang, A. Schaefer, in preparation

Multiplicity ratios of hadrons

- Multiplicity ratio measured at HERMES:

$$R_M^h(z,\nu) = \frac{N_h^A(z,\nu)}{N_e^A(\nu)} \Bigg/ \frac{N_h^D(z,\nu)}{N_e^D(\nu)}$$





HERMES

hep-ex/0012049

hep-ex/0307023

$$\Delta R_M^h \equiv 1 - R_M^h$$

$$\Delta R_M^{\pi^+} \simeq \Delta R_M^{\pi^-} \simeq \Delta R_M^{\pi^0}$$

$$\Delta R_M^{\bar{p}} > \Delta R_M^{p}$$

$$\Delta R_M^{K^-} > \Delta R_M^{K^+}$$

$$\Delta R_M^{h^-} > \Delta R_M^{h^+}$$

Theoretical explanation

- In the constituent quark model:

$$\pi = u\bar{d}\ ,(u\bar{u}-d\bar{d})/\sqrt{2}\ ,d\bar{u}\ ,$$
$$p = uud\ ,\bar{p} = \bar{u}\bar{u}\bar{d}\ ,$$
$$K^{+} = u\bar{s}\ ,K^{-} = \bar{u}s\ .$$

$$\Delta R_M^h(z,\nu) \approx \frac{1}{N}\sum_a \Delta\widetilde{D}_{a\to h}(z,\nu)$$

$$\Delta R_M^{\pi^+}(z,\nu) \simeq \Delta R_M^{\pi^-} \simeq \Delta R_M^{\pi^0}$$
$$\approx \frac{1}{2}(\Delta\widetilde{D}_{q\to h}(z,\nu) + \Delta\widetilde{D}_{\bar{q}\to h}(z,\nu))$$
$$\propto \frac{1}{2}[f_{\bar{q}}^N(x_T) + f_q^N(x_T)]\ .$$

$$\Delta R_M^{p}(z,\nu) \approx \Delta\widetilde{D}_{q\to h}(z,\nu) \propto f_{\bar{q}}^N(x_T)$$
$$\Delta R_M^{\bar{p}}(z,\nu) \approx \Delta\widetilde{D}_{\bar{q}\to h}(z,\nu) \propto f_q^N(x_T)$$

$$\Delta R_M^{\bar{p}}(z,\nu) > \Delta R_M^{p}(z,\nu)$$
$$\Delta R_M^{K^-}(z,\nu) > \Delta R_M^{K^+}(z,\nu)$$

$$\Delta R_M^{h^-} > \Delta R_M^{h^+}$$

Summary(I)

- Heavy quark energy loss induced by gluon radiation is derived in terms of Modified FF with pQCD.

- Two mass effects:

(I) Gluon formation time of the heavy quark is reduced relative to that of a light quark:

Medium size dependence of heavy quark enegy loss is found to change from a linear to a quadratic form when the initial energy and momentum scale are increasing.

(II) Dead-cone effect:

Heavy quark energy loss is significantly suppressed relative to a light quark.

Summary(II)

- Quark-quark double scattering is eA DIS are studied.
- Modification to quark FF in nuclei is different from the modification to anti-quark FF in nuclei.
- This difference may explain the multiplicity ratios of hadrons in nuclei observed at HERMES.

Thank you very much!

Heavy quark Frag. Func.

$$D_{i\to H}(z_H,\mu) = \int_{z_H}^{1} \frac{dz}{z} D_i^Q(z,\mu) D_{Q\to H}(z_H/z),$$

$$D_{c\to D}(z) = \frac{N}{z[1 - z^{-1} - \varepsilon_c/(1-z)]}$$

$$D_Q^Q(z,\mu_0) = \delta(1-z) + \frac{\alpha_s(\mu_0)C_F}{2\pi}\left[\frac{1+z^2}{1-z}\left(\log\frac{\mu_0^2}{M^2} - 2\log(1-z) - 1\right)\right]_+,$$

$$D_g^Q(z,\mu_0) = \frac{\alpha_s(\mu_0)C_A}{2\pi}\left[z^2 + (1-z)^2\right]\log\frac{\mu_0^2}{M^2},$$

$$D_{q,\bar{q},\bar{Q}}^Q(z,\mu_0) = 0.$$