A perturbative approach to medium modification of jet fragmentation

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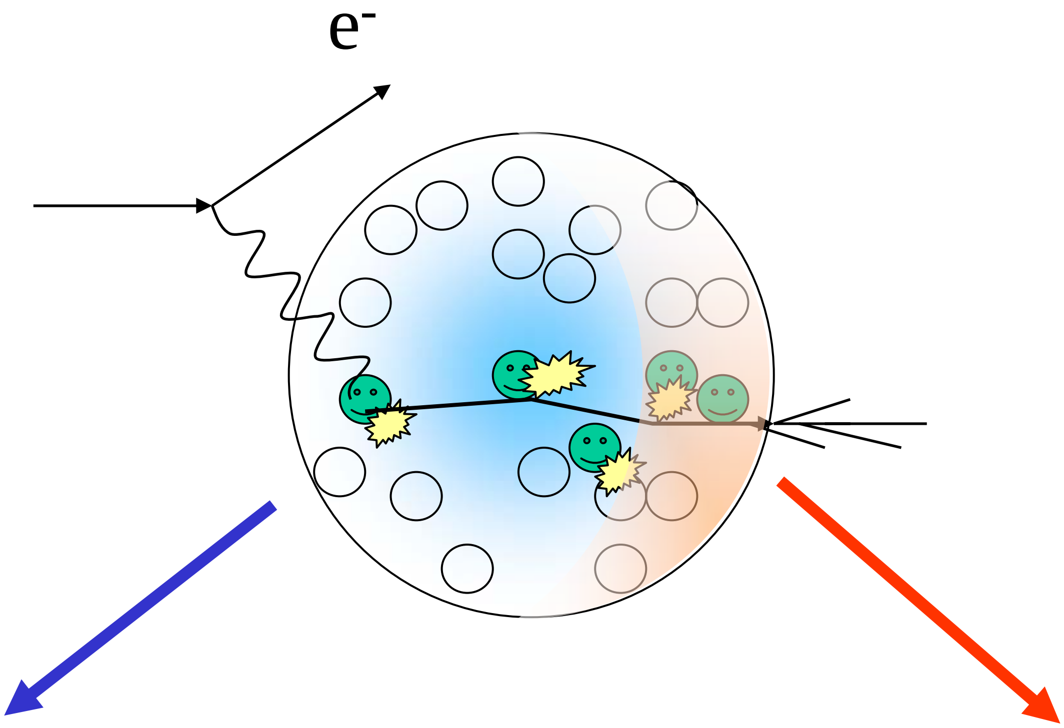
LBNL

Workshop on Parton propagation through strongly interacting systems
ETC, Trento, Sept. 26-7, 2005

Quark scattering or hadron absorption?



Quark propagation and scattering,

Hadronization outside the nuclei

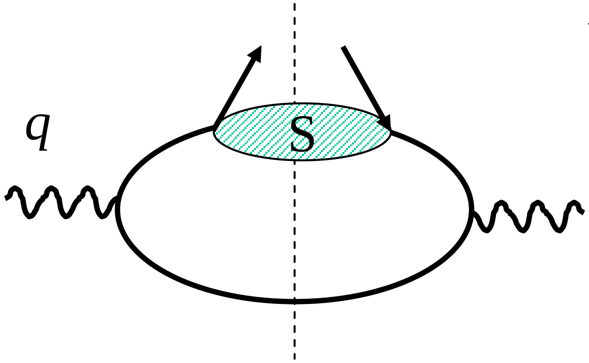
Hadronization inside nuclei

Hadron absorption

Quark Fragmentation Function

$$\sigma_{tot}^{e^+e^-\to h} = \frac{1}{2s}\frac{e^4}{4q^4} L_{\mu\nu}(p,q) W^{\mu\nu}(q)$$

e+e- annihilation



$$W_{\mu\nu}(q) = \sum_X \langle 0 | J_\mu(0) | X \rangle \langle X | J_\nu(0) | 0 \rangle (2\pi)^4 \delta^4(q - p_X)$$

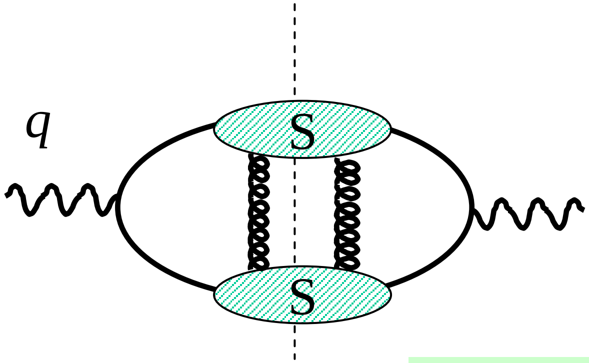
$$= \mathrm{Im}\left\{ i \int d^4 y e^{iq\cdot y} \langle 0 | T\left[J_\mu(y), J_\nu(0) \right] | 0 \rangle \right\}$$

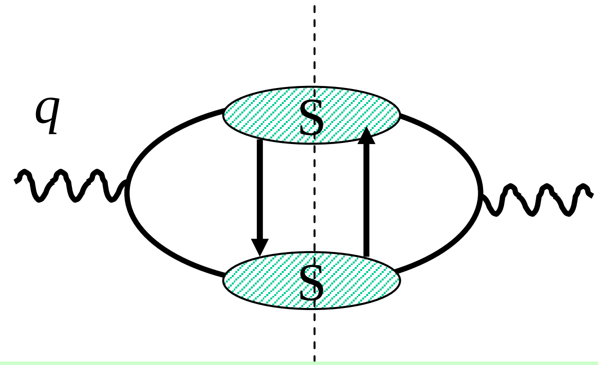
Factorization

$$\frac{d\sigma}{dz} = \sigma_0 \left[D_{q\to h}(z) + D_{\bar{q}\to h}(z) \right]$$

$$D_{q\to h}(z_h) = \frac{z_h}{2} \int \frac{dy^-}{2\pi} e^{-ip_h^+ y^- / z_h} \sum_S Tr\left[\frac{\gamma^+}{2} \langle 0 | \psi_q(0) | p_h, S \rangle \langle p_h, S | \overline{\psi}_q(y^-) | 0 \rangle \right]$$

Color Neutralization





Longitudinal gluon

$$D_{q\to h}(z_h) \propto \sum_S Tr\left[\frac{\gamma^+}{2}\left\langle 0\left|\psi_q(0)\right|p_h,S\right\rangle e^{ig\int_0^y dz\; n\cdot A(z)}\left\langle p_h,S\left|\bar{\psi}_q(y^-)\right|0\right\rangle\right]$$

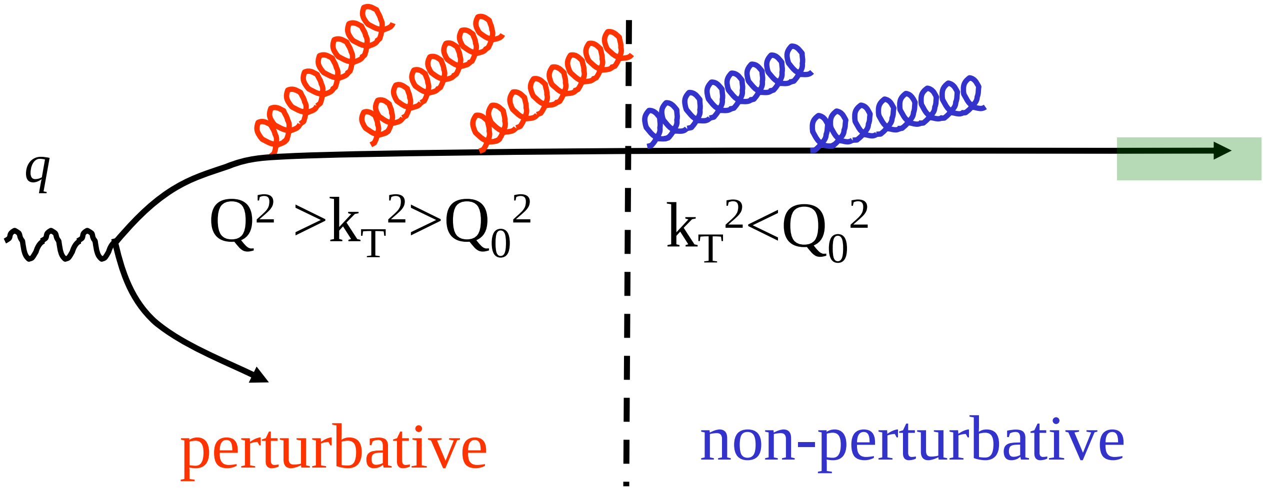
Transverse soft gluon

$$\Delta\sigma \propto \frac{1}{Q^2}\sum_S Tr\left[\frac{\gamma^+}{2}\left\langle 0\left|\psi_q(0)F^{\mu\nu}(y_1)\right|p_h,S\right\rangle\left\langle p_h,S\left|F_{\mu\nu}(y_2)\bar{\psi}_q(y^-)\right|0\right\rangle\right]$$

Quark exchange

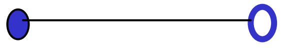
$$\Delta\sigma \propto \frac{1}{Q^2}\sum_S\left[p^+\left\langle 0\left|\psi_q(0)\gamma^+\bar{\psi}_{q'}(y_1)\right|p_h,S\right\rangle\left\langle p_h,S\left|\psi_{q'}(y_2)\gamma^+\bar{\psi}_q(y^-)\right|0\right\rangle\right]$$

Hard vs soft radiation



$$\tau_f \approx \frac{2\omega}{k_T^2}$$

Dynamic "string tension": $$\kappa_{dyn} \equiv \frac{dE}{dx} \approx \frac{C_F}{2\pi}\alpha_S Q_0^2$$

Static string tension: 

Hadronization time: $$\tau_{hadr} \approx \frac{E}{\langle k_T \rangle_h^2} = E\ r_h^2 \qquad <k_T>\sim 1/r_h$$

$$\Delta D_{q\to h}(z_h) = \frac{\alpha_S}{2\pi}\int^{\mu^2}\frac{d\ell_\perp^2}{\ell_\perp^2}\int_{z_h}^{1}\frac{dz}{z}\left[P_{q\to qg}(z)D_{q\to h}\left(\frac{z_h}{z}\right)+P_{q\to qg}(1-z)D_{q\to h}\left(\frac{z_h}{z}\right)\right]$$

Splitting function $$P_{q\to qg}(z) = C_F\left[\frac{1+z^2}{(1-z)_+}+\frac{3}{2}\delta(1-z)\right]$$

DGLAP Evolution II

Binnewies, Kniehl, Kramer 1995

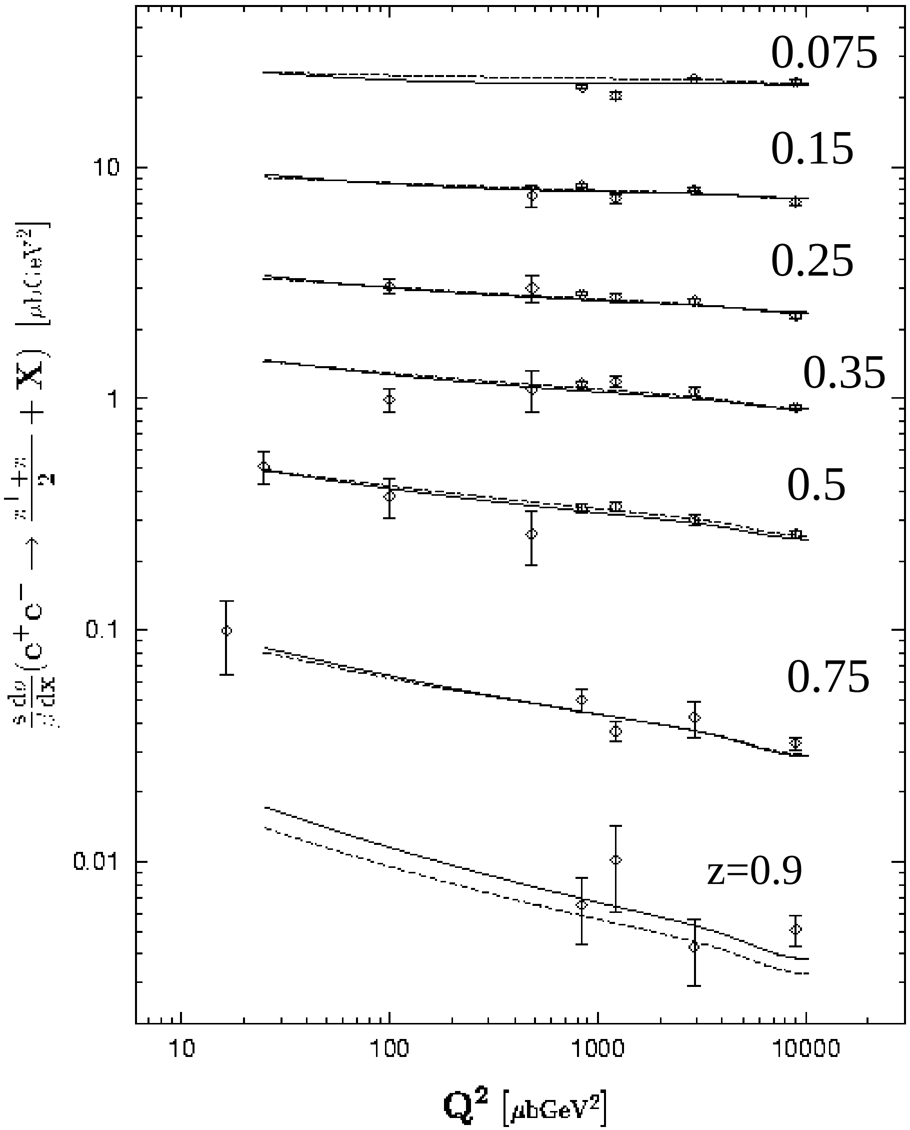
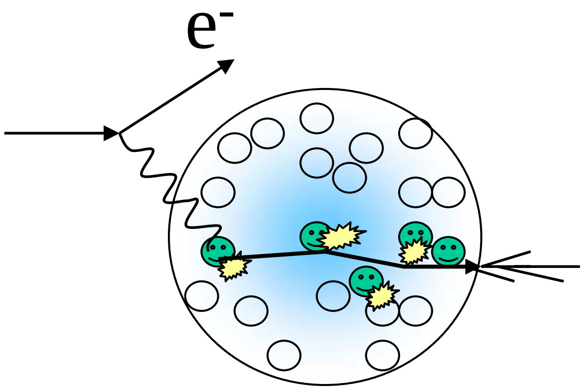
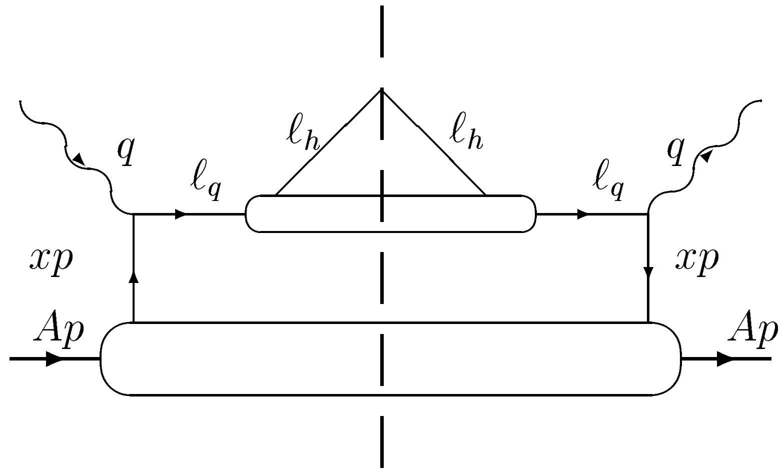


Fig. 10

DIS off Nuclei





$$\frac{dW_{\mu\nu}}{dz_h} = \sum_q \int dx\, f(x) H_{\mu\nu}(x,p,q) D_{q\to h}(z_h)$$

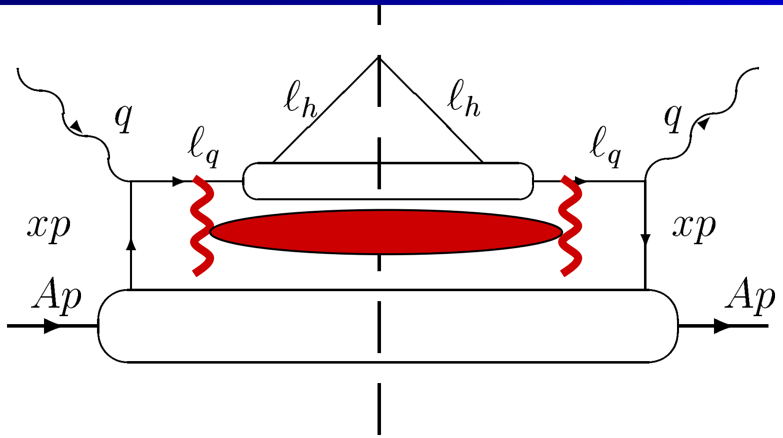
$$f_q(x_B) = \int \frac{dy^-}{2\pi} e^{ixp^+y^-} \frac{1}{2} \langle p | \bar{\psi}(0)\gamma^+ \psi(y^-) | p \rangle$$

$$H_{\mu\nu}(x,p,q) = e_q^2 \frac{1}{2} Tr\left[\gamma \cdot p \gamma_\mu \gamma \cdot (q+xp)\gamma_\nu\right] 2\pi\delta\left[(q+xp)^2\right]$$

Frag. Func.

$$D_{q\to h}(z_h) = \frac{z_h}{2} \int \frac{dy^-}{2\pi} e^{-ip_h^+ y^-/z_h} \sum_S Tr\left[\frac{\gamma^+}{2} \langle 0 | \psi_q(0) | p_h, S \rangle \langle p_h, S | \bar{\psi}_q(y^-) | 0 \rangle\right]$$

Multiple Parton Scattering



$$\tau_h = r_h^{\ 2} E$$

$$\frac{\alpha_S}{\ell_T^4}\left(1 - e^{-ix_L p^+ y_2^-}\right)\left(1 - e^{ix_L p^+ (y_1^- - y^-)}\right)$$

$$\tau_f = \frac{1}{x_L p^+} = \frac{2q^- z(1-z)}{\ell_T^2}$$

Formation time

Multiple Parton Scattering

Generalized factorization:

(LQS'94)

$$W_{\mu\nu}^{D} \propto \int d^2k_T e^{ik\cdot(y_1-y_2)} H_{\mu\nu}^{D}(p,q,k_T)\langle A|\bar{\psi}\gamma^+ A^+(y_1)A^+(y_2)\psi|A\rangle$$

Collinear expansion:

$$H_{\mu\nu}^{D}(p,q,k_T) = H_{\mu\nu}^{D}(p,q,k_T=0) + \partial_{k_T} H_{\mu\nu}^{D}(p,q,k_T=0)k_T$$
$$+\partial_{k_T}^2 H_{\mu\nu}^{D}(p,q,k_T=0)k_T^2 + \cdots$$

Collinear approximation

$$W_{\mu\nu}^{D} \propto \int d^2k_T e^{ik\cdot(y_1-y_2)} H_{\mu\nu}^{D}(p,q,k_T)\langle A|\bar{\psi}\gamma^+ A^+(y_1)A^+(y_2)\psi|A\rangle$$

$$H_{\mu\nu}^{D}(p,q,k_T) = H_{\mu\nu}^{D}(p,q,k_T=0) + \partial_{k_T} H_{\mu\nu}^{D}(p,q,k_T=0)k_T$$
$$+\partial_{k_T}^2 H_{\mu\nu}^{D}(p,q,k_T=0)k_T^2 + \cdots$$

First term

Eikonal →

$$\langle A|\overline{\psi}\gamma^+ \left[ig\int dz\, A^+ - g^2\int dz\int dz' A^+ A^+\right]\psi|A\rangle$$
$$\approx \langle A|\overline{\psi}(0)\gamma^+ \exp\left[ig\int dz\, A^+\right]\psi(z)|A\rangle$$

Double scattering

$$W_{\mu\nu}^{D} \propto \partial_{k_T}^2 H_{\mu\nu}^{D}(p,q,k_T=0)\langle A|\overline{\psi}\gamma^+ F^{+\sigma}F^{+}{}_{\sigma}\psi|A\rangle$$

Modified Fragmentation

Guo & XNW'00

$$\Delta D_{q\to h}(z_h,Q^2) = \frac{\alpha_S}{2\pi}\int_0^{Q^2}\frac{d\ell_\perp^2}{\ell_\perp^4}\int_{z_h}^{1}\frac{dz}{z}\left[\Delta\gamma(z,x_L)D_{q\to h}\left(\frac{z_h}{z}\right)+\cdots\right]$$

Modified splitting functions

$$\Delta\gamma(z,x_L) = \frac{1+z^2}{(1-z)_+}\frac{T^A_{qg}(x,x_L)}{f_q^A(x)}\frac{C_A 2\pi\alpha_S}{N_c}+\cdots(\text{virtual})$$

Two-parton correlation:

$$T^A_{qg}(x,x_L)=\int\frac{dy^-}{2\pi}dy_1^- dy_2^- e^{-ix_B p^+ y^-}\left\langle A\right|\bar{\psi}(0)\frac{\gamma^+}{2}F_\sigma^+(y_1^-)F^{+\sigma}(y_2^-)\psi(y^-)\left|A\right\rangle$$

$$\times\left(1-e^{-ix_L p^+ y_2^-}\right)\left(1-e^{ix_L p^+(y_1^- - y^-)}\right)\theta(-y_2^-)\theta(y^- - y_1^-)$$

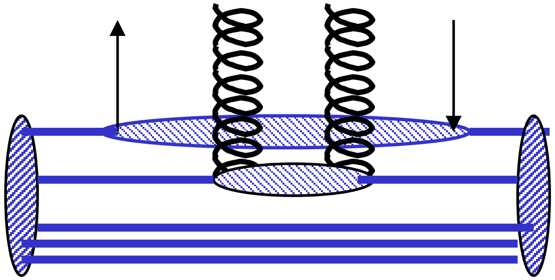
LPM →

Twist Expansion

$$\frac{d\sigma_S}{d\ell_\perp^2} \sim \frac{\alpha_s}{\ell_\perp^2}\int \frac{dy^-}{2\pi} e^{-ix_B p^+ y^-} \langle A|\bar{\psi}(0)\frac{\gamma^+}{2}\psi(y^-)|A\rangle \sim \frac{\alpha_s}{\ell_\perp^2} A f_q(x_B)$$

$$\frac{d\sigma_D}{d\ell_\perp^2} \sim \frac{\alpha_s}{\ell_\perp^4}\int \frac{dy^-}{2\pi}\frac{dy_1^-}{2\pi}\frac{dy_2^-}{2\pi} e^{-ix_B p^+ y^- + ix_T p^+(y_1^- - y_2^-)} \langle A|\bar{\psi}(0)\frac{\gamma^+}{2}F_\sigma^+(y_1^-)F^{+\sigma}(y_2^-)\psi(y^-)|A\rangle$$

$$\sim \frac{\alpha_s}{\ell_\perp^4} A^{4/3} f_q(x_B)\alpha_s x_T G(x_T)$$

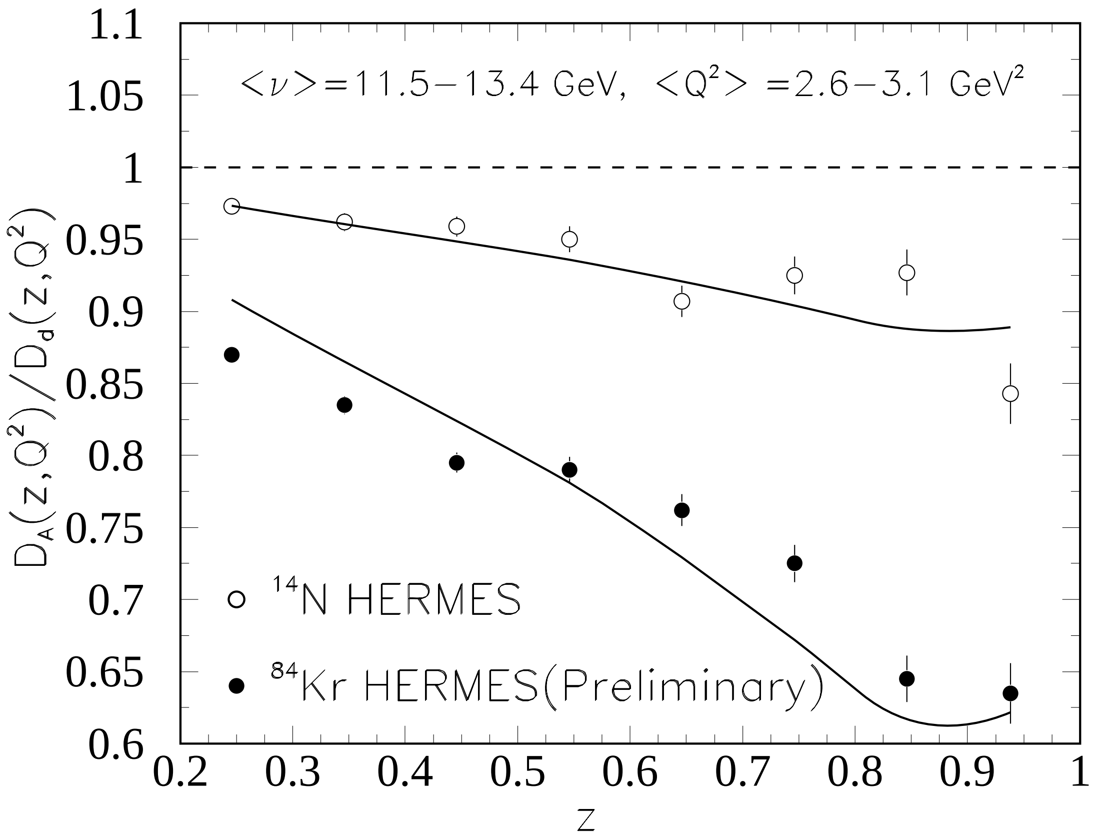


$$\frac{d\sigma_S}{d\ell_\perp^2} + \frac{d\sigma_D}{d\ell_\perp^2} \sim \frac{\alpha_s}{\ell_\perp^2} A f_q(x_B)[1 + c\frac{\alpha_S}{\ell_\perp^2} A^{1/3} x_T G(x_T) + \cdots]$$

$$\text{LPM} \Rightarrow \ell_\perp^2 \geq \frac{Q^2}{A^{1/3}}$$

$$1 + c\frac{A^{2/3}}{Q^2}$$

HERMES data



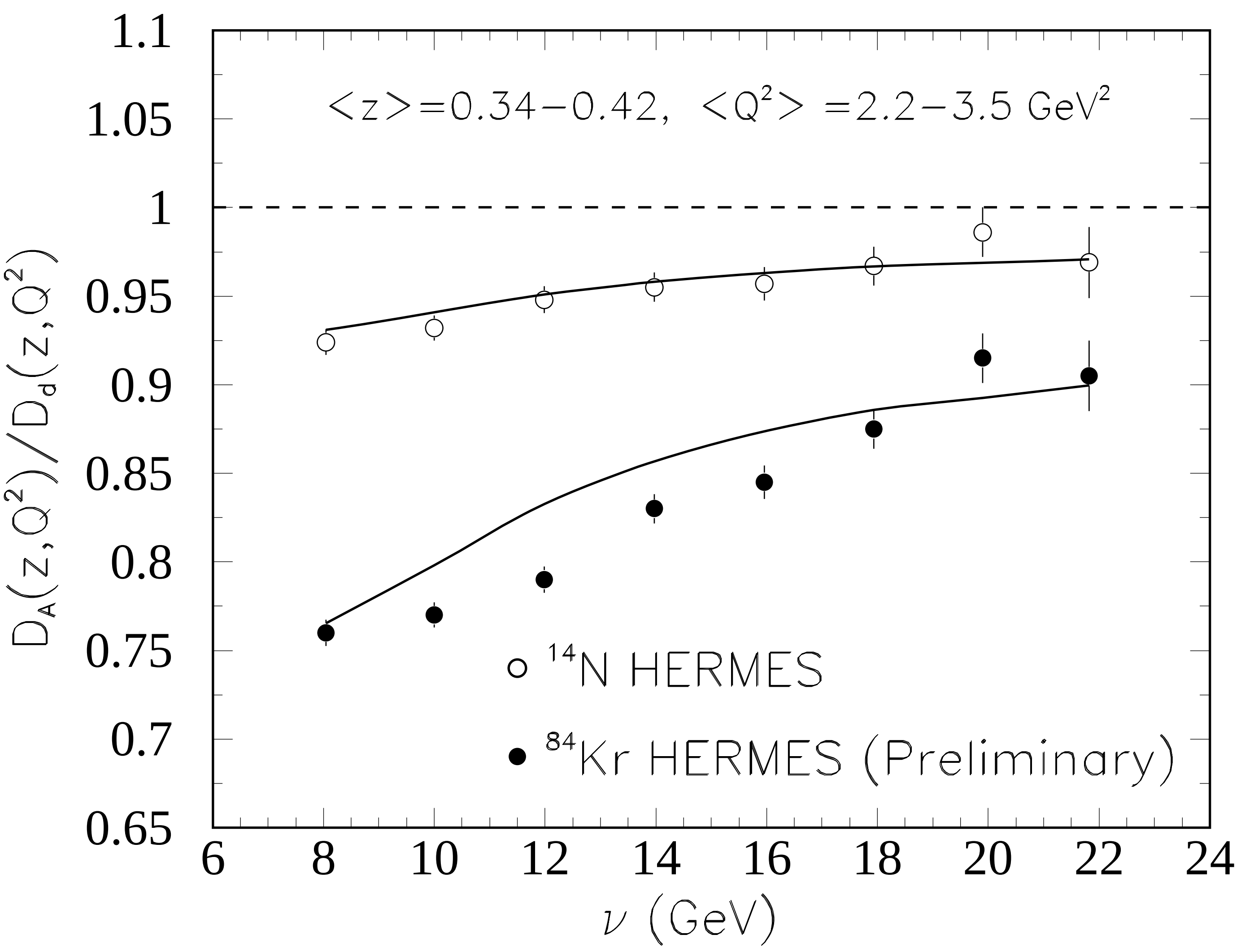
E. Wang & XNW
PRL 2000

$$C\alpha_s^2 \approx 0.00065 \ \text{GeV}^2$$

in Au nuclei

$$\frac{dE}{dx} \approx 0.5 \ \text{GeV/fm}$$

Energy Dependence



Parton Energy Loss

Quark energy loss = energy carried by radiated gluon

$$\langle \Delta z_g \rangle \equiv \int_0^{Q^2} d\ell_T^2 \int_0^1 dz \Delta\gamma(z, \ell_T) z = \int_0^{Q^2} d\ell_T^2 \int_0^1 dz \frac{1+(1-z)^2}{\ell_T^2 \left(\ell_T^2 + \langle k_T^2 \rangle\right)} \frac{C_A \alpha_s^2}{N_c} \frac{T_{qg}^A(x, x_L)}{f_q^A(x)}$$

$$\Delta E = C \alpha_s^2 \frac{C_A}{N_c} m_N r_0^2 A^{2/3} 3 \ln \frac{1}{2x_B}$$

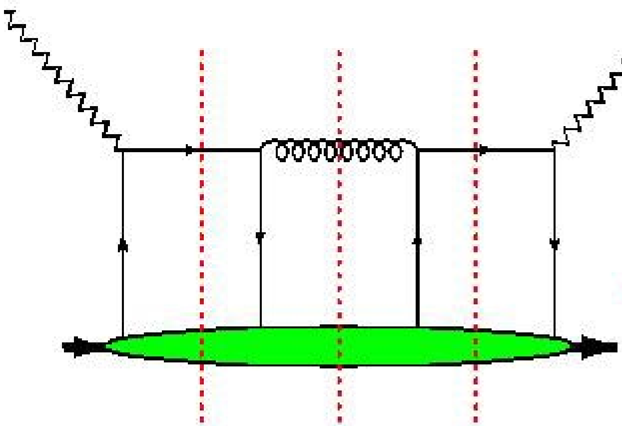
Weak E and Q^2 dependence

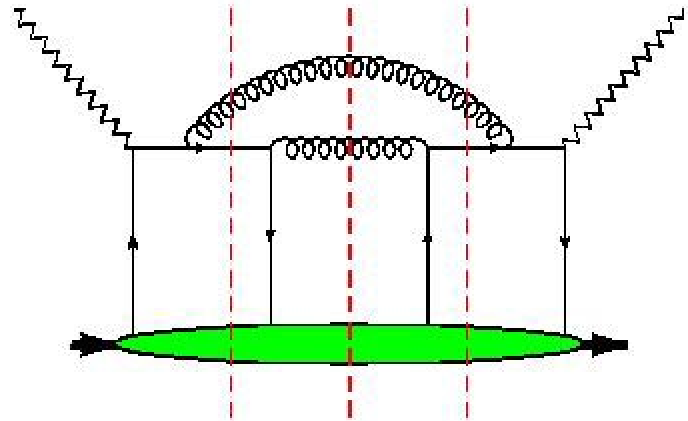
In a generalized case

$$\frac{T_{qg}^A(x, x_L)}{f_q^A(x)} \sim \int dy \mu^2 \sigma_g \rho_g(y) \left[1 - \cos \frac{y}{\tau_f}\right]$$

$$\Delta E = \pi C_a C_A \alpha_s^3 \int_{\tau_0}^{R} d\tau \rho(\tau)(\tau - \tau_0) \ln\left(\frac{2E}{\tau \mu^2}\right)$$

Quark-anti-quark annihilation





$$\Delta D_q(z_h) = \frac{2\pi\alpha_s}{Q^2} x_B \frac{2C_F}{N_c} \frac{T^A_{q\bar{q}}(x)}{f^A_q(x)} \left[D_{g\to h}(z_h) - D_{q\to h}(z_h) \right]$$

$$T^A_{q\bar{q}}(x) \propto \langle A | \bar{\psi}_q(0)\gamma^+\psi_q(y)\bar{\psi}_q(y_1)\gamma^+\psi_q(y_2) | A \rangle \sim f^A_q(x) f^N_{\bar{q}}(x_T)$$

$$\Delta D_{\bar{q}}(z_h) > \Delta D_q(z_h)$$

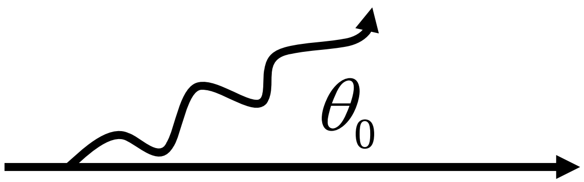
Flavor dependence

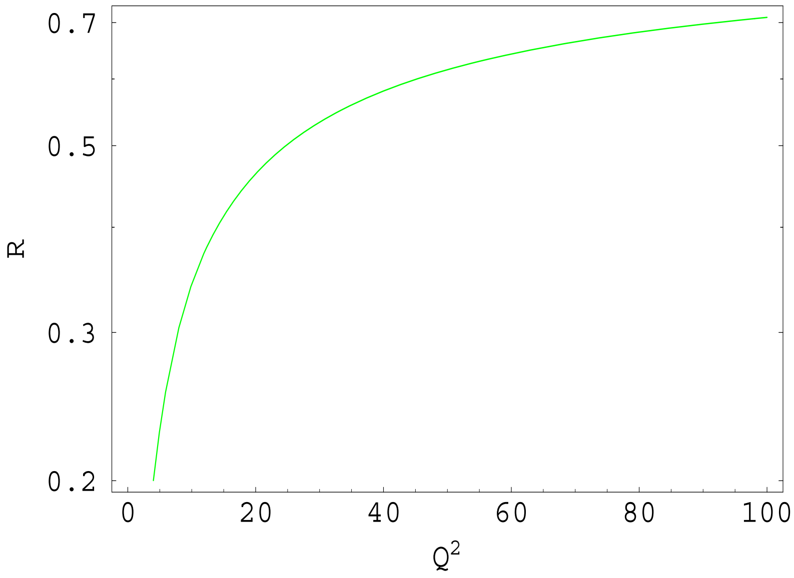
See B. Zhang's talk

Energy Loss of A Heavy Quark

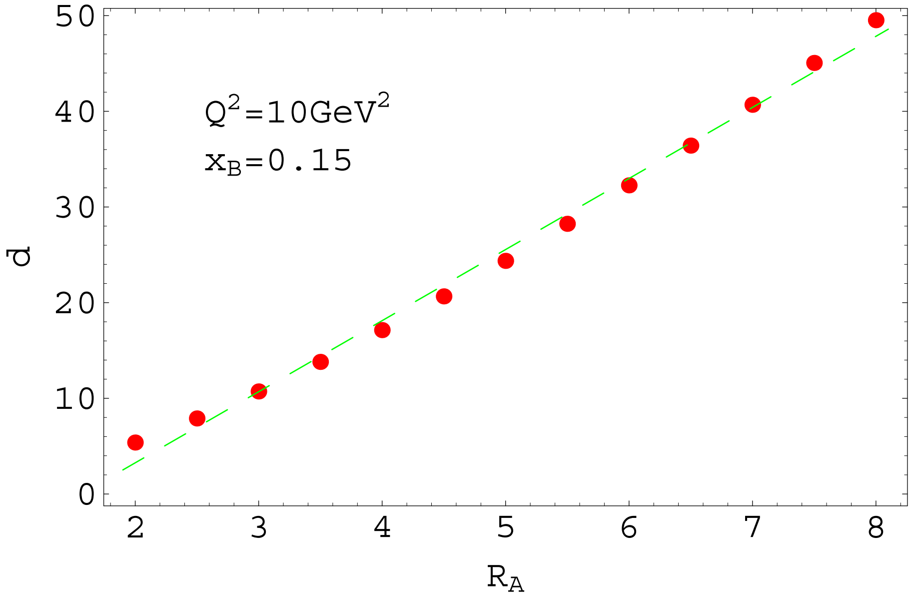
$$\tau_f^H = \frac{1}{1/\tau_f + (1-z)M^2/2zq^-}$$

$$\frac{dN}{d\ell_\perp^2} = \frac{1}{\ell_\perp^2 + (1-z)^2 M^2}$$





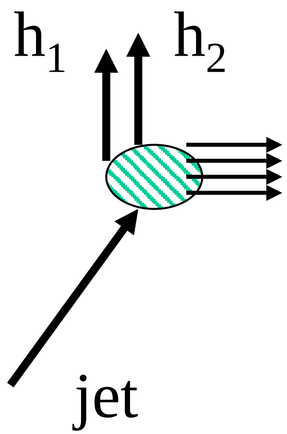
Dead cone effect



See B. Zhang’s talk

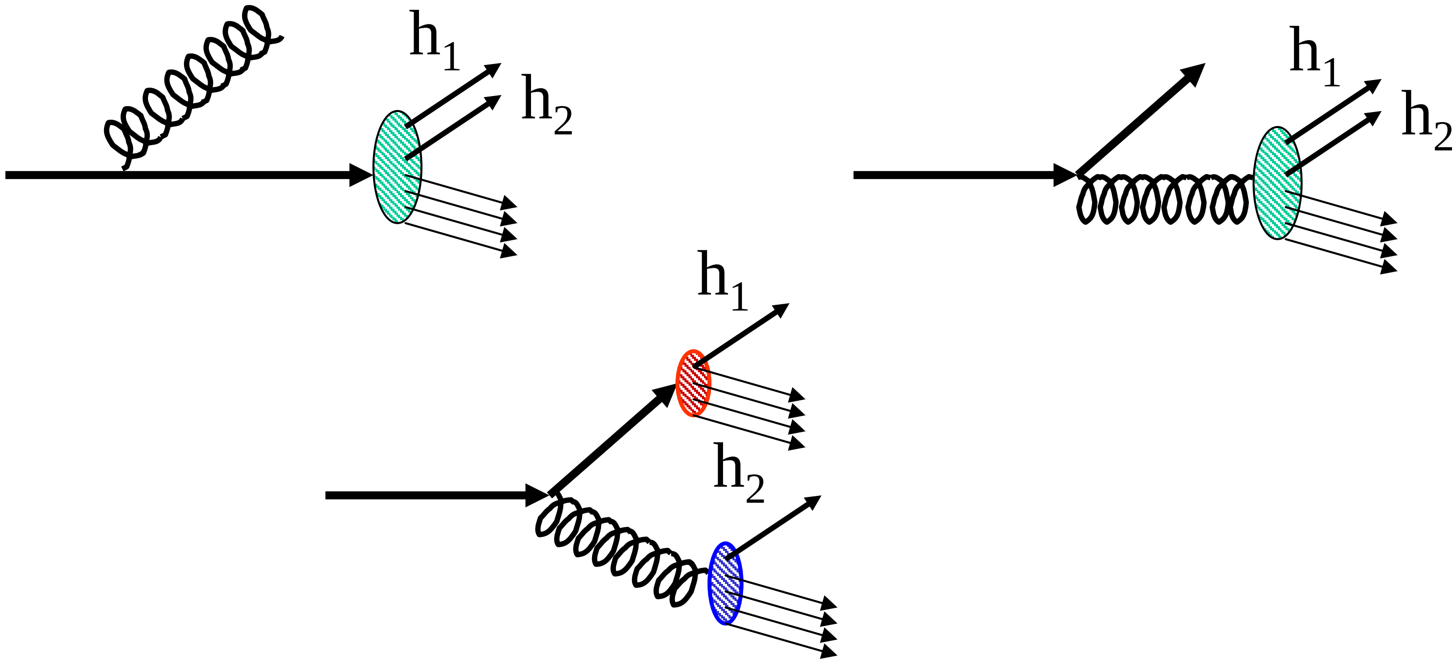
Di-hadron fragmentation function

Majumder & XNW'04



$$D_{q\to h1h2}(z_1,z_2)\propto\sum_S Tr\left[\frac{\gamma^+}{2}\langle 0|\psi_q(0)|p_{h1}p_{h2},S\rangle\langle p_{h1}p_{h2},S|\bar{\psi}_q(y^-)|0\rangle\right]$$

DGLAP for Dihadron Fragmentation

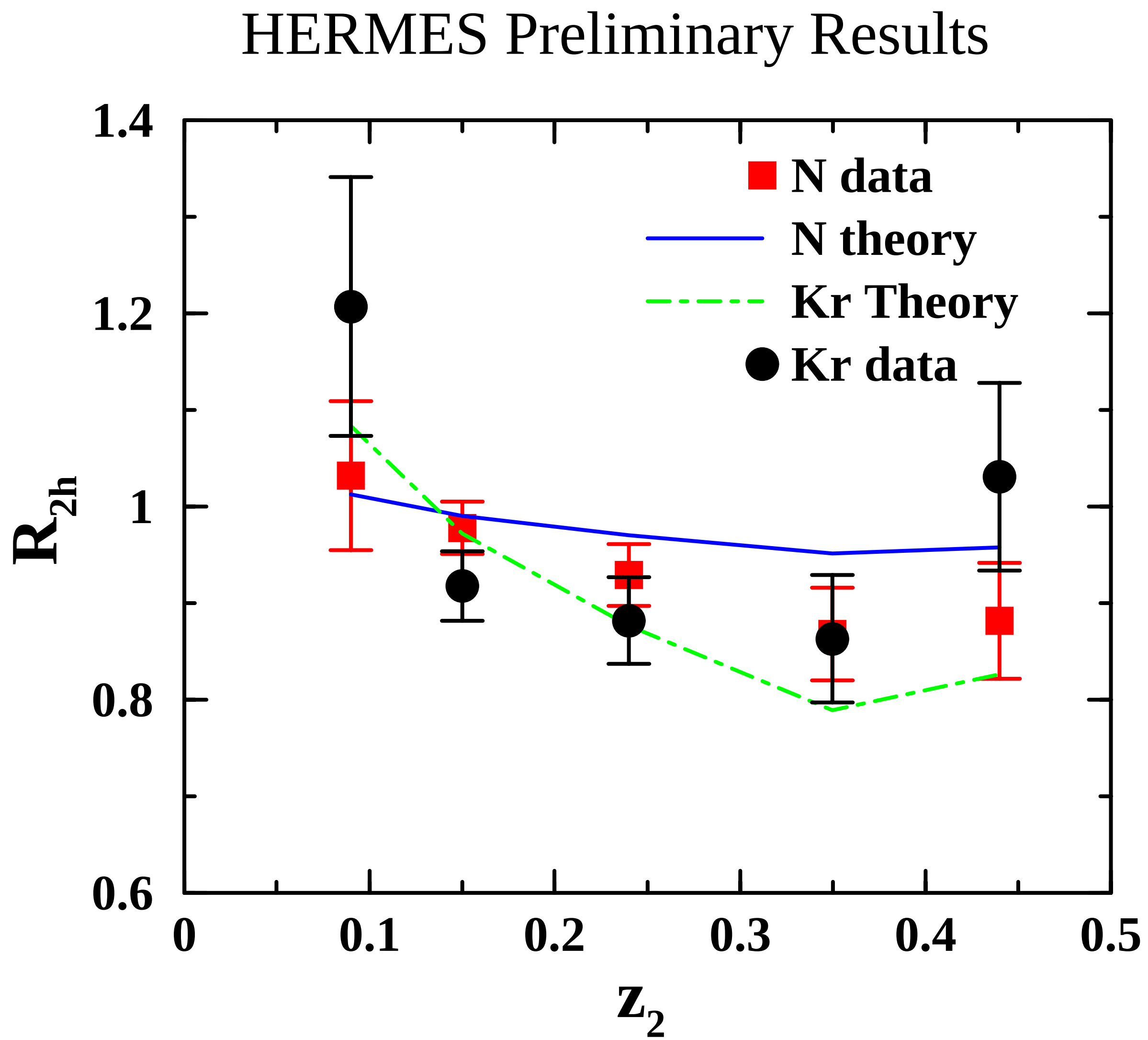


$$\frac{\partial D^q_{h_1h_2}(z_1,z_2,Q^2)}{\partial \ln Q^2} = \int_{z_1+z_2}^{1} \frac{dy}{y^2} P_{q\to qg}(y) D^q_{h_1h_2}\left(\frac{z_1}{y},\frac{z_2}{y},Q^2\right) + (g \to h_1h_2)$$

$$+ \int_{z_1}^{1-z_2} \frac{dy}{y(1-y)} \hat{P}_{q\to qg}(y) D^q_{h_1}\left(\frac{z_1}{y},Q^2\right) D^g_{h_2}\left(\frac{z_2}{1-y},Q^2\right) + (q \leftrightarrow g)$$

Medium Modified Dihadron

$D(z_1,z_2)/D(z_1)$

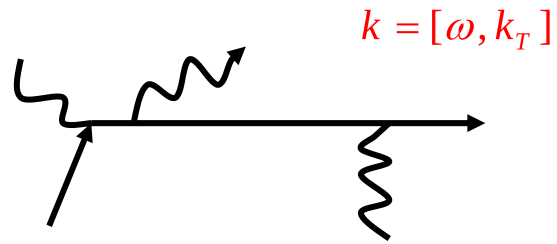


LPM & k_T Correlation

Radiation in vacuum

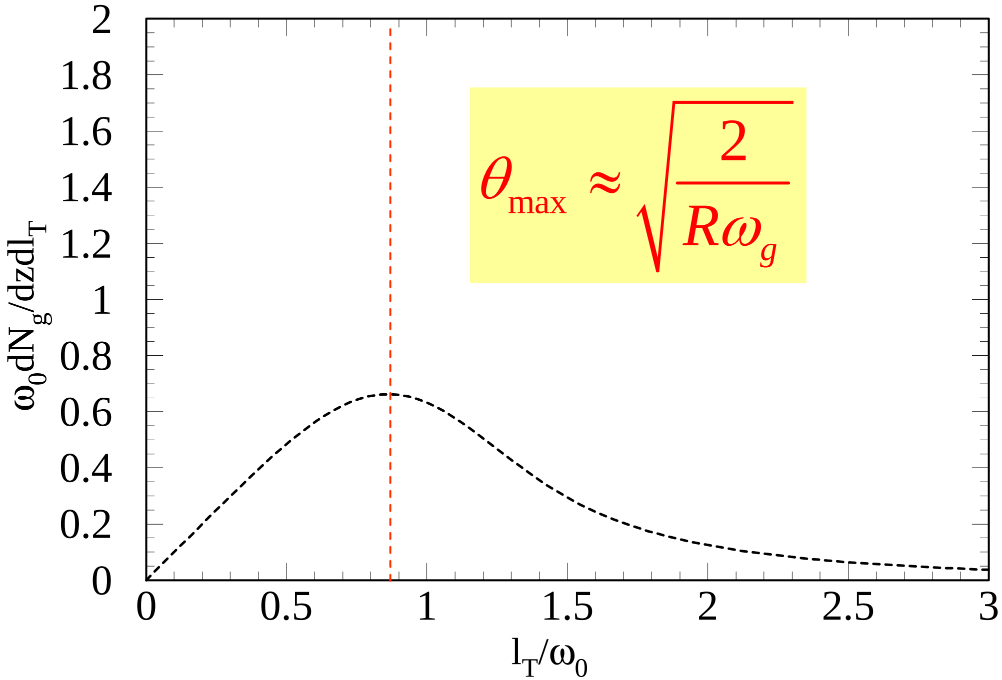
$$\frac{dN_g}{dzdk_T^2} = \frac{\alpha_S}{2\pi} C_F \frac{1+(1-z)^2}{z} \frac{1}{k_T^2}$$

Multiple Scattering in QCD



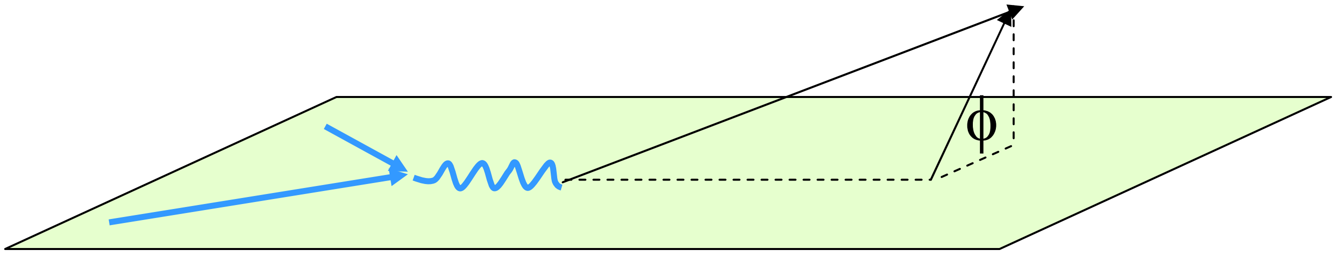
Formation time $\tau_f = \frac{2Ez(1-z)}{k_T^2}$

$$\frac{dN_g}{dzdk_T^2} \propto \frac{1}{k_T^4} \left(1 - e^{-\left(\frac{R}{\tau_f}\right)^2} \right)$$



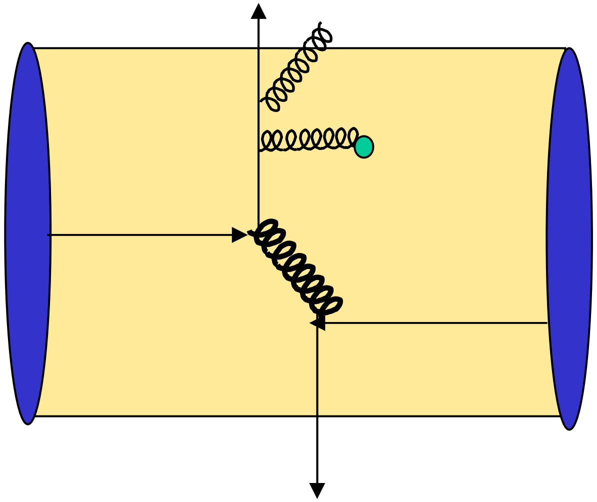
Azimuthal angular distribution

$dN_h/d\phi$



Multiple scattering and induced gluon radiation will modified the azimuthal distribution

Jet Quenching in A+A Collisions



$$\tau_f \approx \frac{2E}{Q^2} = \frac{1}{p_T}$$

Short formation time for the hardest gluon radiation

$$\tau_f \approx \frac{2E}{Q_0^2} = \frac{2p_T}{Q_0^2}$$

Soft radiation still takes a long time

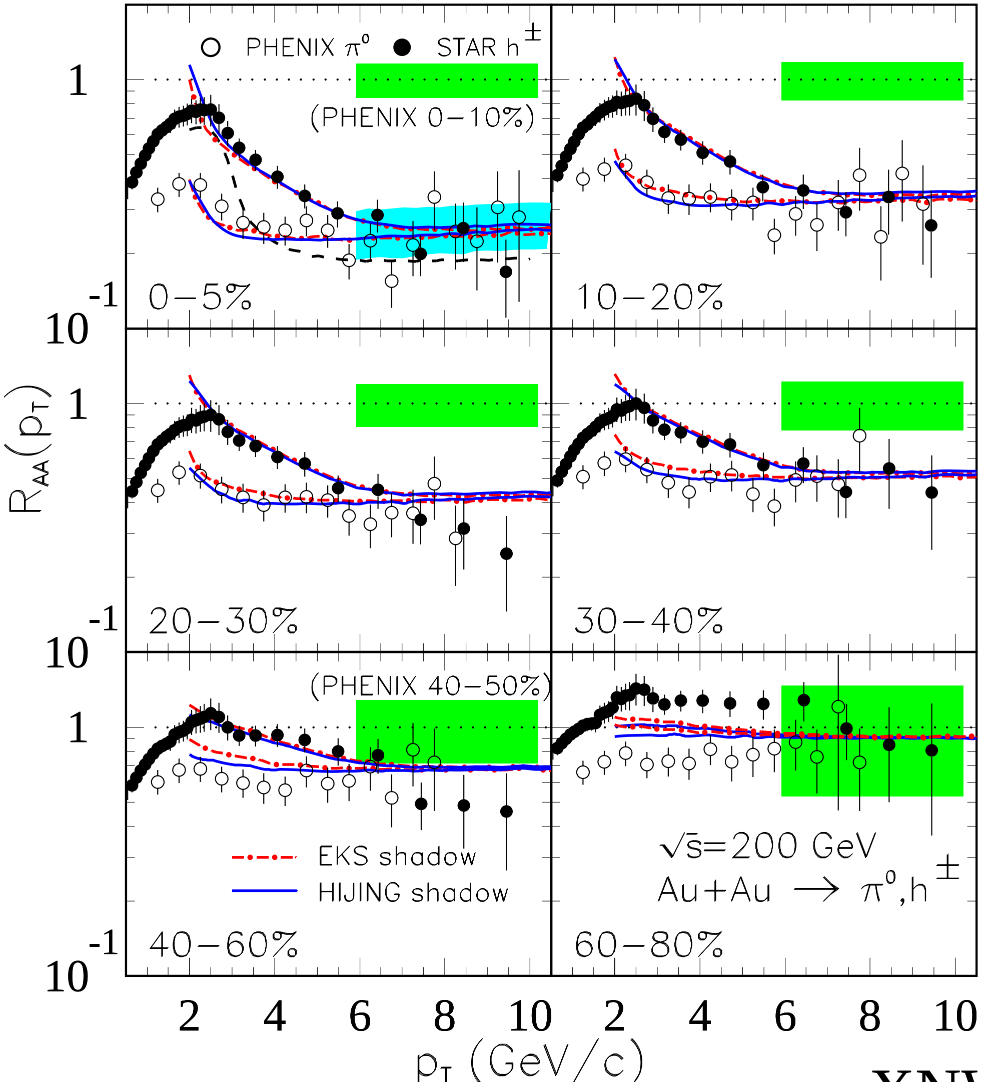
$$\frac{d\sigma_{AB}}{dyd^2p_T} = K\int d^2b \int d^2r_1 d^2r_2 t_A(r_1)t_B(r_2)\sum_{abcd}\int dx_a d^2k_{\perp a} dx_b d^2k_{\perp b}$$

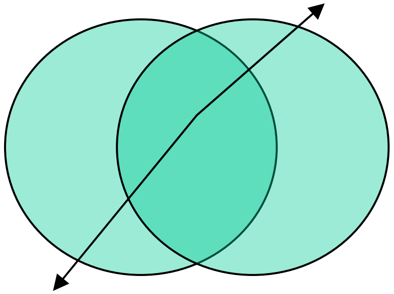
$$f_{a/A}(x_a, k_{\perp a}, r_1) f_{b/B}(x_b, k_{\perp b}, r_2) \frac{d\sigma_{ab\to cd}}{dt} \frac{1}{z_c \pi} \tilde{D}_{h/c}(z_c)$$

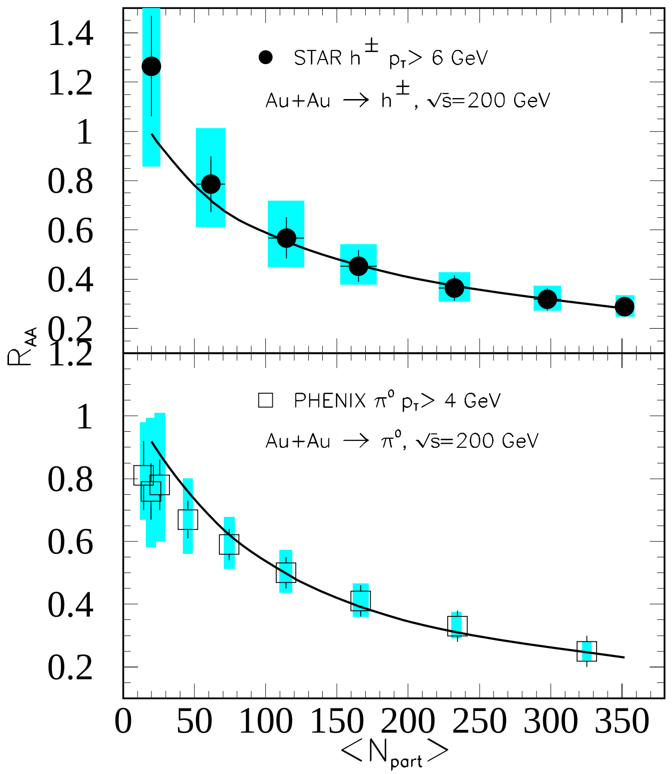
Parton distr. in nuclei & p_T broadening

Modified Frag. Fun.

Jet Quenching at RHIC



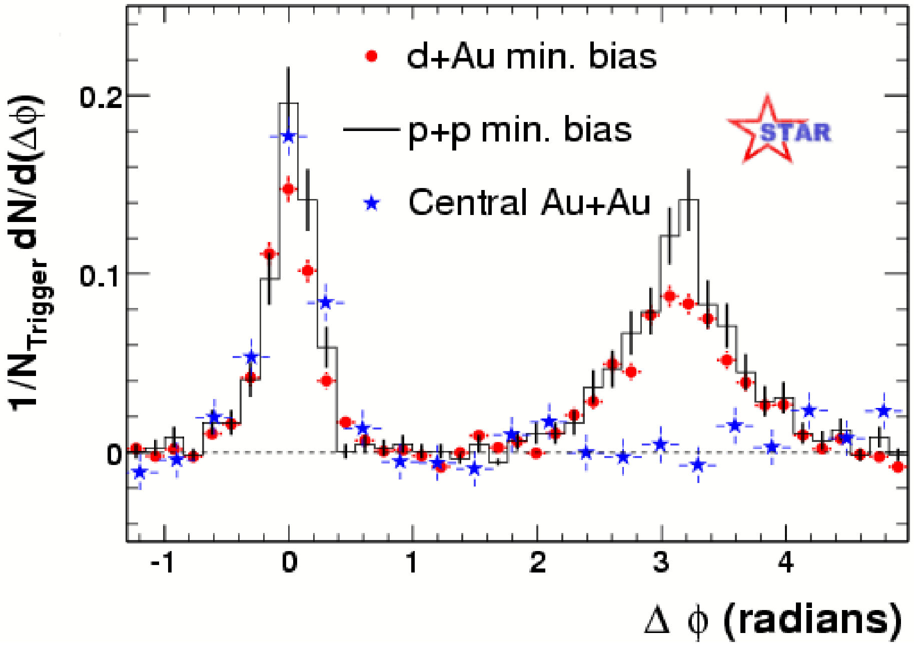


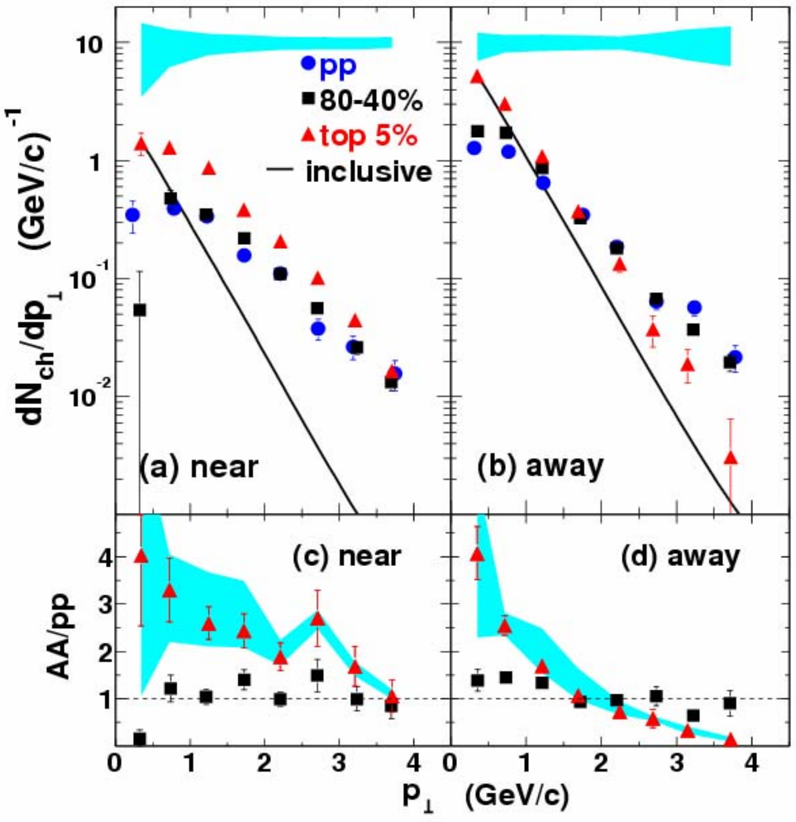


XNW’03

Parton Energy Loss

Same-side jet cone remains the same for large p_T

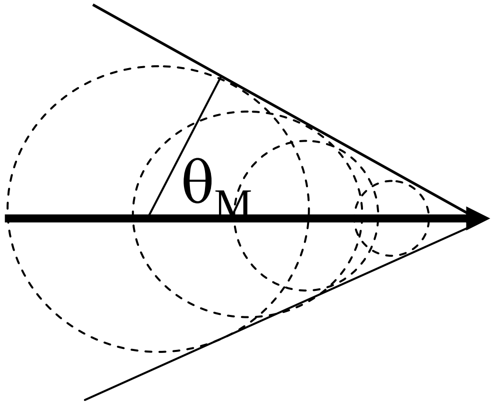




Low p_T enhancement implies energy loss

Hadron rescattering will change the correlation between leading and sub-leading hadrons

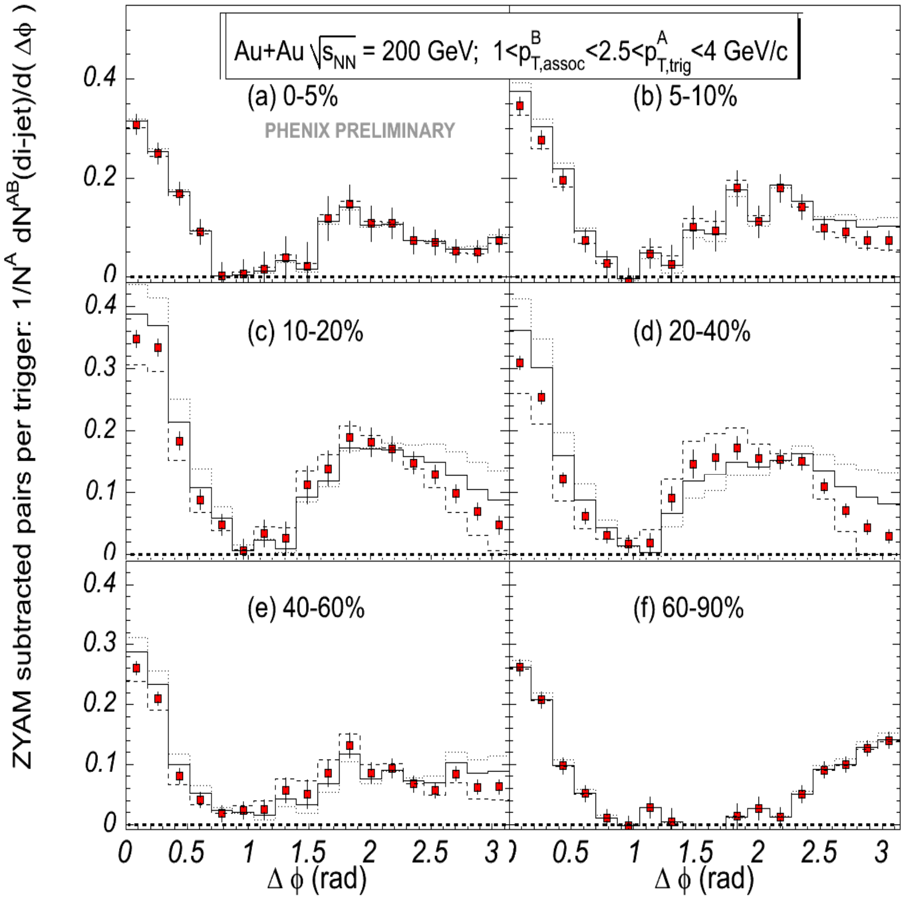
Soft hadrons rings



Shock wave?

$$\cos\theta_M = \frac{1}{c_s}$$

PHENIX



Stoecker'04

Casalderrey-Solana,Shuryak & Teaney '04

LPM & Cherenkov-like Bremsstrahlung

J. Ruppert & B. Muller PLB619(2005)123.
Dremin, JETP(1979), hep-ph/0507167
Dremin (parallel 10a)

$$k_0^2 - k^2 = \Sigma(k), \quad \mathrm{Re}\Sigma(k) < 0$$

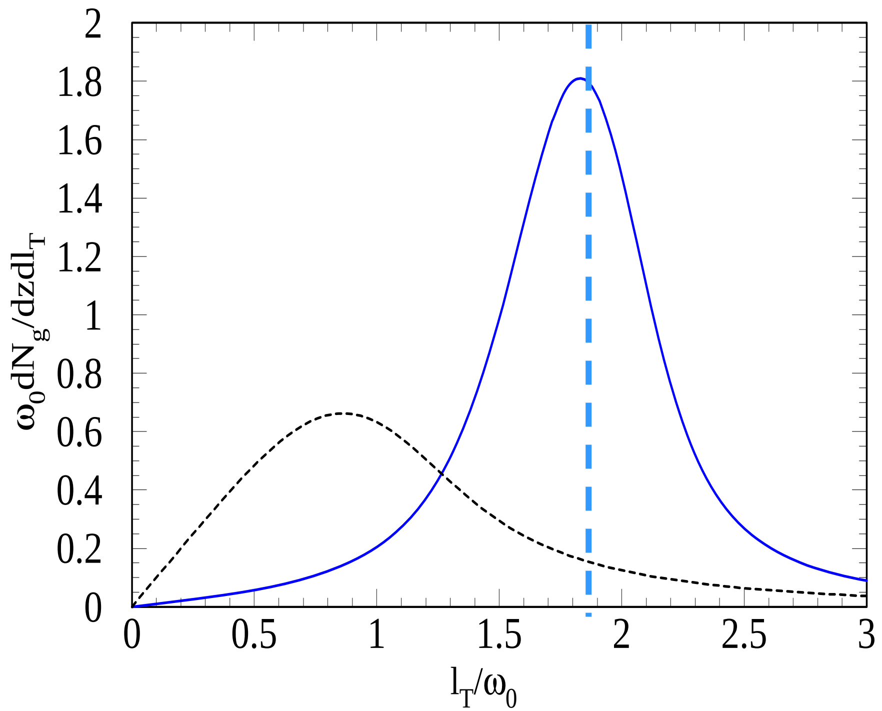
$$\frac{dN_g}{dz dk_T^2} \propto \frac{1}{[k_T^2 + (1-z)\Sigma(k)]^2}\left(1 - e^{-\left(\frac{R}{\tilde{\tau}_f}\right)^2}\right)$$

$$\tilde{\tau}_f = \frac{2Ez(1-z)}{k_T^2 + (1-z)\Sigma(k)}$$

$$k_{T\max}^2 = (1-z)\left|\Sigma(k)\right|$$

Dielectric constant

Majumder & XNW nuth-0507063

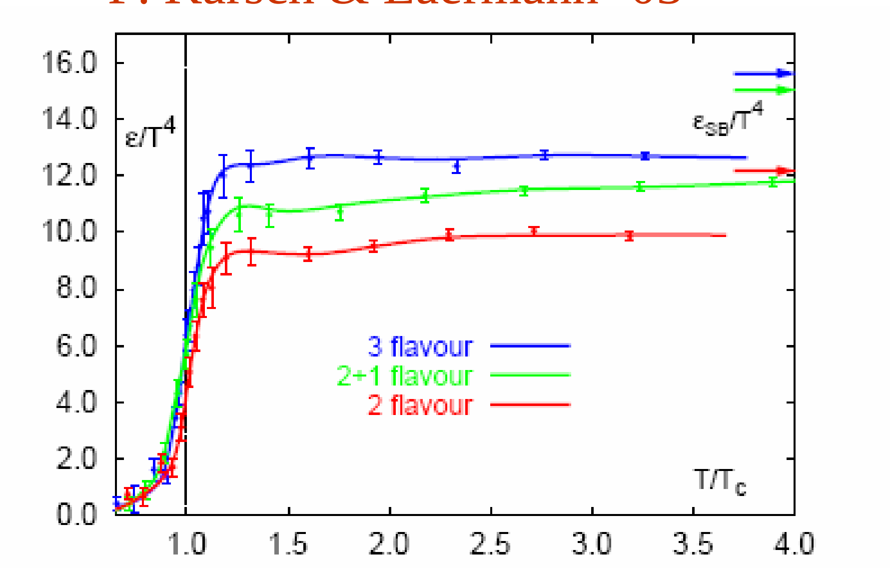


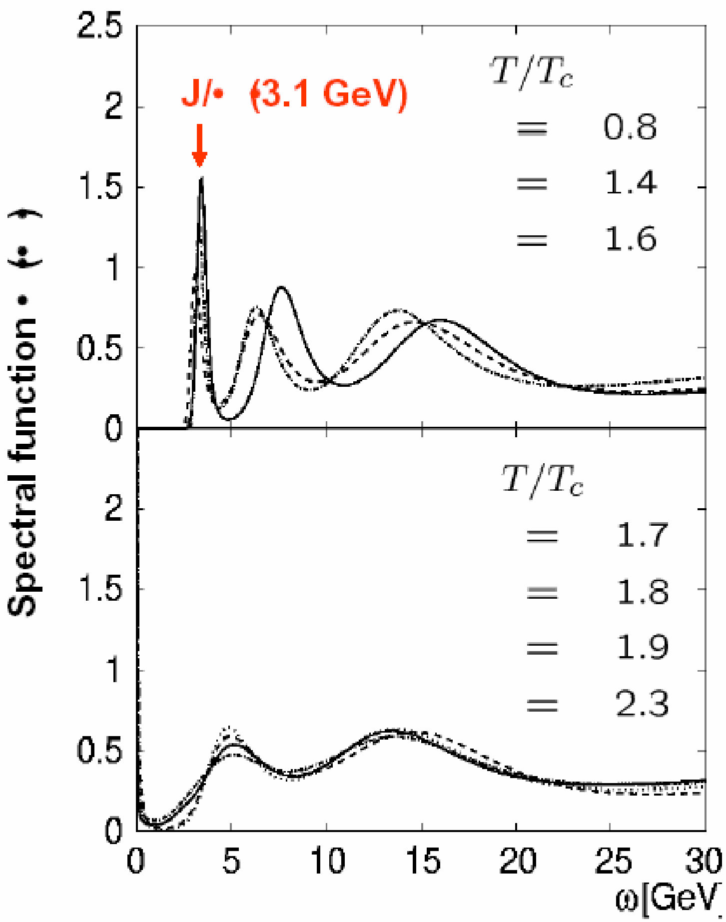
$$\cos^2\theta_c = z + \frac{1-z}{\varepsilon(k)}$$

$$\varepsilon(k) = 1 + \left|\Sigma(k)\right| / k_0^2$$

Resonances in QGP above T_c?

F. Karsch & Laermann '03





J/Ψ survives at T=1.6-2 T_c

Could there be other resonances?

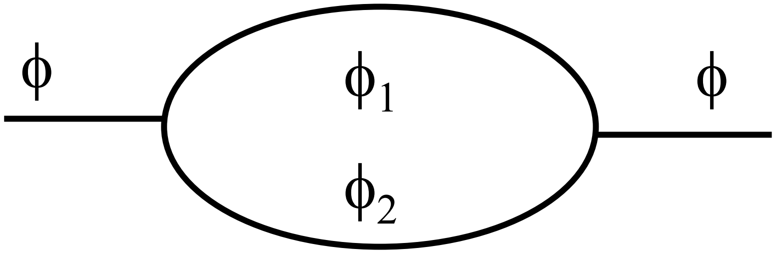
Shuryak & Zahed '04

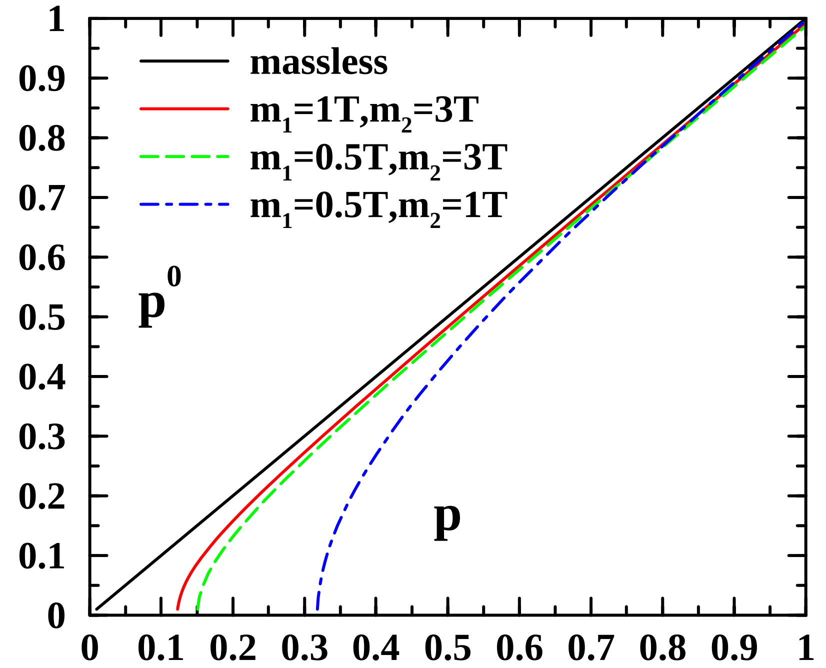
Asakawa &Hatsuda '04
S. Datta, et al '04

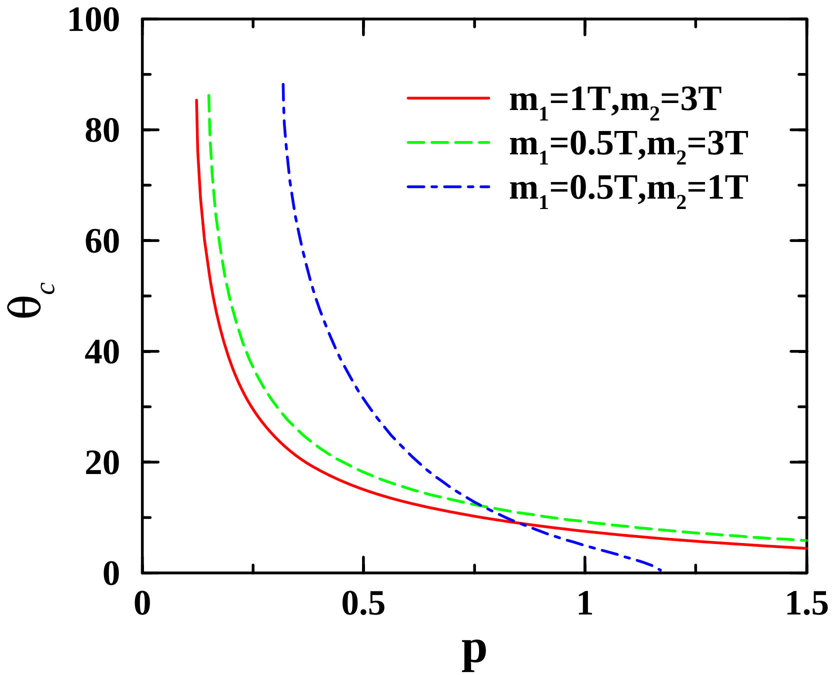
Dielectric Constant in QGP

Koch, Majumder & XNW'05

See Majumder (parallel 3b)



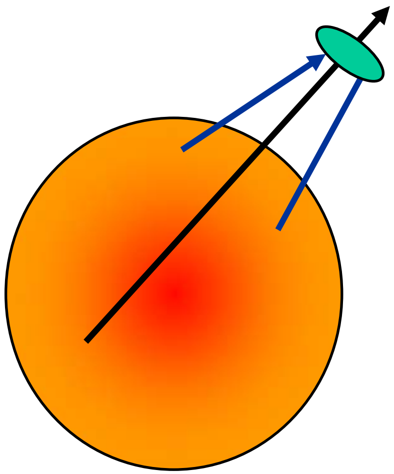


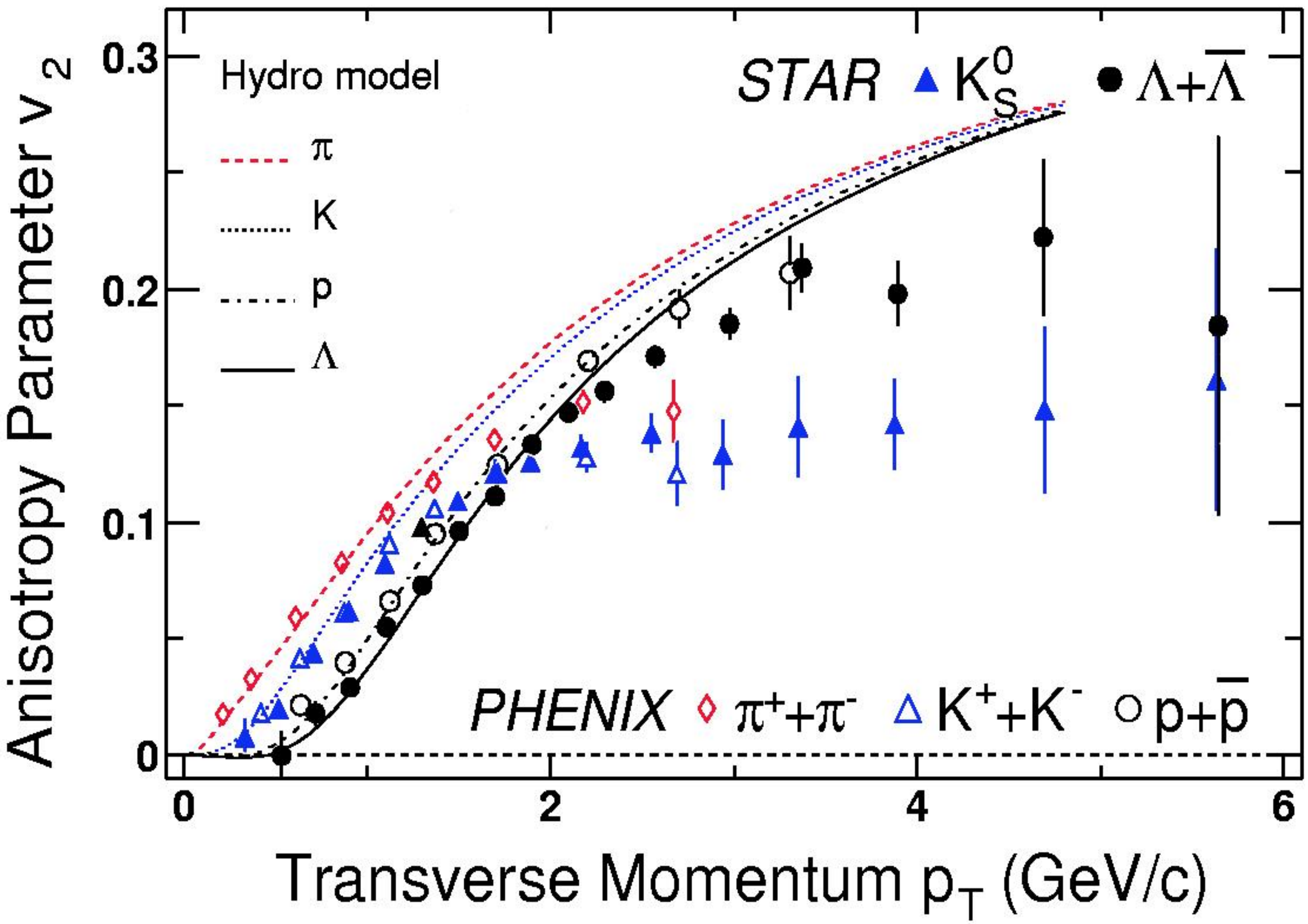


$$\cos\theta_c = \frac{1}{\sqrt{\varepsilon(p)}}$$

Strong *p*-dependence Cherenkov angle

Flavor of Jet Quenching

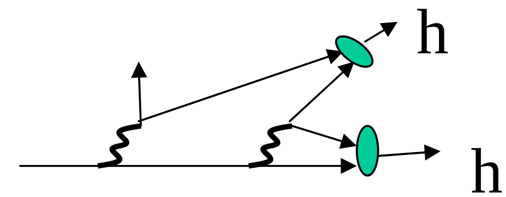




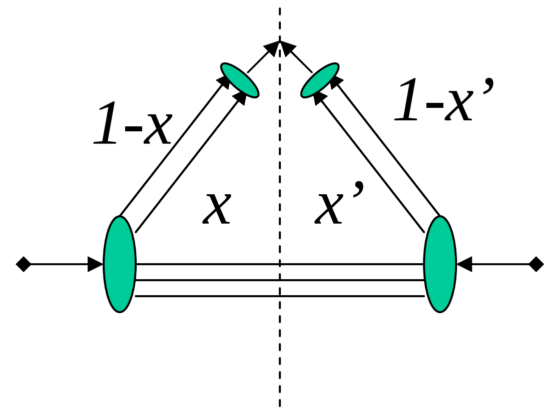
Parton recombination

Parton recombination revisited

$$\left|P_h\right\rangle = \int \prod_{i=1}^{N} \frac{d^2k_{\perp i}}{(2\pi)^3} \frac{dx_i}{2\sqrt{x_i}} \varphi(k_{\perp i}, x_i) \left| k_{\perp 1}, x_1 \cdots ; k_{\perp N}, x_N \right\rangle$$
$$2(2\pi)^3 \delta^{(2)}\left(\sum_i k_{\perp i} - P_{\perp h}\right) \delta\left(1 - \sum_i x_i\right)$$



$$\int \frac{d^2k_\perp}{(2\pi)^3} dx \int \frac{d^2k'_\perp}{(2\pi)^3} dx' \varphi(x, k_\perp) \varphi^*(x', k'_\perp) F(x, x', k_\perp, k'_\perp)$$
$$\approx C \int \frac{d^2k_\perp}{(2\pi)^3} dx \left|\varphi(x, k_\perp)\right|^2 F(x, x, k_\perp, k_\perp)$$



$$D^0_{q\to h}(z_h, Q^2) \approx \int_0^{z_h} dz\, \mathrm{F}_q^{q\bar{q}}(z, z_h - z, Q^2) R^h_{q\bar{q}}(z, z_h - z)$$

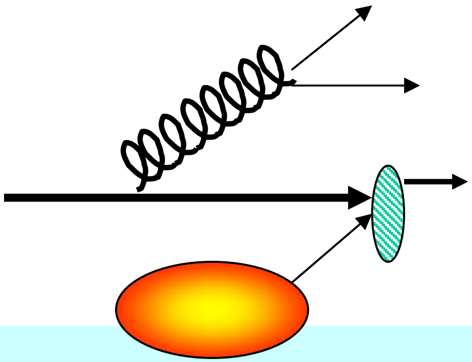
$$R(z_1, z_2) = \left|\varphi(z_1, z_2, k_\perp = 0)\right|^2$$

q-qbar distribution in a jet

Hwa & Yang

Modification due to recombination

Majumder, E.Wang & XNW'04



$$\langle O \rangle \equiv \frac{Tr\left[e^{-\beta\hat{H}} O\right]}{Tr e^{-\beta\hat{H}}}$$

$$D_{q\to h}(z_h, Q^2) \approx D^0_{q\to h}(z_h)$$

$$+\int \frac{dz}{(1-z)^2} F_q^{\bar{q}}((1-z)z_h, Q^2)\, f_{\bar{q}}^{th}(z) R^h_{q\bar{q}}(z)$$

Hwa &Yang
Fries et al
Greco&Ko

$$+V\int \frac{d^2P_{\perp h}}{2P^+(2\pi)^3}\int \frac{d^2q_\perp}{(2\pi)^3}\int_0^1 dx\, f_q(q_\perp, x)\, f_{\bar{q}}(P_{\perp h} - q_\perp, 1-x) R^h_{q\bar{q}}(x, q_\perp)$$

$$F_q^{\bar{q}q}(z_1, z_2, Q^2)$$

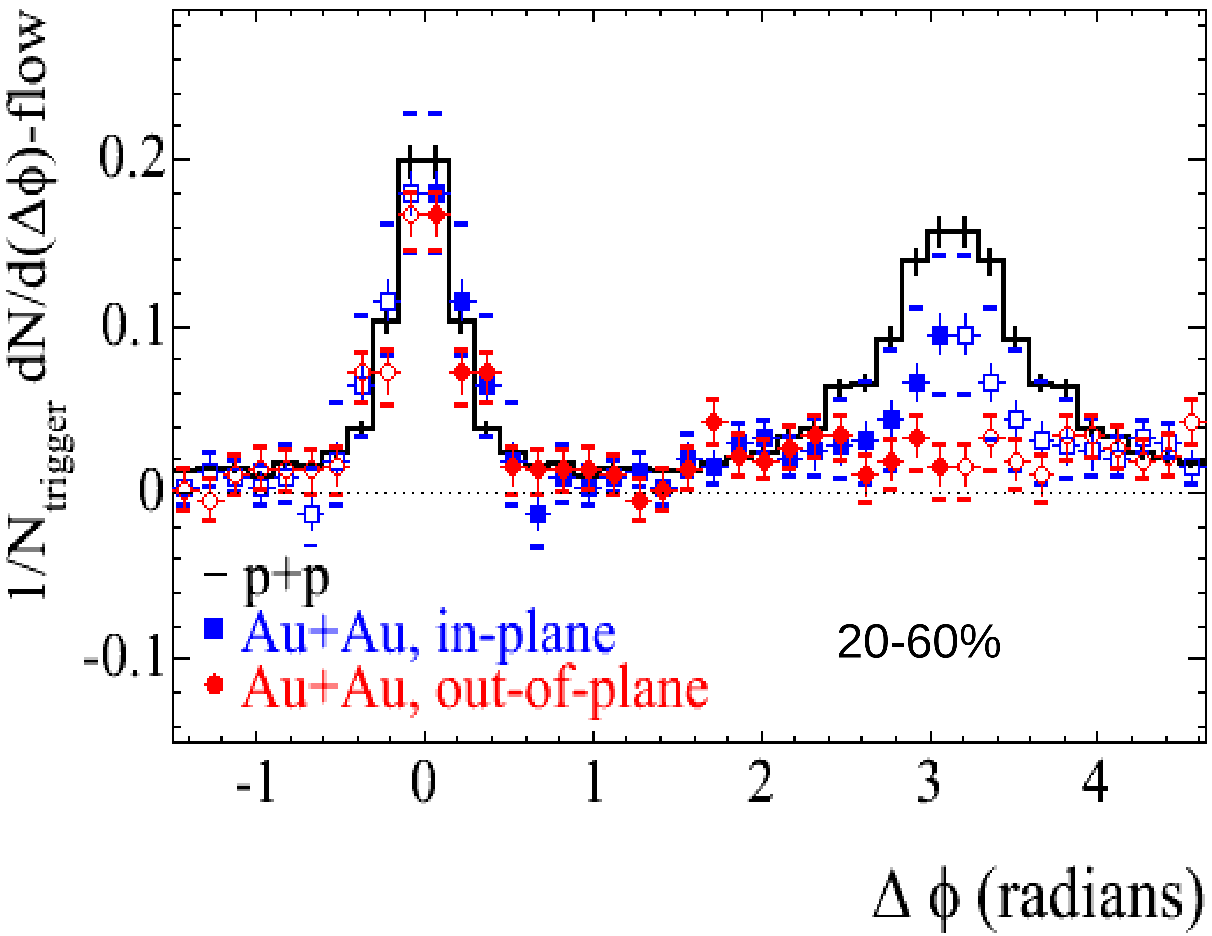
2-quark distribution

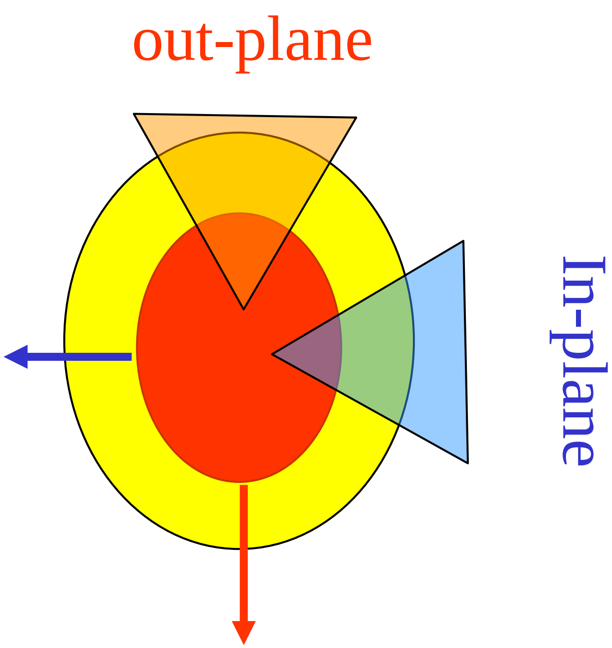
$$F_q^{\bar{q}}(z, Q^2)$$

Single quark distribution

Prospective for Jet Quenching I

Azimuthal Mapping of jet quenching

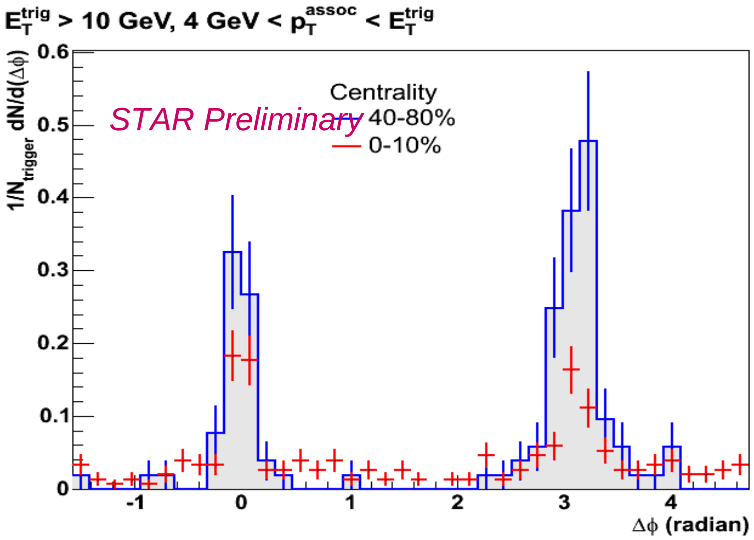




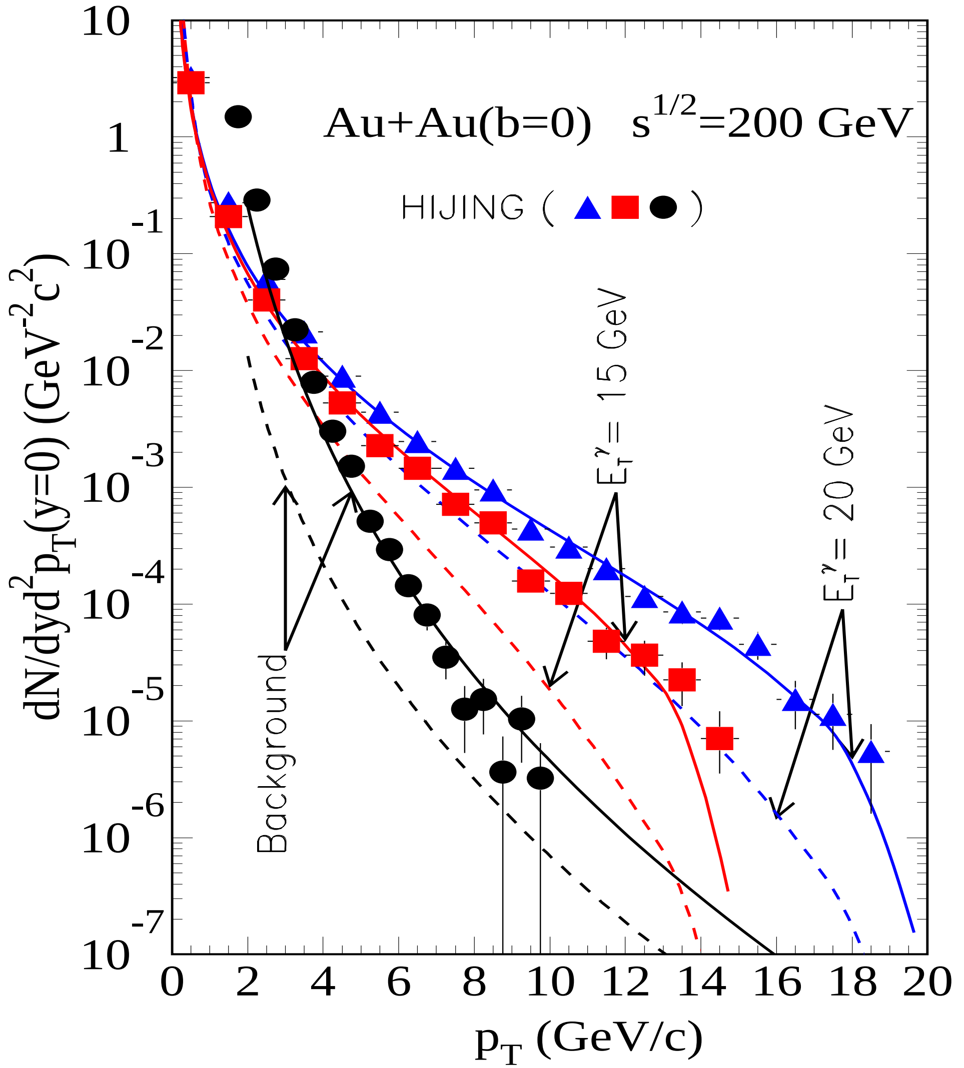
Prospective for Jet Quenching II

γ-jet Events

- No-trigger bias
  - Initial energy
  - Surface emission
  - Correlation background due to v2



XNW, Huang & Sarcevic,PRL77(96)231



Conclusions

- Leading hadrons suppressed in DIS eA, agrees well with multiple parton scattering
- Hadron absorption likely at lower energies
- Initial gluon density in Au+Au is about 30 times higher than cold nuclei
- Multiple hadron correlations critical measurements

Average Formation Time

$$\tau_f = \frac{2Ez(1-z)}{k_T^2}$$

$$\frac{dN_g}{dzdk_T^2} = \frac{\alpha_S}{2\pi} C_F \frac{1+(1-z)^2}{z} \frac{1}{k_T^2}$$

$$\left\langle \tau_f \right\rangle = \frac{1}{dN_g / dz} \int_{Q_0^2}^{z(1-z)Q^2} dk_T^2 \frac{2Ez(1-z)}{k_T^2} \frac{dN_g}{dzdk_T^2}$$

$$= \frac{2Ez(1-z)}{\ln(z(1-z)Q^2 / Q_0^2)} \left[\frac{1}{Q_0^2} - \frac{1}{z(1-z)Q^2} \right]$$