



# Parton Energy Loss and its Application to Hadronic Reactions



## **ECT\* Seminar, Trento, Italy**

October 6, 2005





## Radiative energy loss and jet quenching:

- Theoretical derivation to first order in opacity
- Large angle emission the death of the "dead cone"
- Derivation to all orders in opacity (reaction operator)
- Implementation in the perturbative QCD formalism

# Nuclear modification of (di-)hadrons:

- Energy dependence of the inclusive quenching
- Modification of the yields (energy redistribution)
- Modification of the large angle correlations
- Dijet attenuation

# Conclusions:









# **Vacuum Radiation**





If interested in the small angle small frequency behavior

$$\frac{dN^{g}}{d\omega d\sin\theta * d\delta} \propto \left|M_{c}\right|^{2}$$

 $\frac{dN^{g}_{vac}}{d\omega d\sin\theta^{*}d\delta} \approx \frac{C_{R}\alpha_{s}}{\pi^{2}} \frac{1}{\omega\sin\theta^{*}}$ 

- Both collinear and infrared divergent
- Collinear persists. At fixed order requires subtraction in the PDFs and FFs



## For massive quarks - "dead cone effect"

Takes care of the collinear

$$\frac{dN^{g}_{vac}}{d\omega d\sin\theta^{*}d\delta} \approx \frac{C_{R}\alpha_{s}}{\pi^{2}} \frac{\sin\theta^{*}}{\omega(\sin^{2}\theta^{*} + M^{2}/E^{2})}$$

Cuts part of phase space  $0 \le \theta^* \le M / E$ 



# **Instructive Example**







$$i(-i) = 1 \qquad i(i) = -1 = \cos(\pi)$$

$$\frac{dN^{g}_{med}}{d\omega d\sin\theta * d\delta} \propto \left( \left| M_{a} \right|^{2} + 2\operatorname{Re} M_{b}^{*}M_{c} \right) + \dots$$



Solution to first order in the mean # of scatterings

$$\frac{dN^{g}_{med}}{d\omega d\sin\theta^{*}d\delta} \approx \frac{2C_{R}\alpha_{s}}{\pi^{2}} \int_{z_{0}}^{L} \frac{d\Delta z}{\lambda_{s}(z)} \int_{0}^{\infty} dq_{\perp}q_{\perp}^{2} \frac{1}{\sigma_{el}} \frac{d\sigma_{el}}{d^{2}q_{\perp}}$$

$$\times \int_{0}^{2\pi} d\alpha \frac{\cos\alpha}{(\omega^{2}\sin^{2}\theta^{*} - 2q_{\perp}\omega\sin\theta^{*}\cos\alpha + q_{\perp}^{2})}$$

$$\times \left[1 - \cos\frac{(\omega^{2}\sin^{2}\theta^{*} - 2q_{\perp}\omega\sin\theta^{*}\cos\alpha + q_{\perp}^{2})\Delta z}{2\omega}\right]$$
I.V., hep-ph/0501255

Ivan Vitev, LANL

**October 6, 2005** 



# **Angular and Frequency Behavior**





• The small angle  $\theta^* \to 0$  and small frequency  $\omega \to 0$  behavior of the radiative spectrum is under perturbative control



# **Medium-Induced Bremsstrahlung**









• The calculation of the elastic case (transverse momentum diffusion) was easy since we organized the opacity series as a solution to the Reaction Operator (Although some limiting cases may have been guessed)

$$\mathbf{R} = \left\{ \begin{array}{c} z_n & z_n \\ \vec{q}_n, a_n \\ \vec$$

Acquire broadening:  $dN^{i}(k_{\perp}) = \delta^{2}(k_{\perp})$ 

$$\frac{d\sigma_{el}(b)}{d^2q} = \frac{\mu b}{4\pi^2} K_1(\mu b) \approx \frac{1}{4\pi^2} \left( 1 - \frac{\mu^2 b^2}{2} \xi + O(b^3) \right)$$
$$dN^f(k_\perp) = \frac{1}{2\pi} \frac{e^{-\frac{k_\perp^2}{\chi\mu^2\xi}}}{\chi\mu^2\xi}, \ \left\langle \Delta k_\perp^2 \right\rangle = 2\chi\mu^2\xi$$
$$\text{Were} \quad \xi = \log 2 / (1.08\,\mu b)$$

Obviously the Cronin Effect



• For our current purpose - elastic unitarity

Ivan Vitev, LANL

October 6, 2005



# Medium Induced Non-Abelian Radiative Spectrum



## The goal:

- Landau-Pomeranchuk-Migdal destructive interference effect in QCD
- Incorporates finite kinematics and small number of scatterings
- Applicable for realistic systems

## The idea:



Iterative solution M.Gyulassy, P.Levai, I.V., Nucl.Phys.B594 (2001); Phys.Rev.Lett.85 (2000)

## The constraint:

Inconsistent with large number of scatterings approximation



October 6, 2005



# **Direct Insertion Operator**



$$\omega_{0} - \frac{\mathbf{k}^{2}}{2\omega}, \ \omega_{i} - \frac{(\mathbf{k} - \mathbf{q}_{i})^{2}}{2\omega}, \ \omega_{(ij)} = \frac{(\mathbf{k} - \mathbf{q}_{i} - \mathbf{q}_{j})^{2}}{2\omega}, \ \omega_{(i...j)} = \frac{(\mathbf{k} - \sum_{m=i}^{j} \mathbf{q}_{m})^{2}}{2\omega}$$

$$E^{+} \gg k^{+} \gg \omega_{(i...j)} \gg \frac{(\mathbf{p} + \mathbf{k})^{2}}{E^{+}}$$

$$H = \frac{\mathbf{k}}{\mathbf{k}^{2}}, \qquad C_{(i_{1}i_{2}...i_{m})} = \frac{(\mathbf{k} - \mathbf{q}_{i_{1}} - \mathbf{q}_{i_{2}} - ... - \mathbf{q}_{i_{m}})}{(\mathbf{k} - \mathbf{q}_{i_{1}} - \mathbf{q}_{i_{2}} - ... - \mathbf{q}_{i_{m}})^{2}},$$

$$H = \frac{\mathbf{k}}{\mathbf{k}^{2}}, \qquad C_{(i_{1}i_{2}...i_{m})} = \frac{(\mathbf{k} - \mathbf{q}_{i_{1}} - \mathbf{q}_{i_{2}} - ... - \mathbf{q}_{i_{m}})}{(\mathbf{k} - \mathbf{q}_{i_{1}} - \mathbf{q}_{i_{2}} - ... - \mathbf{q}_{i_{m}})^{2}},$$

$$B_{i} = \mathbf{H} - \mathbf{C}_{i}, \qquad B_{(i_{1}i_{2}...i_{m})} = \frac{(\mathbf{k} - \mathbf{q}_{i_{1}} - \mathbf{q}_{i_{2}} - ... - \mathbf{q}_{i_{m}})}{(\mathbf{k} - \mathbf{q}_{i_{1}} - \mathbf{q}_{i_{2}} - ... - \mathbf{q}_{i_{m}})^{2}},$$

$$Gunion-Bertsch \qquad B_{i} = \frac{\mathbf{k}_{\perp}}{\mathbf{k}_{\perp}^{2}} - \frac{\mathbf{k}_{\perp} - \mathbf{q}_{i_{\perp}}}{(\mathbf{k}_{\perp} - \mathbf{q}_{i_{\perp}})^{2}}$$

$$= a_{n}A_{i_{1}...i_{n-1}}(x, \mathbf{k}, c) = (a_{n} + \hat{S}_{n} + \hat{B}_{n})A_{i_{1}...i_{n-1}}(x, \mathbf{k}, c)$$

$$= a_{n}A_{i_{1}...i_{n-1}}(x, \mathbf{k}, c) + e^{i(\omega_{0} - \omega_{n})z_{n}}A_{i_{1}...i_{n-1}}(x, \mathbf{k}, c) + e^{i(\omega_{0} - \omega_{n})z_{n}}A_{i_{1}...i_{n-1}}(x, \mathbf{k}, - \mathbf{q}_{n}, [c, a_{n}]) - (-\frac{1}{2})^{N_{v}}(A_{i_{1}...i_{n-1}})B_{n} e^{i\omega_{0}z_{n}}[c, a_{n}]T_{cl}(A_{i_{1}...i_{n-1}})$$

$$A unity + shift + BG$$

October 6, 2005







$$\mathcal{A}_{i_1\cdots i_{n-1}}(x,\mathbf{k},c) = -\frac{C_R + C_A}{2} \mathcal{A}_{i_1\cdots i_{n-1}}(x,\mathbf{k},c)$$
$$-e^{i(\omega_0-\omega_n)z_n} a_n \mathcal{A}_{i_1\cdots i_{n-1}}(x,\mathbf{k}-\mathbf{q}_n,[c,a_n])$$
$$-\left(-\frac{1}{2}\right)^{N_v} \frac{C_A}{2} \operatorname{B}_n e^{i\omega_0 z_n} c a_{n-1}^{i_{n-1}} \cdots a_1^{i_1}$$

$$\hat{V}_{n} = -\frac{1}{2}(C_{A} + C_{R}) - a_{n}(\hat{S}_{n} + \hat{B}_{n}) = -a_{n}\hat{D}_{n} - \frac{1}{2}(C_{A} - C_{R})$$

 $\hat{V}_n \sim \hat{D}_n$  suggests huge cancellations at the probability level

Key Identity for Calculating Induced Gluon Radiation

**Example:** 
$$\hat{R} = D^{\dagger}D + V^{\dagger} + V = D^{\dagger}D - aD^{\dagger} - aD - (C_A - C_R) = (D^{\dagger} - a)(D - a) - C_A$$

October 6, 2005



# Medium Induced Non-Abelian Radiative Spectrum



$$\sum_{n=1,2,\dots} x \frac{dN^g}{dx \, d^2 \mathbf{k}} = \frac{C_R \alpha_s}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{i=1}^n \left( \int_{z_{i-1}}^{\infty} dz_i \int d^2 \mathbf{q}_i \left[ \frac{d^2 \sigma_g(z_i)}{d^2 \mathbf{q}_i} - \sigma_g(z_i) \delta^2(\mathbf{q}_i) \right] \right)$$
$$\times \rho_n(z_1, \cdots, z_n) \left( -2 \operatorname{C}_{(1,\cdots,n)} \cdot \sum_{m=1}^n \operatorname{B}_{(m+1,\cdots,n)(m,\cdots,n)} \times \left[ \cos\left(\sum_{k=2}^m \omega_{(k,\cdots,n)} \Delta z_k\right) - \cos\left(\sum_{k=1}^m \omega_{(k,\cdots,n)} \Delta z_k\right) \right] \right),$$

 $\rho_n(z_1,\cdots,z_n) = n! \,\rho_0^n \,\,\theta(L-z_n)\theta(z_n-z_{n-1})\cdots\theta(z_2-z_1)$ 

where

$$\omega_{(j,\dots,n)} = \frac{(\mathbf{k} - \mathbf{q}_{\mathbf{j}} - \dots - \mathbf{q}_{\mathbf{n}})^2}{2xE}$$

$$\mathbf{C}_{(j,\cdots,n)} = \frac{\mathbf{k} - \mathbf{q}_{\mathbf{j}} - \cdots - \mathbf{q}_{\mathbf{n}}}{(\mathbf{k} - \mathbf{q}_{\mathbf{j}} - \cdots - \mathbf{q}_{\mathbf{n}})^2}$$

 $\mathbf{B}_{(j+1,\cdots,n)(j,\cdots,n)} = \mathbf{C}_{(j+1,\cdots,n)} - \mathbf{C}_{(j,\cdots,n)}$ 

Inverse formation times  $\rho_0 = N_s / L A_\perp$ 

Color current propagators

October 6, 2005



# **Numerical Results**





- It is the interference between the hard vacuum radiation and one scattering that sets the overall scale for the intensity spectrum and the mean energy loss.
- Higher order corrections provide an oscillating and relatively quickly converging series

• We look at the radiative spectrum and the mean energy loss from a different perspective – what are the higher order corrections?

## GLV, Nucl.Phys.B 594 (2001)





14



The energy loss measures the line weighted integral through the expanding gluon density

$$\left< \Delta E \right> \propto I_1 = \int_{t_0}^{\infty} dt \ t \rho(t, \vec{x}_0 + \hat{v}(t - t_0))$$

In contrast,  $\mathbf{k}_{\mathrm{T}}\text{-}\text{broadening}$  measures the zeroth order moment

 $\left\langle \Delta k_T^2 \right\rangle \propto I_0 = \int_{t_0}^{\infty} dt \ \rho(t, \vec{x}_0 + \hat{v}(t - t_0))$ 

#### M.Gyulassy, I.V., X.N.Wang, Phys.Rev.Lett.86 (2001)



## $\mu^2/\lambda$ - transport coefficient

 $dN^{g}/dy$  - effective gluon rapidity October 6, 2005 density

Linear Regime: "Thin Plasma"  $Z(x,z) \ll 1 \implies x_c \equiv \frac{\mu^2(z)}{2E}(z-z_0) \ll x \leq 1$ 100  $dN^{g}/dy = 650$ iN<sub>g</sub>/dx, xdN<sub>g</sub>/dx xdN\_/dx 10 E<sub>iet</sub> = 10 GeV (a) 0.1 0.01 0.1  $x = \omega / E$ 1.2  $dN^{g}/dy = 650-800$ Gluons Quarks <mark>9.8</mark> 9.6 9.0 0.4 0.2 0 5 10 15 20 0 E<sub>iet</sub> [GeV]

Numerically slow  $\Delta E / E$  dependence



J.Collins, D.Soper, G.Sterman,

x<sub>a</sub>P

x<sub>b</sub>P<sup>\*</sup>

Adv.Ser.Dir. 5 (1988)





**QCD** factorization

• To LO (2 to 2 scattering) - single and double inclusive hadron production

Can also incorporate Cronin effect:  $\int d^2 k_T f_{med}(k_T)$ 

$$\begin{aligned} \frac{d\sigma_{_{NN}}^{h_1}}{dy_1 d^2 p_{_{T1}}} &= \sum_{abcd} \int_{x_a \min}^1 dx_a \int_{x_b \min}^1 dx_b \phi(x_a) \phi(x_b) \frac{\alpha_s^2}{(x_a x_b S)^2} \left| \overline{M}^{-2}{}_{ab \to cd} \right| \frac{D_{h_1/c}(z_1)}{z_1} \\ \frac{d\sigma_{_{NN}}^{h_1 h_2}}{dy_1 dy_2 d^2 p_{_{T1}} d^2 p_{_{T2}}} &= \frac{\delta(\Delta \varphi - \pi)}{p_{_{T1}} p_{_{T2}}} \sum_{abcd} \int_{z_1}^1 dz_1 \frac{D_{h_1/c}(z_1)}{z_1} D_{h_2/d}(z_2) \frac{\phi(\overline{x}_a) \phi(\overline{x}_b)}{\overline{x}_a \overline{x}_b} \frac{\alpha_s^2}{S^2} \left| \overline{M}^{-2}{}_{ab \to cd} \right| \end{aligned}$$



# **Calibrated Probes**







$$\begin{split} D_{h_1/d}(z_1) &\to \frac{1}{1-\varepsilon} D_{h_1/d} \left( \frac{z_1}{1-\varepsilon} \right) & \text{Quenched parent parton} \\ \\ \frac{\text{Feedback gluons}}{z_1 \int_0^1 \frac{dz_g}{z_g}} D_{h_1/d}(z_g) \frac{dN^g}{d\omega} \end{split}$$

• Use energy conservation to verify the fragmentation sum rule

October 6, 2005



# **Probabilistic Interpretation**



## We calculated the single inclusive spectrum $\omega_{pl} \sim \mu \sim gT$ Several extra gluons. The plasmon frequency forces the radiation in fewer higher-frequency gluons $P(\varepsilon, E) = \sum_{n=0}^{\infty} P_n(\varepsilon, E) \qquad \frac{\Delta E}{E} = \int d\varepsilon \, \varepsilon P(\varepsilon, E)$ $P_{n+1}(\varepsilon, E) = \frac{1}{n+1} \int_{x}^{1-x_0} dx_n \ \rho(x_n, E) P_n(\varepsilon - x_n, E)$ 10 $P_1(\varepsilon, E) = e^{-\langle N_g \rangle} \rho(\varepsilon, E)$ Ρ(ε) The probability can be constructed iteratively from the calculated gluon spectra

 $\int_{0}^{1} P(\varepsilon) d\varepsilon = 1, \quad \int_{0}^{1} P(\varepsilon) \varepsilon \ d\varepsilon = \frac{\Delta E}{E}$ 

The normalization of  $\int_{0}^{0} P(\varepsilon) d\varepsilon = 1$  in the interval  $\varepsilon \in [0,1]$  ensures  $\int_{0}^{0} \Delta E < E$ 

M.Gyulassy, P.Levai, I.V., Phys.Lett.B 538 (2002)

October 6, 2005

 $dN^{g}/dy = 650-800$ 15 5 10 20 E<sub>iet</sub> [GeV]  $dN^{9}/dy = 650$ Gluons Quarks E<sub>iet</sub> = 10 GeV (d) 0.1 0.01 0.1  $\epsilon = \Sigma_{,\omega}/E$ 

• So far exclusive gluon distributions have not been calculated.



# **Single Inclusive Quenching**





I.V., M.Gyulassy, Phys.Rev.Lett. 89 (2002)

• Room for improvement at small and moderate  $\textbf{p}_{\text{T}}$ 



 $\pi R^2 = 120 \ fm^2, \ \tau_0 = 0.5 \ fm$ 

**20 times** the critical energy density for deconfinement

October 6, 2005







October 6, 2005

20

**Ivan Vitev** 



# Modification of the Jet-like Correlations





- Attenuation (disappearance) of the away-side correlation function
- Dependence relative to the reaction plane

$$R_{AA}^{h_{1}h_{2}}(p_{T}) = \frac{d^{2}\sigma^{AA} / dp_{T1}dp_{T2}d\eta_{1}d\eta_{2}}{\langle N_{bin} \rangle d^{2}\sigma^{NN} / dp_{T1}dp_{T2}d\eta_{1}d\eta_{2}}$$

$$C_{2}(\Delta\phi) = \frac{1}{N_{trig}} \frac{dN^{h_{1}h_{2}}_{dijet}(|y_{1} - y_{2}|)}{d\Delta\phi}$$

$$\approx \frac{A_{Near}(|y_{1} - y_{2}|)}{\sqrt{2\pi\sigma_{Near}}} e^{-\Delta\phi^{2}/2\sigma^{2}_{Near}} + \frac{A_{Far}}{\sqrt{2\pi\sigma_{Far}}} e^{-(\Delta\phi - \pi)^{2}/2\sigma^{2}_{Far}}$$

In triggering on the near side all effects are taken by the away side correlation function  $p_{hh}^{hh} \neq p_{h}^{h} = 1.5$ 

- $R^{h_1h_2} / R^{h_1} \sim 1.5$
- The attenuation of the double inclusive hadron production is between the two naïve limits  $R^{h_1h_2} / R^{h_1} \sim 1$ ,  $R^{h_1h_2} / R^{h_1} \sim 2$





# Numerical Results for 62 GeV Correlations



Ivan Vitev, LANL



- Even passage through the center is not effective any more
- Only 25 50 % more suppression for dihadrons

I.V., Phys.Lett.B nucl-th/0404052



October 6, 2005



# **Dijets from K. Zapp**





October 6, 2005



# **More Realistic Simulations**







# **Angular Di-Hadron Distribution**





 $\sigma_{\scriptscriptstyle Far}(AA) > \sigma_{\scriptscriptstyle Far}(pp)$ 

- The width  $\left| \Delta \varphi \pi \right|$  of the large-angle correlations is dominated by medium induced gluon radiation
- Reasessment of the origin of small and moderate p<sub>T</sub> away triggered hadrons

The quenched parton is not wider

**Because:** 

$$\sigma_{Far} \approx \frac{\sqrt{\langle k_T^2 \rangle_{vac}}}{p_{Tc}} \rightarrow \frac{\sqrt{\langle k_T^2 \rangle_{tot}}}{\left[ p_{Tc} / (1 - \varepsilon) \right]}$$

October 6, 2005



# **Results in Au+Au**



#### PHENIX data





Confirmation of a very broad distributions of away-side triggered hadrons





## Hadron production cross section

## • Energy loss

$$\frac{d\sigma^{h}}{dyd^{2}p_{T}} = \sum_{c} \int_{z_{\min}}^{1} dz \, \frac{d\sigma^{c}(p_{c} = p_{T}/z)}{dyd^{2}p_{T_{c}}} \frac{1}{z^{2}} D_{h/c}(z)$$

$$\approx \sum_{c} \frac{d\sigma^{c}(p_{T}/\langle z \rangle)}{dyd^{2}p_{T_{c}}} \frac{1}{\langle z \rangle^{2}} D_{h/c}(\langle z \rangle)$$

$$\approx \sum_{c} \frac{A}{p_{T_{c}}^{n}} \langle z \rangle^{(n-2)} D_{h/c}(z)$$

$$\begin{split} dN^g/dy &\propto dN^h/dy \propto A \propto N_{part} ,\\ L &\propto A^{1/3} \propto N_{part}^{1/3} ,\\ A_{\perp} &\propto A^{2/3} \propto N_{part}^{2/3} . \end{split}$$

## **Combine them to find:**

## Nuclear modification

#### Natural variables

$$R_{AB} = \frac{1}{N_{AB \ col}} \frac{d\sigma_{AB}^{h}/dyd^{2}p_{T}}{d\sigma^{h}/dyd^{2}p_{T}}$$
$$= (1-\epsilon)^{n-2} = \left(1 - \frac{k}{n-2}N_{part}^{2/3}\right)^{n-2}$$

$$\ln R_{AA} = -kN_{part}^{2/3}$$

#### October 6, 2005





• Peripheral reactions are normalized to  $R_{AA} = 1$ 

	Species	$^{9}Be$	$^{16}O$	$^{28}Si$	${}^{32}S$	$^{56}Fe$	$^{64}Cu$	$^{197}Au$	$^{208}Pb$	$^{238}U$
$\left[ \right]$	$b \; [fm]$	1	1	1.5	1.5	2	2	3	3	3
	$N_{part}^{2/3}$	5	8	12	14	20	22	48	50	55

- Good for qualitative and even quantitative description of the quenching
- For central Cu+Cu  $R_{AA} = 0.5$

• Will do a better job if fixed to the central bin



October 6, 2005





# • Absolte scale comparison for Au+Au and Cu+Cu

- $R_{AA} = 0.4 0.5$
- Within 20 % of an analytic guess

## • Absolte scale comparison for Au+Au and Cu+Cu



October 6, 2005





The slide I forgot to put but is critical to see how E is redistribured

Define a measure for nuclear modifications to di-hadron correlations:

$$\boldsymbol{R}_{AA}^{(2)} = \frac{d\sigma_{AA}^{h_1h_2} / dy_1 dy_2 dp_{T1} dp_{T2}}{\left< N_{bin} \right> d\sigma_{pp}^{h_1h_2} / dy_1 dy_2 dp_{T1} dp_{T2}}$$

P<sub>T1</sub> trigger:

- Fix the energy
- Ensure high Q<sup>2,</sup>
- Minimize the effect on the near side
- Maximize the effect on the away side
- The redistribution of the energy is a parameter free prediction
- For large energy loss the radiative gluons dominate to unexpectedly high  $p_{T2} \sim 10 \ GeV$



#### J.Adams et al., nucl-ex/0501016



## Now replaced by the 4 figures I sent and pQCD II added



**October 6, 2005** 





- Energy loss formulation to all orders on the mean number of scatterings. Studied the convergence of the series. Found that numerical calculations of energy loss provide large corrections to the analytic formulas. and verified, it can be used to extract information about the density in the early stages of HIC (jet tomography)
- Jet quenching simulations have successfully predicted the difference in the nuclear modification at SPS and RHIC (LHC t be tested). The extracted e = 15 GeV/fm<sup>3</sup> is well into the deconfinement phase transition
- Showed the first study of large angle gluon emission and large angle hadron production. Implications about the dead cone effect.
- A parameter free description of the redistribution of the lost energy for tagged jets can be obtained in the peturbative approach. The medium parameters only specify -dE (to be improved). Predictions versus p<sub>T</sub> trigger
- Significant broadening of the away side correlations confirmed by PHENIX. Extra theoretical work needed - none of the other models has included any away side distribution of jets. This calculation - approximate way.
- In a realistic calculation  $I_{AA} \sim R_{AA}$ . Has been predicted at least at 2 energies.