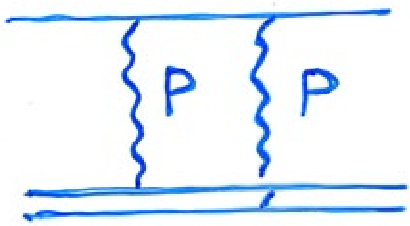
CRONIN EFFECT
AND ENERGY CONSERVATION

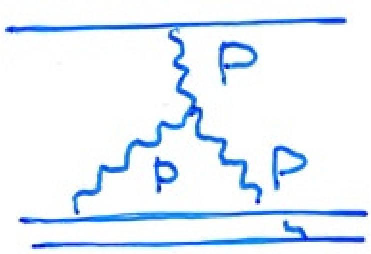
(A. Accardi, E. Cattaruzza and D.T.)

A very general approach to hadron-nucleus interactions is through the Reggeon diagram technique, where at high energies the interaction is described by the exchange of many Pomerons, including

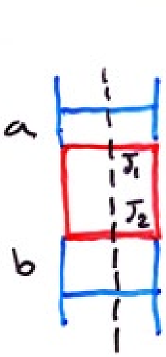
- Independent exchanges with different nucleons

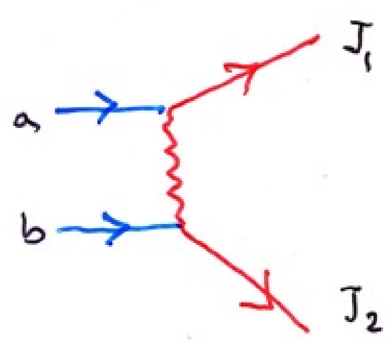


- and multi-Pomeron interactions

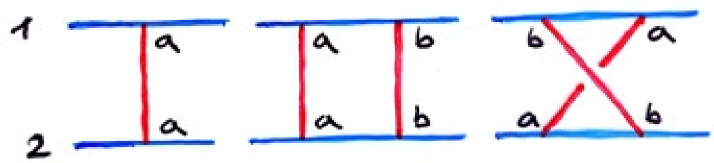


A partonic interaction corresponds to a fluctuation with large p_t inside the structure of an exchanged Pomeron





Lowest order diagrams for a quark-quark interaction,



Color factors:

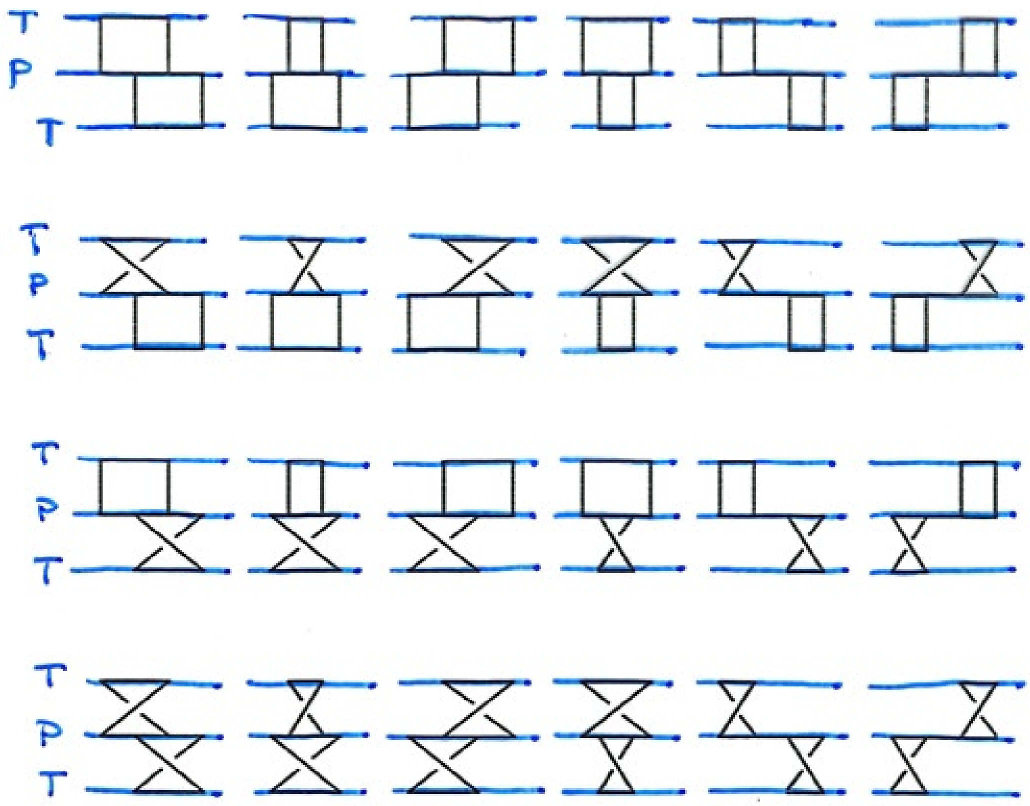
$$
\begin{array}{rl}
T_a^{(1)}T_a^{(2)} & Born-term \\
T_a^{(1)}T_b^{(1)}T_a^{(2)}T_b^{(2)} & box-diagram \\
T_b^{(1)}T_a^{(1)}T_a^{(2)}T_b^{(2)} & crossed-box-diagram
\end{array}
\tag{1}
$$

By introducing the commutators, $[T_a, T_b]$, and the anti-commutators, $\{T_a, T_b\}$, one may write:

$$
\frac{1}{4}\{T_a^{(1)}, T_b^{(1)}\}\{T_a^{(2)}, T_b^{(2)}\} \times (\mathcal{M} + \mathcal{M}')
$$
$$
-\frac{3}{4}T_c^{(1)}T_c^{(2)} \times (\mathcal{M} - \mathcal{M}')
$$

the term with color $T_c^{(1)}T_c^{(2)}$ corrects the propagator of the lowest order Born diagram, is dominantly real and grows as a $\ln s$. The term with the symmetrized color factor is analogous to the abelian case (proportional to $(\mathcal{M} + \mathcal{M}')$) and is mainly imaginary.

Simplest case of a three-body collision:
abelian interaction, massless particles, neglect transverse as compared with longitudinal momentum components whenever possible
Forward amplitude



4! diagrams
Leading contribution:

$$\sum_{\{permut.\}} \delta\Big(\sum_{i=1}^{n}\Big) \frac{1}{\beta_1 + i\epsilon} \cdots \frac{1}{\beta_1 + \beta_2 \ldots + \beta_{n-1} + i\epsilon}$$

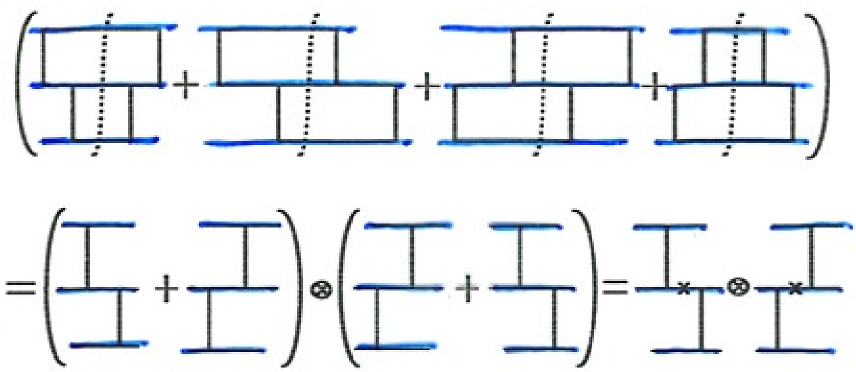
$$= (-2\pi i)^{n-1} \prod_{i=1}^{n} \delta(\beta_i)$$

β_i: scaled light cone components of exchanged momenta in the direction of the target

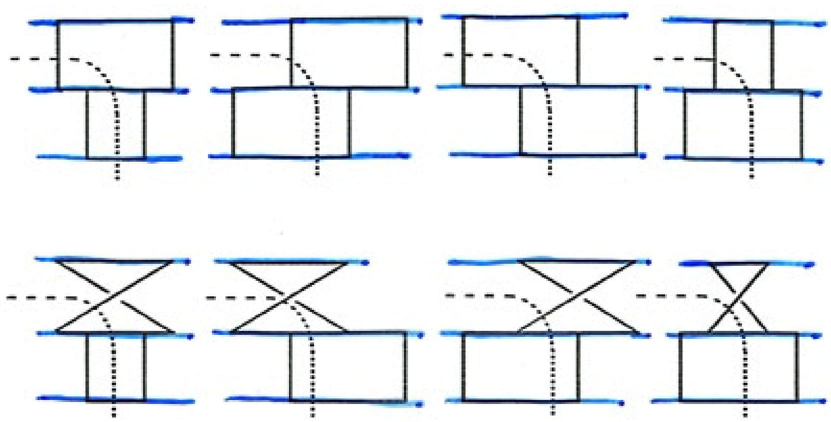
→ All horizontal propagators on mass shell

Analogously for the cuts of the forward amplitude:

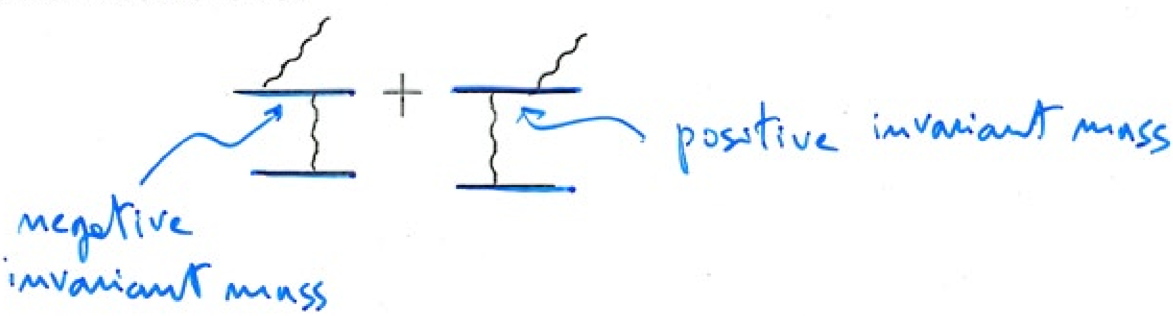
Leading contribution to the two-box cut



Sub-leading contribution to the two-box cut:



destructive interference



As a result one obtains that the leading contribution to the forward amplitude and all its leading cuts are all proportional one to another and the proportionality factors are the AGK weights.

For a leading cut of the $3 \to 3$ amplitude one has moreover

$$A(3 \to 3) \otimes A^*(3 \to 3) = \mid A(2 \to 2) \mid^2 \otimes \mid A(2 \to 2) \mid^2$$

One may argue that the whole semi-hard NA cross section can be expressed in terms of two-body partonic interaction probabilities:

> $\hat{\sigma}_{ij}$: elementary interaction probability between i and j
>
> $\prod_{i=1}^{n} \prod_{j=1}^{m} (1 - \hat{\sigma}_{ij})$: probability of NO interaction between two configurations with n and m partons
>
> $1 - \prod_{i=1}^{n} \prod_{j=1}^{m} (1 - \hat{\sigma}_{ij})$: probability to have at *least one interaction* between two configurations with n and m partons.

Expression analogous to that of the production cross section σ_{NA}^{prod} in the Glauber-Gribov approach ($\sigma_{NA}^{prod} = \sigma_{NA}^{tot} - \sigma_{NA}^{scat}$, where σ_{NA}^{scat}= cross section of all quasielastic processes where the nucleus gets excited to any A^* state, including the elastic)

One needs then to introduce $P_n^A(x_i, b_i)$, the probability of finding the nucleus (and the projectile proton) in a configuration with n partons with fractional momenta $x_1 \ldots x_n$ and transverse coordinates $b_1 \ldots b_n$.

Simplest case (NO correlations)

P_n^A = Poissonian :

$$P_n^A(x_i, b_i) = \frac{1}{n!}\Gamma_A(x_1, b_1)\ldots\Gamma_A(x_n, b_n)e^{-\int dx db \Gamma_A(x,b)}$$

$\Gamma(x, b)$= average number of partons with momentum fraction x and transverse coordinate b

$\Gamma(x, b) = G(x)T(b)$

$G(x)$: nucleon distributions

$T_A(b)$: nuclear thickness function.

Semi-hard cross section

$$\sigma_H^{NA} = \int d^2B \sum_{n,m} P_n^N(x_i, b_i)P_m^A(x_j, b_j - B) \times \left[1 - \prod_{i=1}^{n}\prod_{j=1}^{m}(1 - \hat{\sigma}_{ij})\right] dx_i dx_j d^2b_i d^2b_j$$

B: nuclear impact parameter

Notice that the average number of collisions $\langle N(B) \rangle$ is given gy the single scattering expression (AGK cancellation)

$$\langle N(B) \rangle = \int_{p_t > p_0} dx dx' G_N(x) \sigma(xx') \Gamma_A(x', B)$$

which may be understood as

$$\langle N(B) \rangle = \int dx G_N(x) \langle n_A(x, B) \rangle$$

where $G_N(x)$ is the average number of N-partons and

$$\langle n_A(x, B) \rangle \equiv \int dx' \sigma(xx') \Gamma_A(x', B)$$

the average number of collisions of each N-parton against the nucleus.

Notice that the average number of collisions of a projectile parton is evaluated *without any free parameter.*

The scattering centers of a Pb target, as seen at the LHC by a projectile parton with $x = .1$, for partonic collisions with $p_t > 5GeV$, are shown in the figure. The dots have the size of the partonic cross section. The figure is obtained by projecting in transverse space a random distribution generated inside the volume of a sphere.

The interaction includes both disconnected collisions and rescatterings, neglects parton fusion.

By replacing the average number of collisions one obtains the average number of wounded partons $W_N(x,B)$:

$$\langle n_A(x,B)\rangle \to \Big[1-\exp\{-\langle n_A(x,B)\rangle\}\Big]$$

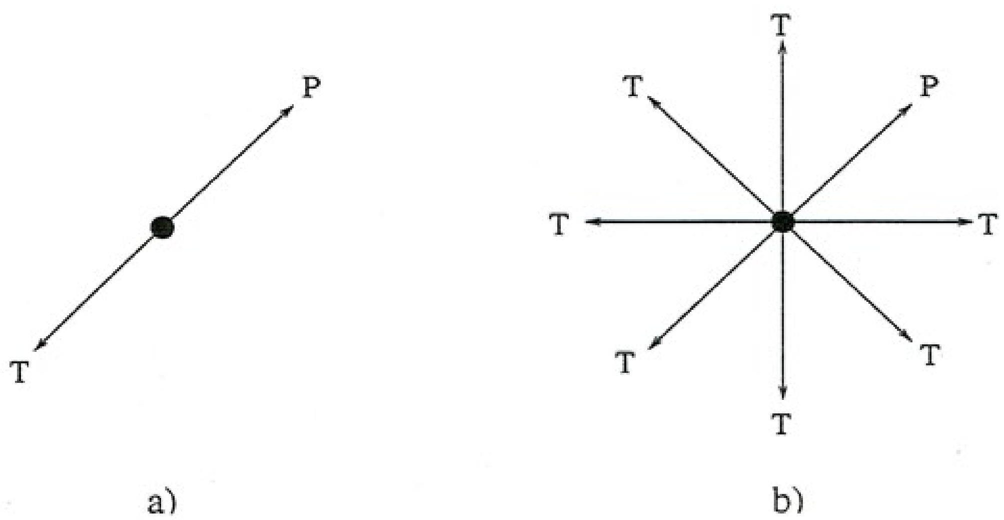
$$W_N(x,B) = G_N(x)\Big[1-\exp\{-\langle n_A(x,B)\rangle\}\Big]$$

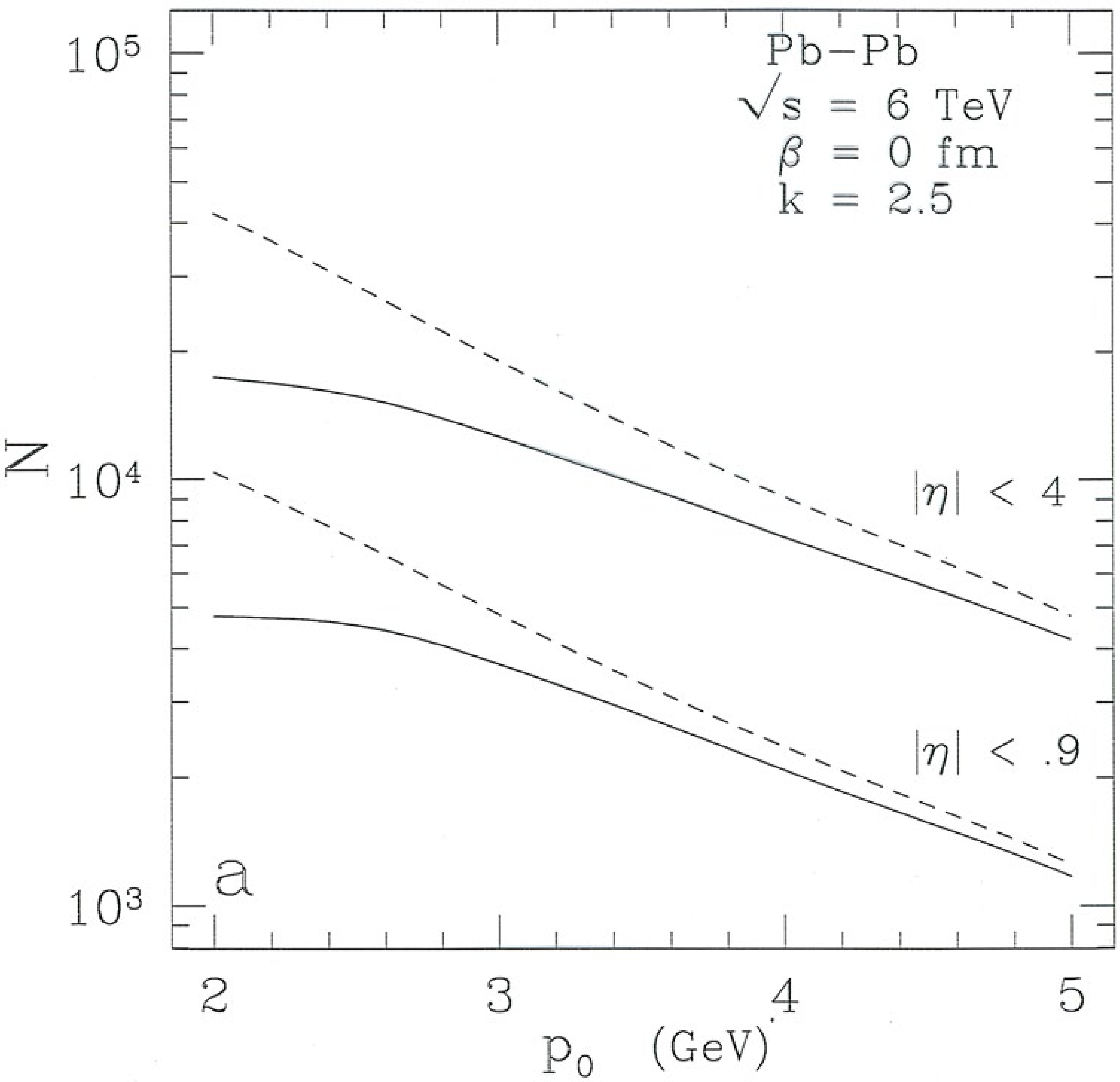
Notice that $W_N(x,B)$ is still meaningful in the black disk limit:

$$W_N(x,B) \to G_N(x)\theta(R_A - B)$$

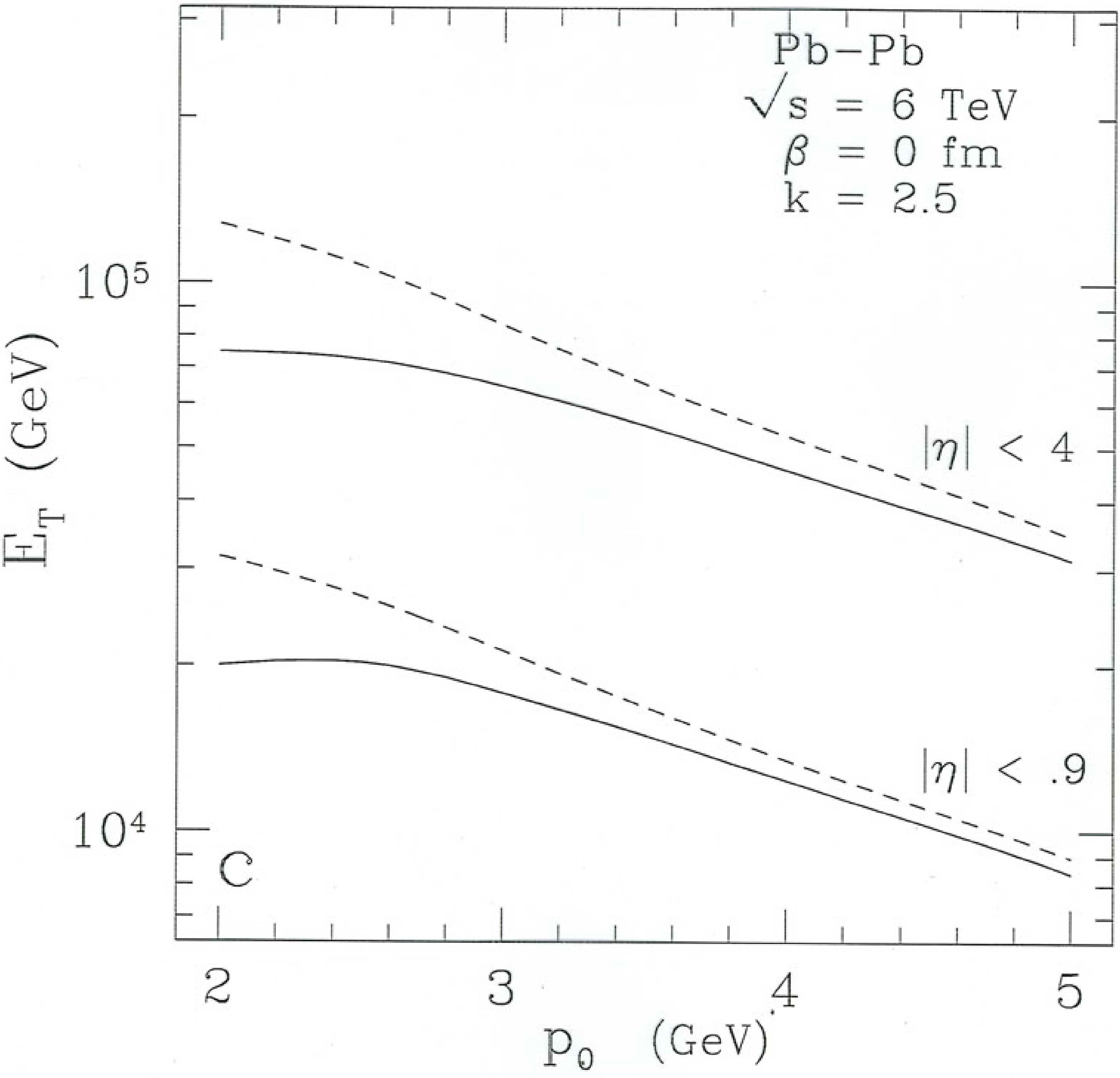
Each wounded parton is a minijet in the final state so that the number of minijets produced is limited by the number of initial state partons.

When $\langle n_A\rangle$ is small the projectile interacts with at most a target parton, when $\langle n_A\rangle$ is large many target partons are involved in the interaction and one recovers the initial state isotropy in transverse space.

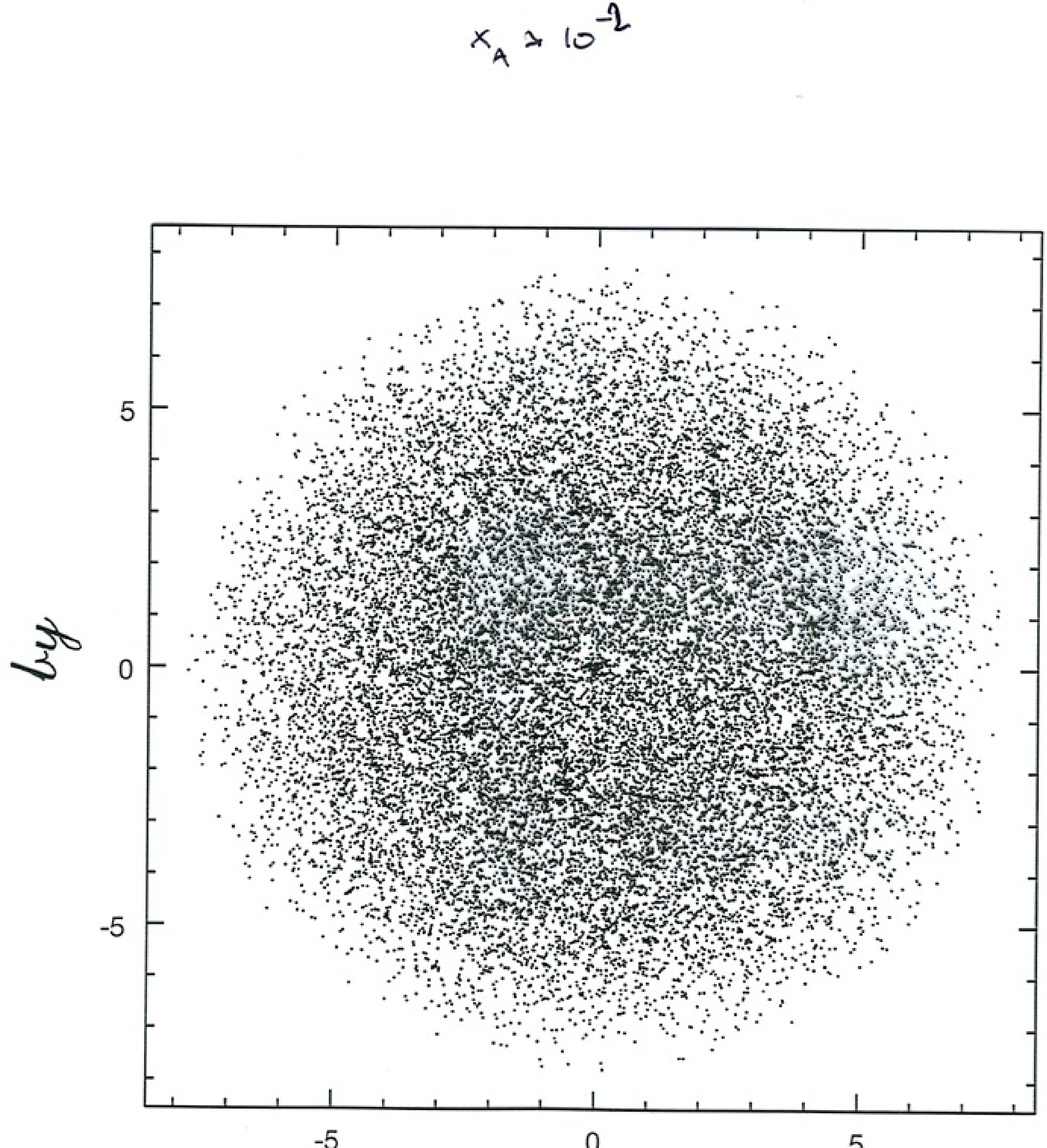




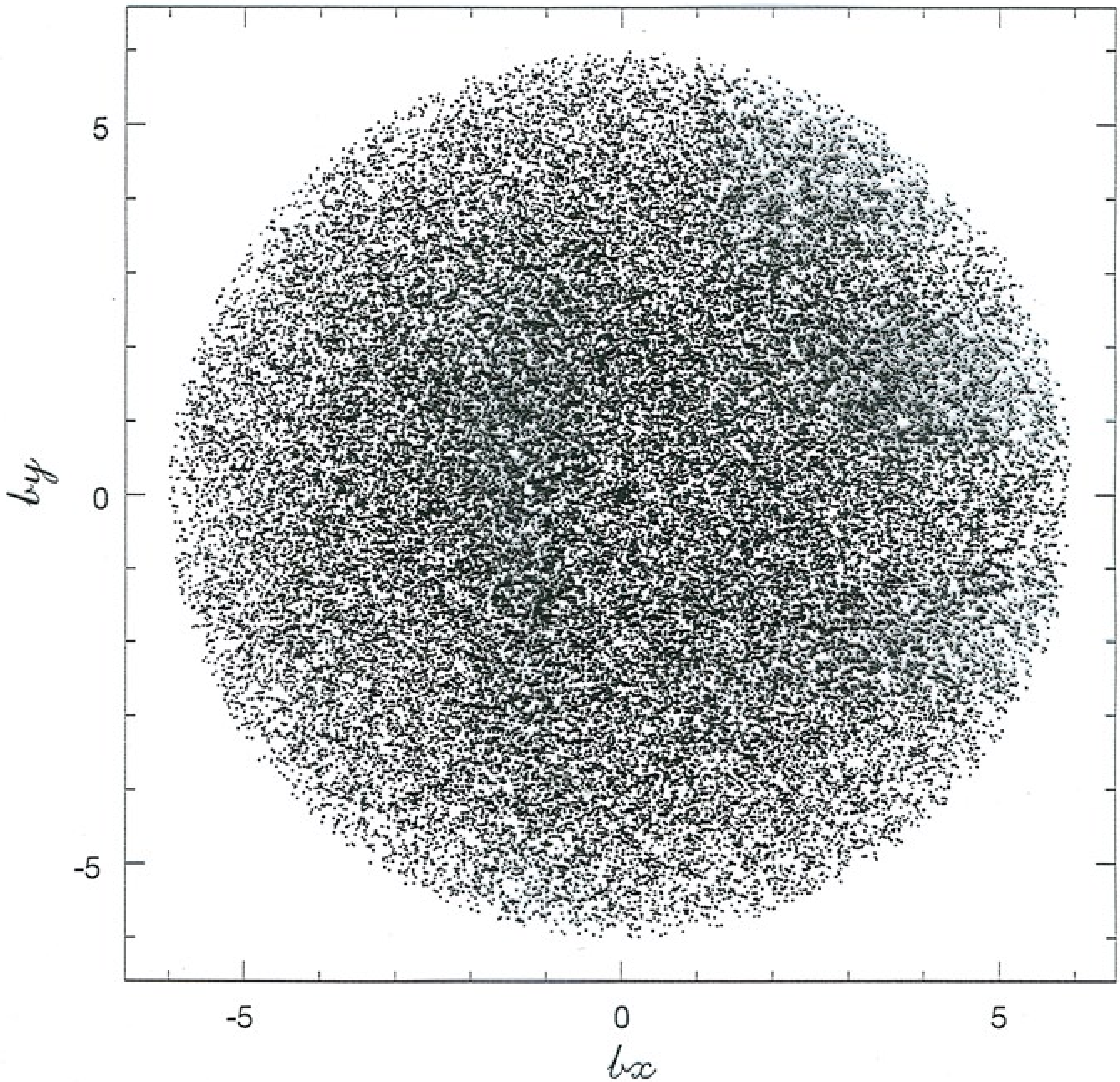
Pb−Pb
√s = 6 TeV
β = 0 fm
k = 2.5
|η| < 4
|η| < .9
a
N
10^5
10^4
10^3
2
3
4
5
p0 (GeV)



Pb-Pb
$\sqrt{s}$ = 6 TeV
β = 0 fm
k = 2.5
$|\eta| < 4$
$|\eta| < .9$
C
10^5
10^4
E_T (GeV)
2
3
4
5
p_0 (GeV)



$x_A > 10^{-2}$
by
5
0
-5
-5
0
5



y=5

$x_A = .1$

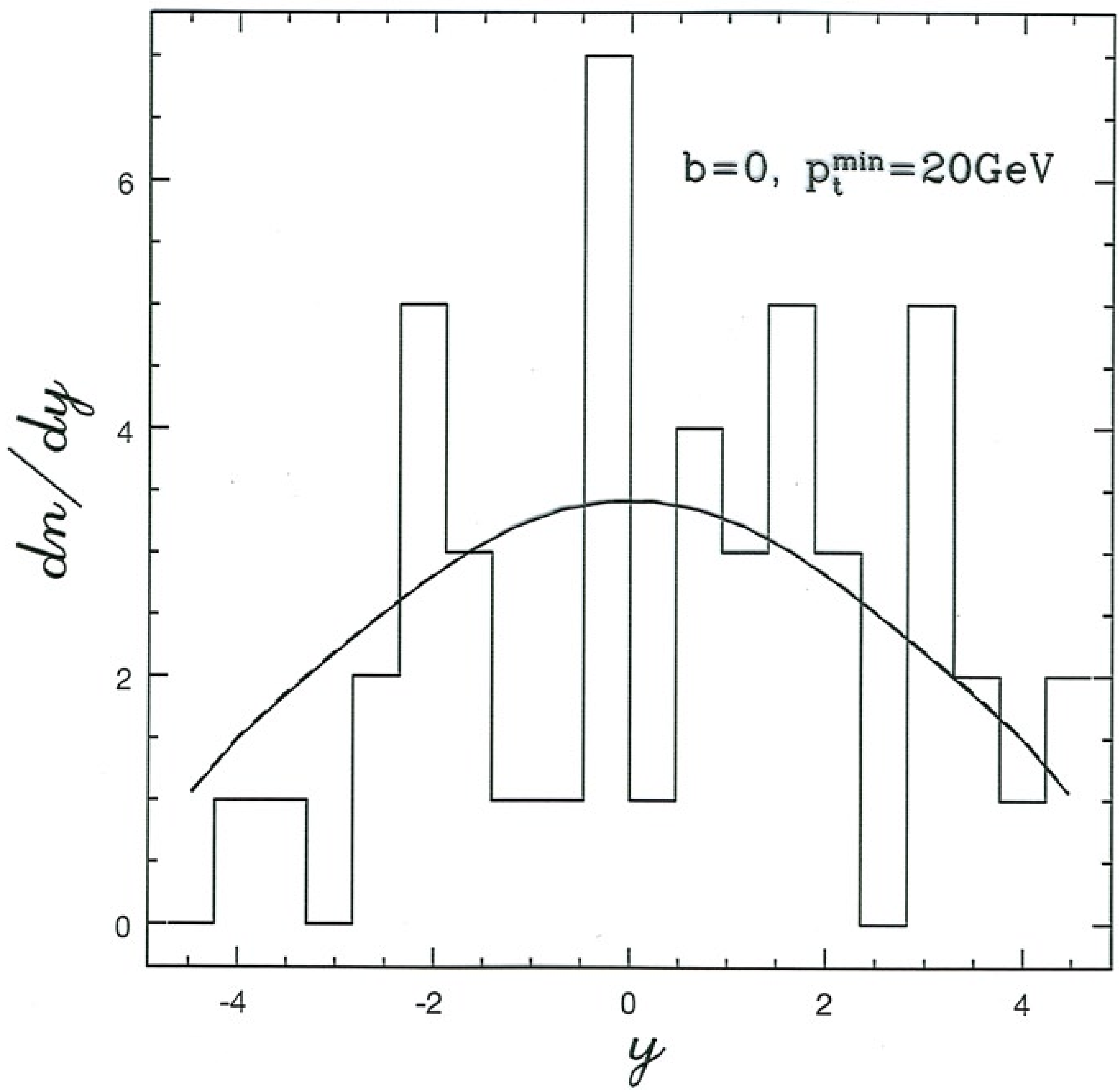


Fig.2a

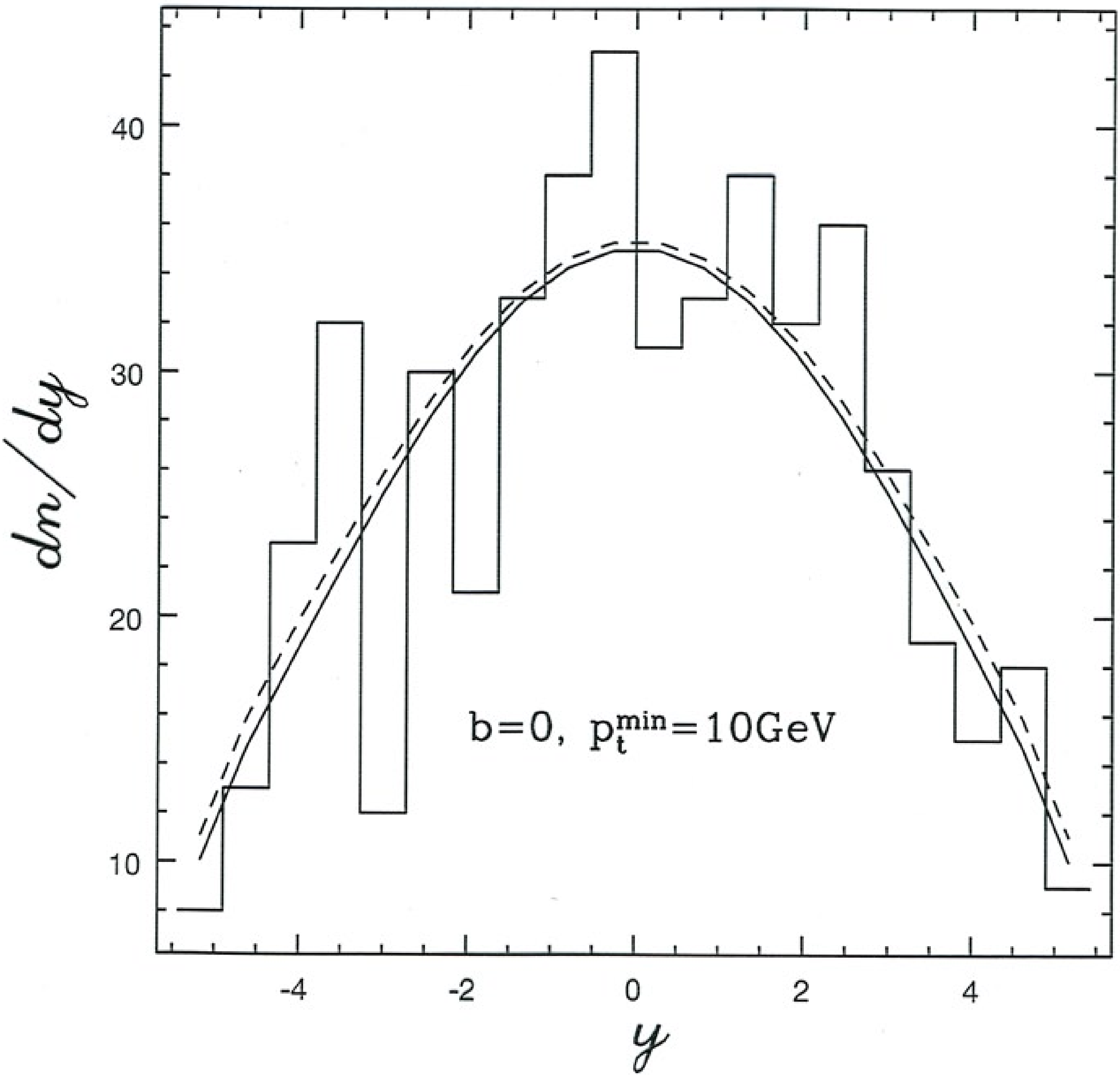


Fig.2b

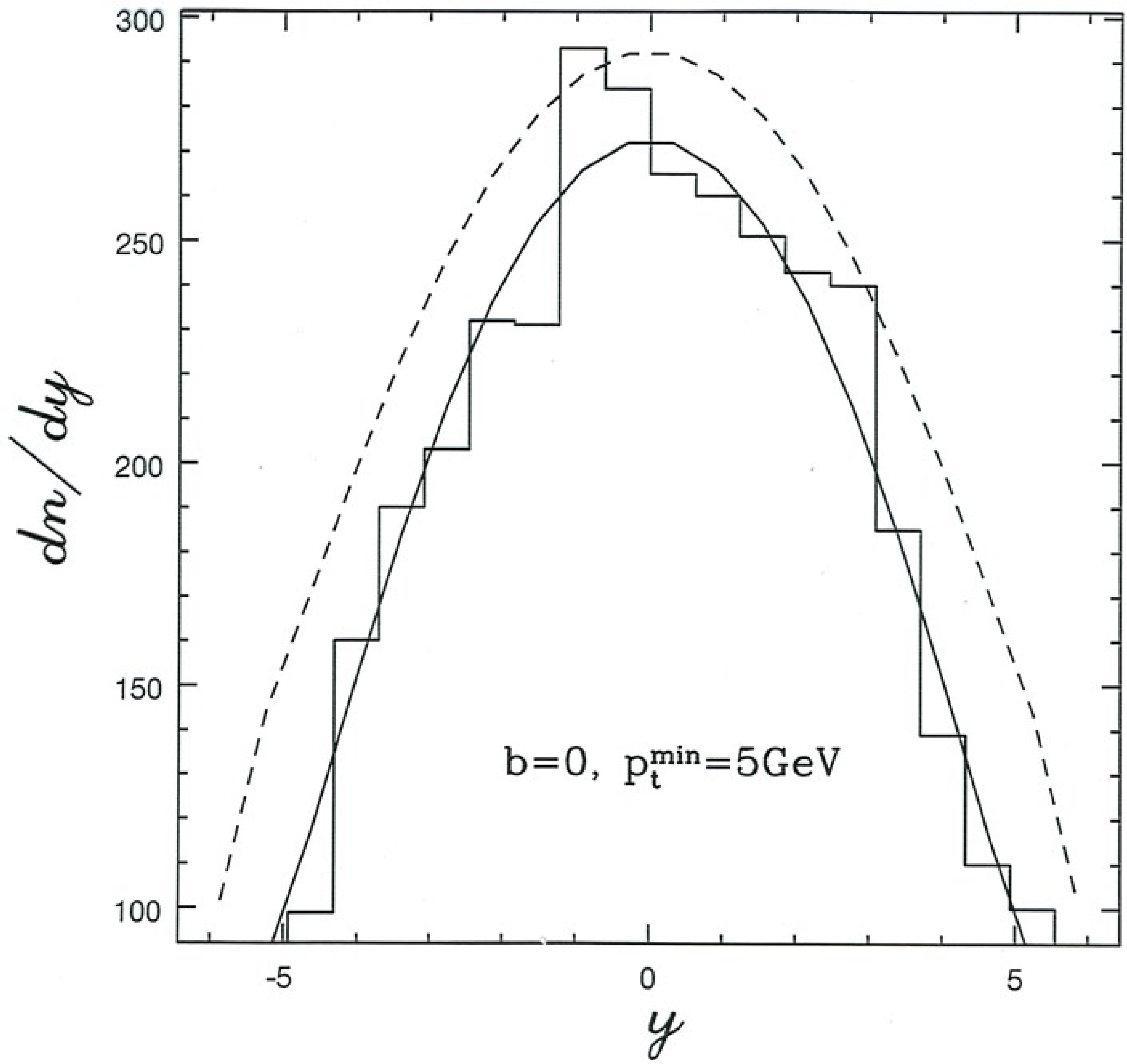
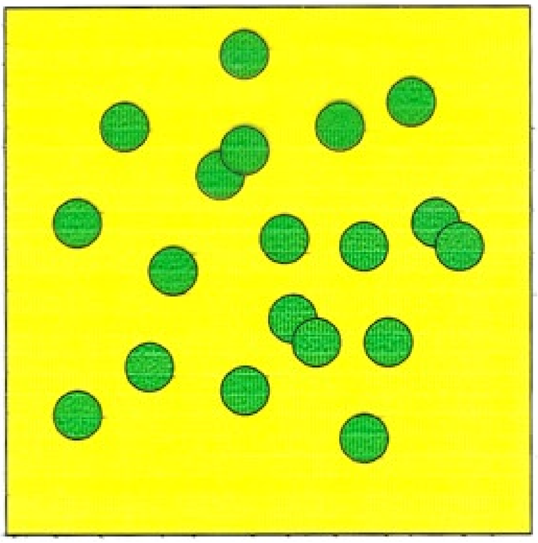
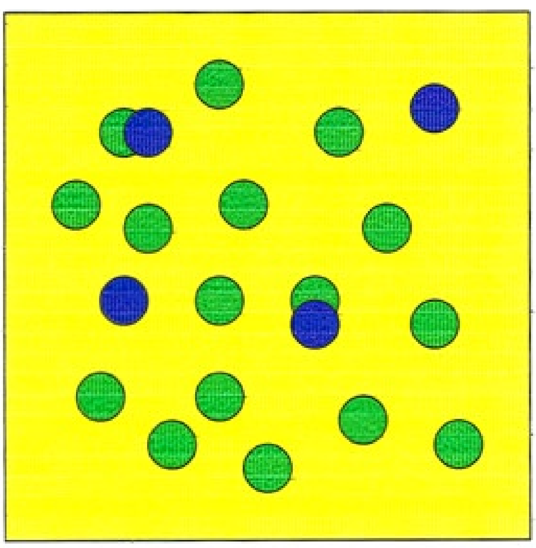


Fig.2c



Clusters of target partons

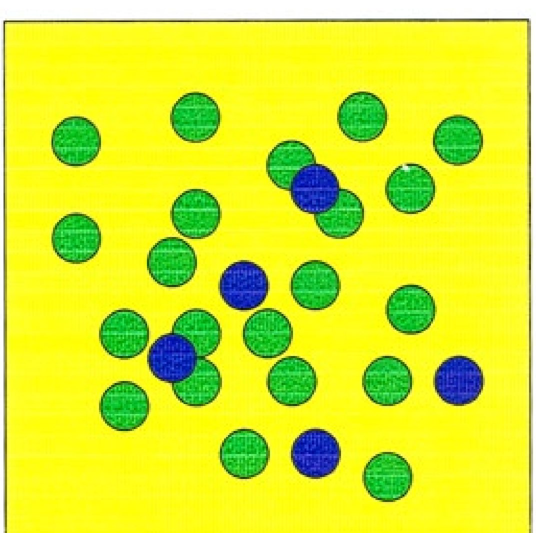
Parton fusion



Projectile partons

Target partons

Disconnected collisions



Rescatterings

A peculiar feature is that disconnected collisions allow to probe the hadron structure at different points at the same time. The case of NA interactions is particularly interesting.

The simplest possibility is the double parton collisions. The process has been searched in pp and $p\bar{p}$ interactions. The cross section in the interaction of two hadrons N and N' is expresses as

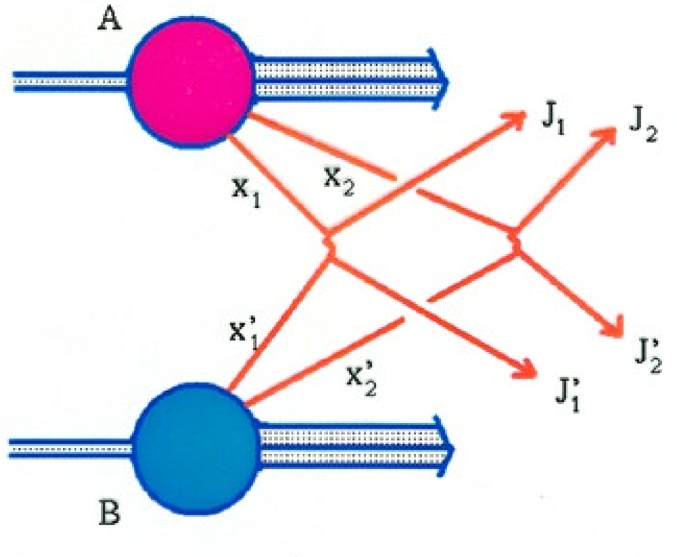
$$\sigma_D = \frac{1}{2}\int \Gamma_N(x_1, x_2; b)\hat{\sigma}(x_1, x_1') \times\hat{\sigma}(x_2, x_2')\Gamma_{N'}(x_1', x_2'; b)dx_1dx_1'dx_2dx_2'd^2b$$

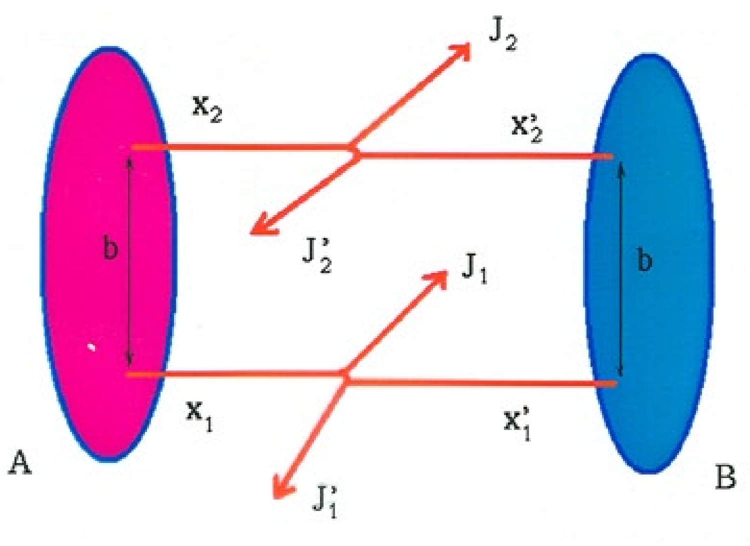
The non perturbative input to the double parton collision process are the two-body parton distribution $\Gamma(x_1, x_2; b)$ which depend on the two fractional momenta x_1 and x_2 of the two partons and on their distance in transverse space b.

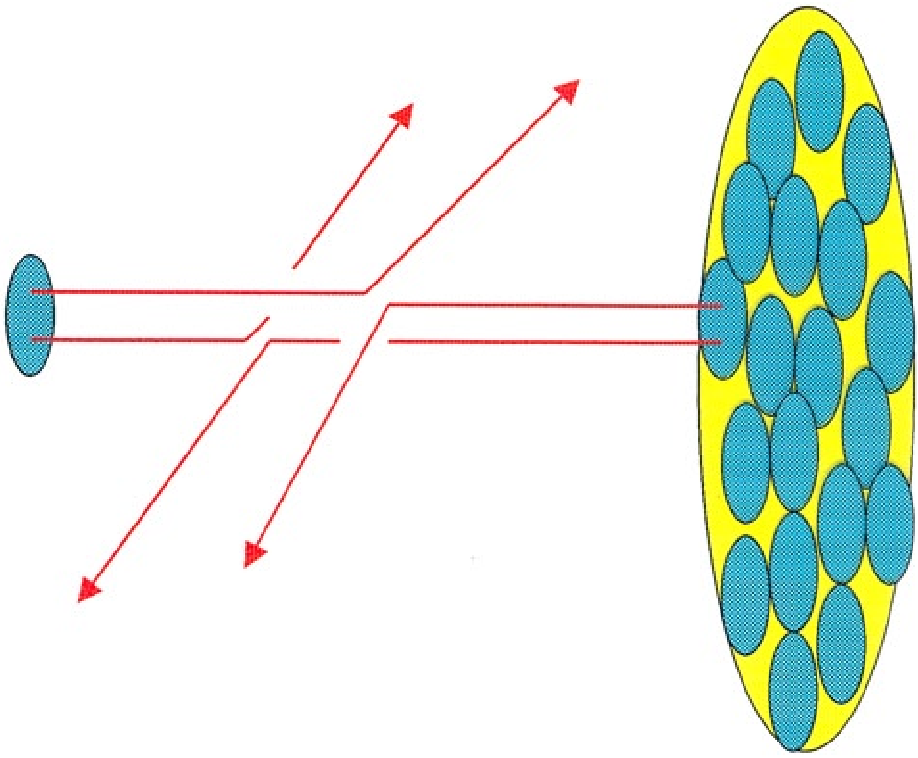
If the two partons are not correlated in x and if the dependence on b can be factorized one may write

$$\Gamma(x_1, x_2; b) = G(x_1)G(x_2)F(b)$$

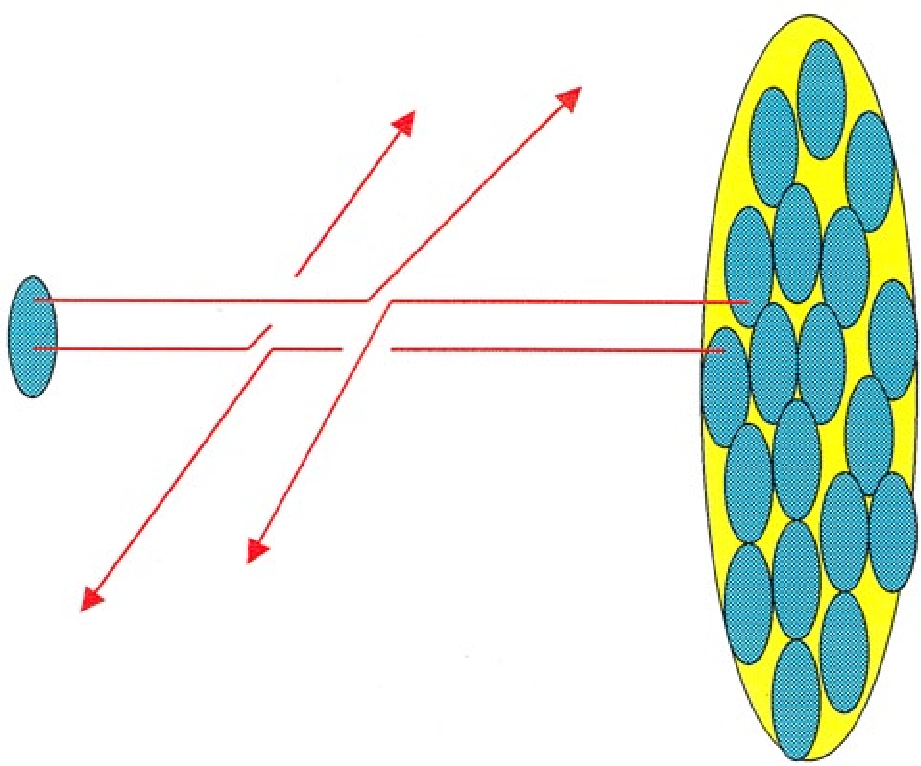
and the inclusive cross section for a double parton collision is expressed, in the case of two indistinguishable parton interactions, as follows:


A
B
J_1
J_2
x_1
x_2
x'_1
x'_2
J'_2
J'_1


J_2
x_2
x'_2
b
J'_2
J_1
b
x_1
x'_1
A
B
J'_1



$\sim A^{1}$



$\sim A^{4/3}$

Expansion of $W_N(x,B)$ in number of collisions

$$W_N(x,B) = G_N(x) \sum_{\nu=1}^{\infty} \frac{\langle n_A(x,B)\rangle^\nu}{\nu!} e^{-\langle n_A(x,B)\rangle}$$

Differential distribution in p_t:

$$\frac{dW_N}{d^2p_t} = G_N(x) \sum_{\nu=1}^{\infty} \int \frac{\prod_{i=1}^{\nu} \Gamma_A(x'_i,B)}{\nu!} e^{-\langle n_A(x,B)\rangle} \times \frac{d\sigma}{d\mathbf{k}_1} \cdots \frac{d\sigma}{d\mathbf{k}_\nu} \delta(\mathbf{k}_1 + \ldots + \mathbf{k}_\nu - \mathbf{p_t}) \prod_{i=1}^{\nu} d\mathbf{k}_i dx'_i$$

Dependence on the cutoff

- One rescattering (all terms of $O(\sigma^2)$)

$$\frac{1}{2} \int \frac{d\sigma}{d\mathbf{k}} \frac{d\sigma}{d(\mathbf{p}_t - \mathbf{k})} d\mathbf{k} - \frac{d\sigma}{d\mathbf{p}_t} \int \frac{d\sigma}{d\mathbf{k}} d\mathbf{k}$$

Power divergences ($\mathbf{k} \to 0$, $\mathbf{k} \to p_t$) are cancelled and one is left with a logarithmic dependence on the cutoff.

If longitudinal and transverse degrees of freedom are decoupled the sum can be done by introducing the Fourier transforms in the transverse variables

$$\frac{dW_N}{d^2p_t} = \frac{1}{(2\pi)^2}\int d^2r e^{ip_t r} G_N(x)\times$$
$$\times\left[e^{T_A(B)[F_A(x,r)-F_A(x,0)]} - e^{-T_A(B)F_A(x,0)}\right]$$

where

$$\Gamma_A(x_i', B) = G_A(x')T_A(B)$$

and

$$F_A(x,b) = \int \frac{d^2p_t}{(2\pi)^2} dx' G_A(x') \frac{d\hat{\sigma}}{d^2p_t} e^{ip_t b}$$

This expression of the cross section is obtained in different approaches (dipole cross-section, color glass)

The expression however is derived with approximate kinematics: the longitudinal momentum of the projectile is conserved in the interaction and final state partons are hence more energetic than the initial ones, the effect being emphasized when the number of re-scatterings grows.

The final p_t spectrum gets hence shifted towards larger transverse momenta and the Cronin ratio is appreciably modified.

When using exact kinematics the series cannot be summed any more and each multiple scattering term has to be evaluated separately.

$$\frac{d\sigma_i}{d^2p_t dy\, d^2b} = \frac{d\sigma_i^{(1)}}{d^2p_t dy\, d^2b} + \frac{d\sigma_i^{(2)}}{d^2p_t dy\, d^2b} + \frac{d\sigma_i^{(3)}}{d^2p_t dy\, d^2b} \tag{1}$$

where

$$\begin{aligned}\frac{d\sigma_i^{(1)}}{d^2p_t dy\, d^2b} &= T_A(b) \sum_j \frac{1}{1+\delta_{ij}} \int d^2q_1\, dx'\, \delta^{(2)}(\bar{q}_1 - \bar{p}_t)\, \hat{\sigma_{ij}}^{(1)}(y, x'; q_1) \\ &\times\ x\left[f_{\frac{i}{p}}(x, Q_{fct})\, f_{\frac{j}{A}}(x', Q_{fct}) + f_{\frac{i}{p}}(x', Q_{fct})\, f_{\frac{j}{A}}(x, Q_{fct})\right]\end{aligned}$$

is the single scattering term (the projectile parton interacts with a single parton of the target and vice-versa),

$$\begin{aligned}\frac{d\sigma_i^{(2)}}{d^2p_t dy\, d^2b} &= \frac{1}{2!}\, T_A(b)^2 \sum_{j,k} \frac{1}{1+\delta_{ij}} \frac{1}{1+\delta_{ik}} \int d^2q_1\, d^2q_2\, dx'\, dx'' \\ &\times\ \left[\delta^{(2)}(\bar{q}_1 + \bar{q}_2 - \bar{p}_t) - \delta^{(2)}(\bar{q}_1 - \bar{p}_t) - \delta^{(2)}(\bar{q}_2 - \bar{p}_t)\right] \\ &\times\ x\, f_{\frac{i}{p}}(x, Q_{fct})\, f_{\frac{j}{A}}(x', Q_{fct})\, f_{\frac{k}{A}}(x'', Q_{fct})\, \hat{\sigma}^{(2)}_{ijk}(y, x', x''; \bar{q}_1, \bar{q}_2)\end{aligned}$$

is the rescattering term (the projectile parton interacts with two different partons of the target), and

$$\begin{aligned}\frac{d\sigma_i^{(3)}}{d^2p_t dy\, d^2b} &= \frac{1}{3!}\, T_A(b)^3 \sum_{j,k,l} \frac{1}{1+\delta_{ij}} \frac{1}{1+\delta_{ik}} \frac{1}{1+\delta_{il}} \int d^2q_1\, d^2q_2\, d^2q_3 \\ &\times\ dx'\, dx''\, dx'''\, \hat{\sigma}^{(3)}_{ijkl}(y, x', x'', x'''; \bar{q}_1, \bar{q}_2, \bar{q}_3) \\ &\times\ \Big[\delta^{(2)}(\bar{q}_1 + \bar{q}_2 + \bar{q}_3 - \bar{p}_t) - \delta^{(2)}(\bar{q}_1 + \bar{q}_2 - \bar{p}_t) - \delta^{(2)}(\bar{q}_2 + \bar{q}_3 - \bar{p}_t) \\ &\times\ -\delta^{(2)}(\bar{q}_3 + \bar{q}_1 - \bar{p}_t) + \delta^{(2)}(\bar{q}_1 - \bar{p}_t) + \delta^{(2)}(\bar{q}_2 - \bar{p}_t) + \delta^{(2)}(\bar{q}_3 - \bar{p}_t)\Big] \\ &\times\ x\, f_{\frac{i}{p}}(x, Q_{fct})\, f_{\frac{j}{A}}(x', Q_{fct})\, f_{\frac{k}{A}}(x'', Q_{fct})\, f_{\frac{l}{A}}(x''', Q_{fct}).\end{aligned}$$

is the double rescattering term (the projectile parton interacts with three different partons of the target).
The subtraction terms, in the rescattering and in the double rescattering terms, are a direct consequence of the implementation of unitarity in the process. The sums run over the different species of target partons. The quantities $f_{\frac{j}{A}}(x', Q_{fct})$ represent the nuclear parton distributions.

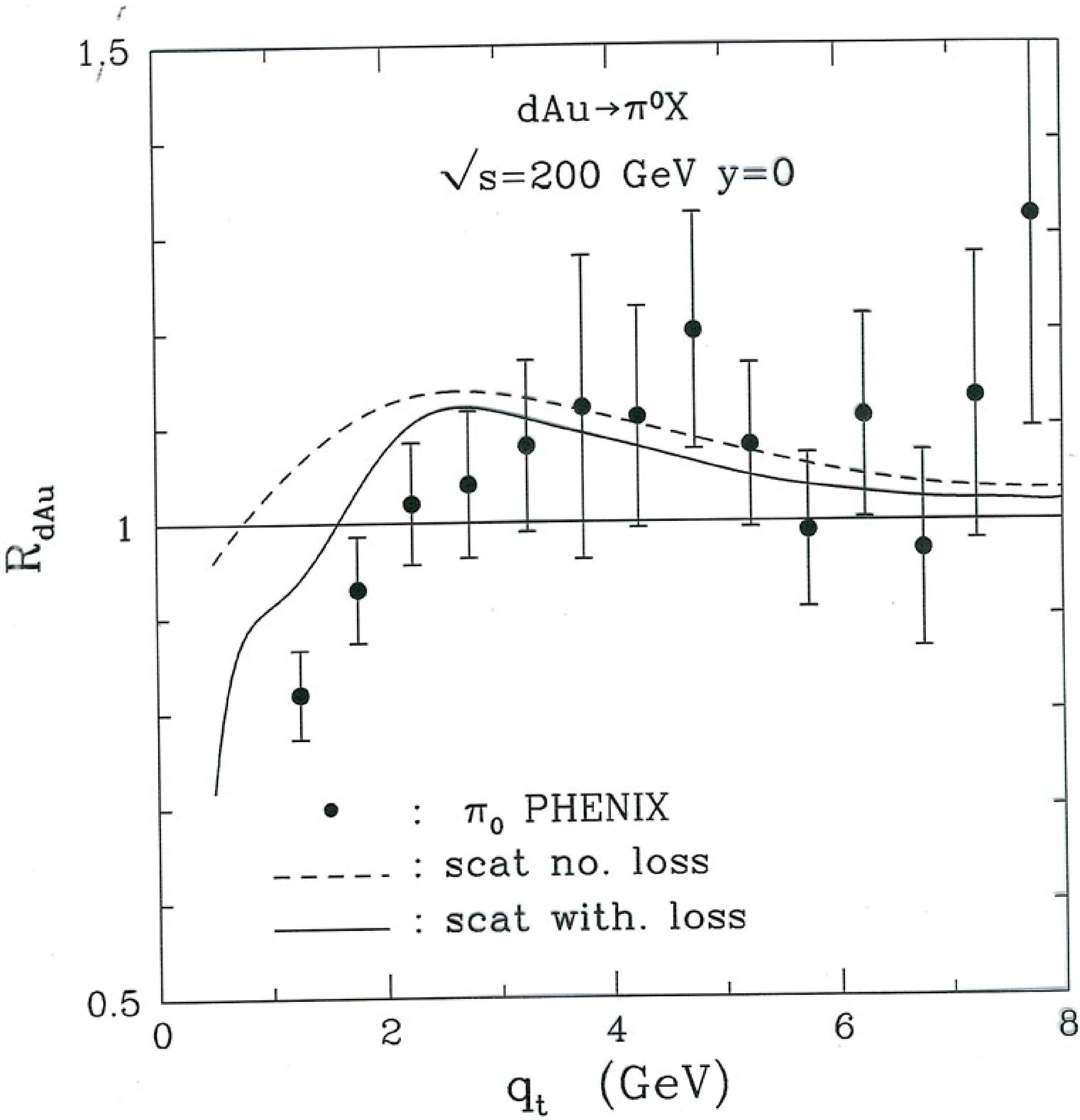
The multiparton interaction cross sections are given by:

$$\hat{\sigma_{ij}}^{(1)}(y, x'; q_1) \quad = k_{factor} \quad \frac{d\hat{\sigma}_{ij}(\hat{s}_1, \hat{t}_1, \hat{u}_1)}{d\hat{t}_1} \Theta(\Phi^{(1)})$$

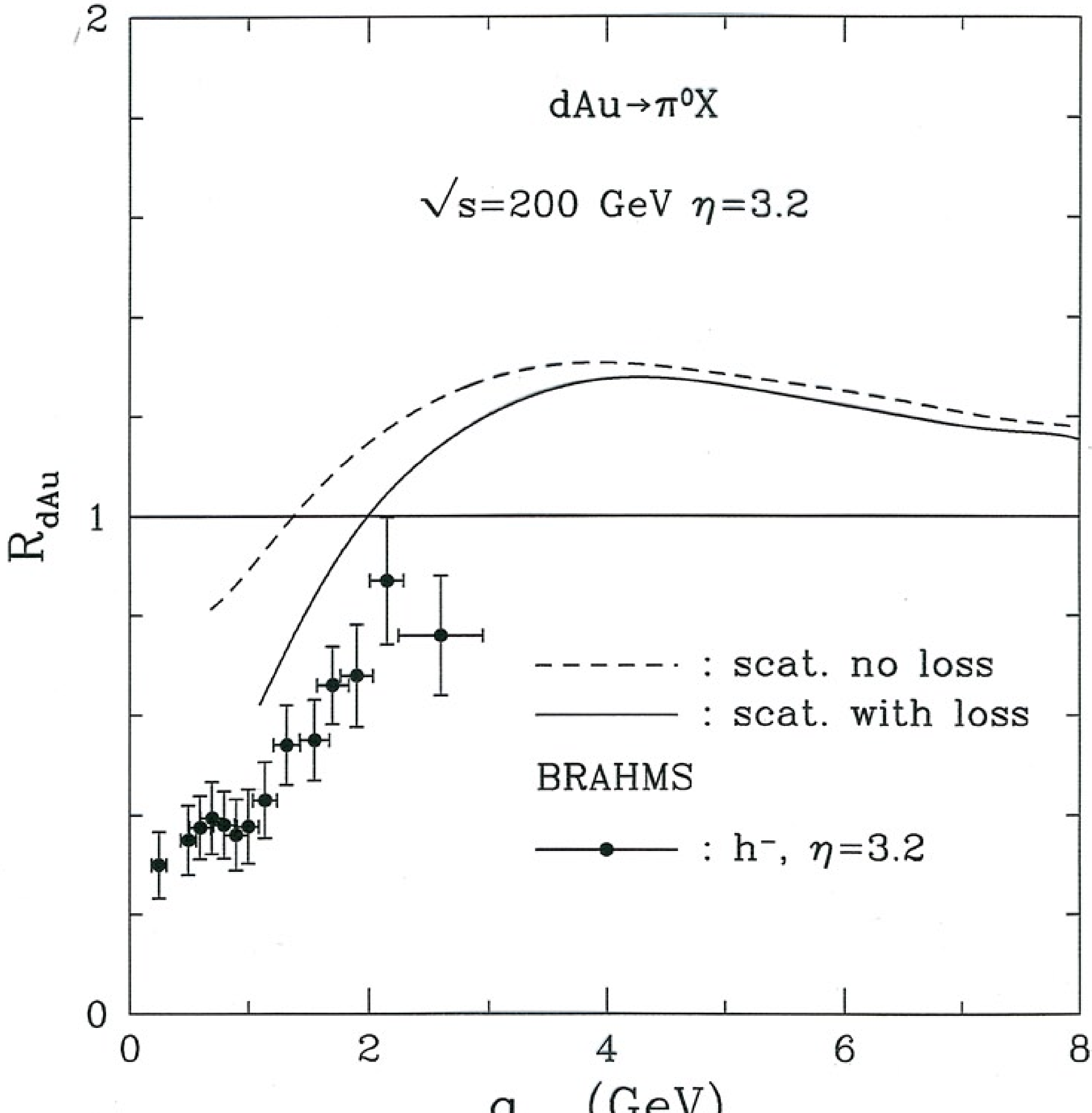
$$\hat{\sigma}^{(2)}_{ijk}(y, x', x''; \bar{q}_1, \bar{q}_2) \quad = \quad (k_{factor})^2 \frac{d\hat{\sigma}_{ik}(\hat{s}_2, \hat{t}_2, \hat{u}_2)}{d\hat{t}_2} \frac{d\hat{\sigma}_{ij}(\hat{s}_1, \hat{t}_1, \hat{u}_1)}{d\hat{t}_1} \Theta(\Phi^{(2)})\, \Theta(\Phi^{(1)})$$

$$\begin{aligned}\hat{\sigma}^{(3)}_{ijkl}(y, x', x'', x'''; \bar{q}_1, \bar{q}_2, \bar{q}_3) \quad &= \quad (k_{factor})^3 \frac{d\hat{\sigma}_{il}(\hat{s}_3, \hat{t}_3, \hat{u}_3)}{d\hat{t}_3} \frac{d\hat{\sigma}_{ik}(\hat{s}_2, \hat{t}_2, \hat{u}_2)}{d\hat{t}_2} \\ &\times \quad \frac{d\hat{\sigma}_{ij}(\hat{s}_1, \hat{t}_1, \hat{u}_1)}{d\hat{t}_1} \Theta(\Phi^{(3)})\, \Theta(\Phi^{(2)})\, \Theta(\Phi^{(1)}),\end{aligned}$$

where the parton$_i$-parton$_j$ differential cross sections $d\hat{\sigma}_{ij}/d\hat{t}$ are evaluated at the lowest order in pQCD by making use of exact kinematics. The kinematical constraints are implemented through the quantities $\Phi^{(i)}$.



dAu→π⁰X
√s=200 GeV y=0
R_dAu
q_t (GeV)
π_0 PHENIX
scat no. loss
scat with. loss
1,5
1
0.5
0
2
4
6
8



dAu→π⁰X
√s=200 GeV η=3.2
: scat. no loss
: scat. with loss
BRAHMS
: h⁻, η=3.2
R_dAu
q_t (GeV)
0
1
2
0
2
4
6
8

SUMMARY

Multiple parton collisions are greatly enhanced in nuclear interactions at high energies.

One may distinguish two different kinds of multiparton interactions: *disconnected collisions* (independent partonic interactions in different points in transverse space) and *rescatterings* (three or more partons interact at the same point in transverse space)

All these interactions may be taken into account in a simplest model where unitarity and the AGK cutting rules are explicitly implemented

A resulting feature is a less singular behavior as a function of the cutoff.

The deformation of the transverse spectrum corresponds to the Glauber picture of independent multiple scatterings.

The steepness of the parton distributions at small x does not allow to use the standard approximation of conserving the longitudinal momentum component of the projectile during the interaction.

Using exact kinematics a reasonable agreement with experimental observations is obtained both at small and at large rapidity values.