# Scaling properties of high-$p_T$ hadron production in heavy ion collisions

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# Introduction

- Scale invariance and asymptotic freedom in QCD lead to power-like behavior of the inclusive cross section at high $p_T$:

$$p_T^n E \frac{d^3\sigma(AB \to hX)}{d^3p} = F(x_T, y), \quad x_T = \frac{2p_T}{\sqrt{S}}. \tag{1}$$

  This is the analog of Bjorken scaling in DIS.

- The power-law (instead of exponential) falloff of inclusive cross sections at large $p_T$ is evidence of pointlike hadron substructure.

- Knowledge of $n$ allows one to learn about production mechanism of high $p_T$ particles.

- Need to understand hadron production in $pp$ before investigating nuclear collisions.

- From nuclear modifications of the $p_T$ dependence, one can obtain information about energy loss.

# Dimensional counting rules

- Assume that the hadronic cross section can be written in factorized form, even for higher twist processes:

$$d\sigma(h_a h_b \to hX) = \sum_{abc} G_{a/h_a}(x_a) G_{b/h_b}(x_b) dx_a dx_b \frac{1}{2\hat{s}} |A_{fi}|^2 dX_f D_{h/c}(z_c) dz_c. \tag{2}$$

- Dimensional analysis:
  - Normalize one-particle states to dimension *length*, $\langle p|p'\rangle = 2E_p(2\pi)^3\delta^{(3)}(\vec{p} - \vec{p}')$.
  - Then, the (partonic) $S$-matrix has dimension $length^{n_a}$, where $n_a = n_{in} + n_{out}$ is the number of partons participating in the reaction
  - Because of

$$S_{fi} = \delta_{fi} + i(2\pi)^4\delta^{(4)}(\sum p_{in} - \sum p_{out})A_{fi}. \tag{3}$$

    The partonic matrix element $A_{fi}$ has dimension $length^{n_a-4}$, and the hard matrix element squared divided by the flux factor $2\hat{s}$ has dimension

$$n = 2n_a - 4. \tag{4}$$

- Intuitively, the larger the number of quarks that need to change direction, the steeper the cross section falls off.
- Idea: translate mass dimension of $A_{fi}$ into power law for $Ed^3\sigma/d^3p$.
  Blankenbecler, Brodsky, Gunion PRD18,900(1978)

## $x_T$ scaling

- The inclusive reaction $AB \to hX$ has 3 independent kinematic invariants, $S$, $T$ and $M_X^2$.
- Investigate $Ed^3\sigma/d^3p$ at fixed values of $x_1 = -U/S$ and $x_2 = -T/S$, or alternatively fixed

$$y = \frac{1}{2}\ln\left(\frac{x_1}{x_2}\right), \tag{5}$$

$$x_R = x_1 + x_2 = 1 - M_X^2/S = \frac{2|\vec{p}_{cm}|}{\sqrt{S}} \tag{6}$$

This requires measurements at different energies.

- Then only one dimensionful invariant exists so that

$$p_T^n E\frac{d^3\sigma(AB \to hX)}{d^3p} = F(x_R, y) \Leftrightarrow \sqrt{S}^n E\frac{d^3\sigma(AB \to hX)}{d^3p} = G(x_R, y) \tag{7}$$

with $n = 2n_a - 4$.

- This is often referred to as $x_T$ scaling, because at $y = 0$ one has $x_R = x_T = 2p_T/\sqrt{S}$.
- Note that $x_{1,2}$ are different from the momentum fractions $x_{a,b}$ in the factorization ansatz.

## Examples of counting rules

- $2 \to 2$ partonic scattering $\Rightarrow n_a = 4$, $n = 2n_a - 4 = 4$ so that

$$E\frac{d^3\sigma(AB \to hX)}{d^3p} \propto \frac{F(x_R, y)}{p_T^4} \tag{8}$$

- $uu \to p\bar{d}$, i.e. direct proton production with $n_a = 1 + 1 + 3 + 1 = 6$, $n = 8$

$$E\frac{d^3\sigma(AB \to pX)}{d^3p} \propto \frac{f_p^2}{p_T^8} F(x_R, y). \tag{9}$$

The dimensionful factor $f_p$ reflects the physics of the proton distribution amplitude. The distribution amplitude of a hadron has dimension mass for mesons and mass squared for baryons, e.g. the pion distribution amplitude is normalized to $f_{\pi^+} = 130$ MeV.

- In the limit $x_R \to 1$, the missing mass approaches 0 and all fields participate, $n_a = 4 \cdot 3 = 12$,

$$E\frac{d^3\sigma(pp \to p(X = p))}{d^3p} \propto \frac{f_p^8}{p_T^{20}} f(y). \tag{10}$$

Scaling laws for exclusive processes are experimentally well confirmed.

## Relation to Bjorken scaling

- In the limit $Q^2 \to \infty$ at fixed $x = \frac{Q^2}{Q^2+M_X^2}$, the DIS structure function depends only on $x$.
- However, for $Q^2 \to \infty$ and $M_X^2$ fixed (i.e. $x \to 1$), one has

$$F_2^p(M_X^2, Q^2) \propto (1-x)^{2n_s-1} = \left(\frac{M_X^2}{Q^2+M_X^2}\right)^3 \sim \frac{1}{Q^6}. \quad (11)$$

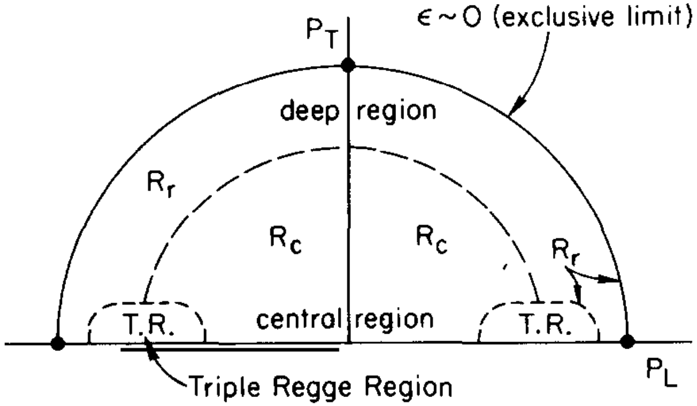
  That way, $F_2^p$ smoothly matches onto the elastic formfactor,

$$(1-x)F_2^p(x, Q^2) \to G(Q^2) \propto \frac{1}{Q^8}. \quad (12)$$

  This is still the case if QCD evolution is included. DGLAP evolution turns off at $x \to 1$.
- The analog of the Bjorken limit in inclusive hadron production is $p_T \to \infty$ at $x_R$ and $y$ fixed.
- At fixed hadronic cm. energy, large $p_T$ does not imply that higher twists disappear.
- Note, the $p_T$ dependence of $Ed^3\sigma/d^3p$ at fixed $\sqrt{S}$ is in general much steeper than at fixed $x_R$ and $y$, because structure and fragmentation functions are probed at different momentum fractions.
- $x_T$-scaling removes this effect and the $p_T$ dependence of the partonic process is unveiled.

## Forward physics and the exclusive limit

- In the exclusive limit $\epsilon = 1 - x_R = M_X^2/S \to 0$, only valence quarks are important.




- $x_T = \frac{2p_T}{\sqrt{S}} = 2\sqrt{x_1 x_2}$, $x_F = \frac{2p_L}{\sqrt{S}} = x_1 - x_2$, $x_R = \frac{2|\vec{p}|}{\sqrt{S}} = x_1 + x_2 = \sqrt{x_F^2 + x_T^2}$
- At very large $\sqrt{S}$ the two kinematical domains of
  - $x_2 = -T/S \ll 1$ (Regge theory applies)
  - $p_T \gg \Lambda_{QCD}$ (pQCD applies)

  overlap.
- Still, the gluon density approaches 0 like $(1 - x_R)^5$ toward the exclusive limit. This behavior is expected from QCD factorization.

## Suppression of gluon radiation at large $x_R$

- The simple spectator counting rules $(1-x)^{2n_s-1}$ receive corrections from QCD evolution.
- Gluon radiation off a constituent (quark, gluon, diquark, intrinsic meson ...) changes the large $x$ behavior of the distribution function to

$$G_{(a/A)}(x_a, p_T) = (1-x_a)^{2n_s-1+\xi(p_T)}, \tag{13}$$

$$\xi(p_T) = \frac{C_R}{\pi}\int_{k^2_{x_a}}^{p_T^2}\frac{dk_\perp^2}{k_\perp^2}\alpha_s(k_\perp^2) = \frac{4C_R}{\beta_0}\ln\frac{\ln(p_T^2/\Lambda^2_{QCD})}{\ln(k^2_{x_a}/\Lambda^2_{QCD})}, \tag{14}$$

where $\beta_0 = 11 - 2N_F/3$ and $C_R$ is the total color charge squared of the constituent, i.e. $C_R = 4/3$ for quarks, $C_R = 3$ for gluons, $C_R = 10/3$ for sextet diquarks and $C_R = 0$ for color-neutral objects.

- Important: large $x$ parton must be far off shell with virtuality

$$k^2_{x_a} = \frac{p_T^2}{1-x_a}. \tag{15}$$

Hence, QCD evolution turns itself off at large $x$ and spectator counting rules become exact.

- This "self-healing" of evolution is necessary to smoothly match the DIS structure function $F_2(x, Q^2)$ onto the elastic formfactor $G(Q^2) \propto 1/Q^8$ in the limit $Q^2 \to \infty$ at fixed $S_{\gamma^* p}$.

# Scaling violations of dimensional counting rules

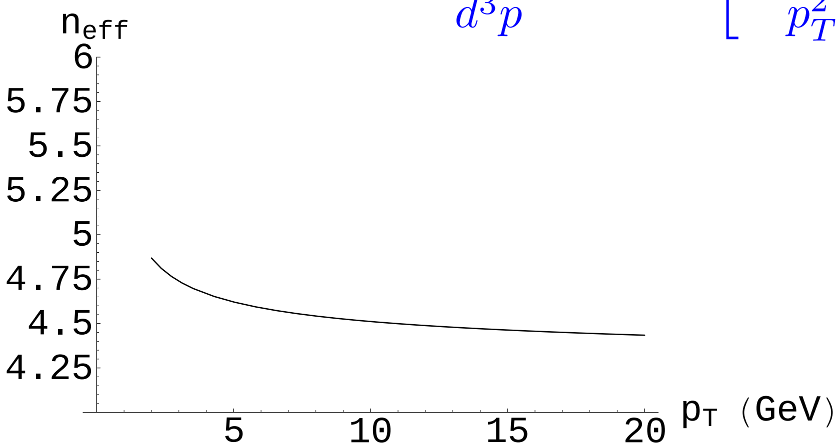
- QCD is only approximately scale invariant. Structure and fragmentation functions are logarithmically scale-dependent. So is the strong coupling constant $\alpha_s(p_T)$.
- Scaling violations lead to corrections to the nominal power laws. The effective exponent

$$n_{eff}(p_T) = -\frac{d\ln E\frac{d^3\sigma(AB\to hX)}{d^3p}}{d\ln(p_T)} \tag{16}$$

is now a slowly varying function of $p_T$.

- The invariant cross section behaves approximately as

$$E\frac{d^3\sigma(AB\to hX)}{d^3p} \sim \left[\frac{\alpha_s(p_T^2)}{p_T^2}\right]^{n_a-2} \frac{(1-x_R)^{2n_s-1+3\xi(p_T)}}{x_R^{\lambda(p_T)}} f(y) \tag{17}$$



## Experimental results: Fixed target energies

- The Chicago-Princeton collaboration measured high-$p_T$ inclusive hadron production in the SPS energy range PRD19,764(1979).
- Data for $p + p \to (\pi^+ + \pi^-)/2 + X$ behave as

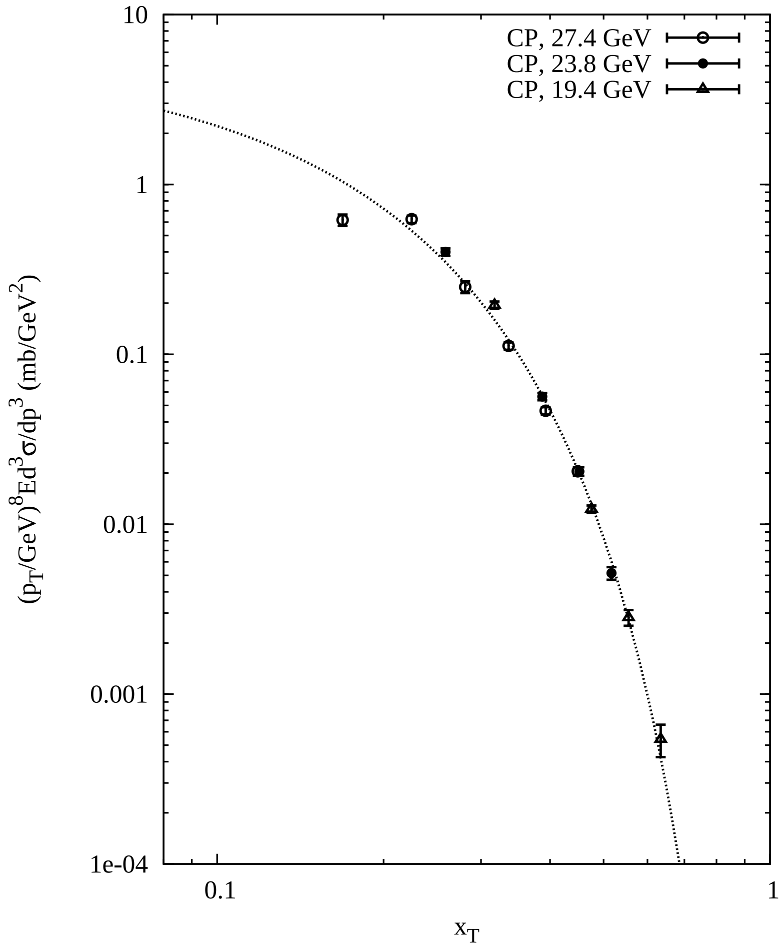
$$p_T^8 E \frac{d^3\sigma}{d^3p} \propto (1 - x_T)^9,$$

  suggesting a higher twist mechanism even at $p_T \sim 5$ GeV.

- Possible explanation: The hard subprocess $q + (q\bar{q}) \to q + \pi$ has $n_a = 1 + 2 + 1 + 2 = 6$ and $n_s = 2 + 3 = 5$, yielding the observed scaling properties. Blankenbecler et al. PRD18,900(1978)
- Protons: $(p + p \to p + X)$

$$p_T^{11.7} E \frac{d^3\sigma}{d^3p} \propto (1 - x_T)^{6.8}.$$

  Consistent with $q + (qq) \to M + p$ and $q + (qqq) \to q + p$.

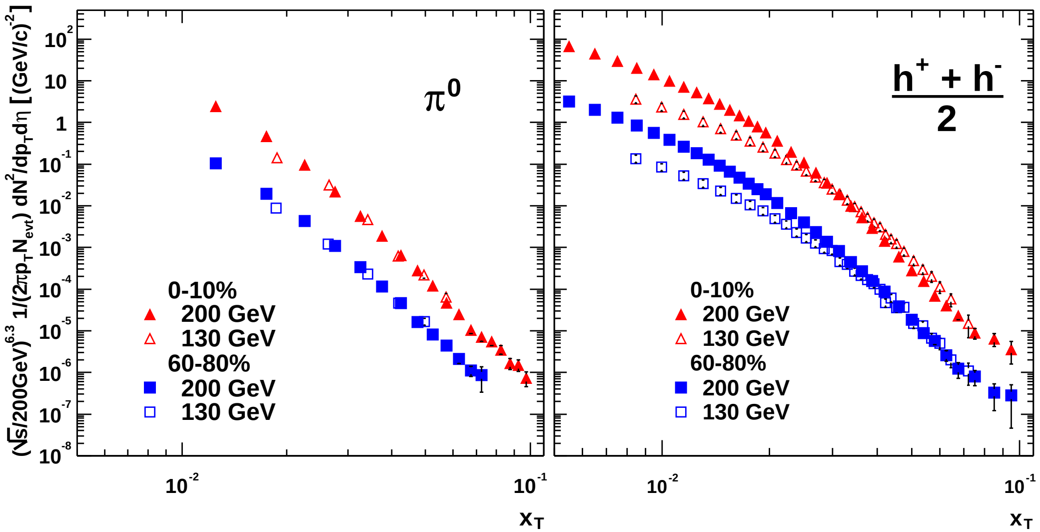


## Experimental results: Collider energies

- PHENIX analysis of $x_T$-scaling between $\sqrt{S} = 130$ GeV and 200 GeV PRC69,034910(2004):
- Charged hadron and $\pi^0$ production go like

$$E\frac{d^3N}{d^3p} \propto \frac{1}{p_T^{6.3\pm 0.5}},$$

which is slightly steeper than leading twist including scaling violations ($n_{eff} \approx 5$).




- Tevatron jet data: $n_{exp} = 4.45$ for $0.15 \leq x_T \leq 0.3$ between $\sqrt{S} = 630$ GeV and 1800 GeV.
- For $2 \rightarrow 1$ gluon fusion, one would expect $p_T^{-2}$. This is a possible signature of CGC at LHC.
- Conclusion: the mechanism of high-$p_T$ hadron production changes from fixed target to collider energies. $\Rightarrow$ Nuclear effects are expected to change as well.

## Nuclear effects

- Fixed target data suggest that pions are not produced by parton fragmentation.
- Instead, a small color neutral object of size $1/p_T^2$ is produced in the hard reaction, which then evolves into a pion.
- Color neutral objects are less affected by the medium than colored partons.
- The dipole interacts with inelastic cross section $\sigma_{in} \sim \pi/p_T^2 \approx 0.1$ mb. The mean free path in is thus of order

$$\lambda_{free} \approx \begin{cases} 450 \text{ fm } \rho_A = 0.16 \text{ fm}^{-3} \\ 45 \text{ fm } \rho_A = 1.6 \text{ fm}^{-3} \end{cases} \tag{18}$$

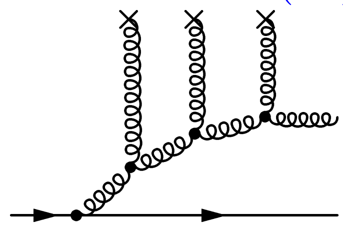
- In this scenario, no quenching of pion spectra would occur at SPS. This mechanism can be verified directly by imposing exclusion cuts around a high $p_T$ pion.
- In addition, if protons are produced by such a mechanism (at SPS or RHIC), $R_{CP} \approx 1$ for protons. At RHIC, this results in an apparent proton enhancement in $AB$ collisions.

## BDMPS-Z approach to medium induced gluon radiation

- See Baier, Schiff, Zakharov, Ann. Rev. Nucl. Part. Sci. 50:37-69,2000
- LPM effect: The Bethe-Heitler (BH) formula for bremsstrahlung,

$$\frac{dI_{BH}}{d\omega dz} \sim \alpha_s C_R \frac{1}{\omega \lambda_{free}}, \tag{19}$$

  is modified for very energetic particles due to coherent rescattering.



- In QCD, in leading log(x) approximation, only the radiated gluon rescatters:
- The time needed to radiate a gluon (coherence time) can be estimated from the uncertainty relation, $t_c \approx \omega/k_T^2$.
- During that time, the gluon passes by $N_{coh} = \sqrt{\omega/\omega_{BH}}$ scattering centers, which act as one single scattering center.
- Coherence effects are relevant for frequencies $\omega > \omega_{BH} \approx \langle k_T^2 \rangle \lambda_{free} \sim$ few-hundred MeV.
- Consequently, the gluon spectrum is modified,

$$\frac{dI_{LPM}}{d\omega dz} \sim \frac{1}{N_{coh}} \frac{dI_{BH}}{d\omega dz} \sim \alpha_s C_R \frac{\sqrt{\hat{q}}}{\omega^{3/2}}, \quad \hat{q} = \frac{\omega_{BH}}{\lambda_{free}^2}. \tag{20}$$

# Estimate of the BDMPS transport coefficient

- The transport coefficient $\hat{q}$ and the dipole cross section $\sigma_{q\bar{q}}(r_T^2) = Cr_T^2$ are both related to the average color-field strength $\langle F^2 \rangle$ in the medium JR, PLB557,184(2003),

$$C = \frac{\pi^2}{3}\alpha_s\langle F^2\rangle \tag{21}$$

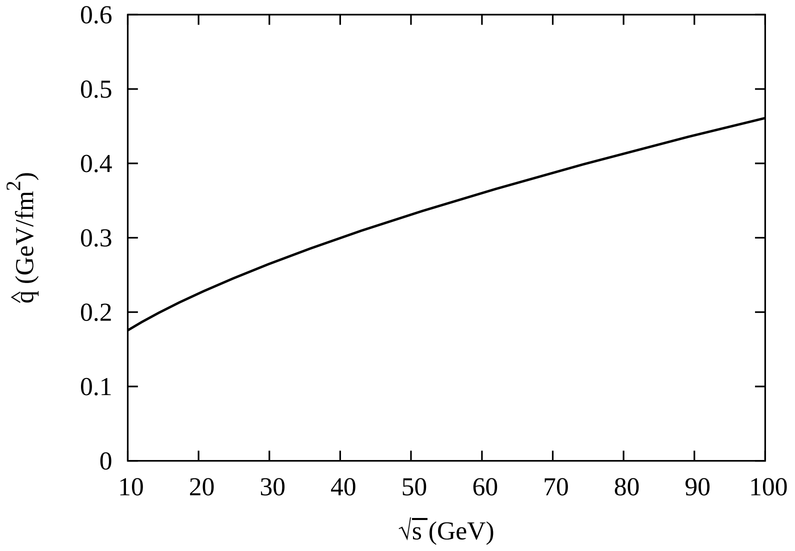
$$\hat{q} = 2\rho_A\frac{\pi^2}{3}\alpha_s\langle F^2\rangle \tag{22}$$

- The dipole approach has a highly developed and successful phenomenology in DIS, Drell-Yan, heavy flavor production, total hadronic cross sections, color transparency . . . .

- Use KST parameterization of $\sigma_{q\bar{q}}$ to determine $\hat{q}$. Kopeliovich et al. PRD62,054022(2000)

- Higher order corrections make $\hat{q}$ weakly energy dependent, $\hat{q} \propto E^{0.08}$.

- Note: this estimate is for cold nuclear matter and works only at high energies.



# The transport coefficient in heavy ion collisions

- In HIC, a medium with high energy density is created. Bjorken's estimate of the initial energy density at RHIC yields

$$\epsilon_{Bj} = \frac{\langle m_T \rangle}{\pi R_A^2 \tau_0} \left( \frac{dN}{dy} \right)_{y=0} \approx 10\,\text{GeV}/\,\text{fm}^3 \approx 60 \epsilon_{cold} \tag{23}$$

at initial time $\tau_0 = 0.5$ fm.

- Because of the expansion of the medium, the hard parton sees an averaged transport coefficient,

$$\hat{q}^{med} = \frac{2\hat{q}}{L^2} \int_{\tau_0}^{\tau_0 + L} d\tau (\tau - \tau_0) \frac{\tau_0}{\tau}. \tag{24}$$

Salgado, Wiedemann, PRL89,092303(2002)

- The averaged transport coefficient is then

$$\hat{q}^{med} \approx 10 \hat{q}^{cold} \approx 2\,\text{GeV}/\,\text{fm}^2. \tag{25}$$

# Medium induced energy loss

- Distinguish 3 different regimes:

1. $E < \omega_{BH} \sim$ few-hundred MeV: Bethe Heitler applies,

$$-\left(\frac{dE}{dz}\right)_{BH} \sim \int^{E} d\omega\omega \frac{\alpha_s C_R}{\lambda_{free}\omega} \sim \frac{\alpha_s C_R E}{\lambda_{free}}. \tag{26}$$

2. $\omega_{BH} \ll E \ll \omega_{LPM} = \hat{q}L^2 \sim \begin{cases} 5 \text{ GeV (cold)} \\ 50 \text{ GeV (hot, expanding medium)} \end{cases}$:

$$-\left(\frac{dE}{dz}\right)_{LPM_1} \sim \int^{E} d\omega \frac{\alpha_s C_R \sqrt{\hat{q}}}{\sqrt{\omega}} \sim \alpha_s C_R \sqrt{\hat{q}E}. \tag{27}$$

This is the same $E$ dependence as for the LPM effect in QED.

3. $\omega_{LPM} \ll E$:

$$-\left(\frac{dE}{dz}\right)_{LPM_2} \sim \int^{\omega_{LPM}} d\omega \frac{\alpha_s C_R \sqrt{\hat{q}}}{\sqrt{\omega}} \sim \alpha_s C_R \hat{q} L. \tag{28}$$

There is an additional contribution $\sim \alpha_s C_R E/L$, where the entire target acts as a single scattering center.

## Energy loss and inclusive hadron production

- Discovery of RHIC: high-$p_T$ hadron spectra in central nucleus-nucleus collisions are strongly suppressed.
- Medium modified cross section for production of a high-$p_T$ parton,

$$\frac{d\hat{\sigma}^{med}}{d\hat{p}_T^2} = \int d\epsilon P(\epsilon) \frac{d\hat{\sigma}^{vac}}{d\hat{p}_T^2}(\hat{p}_T + \epsilon). \tag{29}$$

- With $d\hat{\sigma}^{vac}/d\hat{p}_T^2 \propto \hat{p}_T^{-6}$, a small energy loss can result in a large suppression,

$$\frac{1}{N_{coll}} \frac{d^2\sigma(AB \to hX)}{dy dp_T^2} = \frac{d^2\sigma(pp \to hX)}{dy dp_T^2} Q(p_T). \tag{30}$$

  In experiment (AuAu at $\sqrt{S} = 200$ GeV): $Q(p_T) \approx 0.2$ for $h = \pi^0$ up to $p_T = 20$ GeV.

- At fixed $\sqrt{S}$, one has additional suppression due to structure and fragmentation functions,

$$\frac{d \ln(E d^3\sigma(\sqrt{S} = 200\,\text{GeV})/d^3p)}{d \ln p_T} \approx -8. \tag{31}$$

  20% energy loss $\Rightarrow$ 80% suppression (i.e. $Q(p_T) \approx 0.2$).

## Quenching weights

- Numerical results for quenching weights $Q(p_T)$ strongly depend on gluon radiation in the few-hundred MeV range.

- Idea: investigate $x_T$ scaling to find out if the medium has modified the $p_T$ dependence of the microscopic process,

$$n^{med} = n^{vac} - \frac{d\ln Q(p_T)}{d\ln p_T} \tag{32}$$

- Following BDMS, JHEP09(2001)033, expand the logarithm of the cross section to obtain

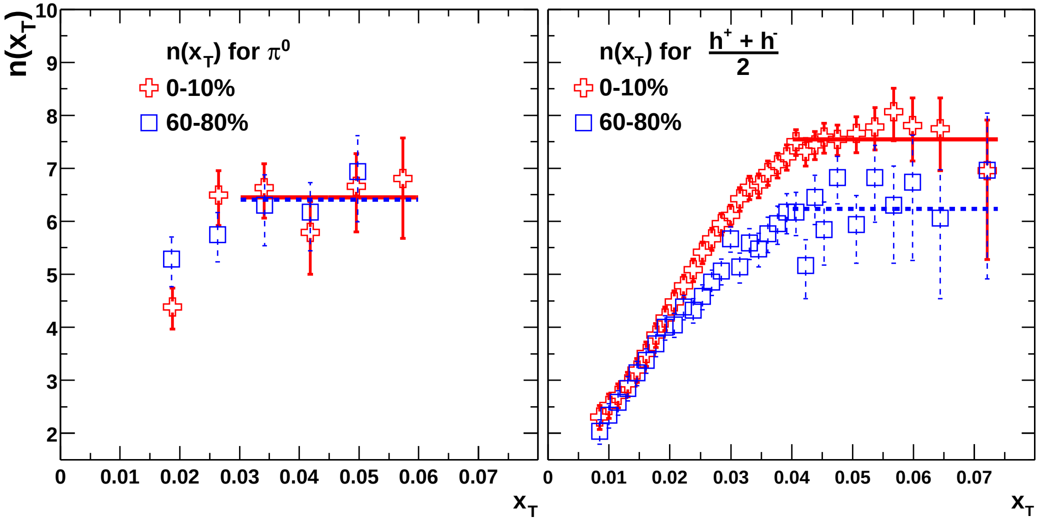
$$Q(p_T) \approx \int d\epsilon P(\epsilon) \exp\left(-\frac{\epsilon}{p_T} n^{vac}\right) = \exp\left(-\frac{n^{vac}}{p_T}\int_0^\infty d\omega e^{-n^{vac}\omega/p_T}\int_{\omega'}^{\omega'_{max}} d\omega' \frac{dI}{d\omega'}\right) \tag{33}$$

- Eventually, one arrives at the result

$$\Delta n \approx -\frac{d\ln Q(p_T)}{d\ln p_T} \approx \int d\omega \frac{dI}{d\omega} \sim \begin{cases} \alpha_s C_R \sqrt{\frac{\omega_{LPM}}{p_T}} & E \ll \omega_{LPM} \\ \alpha_s C_R & E \gg \omega_{LPM} \end{cases} \tag{34}$$

# Experimental results

- PHENIX (PRC69,034910(2004)) found no centrality dependence of $x_T$ scaling for $\pi^0$.
- Apparently, only a small number of gluons is radiated and energy loss is proportional to energy within error bars. In the $\sqrt{\hat{q}E}$ regime, this would require a considerable energy dependence of $\hat{q}$.




- For all charged hadrons, the $p_T$ dependence changes from $p_T^{-6.3\pm0.5}$ to $p_T^{-7.5\pm0.5}$.
- This observation suggests the proton production mechanism $uu \to p\bar{d}$, so that

$$E\frac{d^3\sigma(AB \to pX)}{d^3p} \propto \frac{f_p^2}{p_T^8}. \tag{35}$$

Pions are strongly quenched, protons do not lose energy in the medium.

## Summary

- Scale invariance and asymptotic freedom in QCD lead to power-like behavior of the inclusive cross section at high $p_T$:

$$p_T^n E \frac{d^3\sigma(AB \to hX)}{d^3p} = F(x_R, y). \quad (36)$$

  The power $n(y, x_R) = 2n_a - 4$ is related to the number of active fields.

- Leading twist pQCD: $n_a = n = 4$. Including scaling violations, one finds $n_{eff} \approx 4.5 \dots 5$.

- Scaling properties of inclusive high-$p_T$ meson production change from SPS to RHIC
  - SPS: direct production of small color-neutral objects ($n = 8.5 \pm 0.5$) $\Rightarrow$ no jet quenching
  - RHIC: leading twist pQCD dominates in meson production ($n = 6.3 \pm 0.5$) $\Rightarrow$ strong medium effects

- Nuclear modifications of $x_T$ scaling suggest that a small number of gluons takes away a finite fraction of the projectiles momentum.

- Intermediate $p_T$ protons may be produced instantaneously as small colorless objects. This would explain the absence of nuclear effects in proton production at RHIC.

- Inclusive hadron production must approach its exclusive limit at $x_R \to 1 \Rightarrow n \to 20$.
  - Only valence quarks are important at the largest rapidities, high energy $\neq$ low $x$.
  - Coherence effects reach their maximum around $x_F \approx 0.5$.