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Parton propagation through Strongly interacting Systems
– ECT*, Trento, September 2005 –

Collisional energy loss in the sQGP

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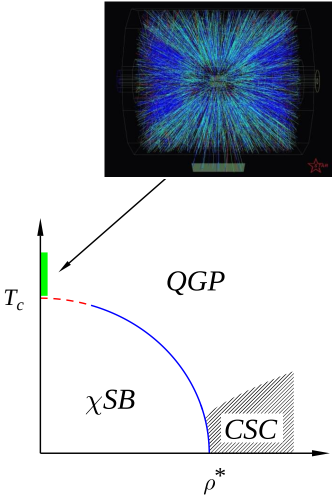
1. Bjorken's estimate
2. Strongly coupled (Q)GP
3. $= 1 + 2$

Why collisional energy-loss?



AIP Bulletin, April 20 2005:

Now, for the first time since starting nuclear collisions at RHIC in the year 2000 and with plenty of data in hand, all four detector groups operating at the lab [BNL] . . . believe that the fireball is a liquid of strongly interacting quarks and gluons rather than a gas of weakly interacting quarks and gluons.

Why collisional energy-loss?

paradigm: radiative energy-loss [BDMPS] dominates collisional energy-loss

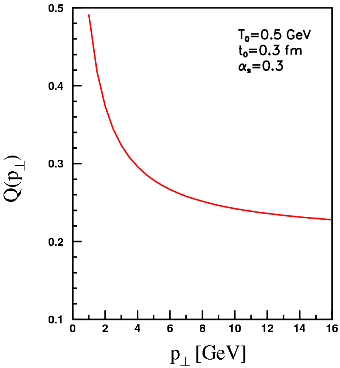
- How realistic are commonly used (pQCD based) input parameters for e-loss estimates?
- 'observed' at RHIC:
  strongly coupled QGP (sQGP)
  - collective phenomena, in line with hydrodynamics
  - fast equilibration, low viscosity
  - large Xsections, large coupling

  $\Downarrow$

  dense liquid $\Leftrightarrow$ collisional e-loss

- Fokker-Planck eqn. with drag and diffusion parameters related to dE_{coll}/dx [Mustafa, Thoma]

  $\Rightarrow$ compatible quenching factors



things are involved ⇒ **simplify**

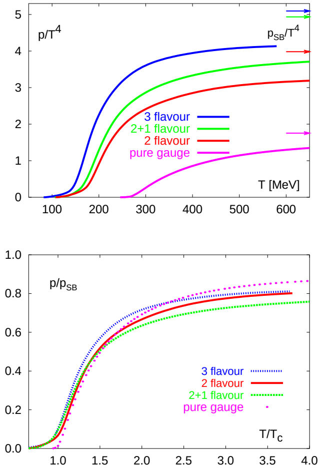
- consider energy-loss of hard jet in static infinite thermalized medium (QGP)

- consider mostly quenched QCD; pQCD expectation *quarks and gluons differ by group factors (coupling Casimirs, d.o.f.)* seen also for large coupling (lattice)

  rescale:

  $T_c^{quench} = 260\,\text{MeV} \to 170\,\text{MeV}$

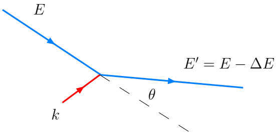
'Universality' in QCD

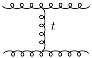


Bjorken's formula

[Bjorken '82] considers energy-loss per length due to elastic collisions

$$\frac{dE}{dx} = \int_{k^3} \underbrace{\rho(k)\,\Phi}_{\text{flux}} \int dt\, \frac{d\sigma}{dt}\, \Delta E$$





small t dominate: $\quad \dfrac{d\sigma}{dt} = 2\pi C_{gg}\,\dfrac{\alpha^2}{t^2}$

$E,\ E' \gg k \sim T$: $\quad t = -2(1-\cos\theta)k\Delta E$

$$\Phi = 1 - \cos\theta$$

divergences – cut-offs: $\quad \displaystyle\int_{t_1}^{t_2} dt\, \frac{d\sigma}{dt}\, \Delta E = \frac{\frac{9}{4}\pi\alpha^2}{k(1-\cos\theta)} \ln\frac{t_1}{t_2}$

screening: $\quad t_2 = -\mu^2$

$\Delta E < \Delta E_{max}$: $\quad t_1 = -2(1-\cos\theta)k\Delta E_{max}$

jet persist: $\Delta E_{max} \approx 0.5E$

Bjorken's formula

$$\frac{dE}{dx} = \frac{9\pi}{4}\,\alpha^2 \int_{k^3} \frac{\rho(k)}{k}\,\ln\frac{(1-\cos\theta)kE}{\mu^2}$$

pragmatically: $(1-\cos\theta) \to 2$

$$\int dk\, k\rho(k) \ln k \to \ln\langle k\rangle \int dk\, k\rho(k)$$

$$\langle k\rangle \to 2T$$

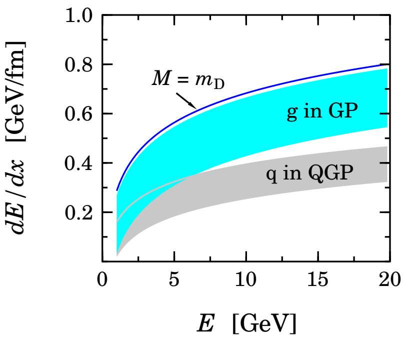
with $\rho(k) = 16 n_b$: $\quad \dfrac{dE_B}{dx} = 3\pi\alpha^2 T^2 \ln\dfrac{4TE}{\mu^2} \qquad \mu^2 \to m_D^2 = 4\pi\alpha T^2$

shortcomings:

1. sloppy k-integral $\leftrightarrow$ $dE/dx < 0$??
2. phenomenological IR cut-off
3. relevant scale for coupling?

collisional energy-loss of hard gluons$(+1)$ and quarks (-1)

$$\frac{dE_B}{dx} = \left(\frac{3}{2}\right)^{\pm 1} \left(1 + \tfrac{1}{6}\, n_f\right) 2\pi\alpha^2 T^2 \ln \frac{4TE}{\mu^2}$$



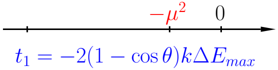
$$\begin{aligned} T &= 300\,\text{MeV} \\ \alpha &= 0.2 \\ \mu &= 0.5 \ldots 1\,\text{GeV} \\ &\sim m_D = \sqrt{4\pi\alpha}\,T \end{aligned}$$

compare to

- 'cold' e-loss $\sim 1\,\text{GeV/fm}$
- radiative e-loss $\Delta E_{rad} \sim E^{\beta}$, $\beta = 0, \frac{1}{2}, 1$

Bjorken's formula – improvements

- orderly t and k integrals



$t_1 \overset{!}{<} -\mu^2$: $\Delta E > 0$
constrains $\int_{k^3}$

limit $ET \gg \mu^2$:

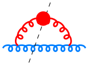
$$\frac{d\tilde{E}_B}{dx} = 3\pi\alpha^2 T^2 \ln \frac{0.64\, TE}{\mu^2}$$

- screening systematically within HTL perturbation theory

$$\frac{dE^\star}{dx} = 3\pi\alpha^2 T^2 \ln \frac{1.27\, TE}{m_D^2}$$

[Braaten, Thoma]

$$\mu_\star^2 \approx 0.5 m_D^2$$

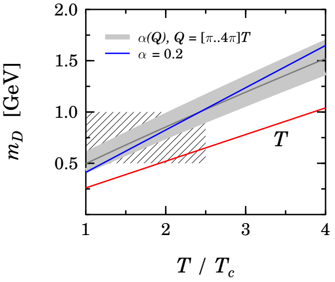


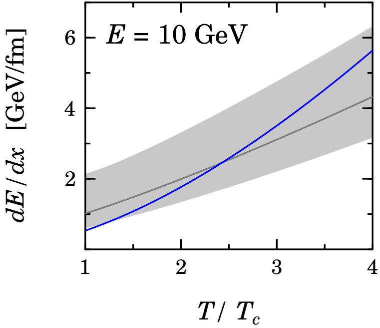
NB: conform with general form of collisional energy loss, in leading-log approximation, related to cut *dressed* 1-loop diagrams [Thoma]

- running coupling

$$\alpha^{\text{pert}}(Q) = \frac{4\pi}{11\ln(Q^2/\Lambda^2)}$$

often: $Q \sim 2\pi T$, $\Lambda = T_c/1.14$





NB: even *conservative* $\alpha(T)$:

$m_D > T$

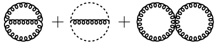
(pQCD: $m_D = gT \ll T$)

what is relevant scale for α?
how reliable is extrapolated pQCD?

Quasiparticle perspective of s(Q)GP

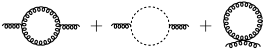
- 2PI formalism: thermodynamic potential in terms of *full* propagator

$$\Omega = \tfrac{1}{2}\,\mathrm{Tr}\left(\ln(-\Delta^{-1}) + \Pi\Delta\right) - \Phi, \quad \Phi =$$



$$+ \ldots$$

$$\Pi = 2\frac{\delta\Phi}{\delta\Delta} =$$



$$+ \ldots$$

- truncation $\Rightarrow$ thermodyn. consistent resummed approximations

  entropy functional of dressed (=quasiparticle) propagator [Riedel, ...]

$$s[\Delta] = -\sum_{i=T,L} d_i \int_{p^4} \frac{\partial n_b}{\partial T}\left(\mathrm{Im}\ln(-\Delta_i^{-1}) + \mathrm{Im}\Pi_i\,\mathrm{Re}\Delta_i\right)$$

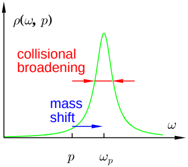
(in Fermi liquid theory: *dynamical quasiparticle entropy*)

Quasiparticle perspective of s(Q)GP

- *dressed propagator* for momenta $p \sim T$ ($d_g = 16$ transverse d.o.f.)
  - *Ansatz*: Lorentzian spectral function corresponds to self-energy

    $$\Pi = m^2 - 2i\gamma\omega$$

    $\gamma \longrightarrow$ transport properties



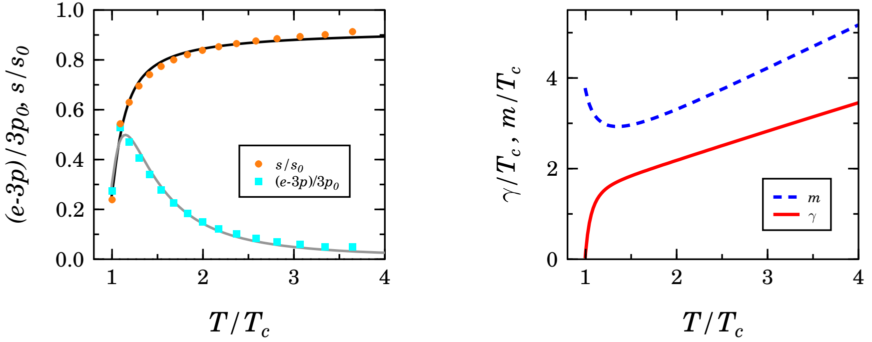
  - quasiparticle mass and (collisional) width parameterized in form of pQCD results (gauge invariant, momentum-independent)

$$m^2 = 2\pi\alpha T^2 \,, \quad \gamma = \frac{3}{2\pi}\,\alpha\, T \ln\frac{c}{\alpha}$$

- to extrapolate to $T \sim T_c$ use effective coupling

$$\alpha(T) = \frac{4\pi}{11\ln(\lambda(T - T_s)/T_c)^2}$$

- QP model [AP] vs. lattice data [Okamoto et al.]



- non-perturbative 'quasiparticles'
  - $m \sim T$ heavy excitations
  - $\gamma \sim T$ short mean free path – except very near T_c (crit. slow down)

- pQCD

$$\alpha^{\text{pert}}(Q) = \frac{4\pi}{11\ln(Q^2/\Lambda^2)}$$

with $Q \sim 2\pi T$, $\Lambda \sim T_c$

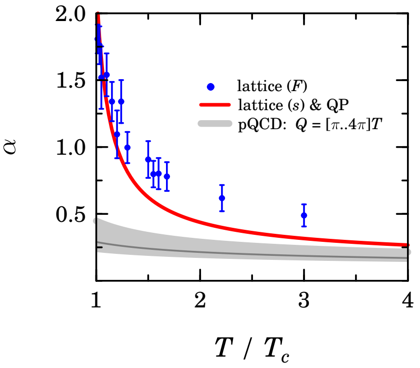
- analyze lattice data
  1. entropy within QP model
  2. static $q\bar{q}$ free energy

$$F(r,T) \to C_r \frac{\alpha}{r}\, exp(-m_D r)$$

[Kaczmarek et al.]

for $T \sim \mathcal{O}(T_c)$

IR enhancement of α



s(Q)GP: cross section

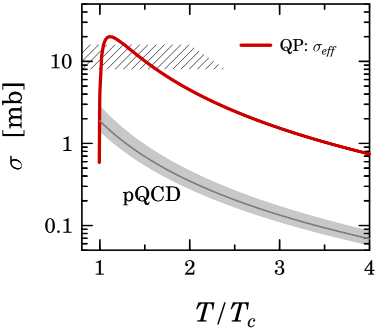
- pQCD & cut-off $m_D^2 = 4\pi\alpha T^2$

$$\sigma_{pert} = \int dt \frac{\frac{9}{2}\pi\alpha^2}{t^2} = \frac{9}{8}\frac{\alpha}{T^2}$$

$$\rightarrow \frac{9}{8}\frac{\alpha(2\pi T)}{T^2} \sim \mathcal{O}(1\,\mathrm{mb})$$

- phenomenology [Molnar, Gyulassy]

$$\sigma_{RHIC} \sim \mathcal{O}(10\,\mathrm{mb})$$



- QP model, from $2 \rightarrow m$ interaction rate d^4N/dx^4 [AP, Cassing]

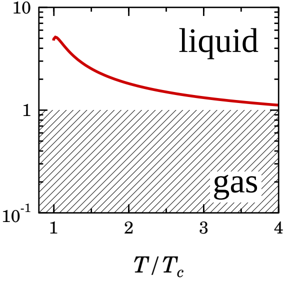
$$\mathrm{Tr}_{p_1,p_2}\left[\frac{2\sqrt{\lambda}}{2\omega_1 2\omega_2}\, n_b(\omega_1)n_b(\omega_2)\,\sigma\,\Theta(P_1^2)\Theta(P_2^2)\right] = \mathrm{Tr}_p\left[\frac{1}{2\omega}\,\gamma\, n_b(\omega)\Theta(P^2)\right]$$

average σ and γ $\Rightarrow$ $\sigma_{eff} = \gamma\,\frac{N_+}{I_2} \sim \mathcal{O}(10\,\mathrm{mb})$

QP-model indicates an almost ideal liquid$^{(\star)}$ [AP, Cassing]

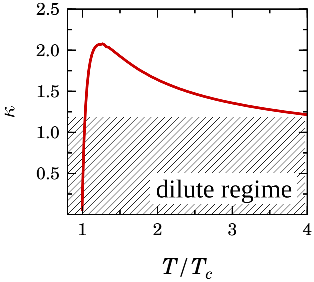
- large plasma parameter

$$\Gamma = 2\,\frac{N_c\alpha}{N^{-1/3}}\,\frac{1}{\langle E_{kin}\rangle}$$



- large percolation measure

$$\kappa_2 = \sigma_{eff}\, N^{2/3}$$



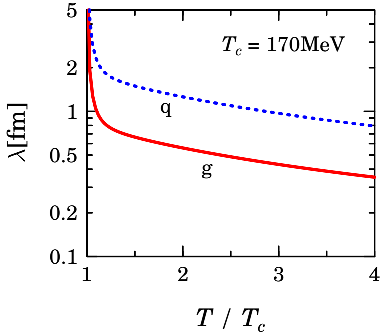
- very low shear viscosity

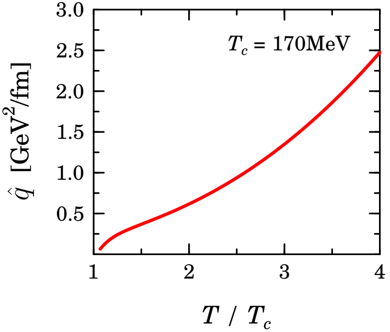
$$\frac{\eta}{s} \approx 0.2 \text{ near } T_c$$

$^{(\star)}$ $\neq$ ideal gas!!!

quantities relevant for radiative energy-loss

- mean free path $\lambda = \gamma^{-1}$
- transport coefficient $\hat{q} = m_D^2/\lambda$





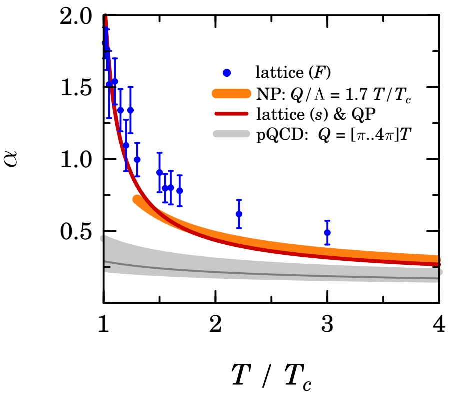
NB: $\gamma^{-1}(p \sim T)$ as a lower estimate for mean free path of hard jet

Can pQCD, by using appropriate scales, be extrapolated to 'near' T_c?

$$\alpha^{\rm pert}(Q) = \frac{4\pi}{11\ln(Q^2/\Lambda^2)}$$

- pQCD: $\dfrac{Q}{\Lambda} \sim \dfrac{2\pi T}{T_c}$
- non-pert. parameterization (aim at $T \gtrsim 1.3\,T_c$):

$$\frac{Q}{\Lambda} \to 1.7\,\frac{T}{T_c}$$



lQCD data [Kaczmarek et al.]

Non-perturbative parameterization: Debye mass

Can pQCD be consistently extrapolated to 'near' T_c?

$$m_D^2 = 4\pi\alpha T^2$$

- pQCD:

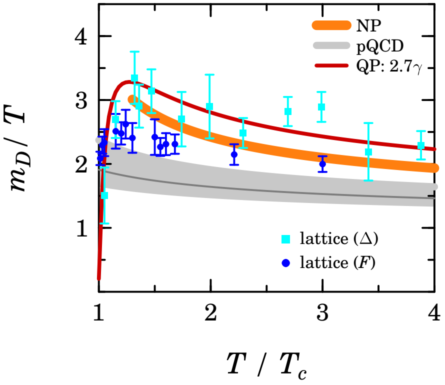
$$\alpha \to \alpha^{\rm pert}(Q \sim 2\pi T)$$

- non-pert. parameterization:

$$\alpha \to \alpha^{\rm NP}(T)$$

- observation within QP model:

$$m_D \approx 2.7\gamma$$



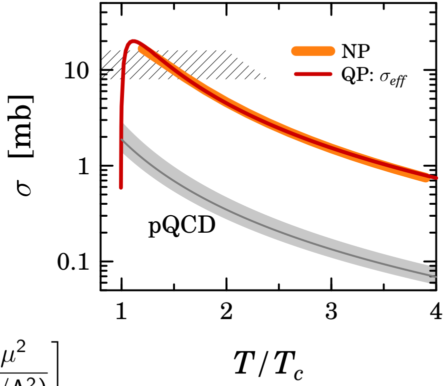
lQCD data [Nakamura et al.], [Kaczmarek et al.]

Can pQCD be consistently extrapolated to 'near' T_c?

$$\sigma = \int^{-\mu^2} dt \, \frac{\frac{9}{2}\pi\alpha^2}{t^2}$$

- pQCD: $\sigma = \frac{9}{8} \frac{\alpha|_{Q\sim 2\pi T}}{T^2}$
- running $\alpha(t) = A/\ln(-t/\Lambda^2)$:

$$\begin{aligned}\sigma &= \frac{9\pi}{2} \int^{-\mu^2} dt \left[\frac{A}{t\ln(-t/\Lambda^2)}\right]^2 \\ &= \frac{9\pi}{2} \frac{A^2}{\Lambda^2} \left[Ei(-\ln(\mu^2/\Lambda^2)) + \frac{\Lambda^2/\mu^2}{\ln(\mu^2/\Lambda^2)}\right] \\ &\to \frac{9\pi}{2} \frac{\alpha^2(\mu)}{\mu^2} [1 + \ldots]\end{aligned}$$



$\mu_{NP} = 0.6 m_D$
$\Lambda_{NP} = 420$ MeV

Non-perturbative parameterization

Indeed, pQCD can consistently be extrapolated to 'near' T_c.

- lattice results for $\alpha(T)$, $m_D(T)$, for $T/T_c \in [1.3, 4]$, consistent with

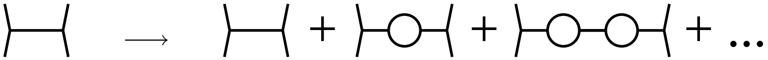
$$\alpha^{NP}(T) = \frac{4\pi}{11 \ln(1.7\, T/T_c)^2}$$

- cross section ($\times 10$) enhancement consistent with

pert. Xsection	$d\sigma/dt \sim \alpha^2/t^2$
running coupling	$\alpha(t) = \dfrac{4\pi}{11 \ln(-t/\Lambda_{NP}^2)}$, $\Lambda_{NP} = 420\,\text{MeV}$
cut-off	$\mu = 0.6 m_D$ (compare to $\mu_\star = 0.7 m_D$)

- relation between $\alpha^{NP}(T)$ and $\alpha(t)$, assuming $\sqrt{|\bar{t}|} = \kappa T$,

$$\kappa = 1.7 \frac{\Lambda_{NP}}{T_c} \approx 2.74 \quad \left(\text{compare to } \langle k \rangle = \frac{\int_{k^3} k \rho(k)}{\int_{k^3} \rho(k)} \approx 2.70\, T\right)$$



$$\frac{\alpha_0}{P^2} \longrightarrow \frac{\alpha}{P^2 - \Pi}$$ (resummation, renormalization)

- vacuum:

$$\Pi = \Pi_{\rm ren} \sim \alpha P^2 \ln(-P^2/\mu^2)$$
$$\Downarrow$$
$$\alpha(P^2) = \frac{4\pi}{11 \ln(-P^2/\Lambda^2)}$$

- medium:

$$\Pi = \Pi_{\rm ren} + \Pi_{\rm mat}$$
$$\Pi_{\rm mat} \sim \alpha T^2 \sim m_D^2$$

IR cut-off: $P \gtrsim m_D$

running important when $m_D \sim gT \sim \Lambda$ (non-pert. regime)

Collisional e-loss with running coupling

Bjorken: $$\frac{dE}{dx} = \int_{k^3} \rho(k)\, \Phi \int dt\, \frac{d\sigma}{dt}\, \Delta E$$

t-integral, with $\alpha(t) = A/\ln(-t/\Lambda^2)$

$$\Phi \int_{t_1}^{t_2} dt\, \frac{d\sigma}{dt}\, \Delta E = -\frac{\frac{9}{4}\pi A^2}{k} \int_{t_1}^{t_2} \frac{dt}{t} \frac{1}{\ln^2(-t/\Lambda^2)}$$

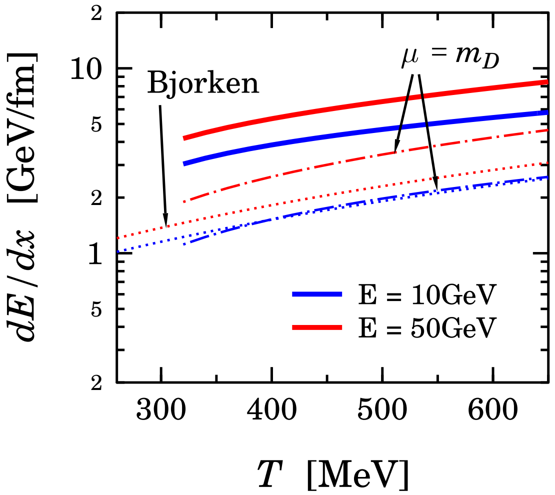
$$= \begin{cases} \dfrac{\frac{9}{4}\pi A}{k} \left[\alpha(\mu^2) - \alpha((1-\cos\theta)kE)\right] \\ \text{constraint} \quad (1-\cos\theta)k \geq \mu^2/E \end{cases}$$

θ-integration leads to logarithmic integrals, $\mathrm{li}(x) = \mathcal{P}\int_0^x dt/\ln(t)$

$$\frac{dE}{dx} = \frac{9A^2}{8\pi} \int_{\bar{k}}^{\infty} dk k \rho(k) \left[\frac{1-\mu^2/(2Ek)}{\ln(\mu^2/\Lambda^2)} + \frac{\Lambda^2}{2Ek}\left(\mathrm{li}\frac{\mu^2}{\Lambda^2} - \mathrm{li}\frac{2Ek}{\Lambda^2}\right)\right], \quad \bar{k} = \frac{\mu^2}{2E}$$

$$= T^2\, F\!\left(\frac{\mu^2}{2TE}, \frac{\mu^2}{\Lambda^2}\right) \to \left.\frac{dE_B}{dx}\right|_{\alpha(\mu)} [1 + \mathcal{O}(\alpha)]$$

- non-perturbative parameterization ($T \gtrsim 1.3\,T_c$)



$$\frac{dE}{dx} = T^2 F\left(\frac{\mu^2}{2TE}, \frac{\mu^2}{\Lambda^2}\right)$$

parameters

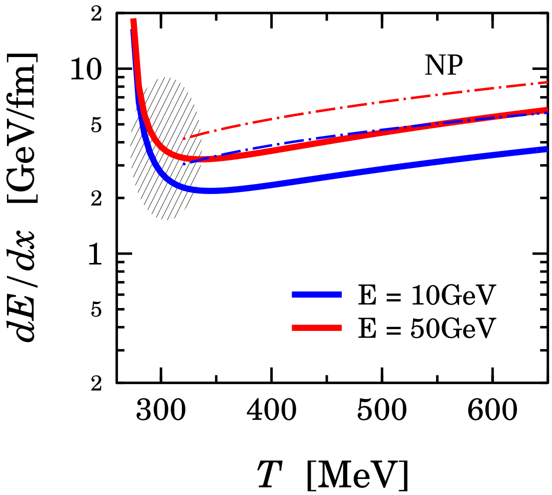
$\Lambda \to \Lambda_{NP}$

$\mu^2 = 0.6 m_D^2$

$m_D^2 \to 4\pi\alpha^{\rm NP}(T)\,T^2$

- lQCD/QP parameterization of Debye mass: $m_D \approx 2.7\gamma$

  small $m_D(T_c)$ ($\sim$ phase transition) $\rightarrow$ **increased energy loss**



$$\frac{dE}{dx} = T^2 F\left(\frac{\mu^2}{2TE}, \frac{\mu^2}{\Lambda^2}\right)$$

parameters

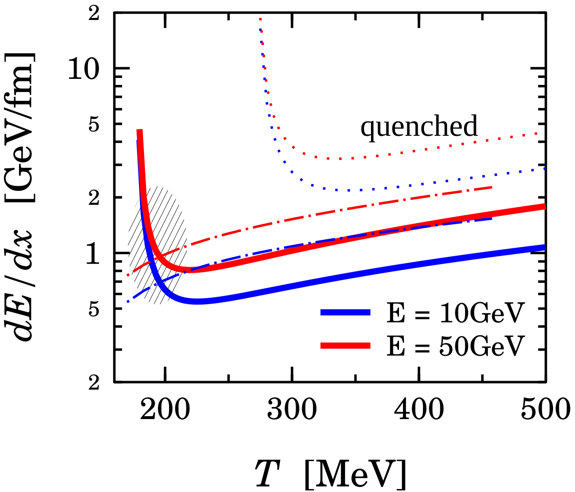
$\Lambda \rightarrow \Lambda_{NP}$

$\mu^2 = 0.6 m_D^2$

$m_D^2 \rightarrow m_{D,eff}^2$

Collisional e-loss in s(Q)GP – numerical results

- unquenching: energy-loss of a quark in the sQGP



$$\frac{dE}{dx} = T^2 F\left(\frac{\mu^2}{2TE}, \frac{\mu^2}{\Lambda^2}\right)$$

parameters

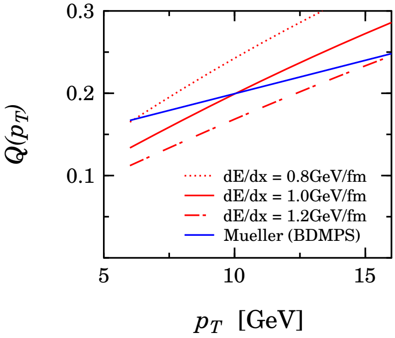
$T_c \rightarrow 170$ MeV

$C_{gg} \rightarrow C_q(1 + n_f/6)$

Is $dE/dx \sim 1\,\mathrm{GeV/fm}$ enough?

assume

- constant dE/dx
- Bjorken dynamics
- quenching factor



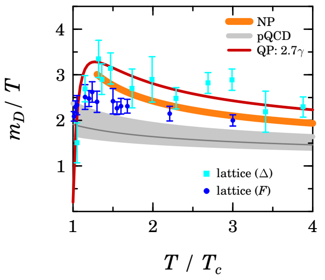
$$\frac{dN}{d^2p_T} = Q(p_T)\frac{dN_0}{d^2p_T} = \frac{1}{2\pi R^2}\int_0^{2\pi} d\phi \int_0^R dr^2 \frac{dN(p_T+\Delta p_T)}{d^2p_T}$$

comparable to [Müller]: BDMPS + transv. profile + Bjorken dynamics

Resumé

- realistic parameters for sQGP

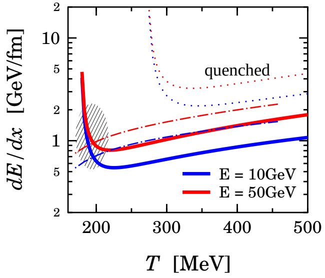
  $\Rightarrow$ **enhanced** $\dfrac{dE_{coll}}{dx}$



- (quasi) critical screening

  $\Rightarrow$ T_c quenching

- does sQGP$^{(\star)}$ quench too much?
  - far/near side jets vs. geometry+delay (talk Cassing)
  - retardation (talk Gossiaux)



$^{(\star)}$strongly **Quenching** (Q)GP