Hadronization via Coalescence

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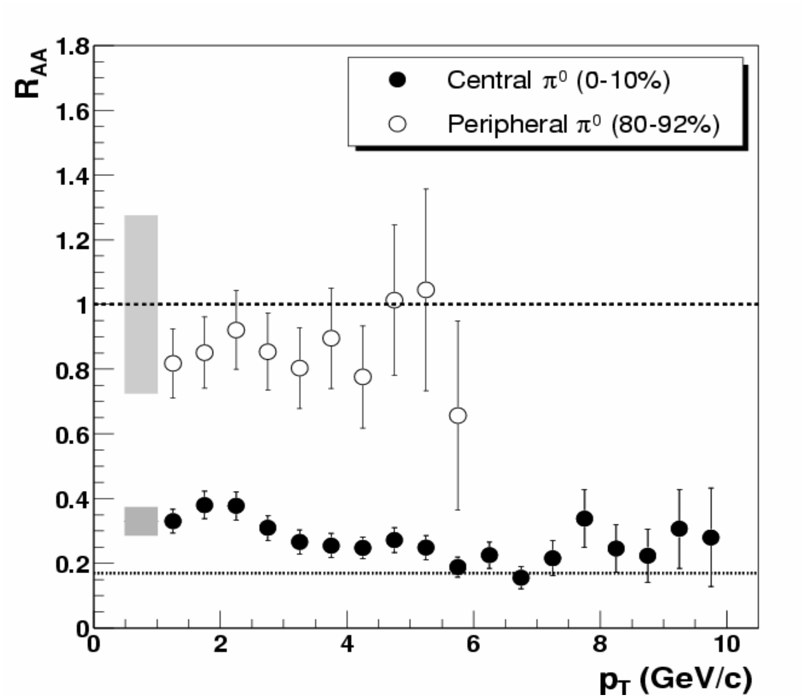
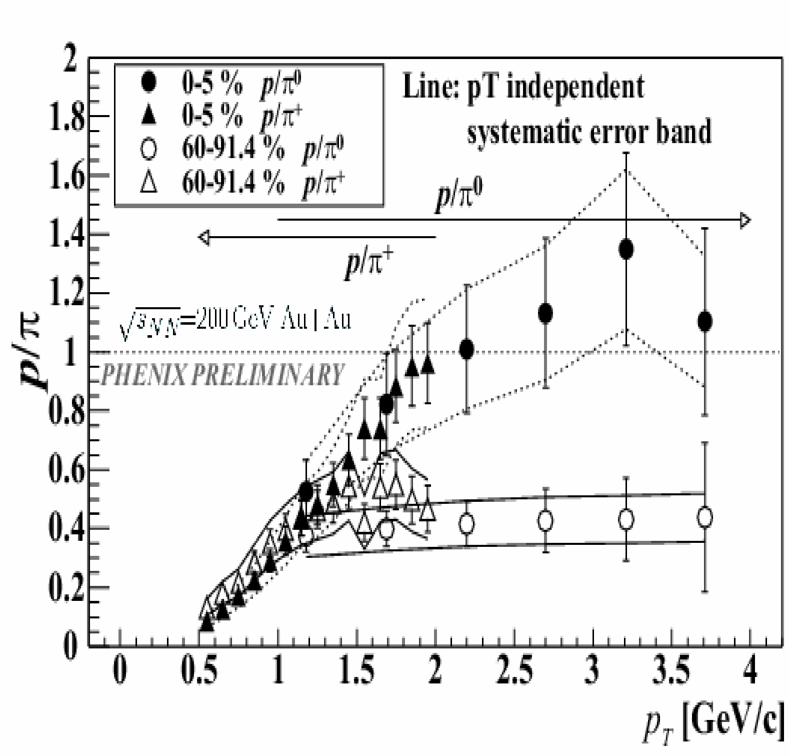
- Introduction
- Quark coalescence
    - Baryon/meson ratio
    - Hadron elliptic flows and quark number scaling
    - Effect of resonance decays
    - Higher Fock states
    - Charm flow
    - Higher-order anisotropic flows
- Coalescence in transport model
- Entropy problem
- Summary

Puzzle: Large proton/meson ratio



PHENIX, nucl-ex/0304022

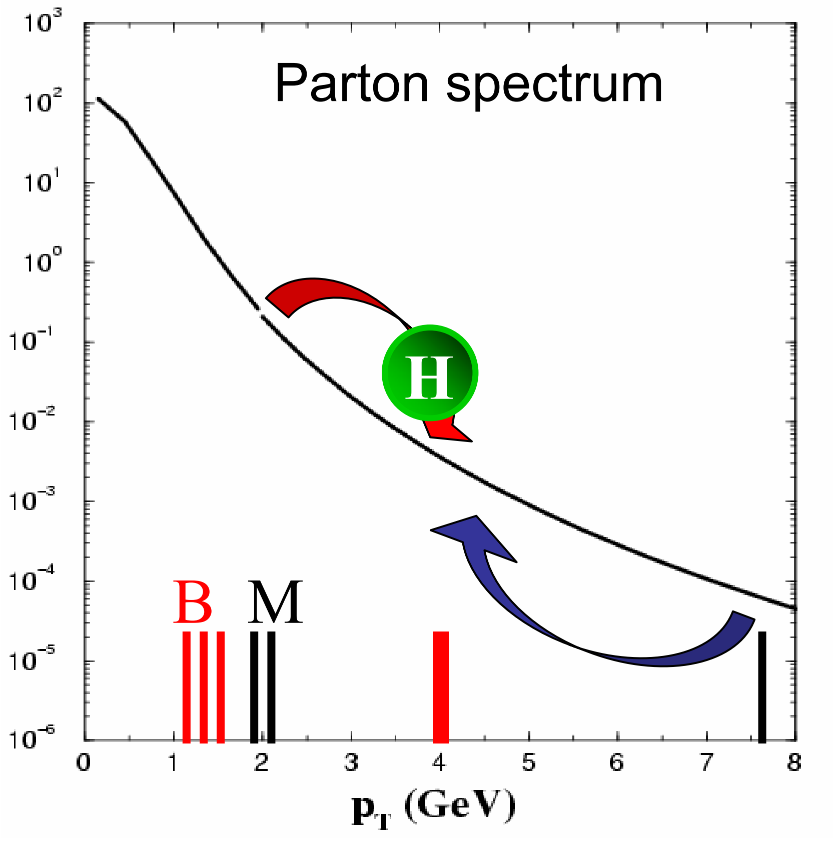
π^0 suppression: evidence of jet quenching before fragmentation



PHENIX, nucl-ex/0212014

- Fragmentation leads to $p/\pi \sim 0.2$
- Jet quenching affects both
- Fragmentation is not the dominant mechanism of hadronization at $p_T < 4\text{-}6$GeV

Coalescence vs. Fragmentation



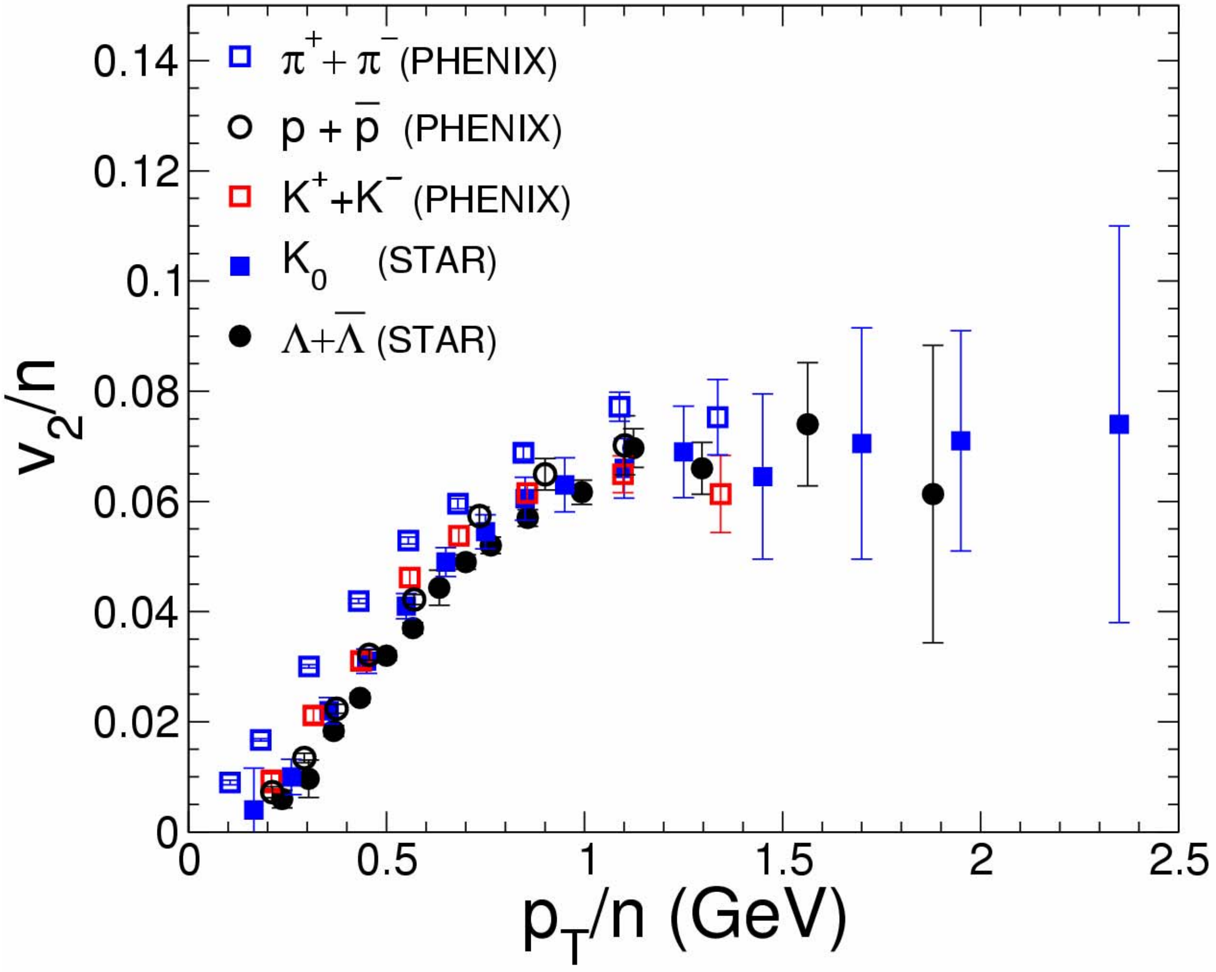
Fragmentation

Leading parton with p_T leads to hadrons of p_h=z p_T with a probability $D_h(z)$, where z<1

Colascence

- partons are already there
- p_h= n p_T ,, n = 2 , 3
- Need to be close in phase space
- Partonic hydro behavior is shifted to higher p_T

Surprise: quark number scaling of hadron elliptic flow



Except pions, $v_{2,M}(p_T) \sim 2\, v_{2,q}(p_T/2)$ and $v_{2,B}(p_T) \sim 3\, v_{2,q}(p_T/3)$ consistent with hadronization via quark recombination

Coalescence model in heavy ion collisions

- Extensively used for light clusters production
- First used for describing hadronization of QGP by Budapest group
- Currently pursued by
  Oregon: Hwa, Yang (PRC 66 (02) 025205), ………
  Duke-Minnesota: Bass, Nonaka, Meuller, Fries (PRL 90 (03) 202303;
  PRC 68 (03) 044902)
  Ohio and Wayne States: Molnar, Voloshin (PRL 91 (03) 092301;
  PRC 68 (03) 044901)
  Texas A&M: Greco, Levai, Rapp, Chen, Ko (PRL (03) 202302;
  PRC 68 (03) 034904)
- Most studies are schematic, based on parameterized QGP parton distributions
- Study based on parton distributions from transport models has been developed by TAMU group (PRL 89 (2002) 152301; PRC 65 (2002) 034904) and is now also pursued by D. Molnar (nucl-th/0406066)

Coalescence model

PRL 90, 202102 (2003); PRC 68, 034904 (2003)

Number of hadrons with n quarks and/or antiquarks

$$\boxed{N_n = g\int\prod_{i=1}^{n} p_i d\sigma_i \frac{d^3p_i}{(2\pi)^3E_i} f_{q,i}(x_i,p_i)\, f_n(x_1,\ldots,x_n;p_1,\ldots,p_n)}$$

Spin-color statistical factor g_M e.g. $g_\pi = g_K = 1/36 \quad g_\rho = g_{K^*} = 1/12$

$$g_p = g_{\bar{p}} = 1/108, \quad g_\Delta = g_{\bar{\Delta}} = 1/54$$

Quark distribution function $f_q(x,p)$

$$\boxed{\int p\cdot d\sigma \frac{d^3p}{(2\pi)^3E} f_q(x,p) = N_q}$$

Coalescence probability function

$$\Delta_x \cdot \Delta_p \geq \hbar$$

$$\boxed{\begin{aligned} f_M(x_1,x_2;p_1,p_2) &= f_2(x_1-x_2;p_1-p_2) \\ &= \exp[(x_1-x_2)^2/2\Delta_x^2] \\ &\times \exp\{[(p_1-p_2)^2-(m_1-m_2)^2]/2\Delta_p^2\} \end{aligned}}$$

For baryons, Jacobi coordinates for three-body system are used.

Monte-Carlo method

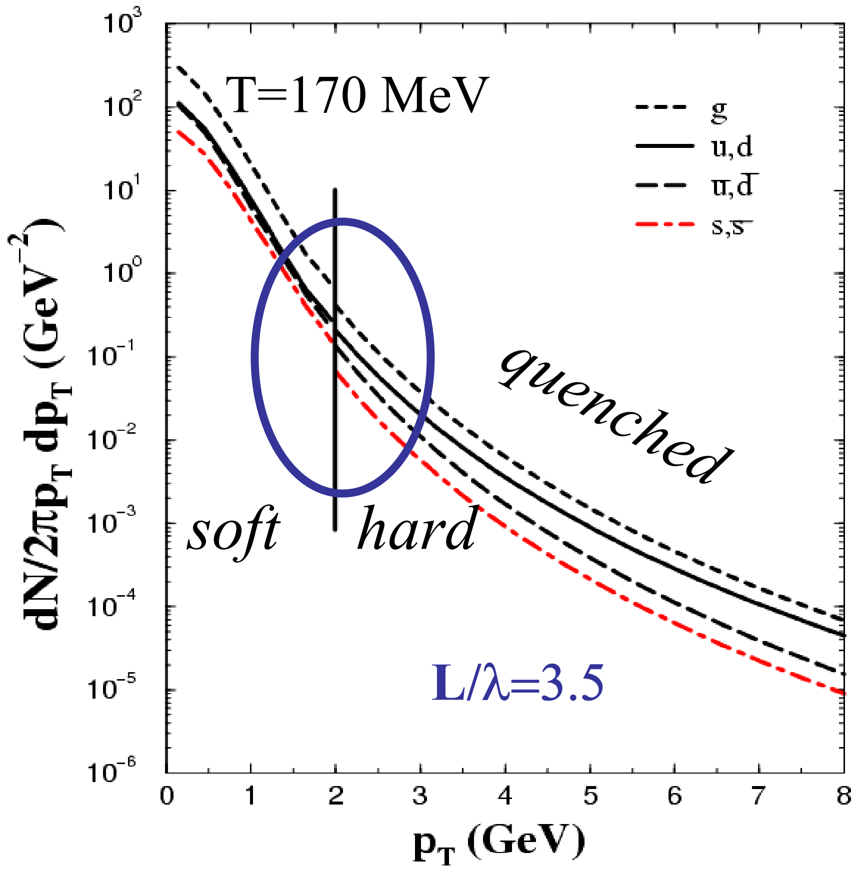
Introduce quark probabilities $P_q(i)$ according to their transverse momentum and spatial distributions

$$\boxed{\begin{aligned}\frac{dN_M}{d^2\vec{p}_T} &= g_M \prod_{i,j} P_q(i) P_{\bar{q}}(j)\, \delta^{(2)}(\vec{p}_T - \vec{p}_{iT} - \vec{p}_{jT}) \\ &\times f_M(x_i, x_j; p_i, p_j)\end{aligned}}$$

$$\boxed{\begin{aligned}\frac{dN_B}{d^2\vec{p}_T} &= g_B \sum_{i\neq j\neq k} P_q(i) P_q(j) P_q(k)\, \delta^{(2)}(\vec{p}_T - \vec{p}_{iT} - \vec{p}_{jT} - \vec{p}_{kT}) \\ &\times f_B(x_i, x_j, x_k; p_i, p_j, p_k)\end{aligned}}$$

Allow to treat all quarks on same footing

Parton transverse momentum distributions



P. Levai *et al.*, NPA 698 (02) 631

- Thermal QGP $p_T \leq 2\,\mathrm{GeV}$
- Power-law minijets $p_T \geq 2\,\mathrm{GeV}$
- Choose $R = 8\ \mathrm{fm}$

$$\tau = 5\ \mathrm{fm}, \quad |y| \leqslant 0.5$$

$$\Rightarrow \quad V \approx 1100\ \mathrm{fm}^3$$

$$N_u = N_d \approx 245, \quad N_s \approx 149$$

$$\left.\frac{dE_T}{dy}\right|_{|y| \leq 0.5} \approx 788\ \mathrm{GeV}$$

Consistent with data (PHENIX)

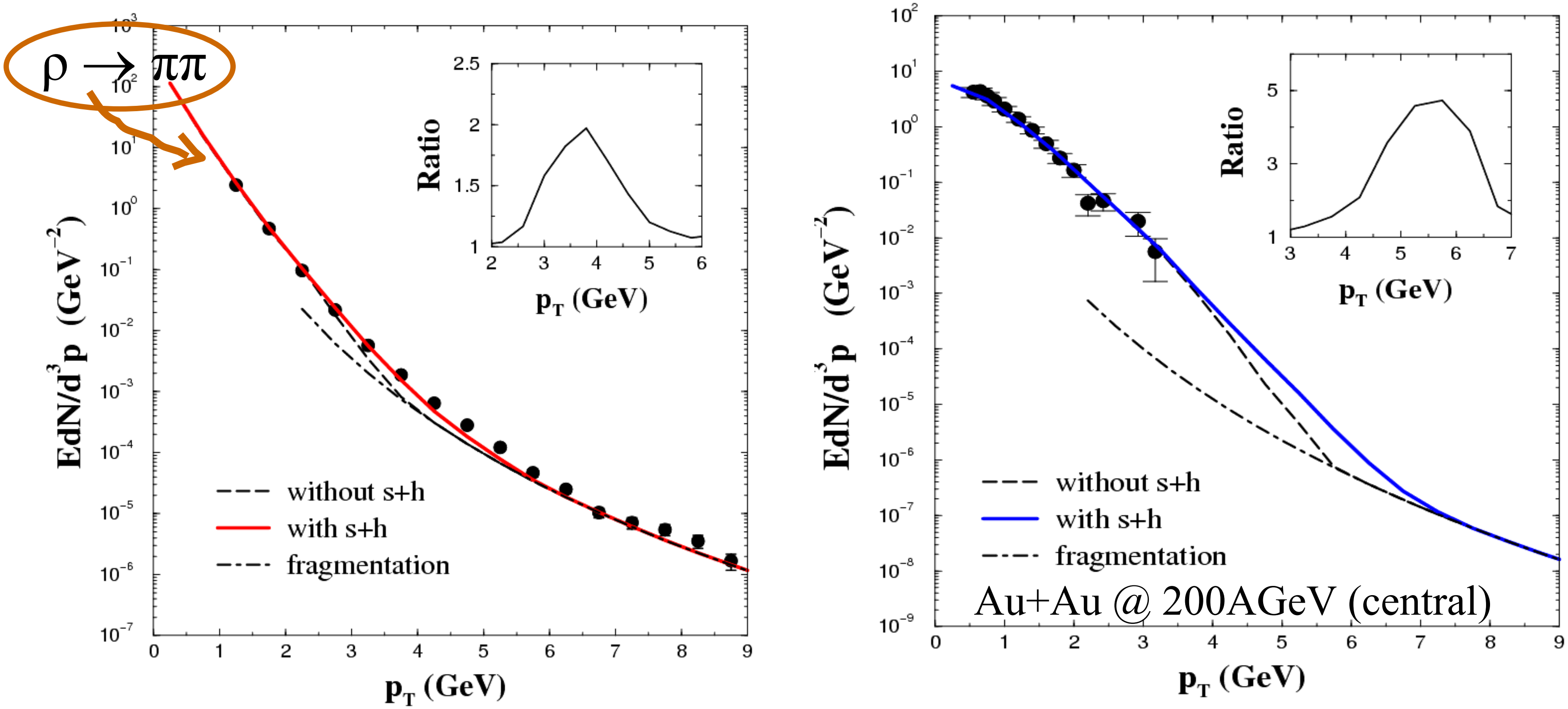
Other inputs and assumptions

- Minijet fragmentation via KKP fragmentation functions (Kniehl, Krammer, Potter, NPB 582, 514 (2000))

$$\frac{dN}{d^2\vec{p}_{had}} = \sum_{jet} \int dz \frac{dN}{d^2\vec{p}_{jet}} \frac{D_{had/jet}(z,Q^2)}{z^2}, \quad z = \frac{p_{had}}{p_{jet}}$$

- Gluons are converted to quark-antiquark pairs with equal probabilities in all flavors.
- Quark-gluon plasma is given a transverse collective flow velocity of β=0.5 c, so partons have an additional velocity v(r)=β(r/R).
- Minijet partons have current quark masses $m_{u,d}$=10 MeV and m_s=175 MeV, while QGP partons have constituent quark masses $m_{u,d}$=300 MeV, m_s=475 MeV (Non-perturbative effects, Levai & Heinz, PRC 57, 1879 (1998))
- Use coalescence radii Δp=0.24 GeV for mesons and 0.36 GeV for baryons

Pion and proton spectra

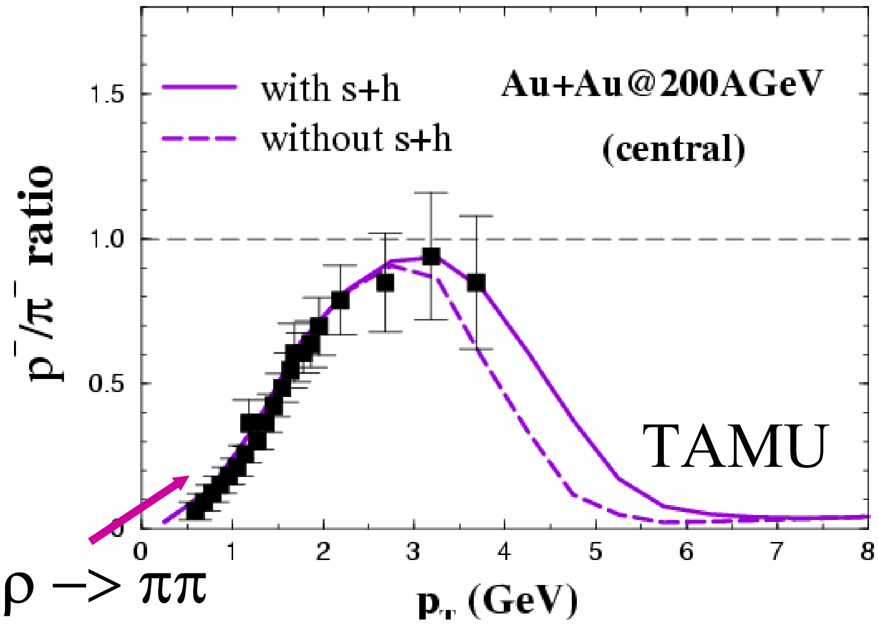


Similar results from other groups

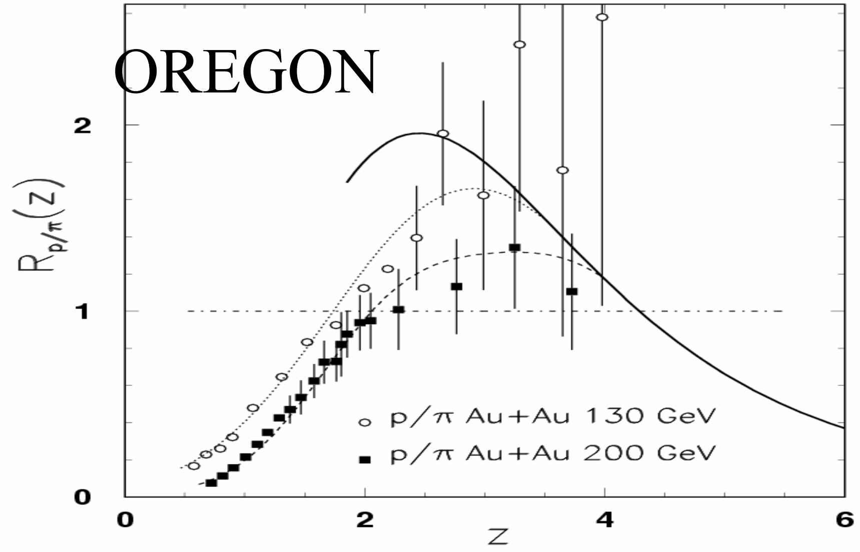
Oregon: parton distributions extracted from pion spectrum

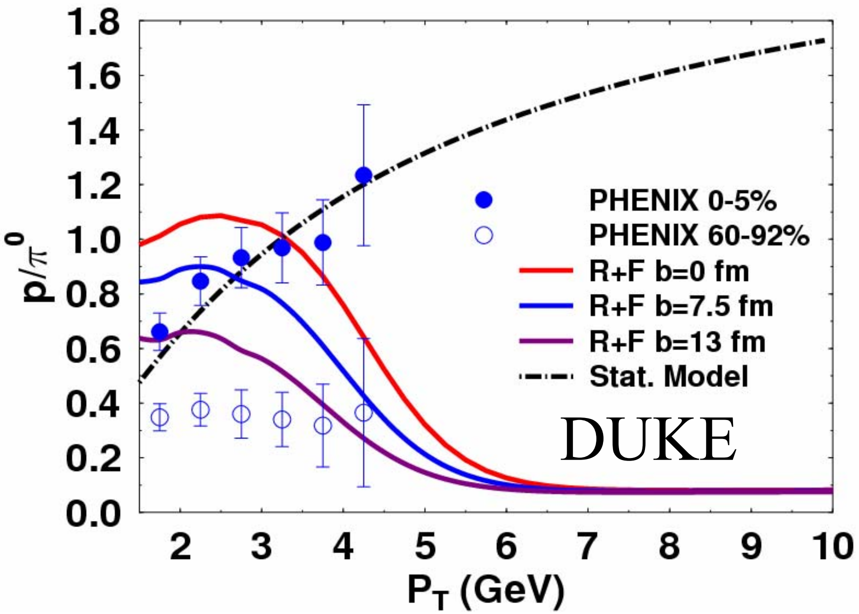
Duke group: no resonances and s+h but use harder parton spectrum

Baryon/Meson ratio



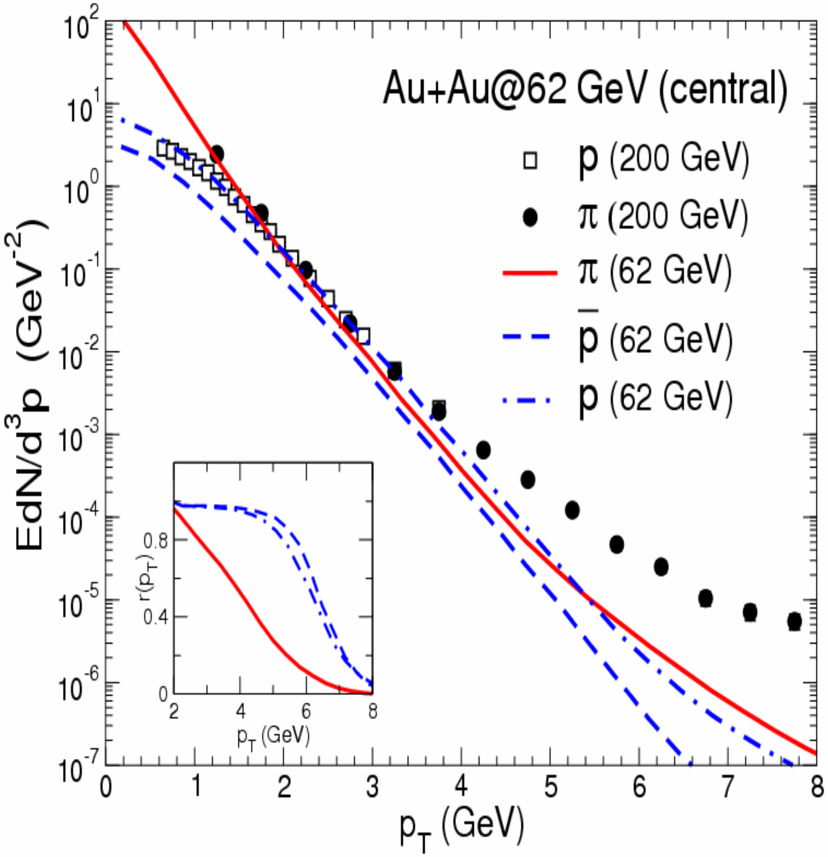
ρ –> ππ

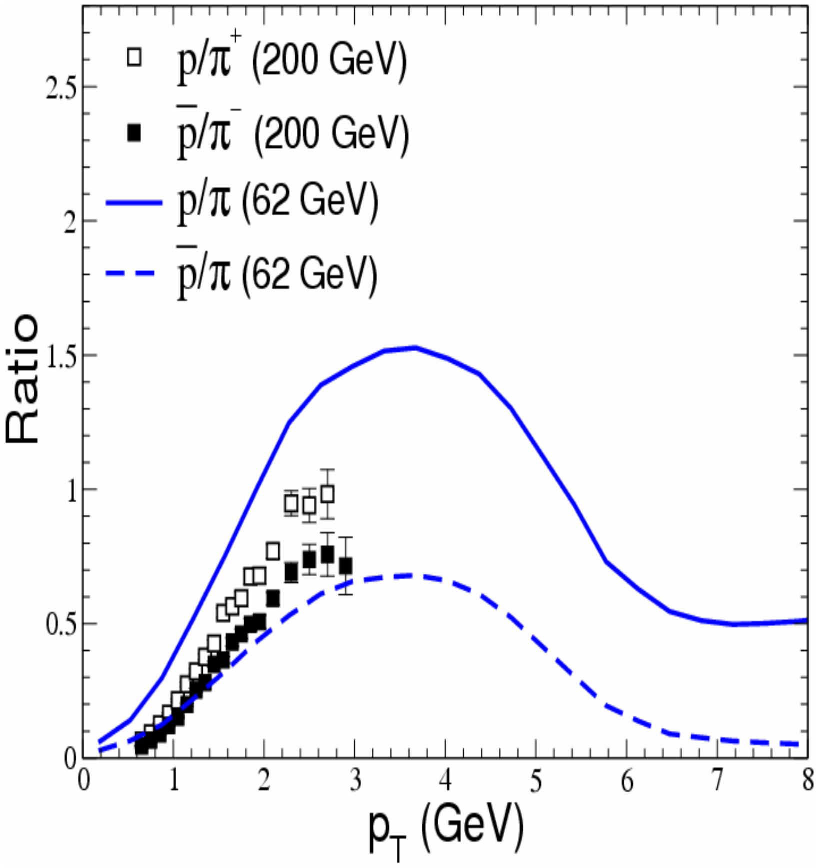




Baryon/meson ratio at lower energy

Greco, Ko & Vitev, PRC 71, 041901(R) (2005)

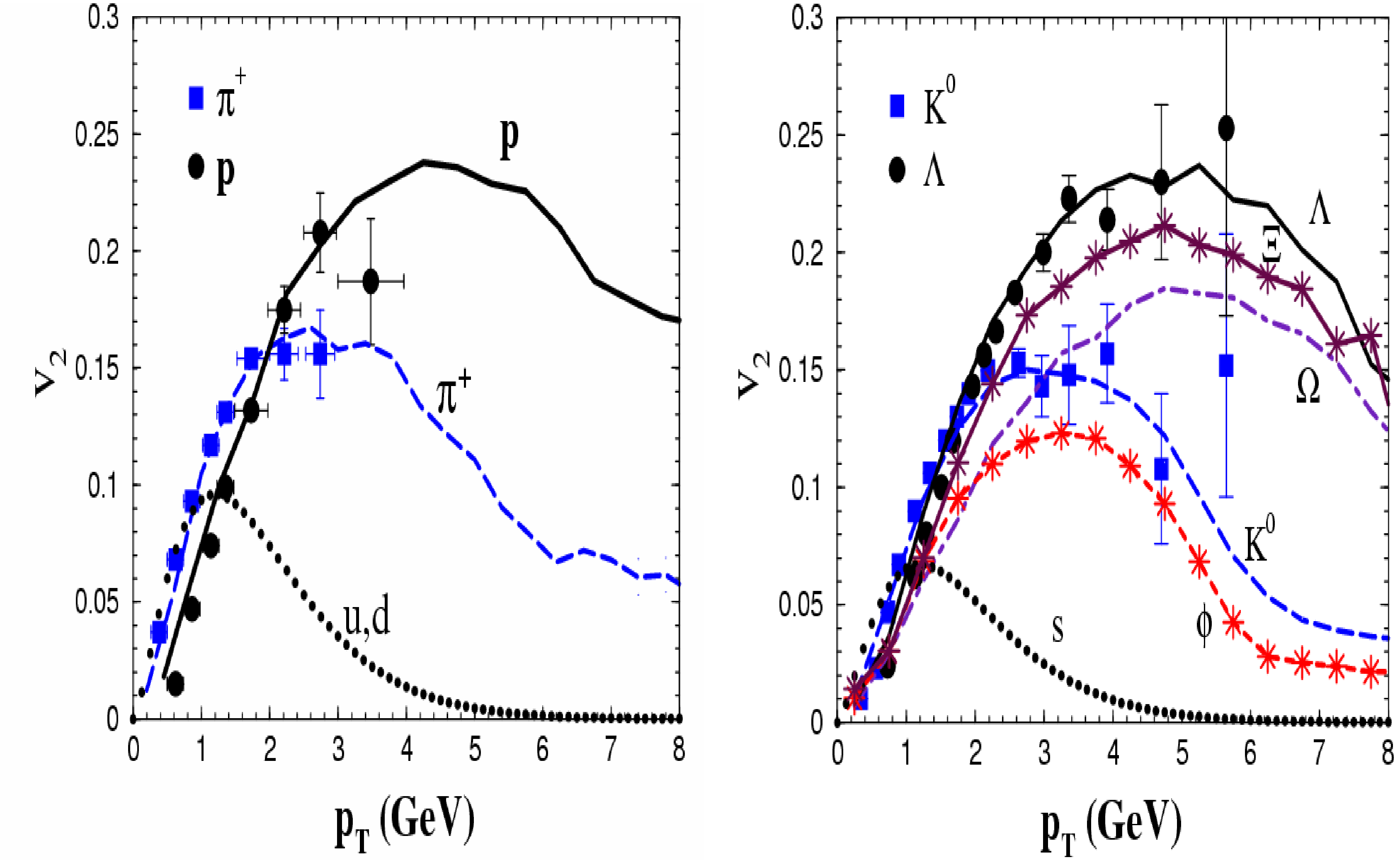




r=fraction due to coalescence

p/π increases by 20% while pbar/π decreases slightly

Elliptic flow



Quark v_2 extracted from pion and kaon v_2 using coalescence model

Naïve quark coalescence model

Only quarks of same momentum can coalescence, i.e., $\Delta p=0$

Quark transverse momentum distribution

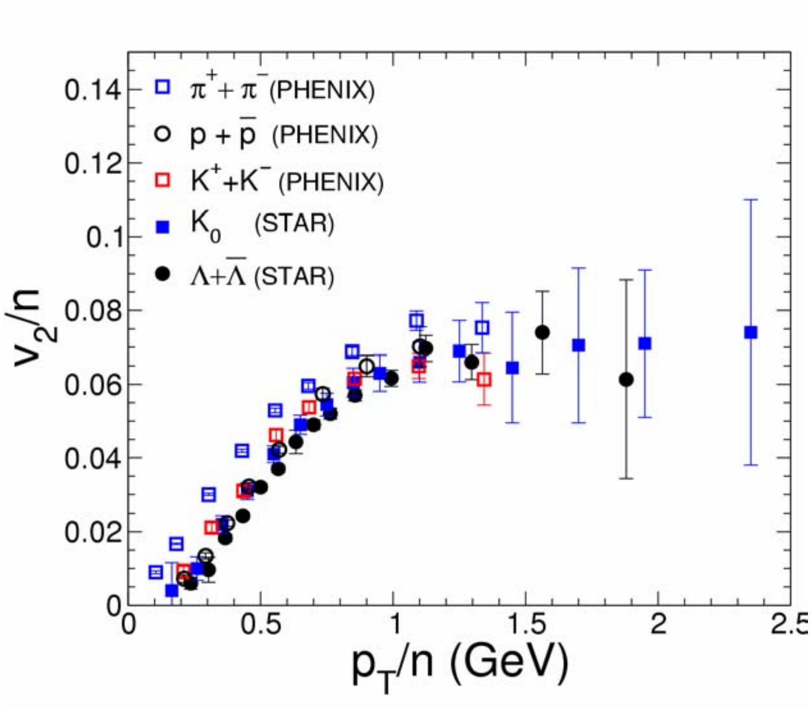
$$\boxed{f_q(p_T) \propto 1 + 2v_{2,q}(p_T)\cos(2\varphi)}$$

Meson elliptic flow

$$\boxed{v_{2,M}(p_T) = \frac{2v_{2,q}(p_T/2)}{1+2v_{2,q}^2(p_T/2)} \approx 2v_{2,q}(p_T/2)}$$

Baryon elliptic flow

$$\boxed{v_{2,B}(p_T) = \frac{3v_{2,q}(p_T/3)}{1+6v_{2,q}^2(p_T/3)} \approx 3v_{2,q}(p_T/3)}$$

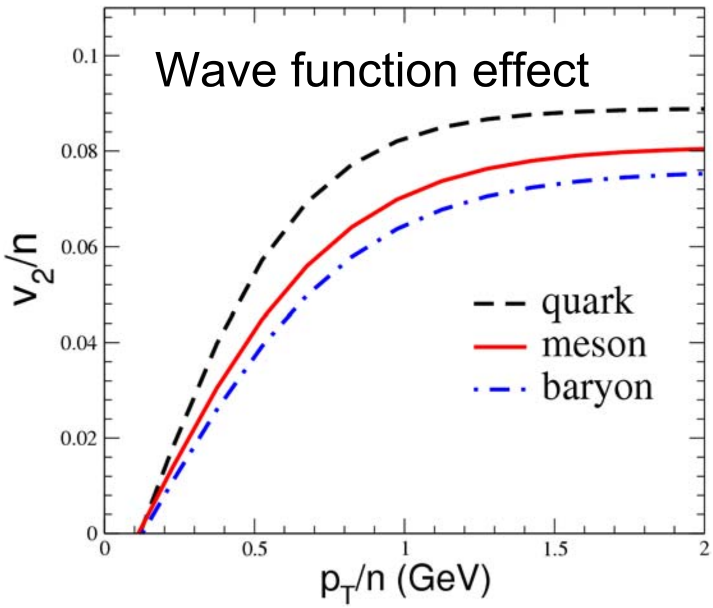


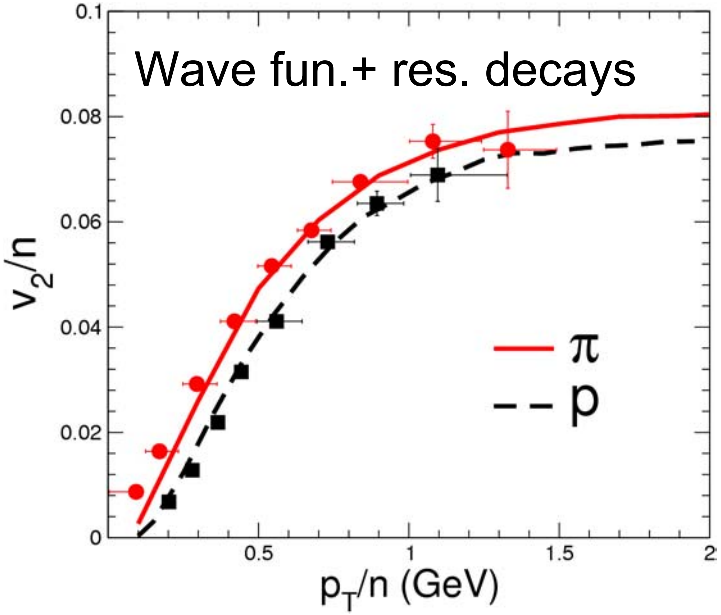
Quark number scaling of hadron v_2 (except pions):

$$\boxed{\frac{1}{n} v_2(p_T/n)}$$

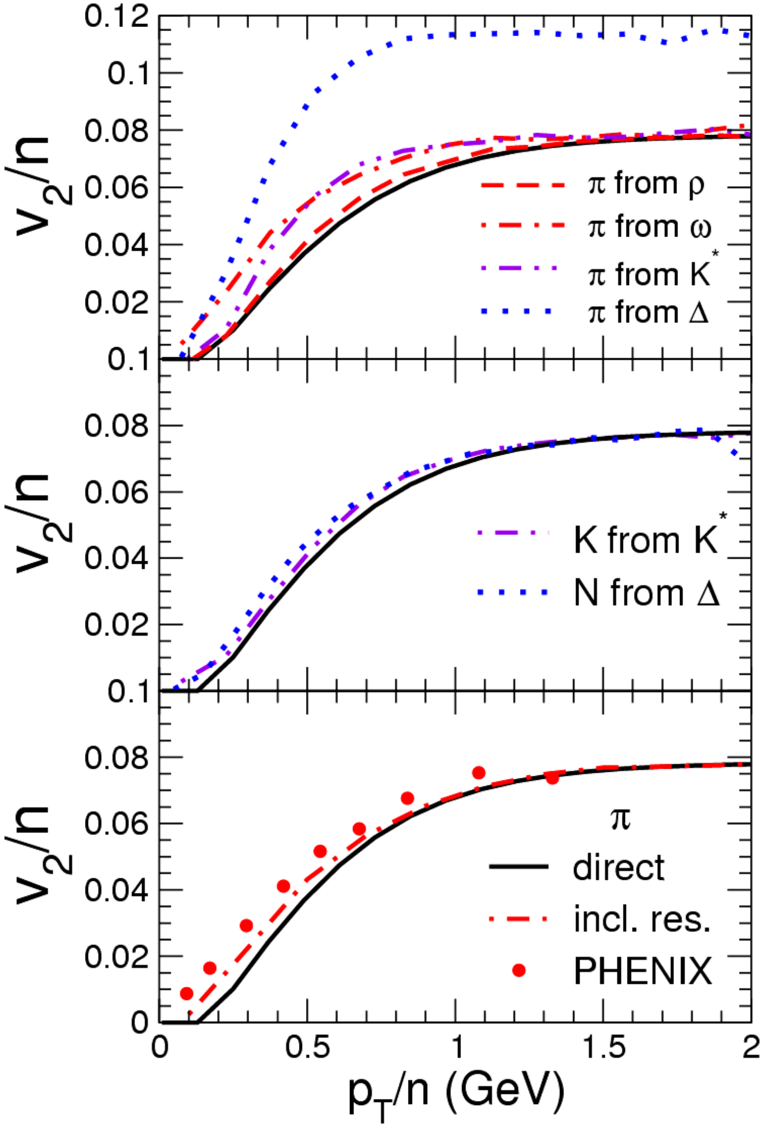
same for mesons and baryons

Effects due to wave function and resonance decays



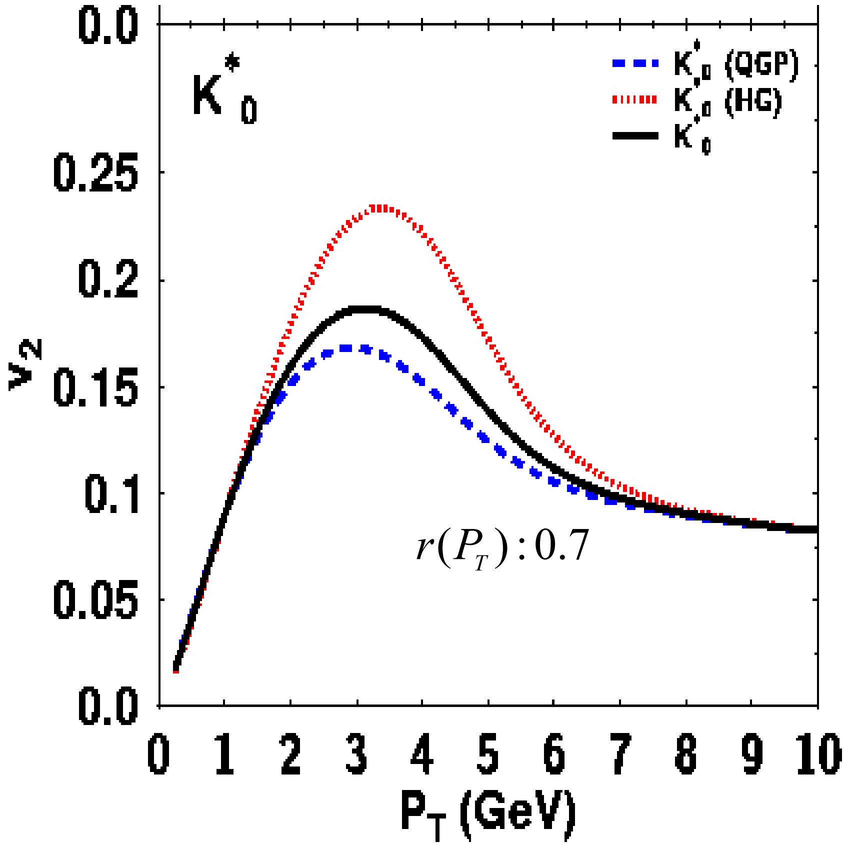


Effect of resonance decays



Elliptic flow of resonances

Nonaka et al, PRC 69, 31902 (04)



- K* produced during hadronization has v_2 given by $v_{2,q}+v_{2,s}$
- K* produced from Kπ scattering has v_2 given by
  $v_{2,\pi}+v_{2,K} \sim 3v_{2,q}+v_{2,s}$
- Observed K* has v_2 given by

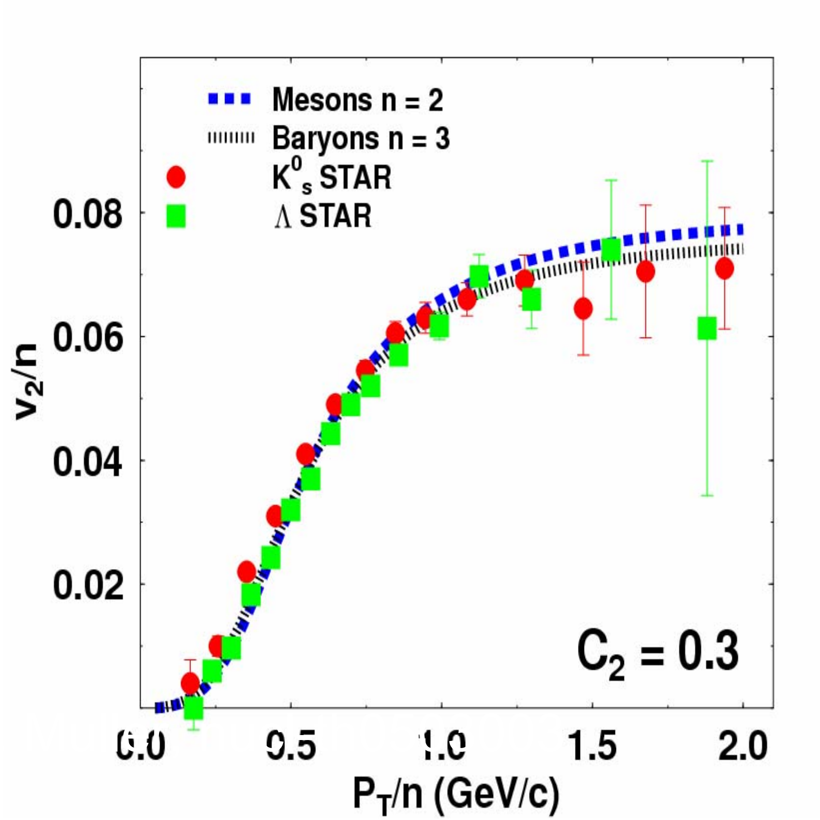
$$v_2^{\text{full}} = r(P_T)v_2^{\text{QGP}} + (1-r(P_T))v_2^{\text{HG}}$$

with $r(p_T)$ depending on K* width and Kπ scattering cross section

Higher Fock States

Meuller, Fries & Bass, PLB 618, 77 (05)

$$\left|M\right\rangle = c_1\left|q_\alpha \bar{q}_\beta\right\rangle + c_2\left|q_\alpha \bar{q}_\beta g\right\rangle + c_3\left|q_\alpha \bar{q}_\beta q_\gamma \bar{q}_\gamma\right\rangle + \ldots$$



$$\mathrm{v}_2^M(p) = \sum_\nu c_\nu^{(M)} n_\nu^{(M)} \mathrm{v}_2\left(p / n_\nu^{(M)}\right)$$

$$\mathrm{v}_2^B(p) = \sum_\nu c_\nu^{(B)} n_\nu^{(B)} \mathrm{v}_2\left(p / n_\nu^{(B)}\right)$$

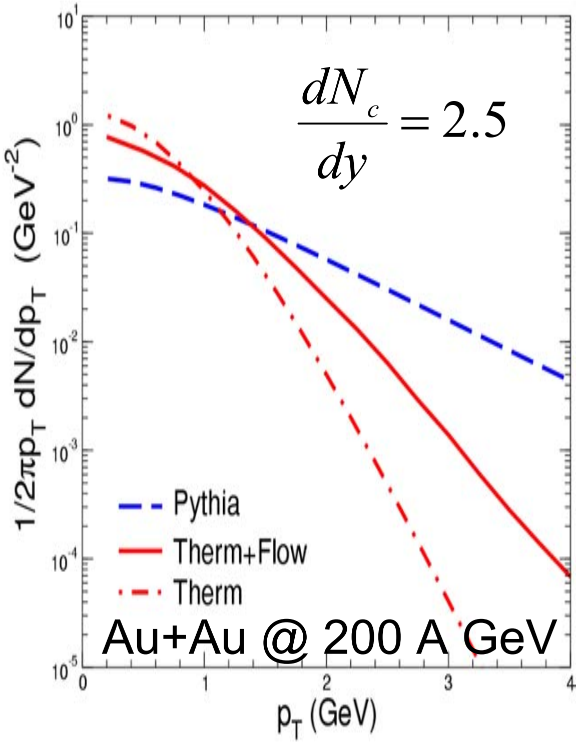
ν: Fock state, n_ν = # of partons

Spectra are also not affected
(at least for $p_T >> m$)

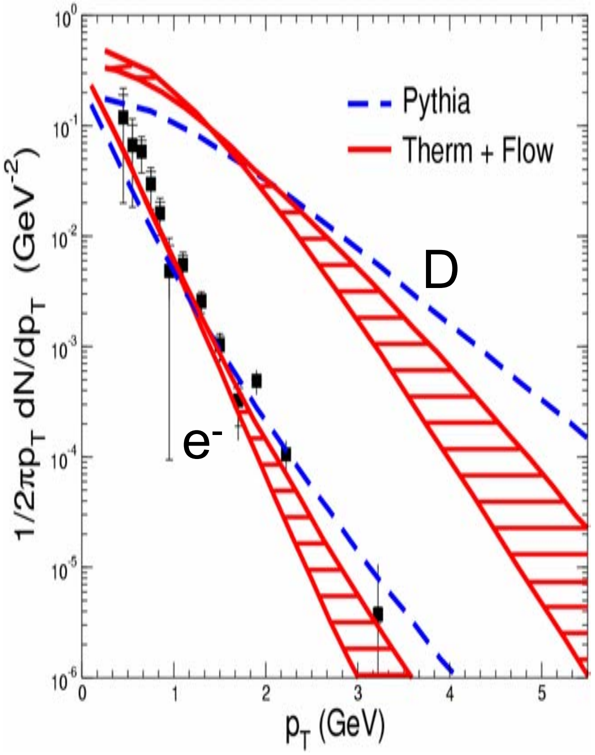
Charm spectra

Greco, Rapp & Ko, PLB595 (04) 202

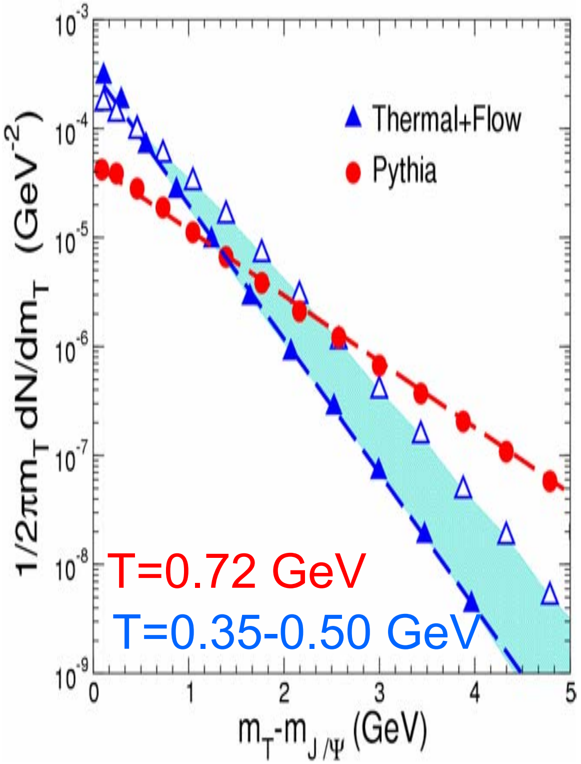
Charm quark



D meson



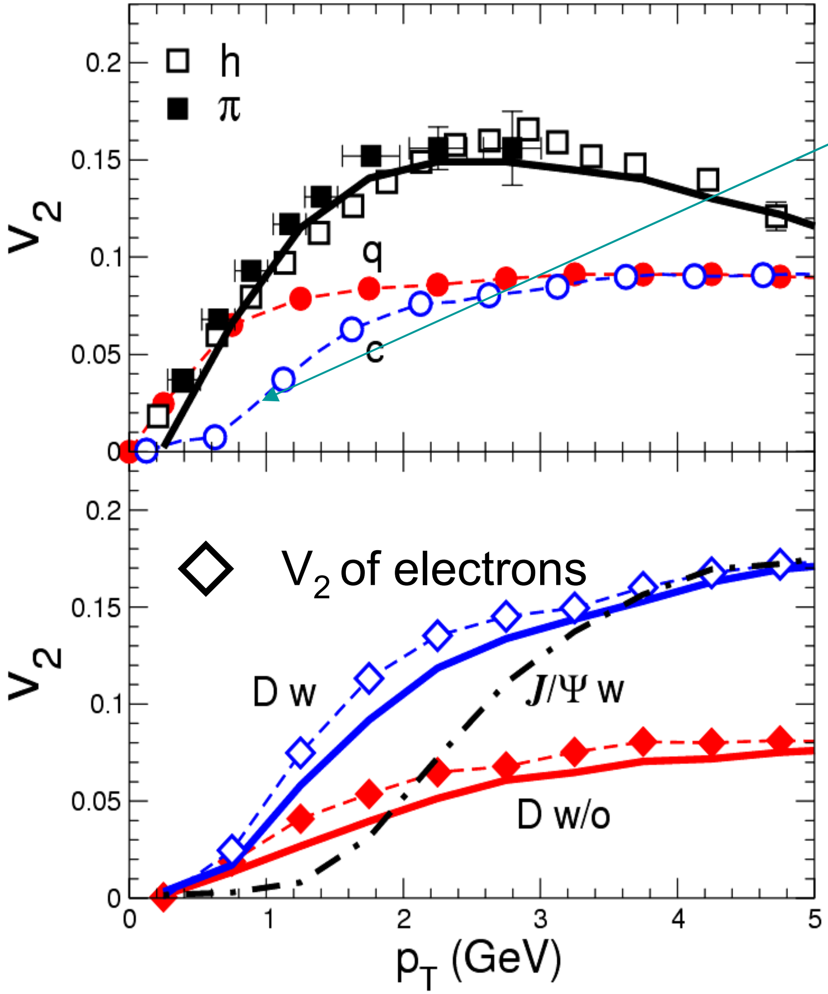
J/ψ



Bands correspond to flow velocities between 0.5 and 0.65

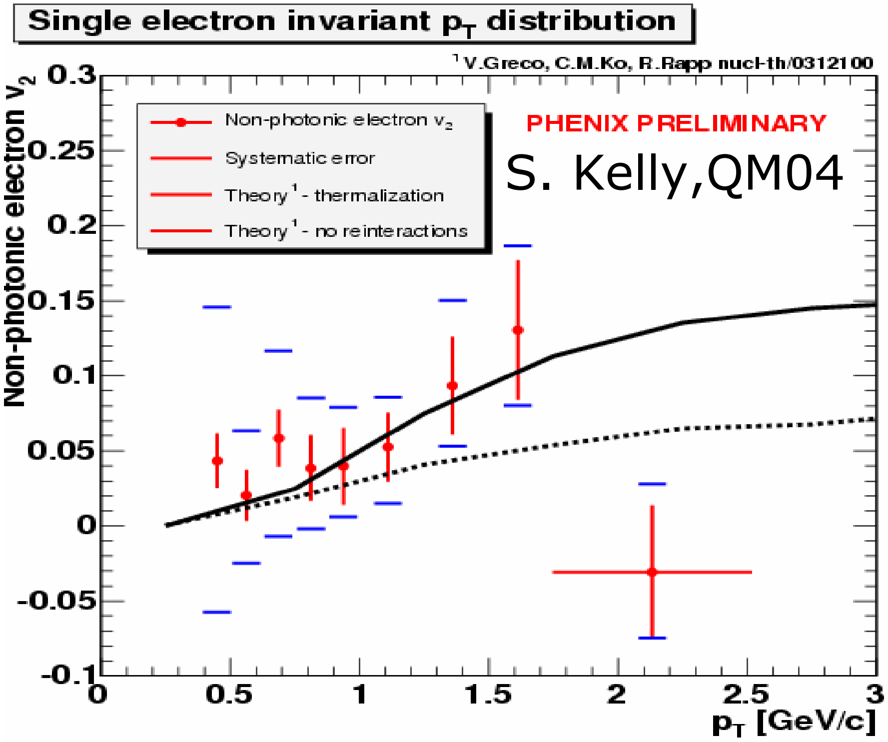
$N_{J/\psi} = 2.7 \cdot 10^{-3}$
$N_{J/\psi} = 0.9 \cdot 10^{-3}$

Charmed meson elliptic flow



Greco, Rapp & Ko, PLB595 (04) 202

Smaller charm v_2 than light quark V_2 at low p_T due to mass effect



S. Kelly,QM04

Data consistent with thermalized charm quark with same v_2 as light quarks

Effect of higher-order parton anisotropic flows

Including 4th order quark flow Kolb, Chen, Greco, Ko, PRC 69 (2004) 051901

$$\boxed{f_q(p_T) \propto 1 + 2v_{2,q}(p_T)\cos(2\varphi) + 2v_{4,q}(p_T)\cos(4\varphi)}$$

Meson elliptic flow

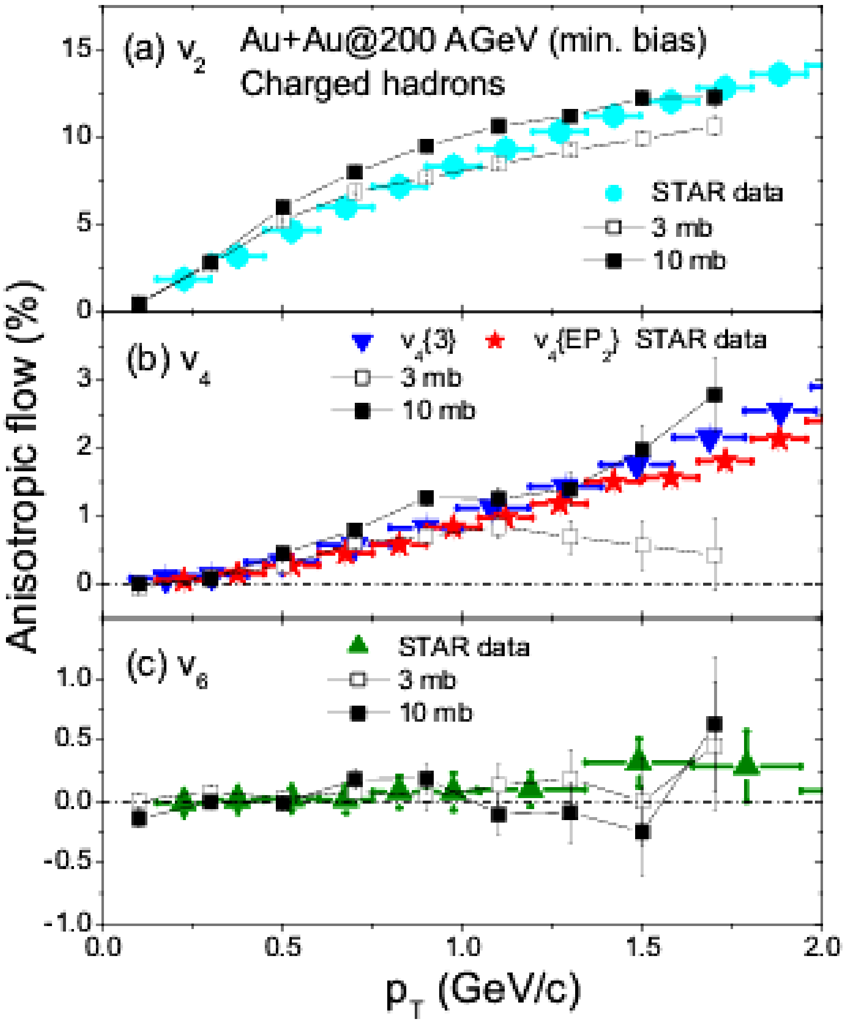
$$v_{2,M} = \frac{2v_{2,q} + 2v_{2,q}v_{4,q}}{1 + 2(v_{2,q}^2 + v_{4,q}^2)}, \quad v_{4,M} = \frac{2v_{4,q} + v_{2,q}^2}{1 + 2(v_{2,q}^2 + v_{4,q}^2)}$$

Baryon elliptic flow

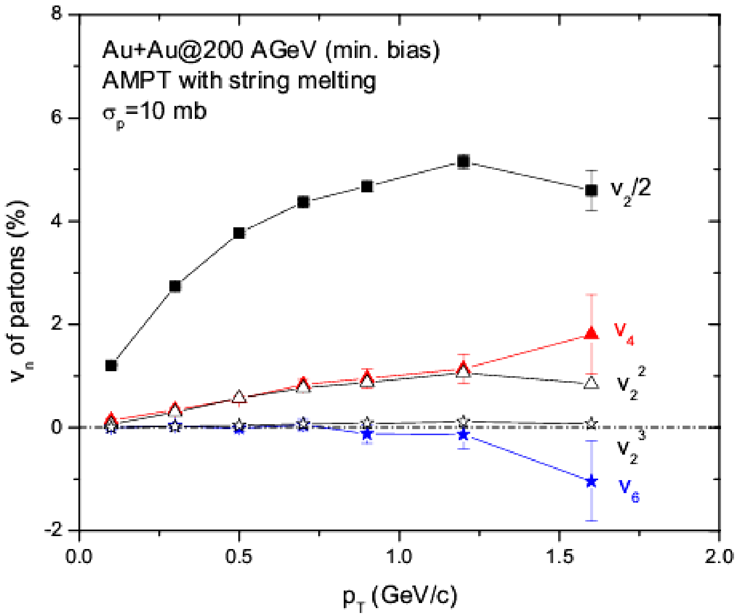
$$v_{2,B} = \frac{3v_{2,q} + 6v_{2,q}v_{4,q} + 3v_{2,q}^3 + 6v_{2,q}v_{4,q}^2}{1 + 6(v_{2,q}^2 + v_{4,q}^2 + v_{2,q}^2 v_{4,q})}, \quad v_{4,B} = \frac{3v_{4,q} + 3v_{2,q}^2 + 6v_{2,q}^2 v_{4,q} + 3v_{4,q}^3}{1 + 6(v_{2,q}^2 + v_{4,q}^2 + v_{2,q}^2 v_{4,q})}$$

$$\Rightarrow \frac{v_{4,M}}{v_{2,M}^2} = \frac{1}{4} + \frac{1}{2}\frac{v_{4,q}}{v_{2,q}^2}, \quad \frac{v_{4,B}}{v_{2,B}^2} = \frac{1}{3} + \frac{1}{3}\frac{v_{4,q}}{v_{2,q}^2}$$

Higher-order anisotropic flows



Data can be described by a multiphase transport (AMPT) model



Parton cascade $\boxed{v_{4,q} \approx v_{2,q}^2}$

Data $\boxed{\frac{v_4}{v_2^2} \approx 1.2 \Rightarrow v_{4,q} \approx 2v_{2,q}^2}$

Hydro gives a ratio of ½
(Borghini & Ollitrault, nucl-th/0506045)

A multiphase transport model

Default: Lin, PaL, Zhang, Li &Ko, PRC 61, 067901 (00);
64, 041901 (01)

- Initial conditions: HIJING (soft strings and hard minijets)
- Parton evolution: ZPC
- Hadronization: Lund string model for default AMPT
  Coalescence model for string melting scenario
- Hadronic scattering: ART

String melting: PRC 65, 034904 (02); PRL 89, 152301 (02)

- Convert hadrons from string fragmentation into quarks and antiquarks
- Evolve quarks and antiquarks in ZPC
- When stop interacting, combine nearest quark and antiquark to meson, and nearest three quarks to baryon,
- Hadron flavors are determined by quarks' invariant mass

Zhang's parton cascade (ZPC)

Bin Zhang, Comp. Phys. Comm. 109, 193 (1998)

$$p^{\mu}\partial_{\mu}f_1(x, p, t) \propto \int dp_2 d\Omega \, |\vec{v}_1 - \vec{v}_2| (d\sigma/d\Omega)(f_1' f_2' - f_1 f_2)$$

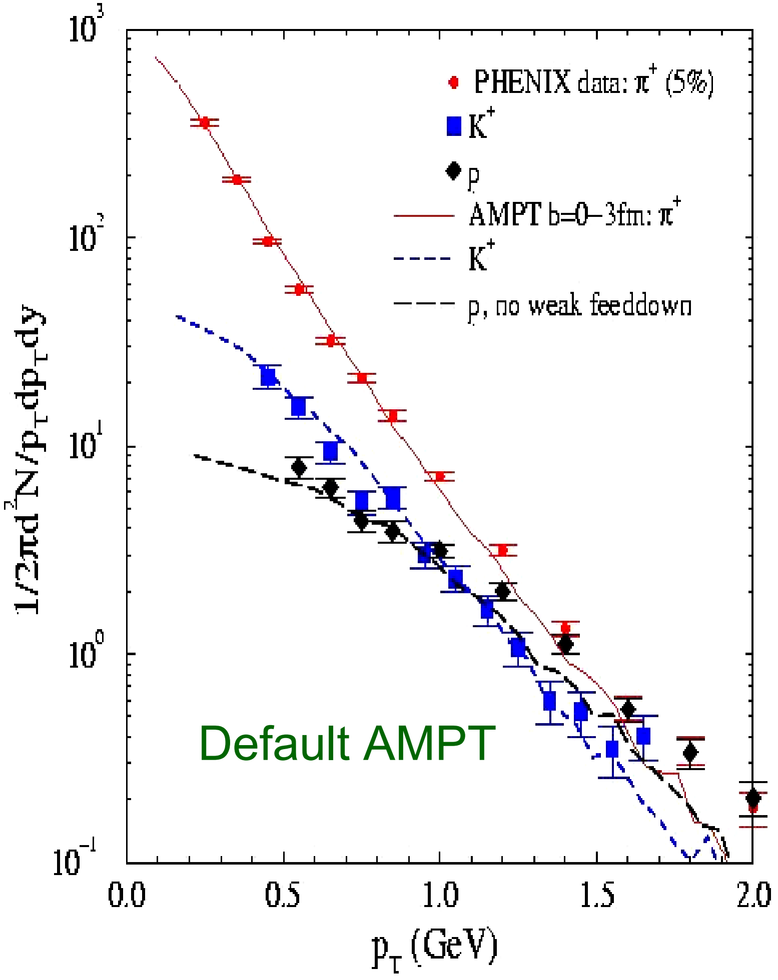
$$\frac{d\sigma}{dt} \approx \frac{9\pi\alpha_s^2}{2(t-\mu^2)^2}, \quad \sigma = \frac{9\pi\alpha_s^2}{2\mu^2}\frac{1}{1+\mu^2/s}$$

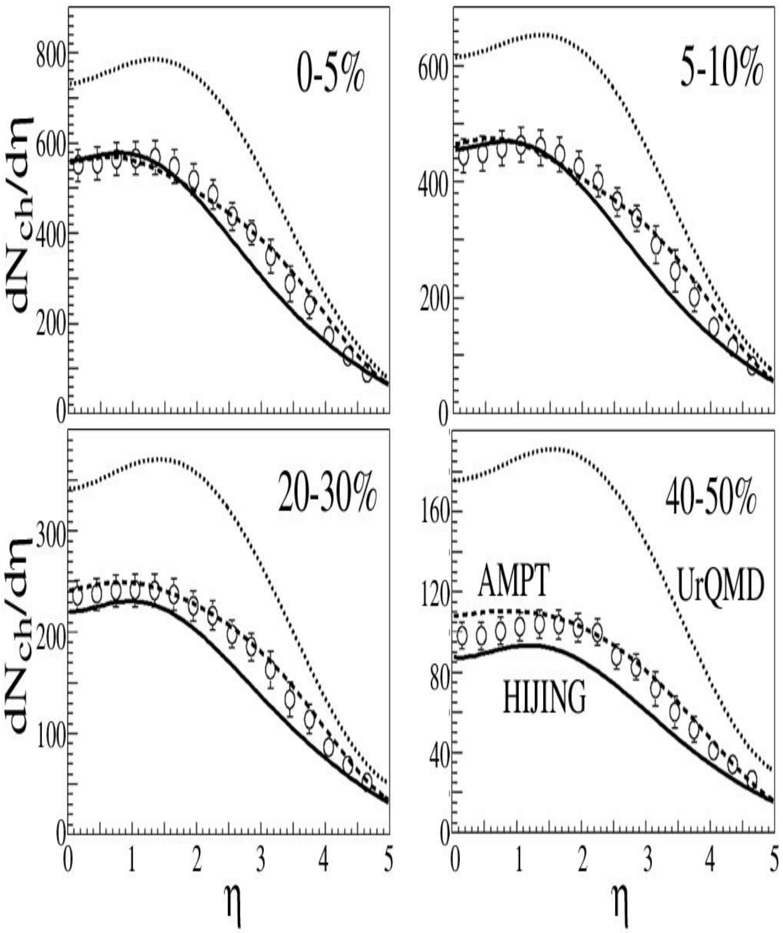
- Using $\alpha_s=0.5$ and screening mass $\mu=gT\approx0.6$ GeV at $T\approx0.25$ GeV, then $\langle s\rangle^{1/2}\approx4.2T\approx1$ GeV, and pQCD gives $\sigma\approx2.5$ mb and a transport cross section

$$\sigma_t \equiv \int d\Omega \frac{d\sigma}{d\Omega}(1-\cos\theta) \approx 1.5\text{mb}$$

- $\sigma=6$ mb → $\mu\approx0.44$ GeV, $\sigma_t\approx2.7$ mb
- $\sigma=10$ mb → $\mu\approx0.35$ GeV, $\sigma_t\approx3.6$ mb

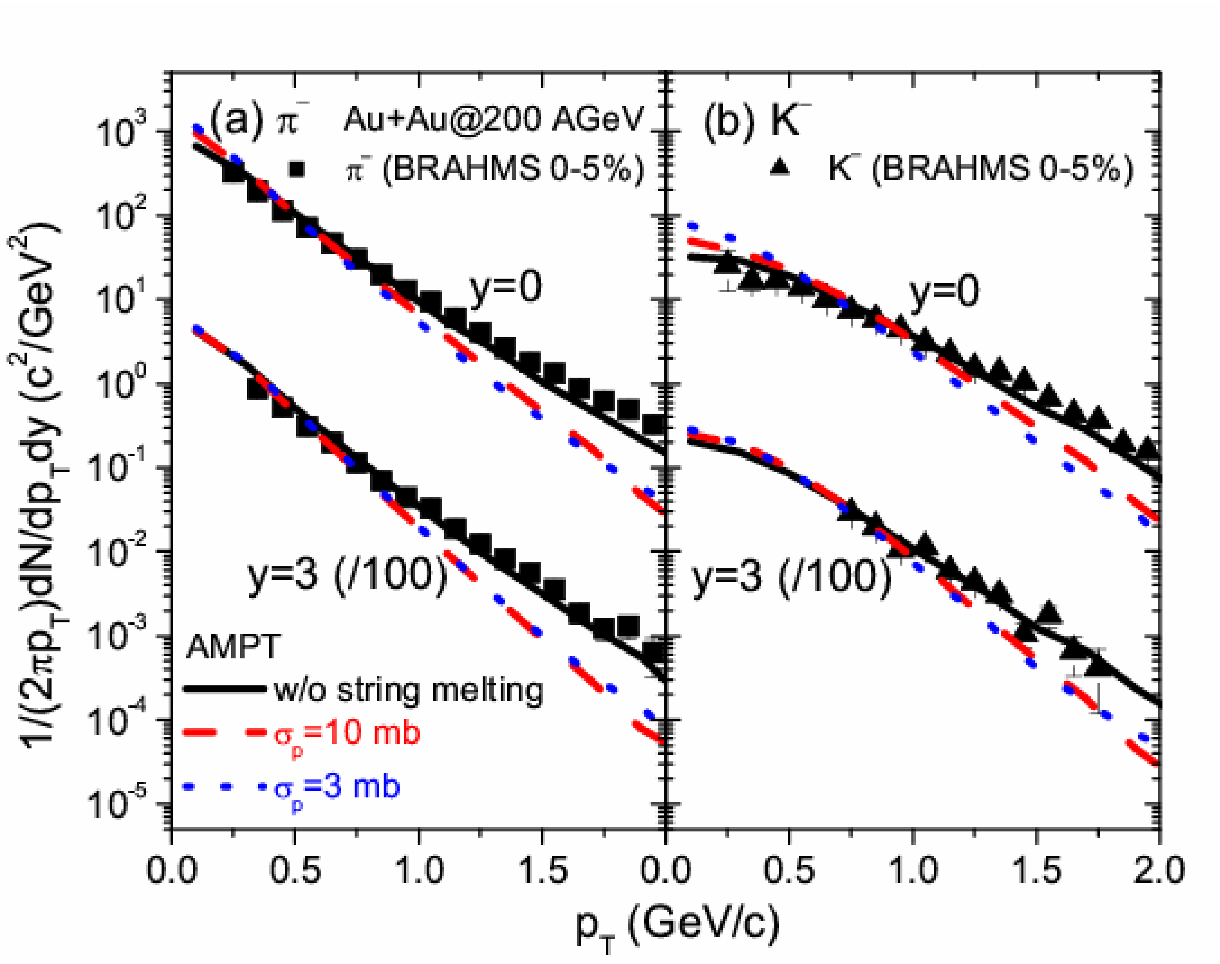
Transverse momentum and rapidity distribution from default AMPT





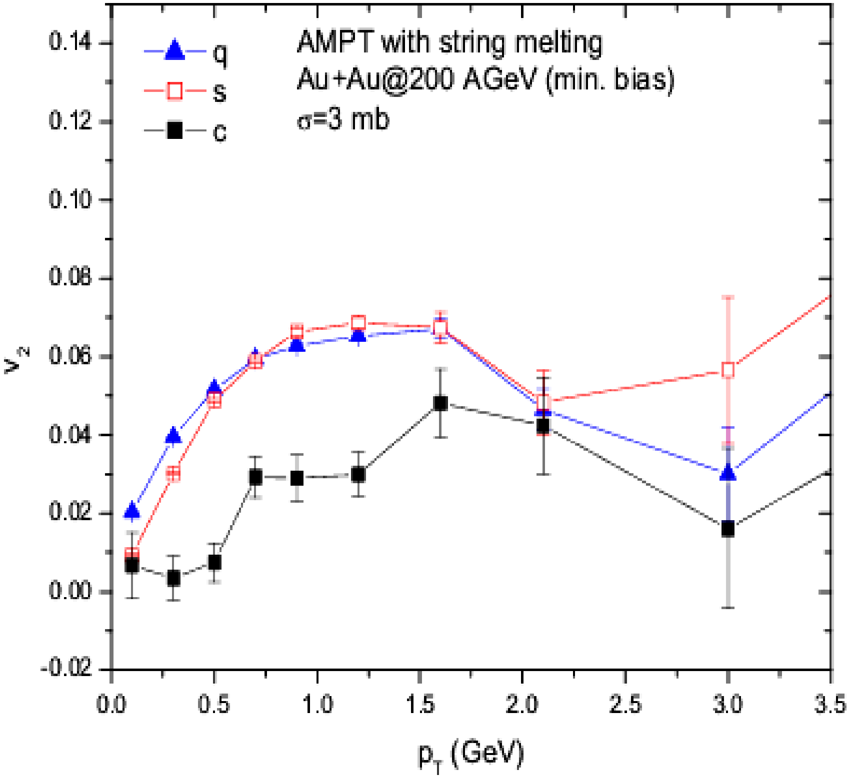
BRAHMS Au+Au @ 200 GeV

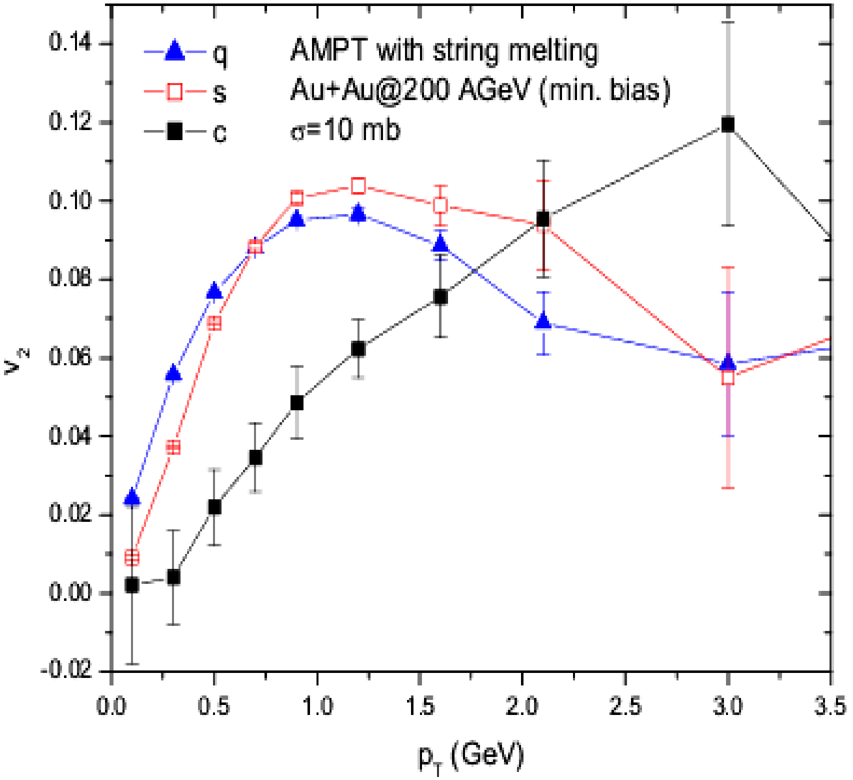
Transverse momentum spectra from AMPT with string melting



- Spectra are softer than in default AMPT as current quark masses are used, whose spectra are less affected by collective radial flow

Quark elliptic flows from AMPT

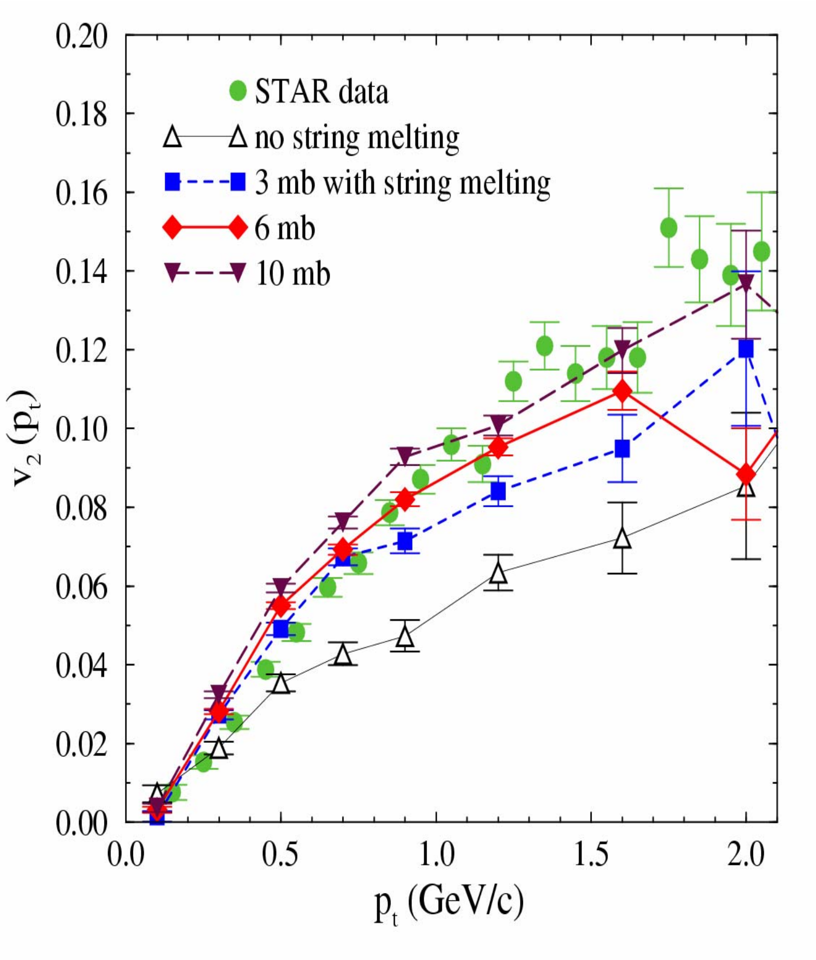


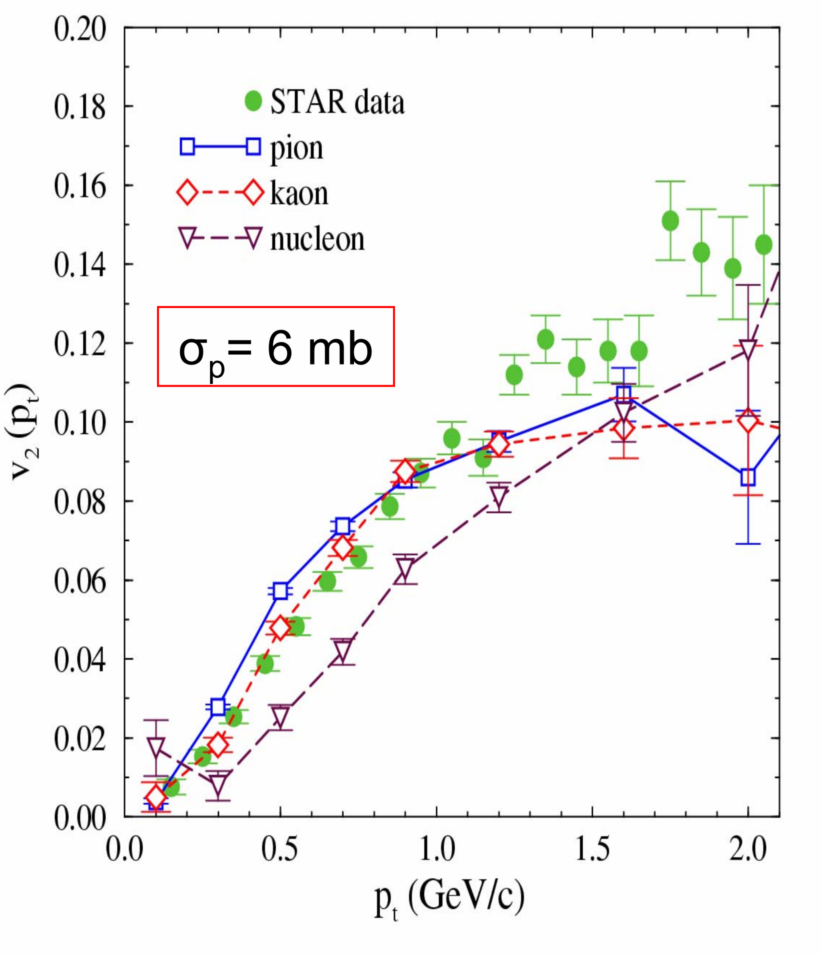


- p_T dependence of charm quark v_2 is different from that of light quarks
- At high p_T, charm quark has similar v_2 as light quarks
- Charm elliptic flow is also sensitive to parton cross sections

Elliptic flow from AMPT

Lin & Ko, PRC 65, 034904 (2002)

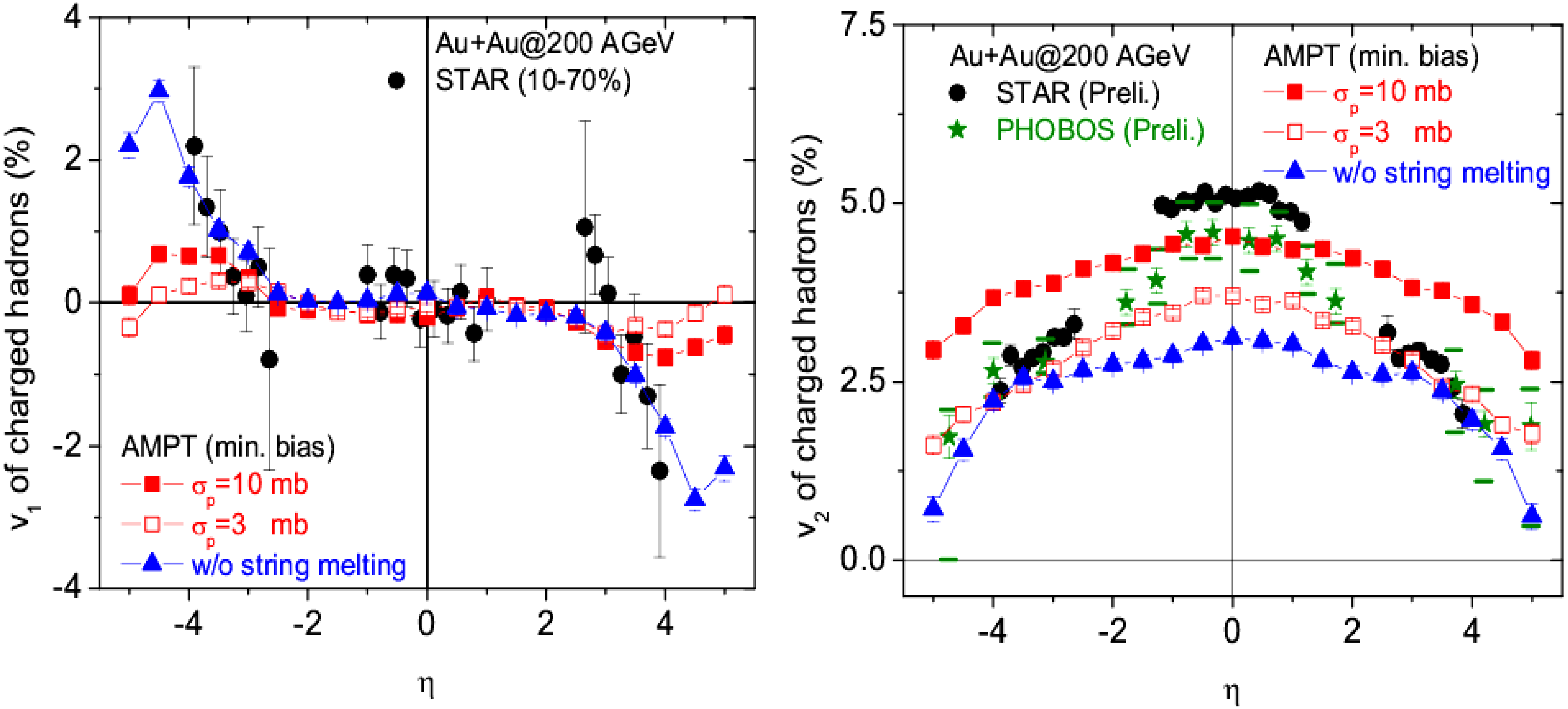




Need string melting and large parton scattering cross section

Pseudorapidity dependence of v_1 and v_2

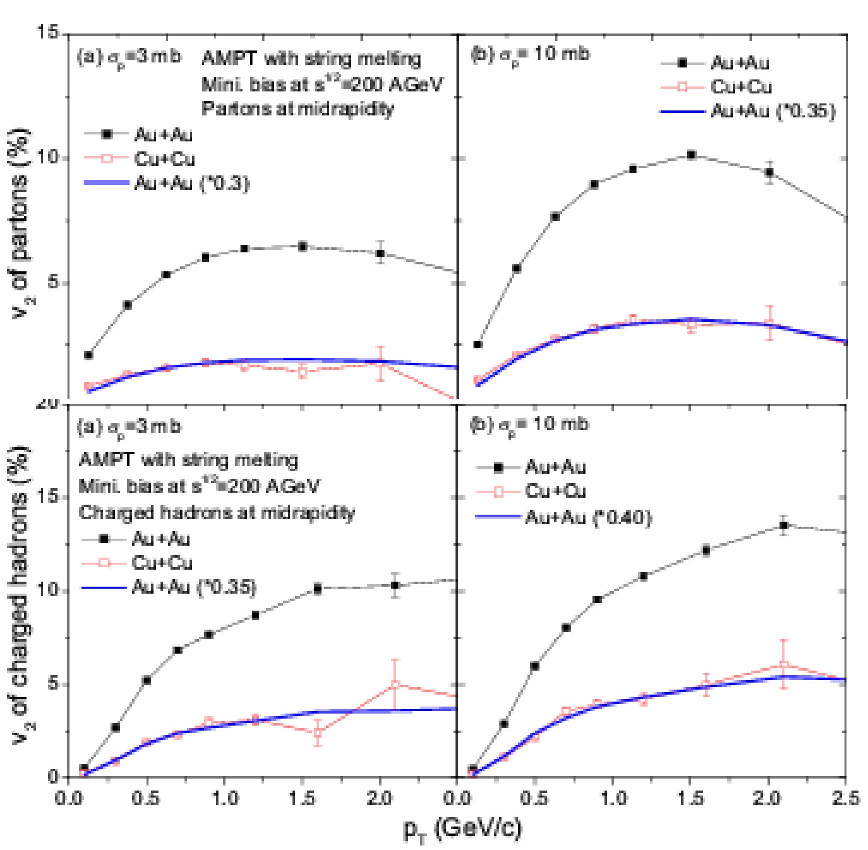
Chen, Greco, Ko & Koch, PLB 605, 95 (2005)



- String melting describes data near mid-rapidity ($|\eta|<1.5$)
- At large rapidity ($|\eta|>3$), hadronic picture works better

System size dependence of elliptic flow

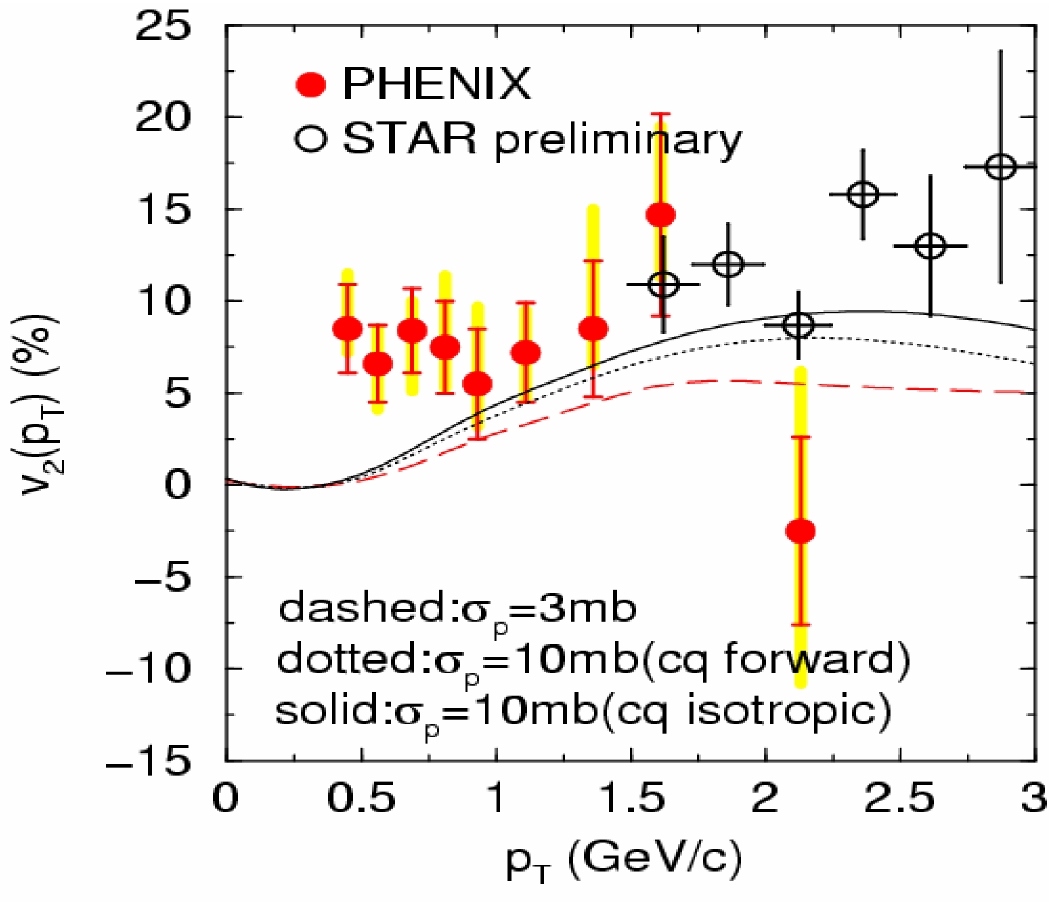
Chen & Ko, nucl-th/0505044



Ratio of elliptic flow is ~ 1/3 and scales with the size of colliding systems (~ product of ratios of initial eccentricity (~ ½) and energy density ~2/3)

Charmed meson elliptic flow from AMPT

Zhang, Chen & Ko, PRC 72, 024906 (05)



Current light quark masses are used in AMPT. With constituent masses will enhance the charmed meson elliptic flow.

Entropy

For non-relativistic system

$$\boxed{S = N\left[\frac{5}{2} - \frac{\mu}{T}\right] = N\left[\frac{5}{2} + \frac{m}{T} - \log\left(\frac{N}{N_{th}}\right)\right]}$$

For g→π in duality picture with $m_g = m_\pi = 0$ and gluon in equilibrium

$$S_g = 2.5\,N_g \qquad S_\pi = N_g\left[\frac{5}{2} - \log\left(\frac{g_g}{g_\pi}\right)\right] \cong 0.8 N_g$$

70% decrease

Coalescence model

$$S_{QGP} \approx 4870 \qquad S_{Had} \approx 4080$$

16% decrease

But energy is not conserved ΔE/E~ 18%

Need to take into account binding effect such as treated approximately in AMPT by considering invariant mass of coalescence quarks

Summary

- Quark coalescence can explain observed
  Large baryon/meson ratio at p_T~ 3GeV
  Quark number scaling of hadron v_2
  → signature of deconfinement?
- Coalescence of minijet partons with thermal partons is significant
  → medium modification of minijet fragmentation.
- Scaling violation of pion v_2 can be explained by resonance decays.
- Coalescence of thermalized charm quarks can explain preliminary charmed meson spectrum and v_2 as well as J/ψ yield.
- Required quark v_2 is consistent with that from parton cascade with large parton cross section (~10 mb).
- Appreciable parton v_4 is seen in parton cascade.
- Entropy violation (~16%) is not as large as one naively thinks and is related to energy violation of similar magnitude