Energy Loss in Hot QGP

Sangyong Jeon – Presenting

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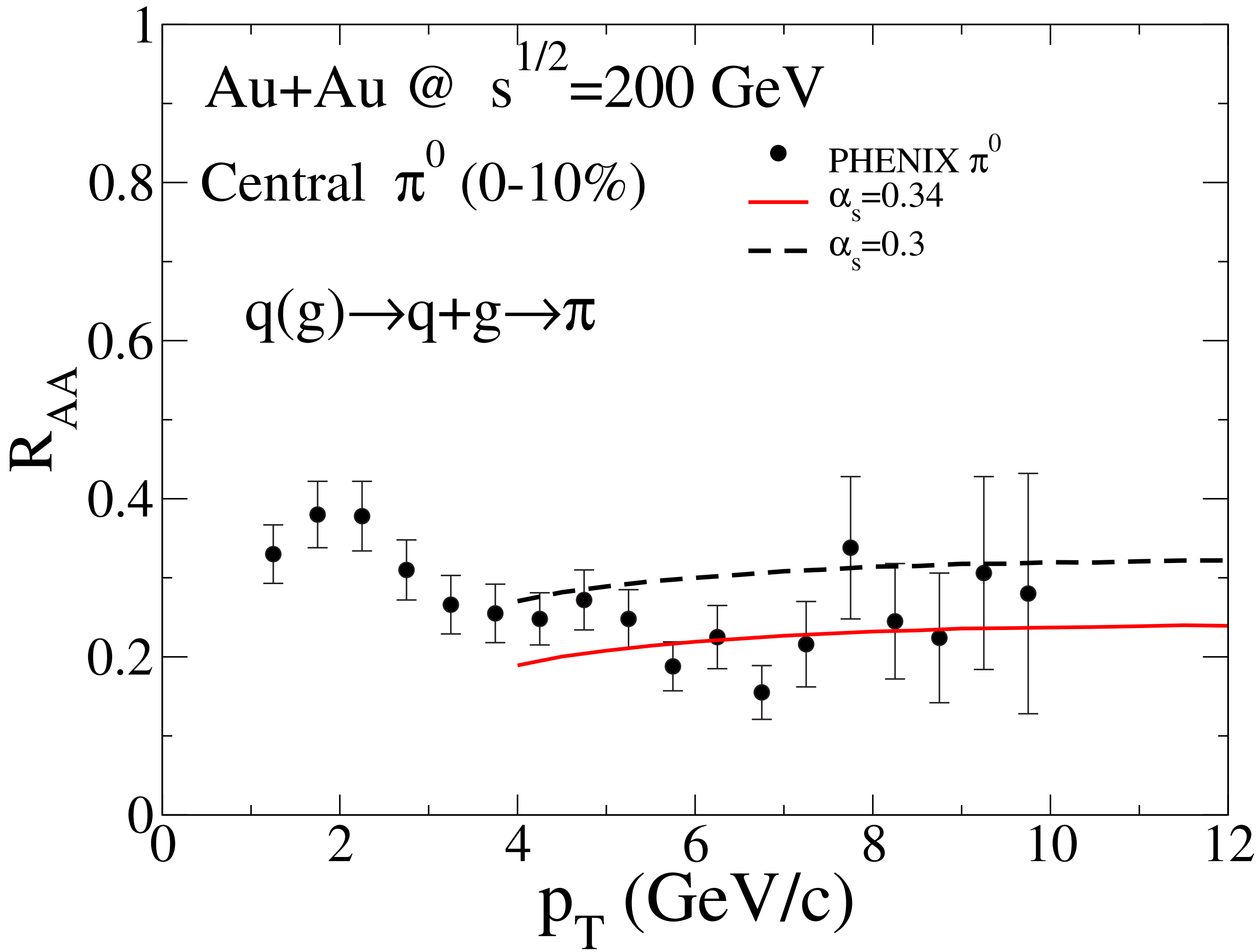
RBRC

with S.Turbide, C.Gale and G.Moore

Where we want to go:



and much more!

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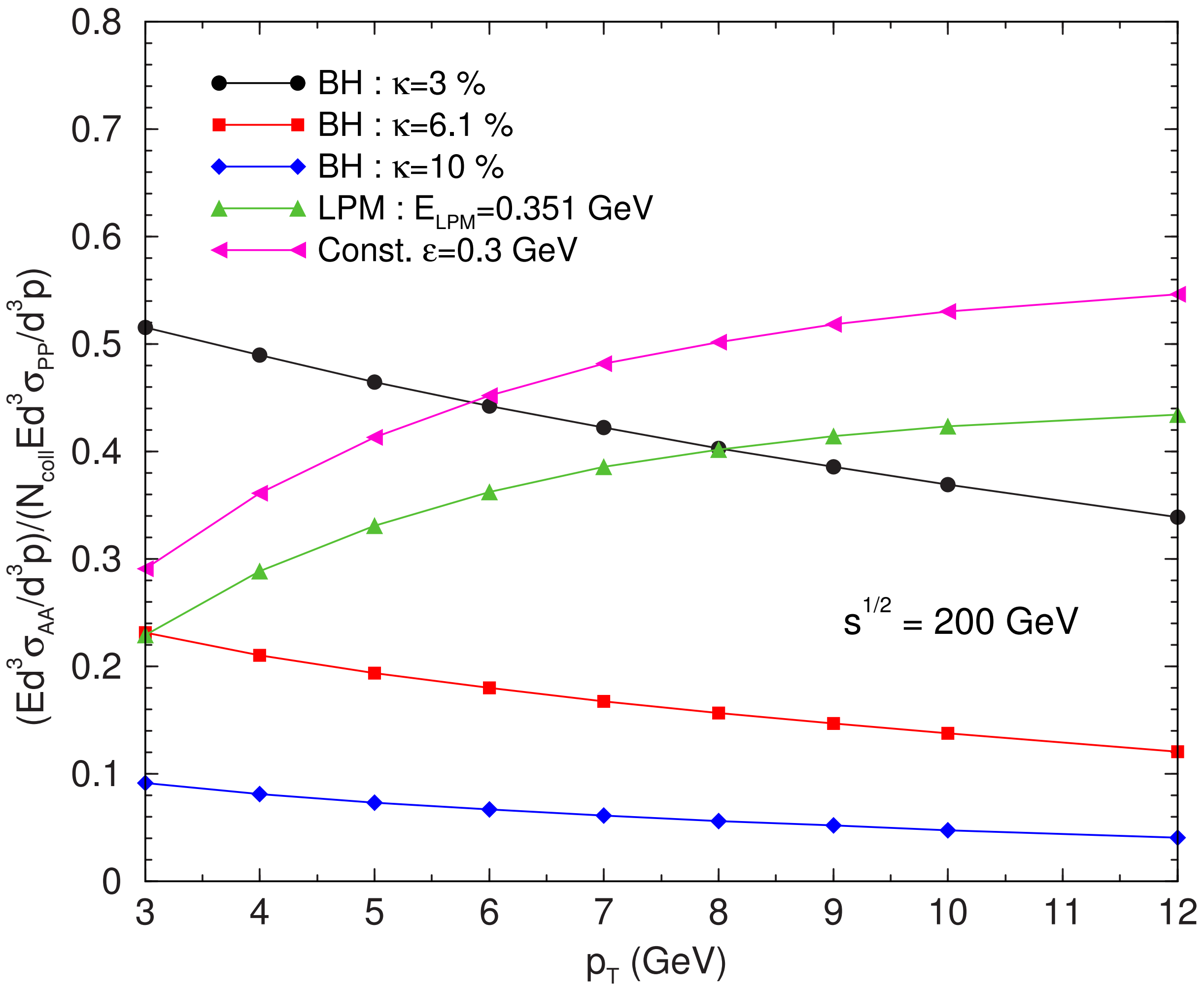
(Major) Differences with others

- Medium is dynamic. – Absorption, $q\bar{q}$ annihilation included.
- Loss rate is good for all $p_T > T$.
- We solve the time evolution equation.

Little Detour



S.Jeon, J.Jalilian-Marian and I.Sarcevic, QM02.

"High" p_t Spectra

PHOBOS – $\frac{dN}{dp_t^2} \sim$ # part.'s
NOT # coll.'s ?

PHENIX: $R_{AA} \approx$ const.
p_t: 4 → 8 GeV

$\left.\frac{dN}{dp_t^2}\right|_{NLO}$ ⊂ $\frac{\Delta E}{E} \sim .07/\text{scatt.'s}$

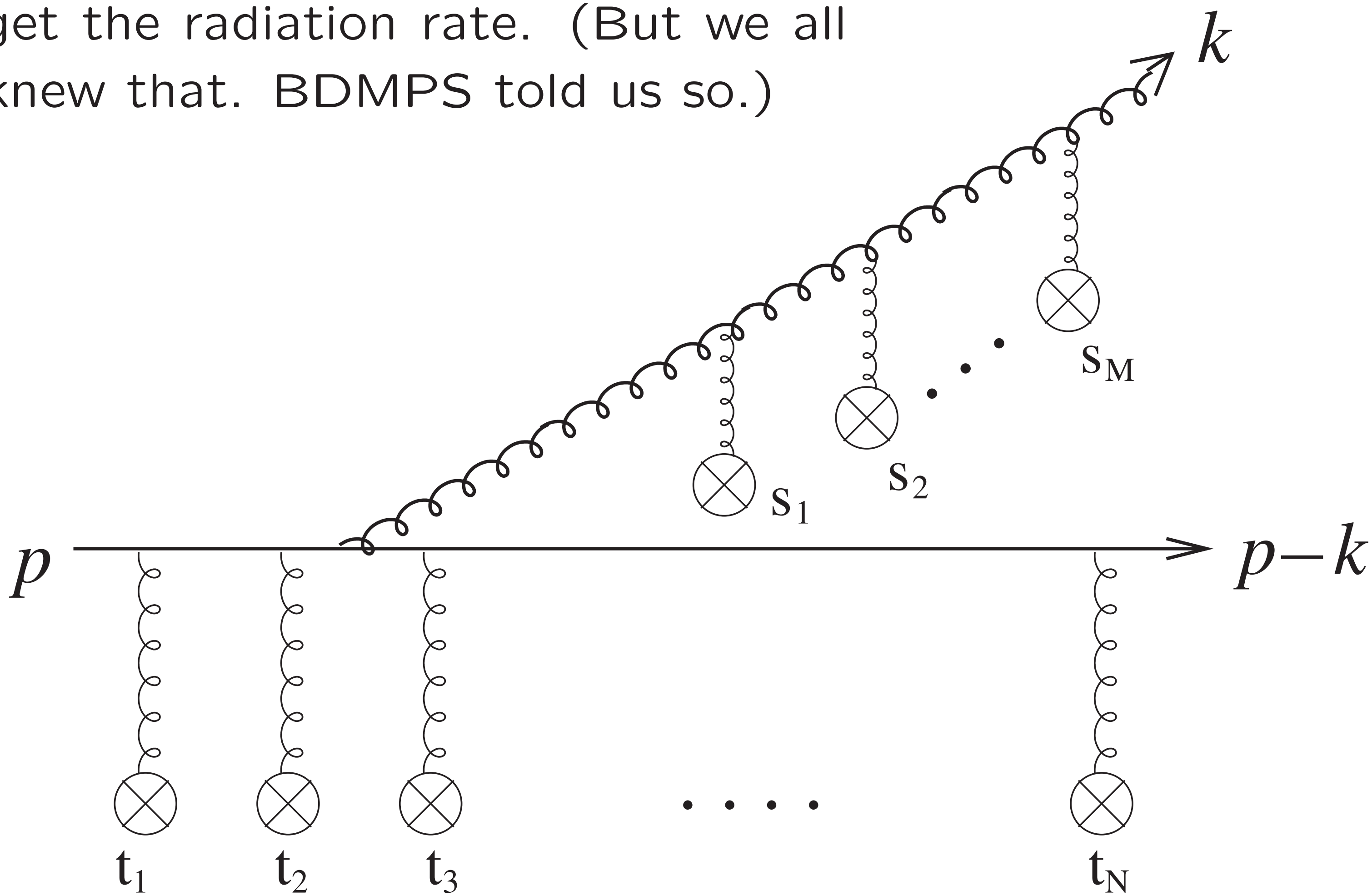
← WRONG! Jeon, Jalilian-Marian, Sarcevic

Right: $\Delta E \sim \sqrt{E}$ Baier, Dokshitzer, Mueller, Schiff

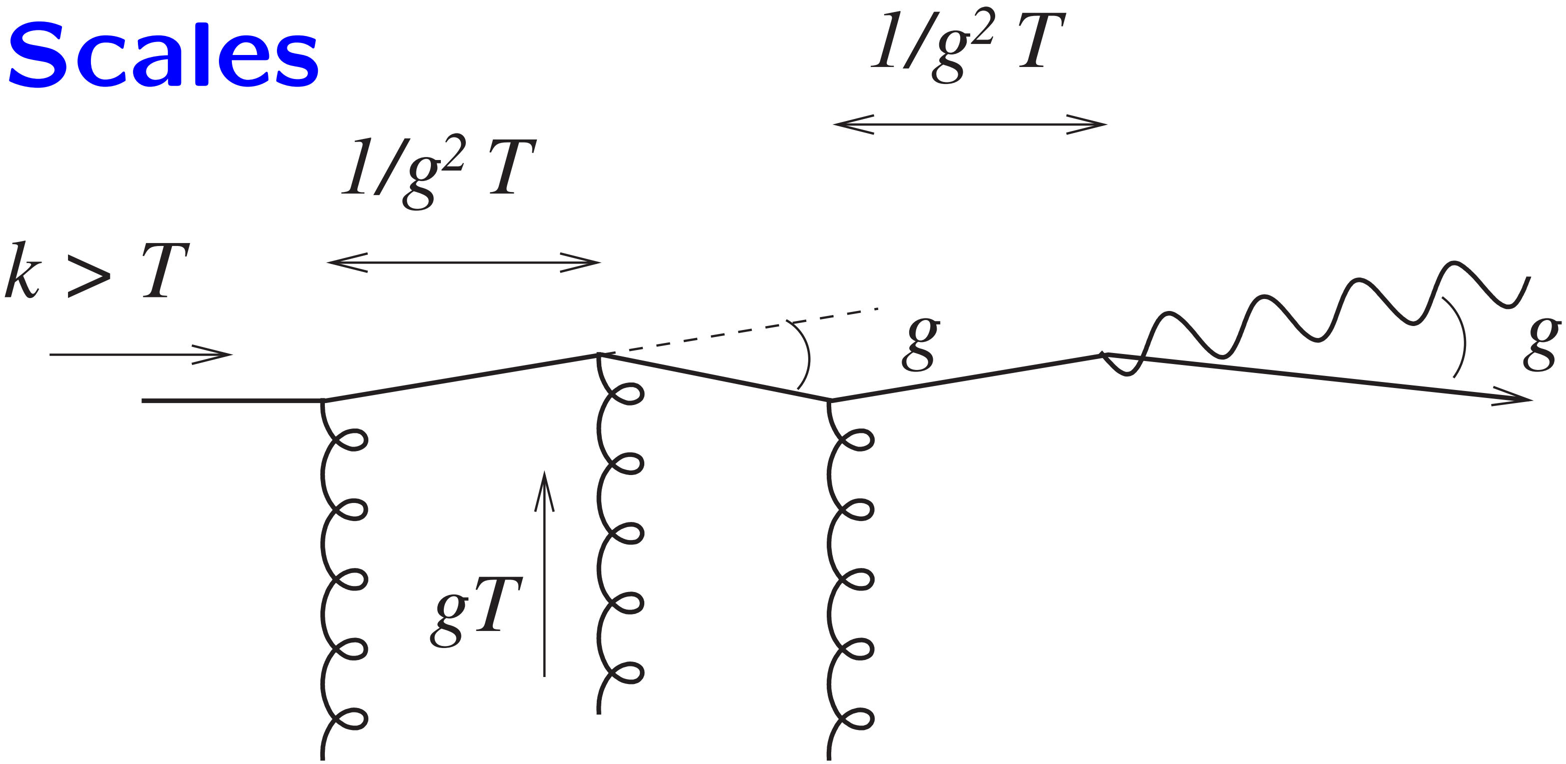
Pisarski, QM02 Summary.

Radiations in QED and QCD

Amplitude to radiate: Need to sum over all N and all M and all possible radiation points. Then square it to get the radiation rate. (But we all knew that. BDMPS told us so.)



Scales

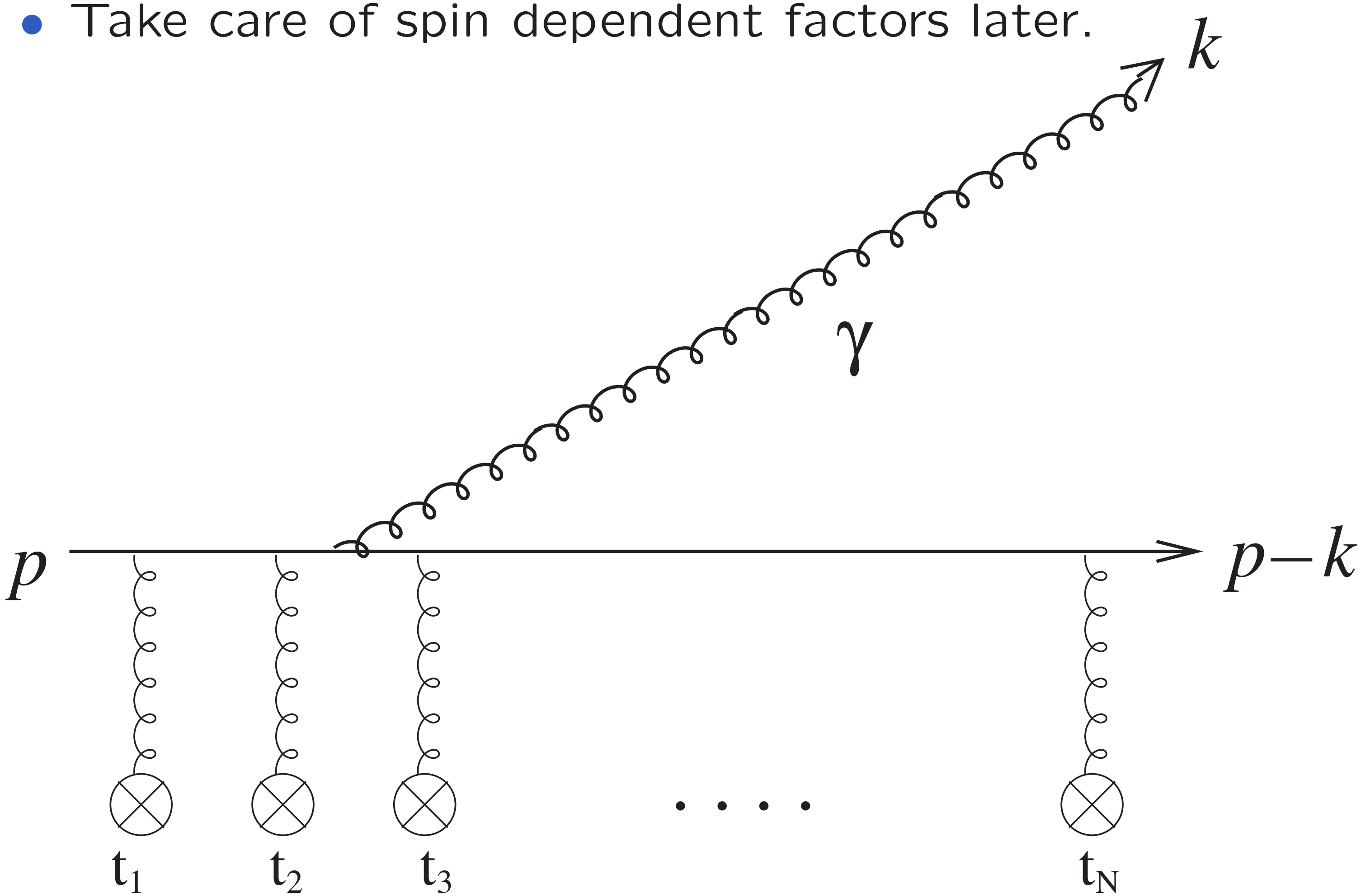


Reason to resum:

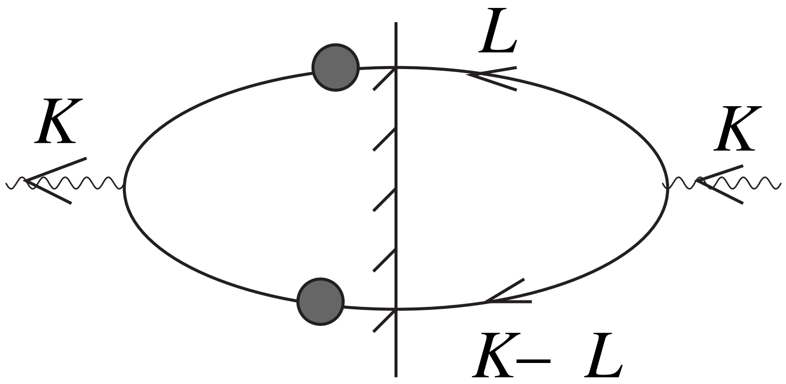
1. Radiation angle is g. Hence the transverse speed is g.
2. gT kick makes the size of the parton $1/gT$.
3. It takes $(1/gT)/g = 1/g^2T$ to get separated.
4. But that's just the mean free path for the next gT kick!
5. Another way of saying LPM matters.

Photon Radiation with scalar quarks

- Will need it later for photon production.
- Simpler problem to solve.
- Take care of spin dependent factors later.



Diagrams to sum



Choose a frame such that $K = (k, 0, 0, k)$.
$k = O(T)$.

To estimate, use the HTL resummed propagator for $K - L$ line but use free propagator for the L line. $n_B(O(T)) \sim O(1)$.

$$\Sigma_I^{\mu\nu} \sim \alpha_{EM} \int d^4L\, (2L-K)^\mu (2L-K)^\nu \delta(L^2 - m_T^2) \rho(L-K)$$
$$\sim \alpha_{EM} \int \frac{d^3l}{E_l} (2L-K)^\mu (2L-K)^\nu \rho(L-K)$$

Must project with

$$\hat{P}_T^{\mu\nu} = g^{\mu\nu} - K^\mu K^\nu / K^2$$

One-loop Cont.

Two regions dominate:

1. Hard momentum $L_\mu = O(T)$

   Every quantity is $O(T)$ except

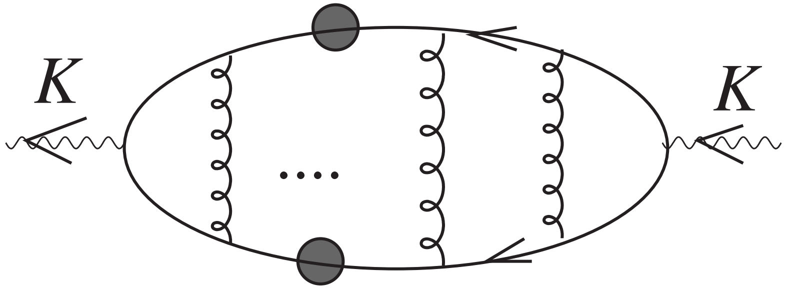
$$\rho(K-L) \sim \frac{\Sigma_I}{T^4} = O(g^2)$$

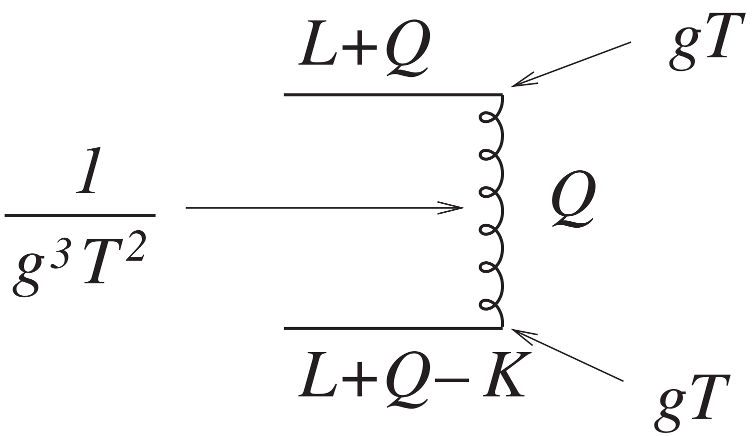
   $\Longrightarrow \Sigma_{I,1} = O(\alpha_{EM}\alpha_s T^2)$

2. Co-linear with K: $L = (O(T), O(gT), O(gT), O(T))$ – Pinching pole contribution

$$\begin{aligned}\Sigma_{I,1} &\sim \alpha_{EM} g^2 T^2 (gT)^2 \frac{\Sigma_I(L-K)}{|(L-K)^2 - m_T^2|^2 + \Sigma_I(L-K)^2} \\ &\sim \alpha_{EM} g^2 T^2 (gT)^2 \frac{\Sigma_I(L-K)}{|2L\cdot K|^2 + \Sigma_I(L-K)^2} \\ &= O(\alpha_{EM}\alpha_s T^2)\end{aligned}$$

Ladder Diagrams





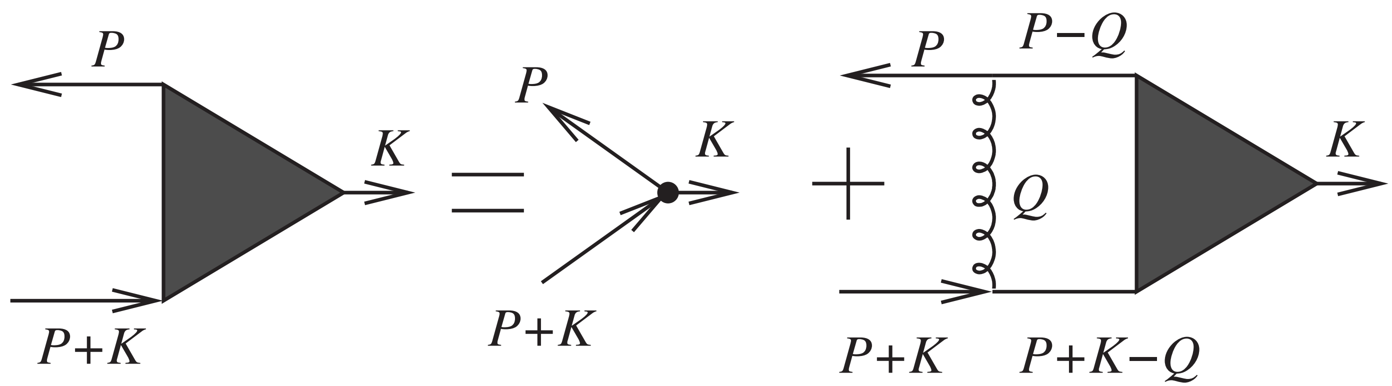
Consider a rung $Q = O(gT)$. Each vertex gT. The gluon propagator is $1/g^3T^2$ (HTL, Bose enhanced). So a rung is $O(1/g)$.

Phase space integral goes like

$$d^3q dq^0 \delta((L+Q)^2 - m_T^2) G(L+Q-K) \sim \frac{g^3T^3}{g^2T^3} \sim g$$

Altogether, adding one more rung is $O(1)$. $\Longrightarrow$ Must resum.

Schwinger-Dyson Equation



After much analysis, simplification: Final results

$$2\mathbf{p}_\perp = i\delta E\,\mathbf{f}(\mathbf{p}_\perp; p_{||}, \mathbf{k}) + g^2 C_R \int_Q 2\pi\delta(q^0 - q_{||}) \frac{m_D^2}{\mathbf{q}_\perp^2(\mathbf{q}_\perp^2 + m_D^2)} \times \Big[\mathbf{f}(\mathbf{p}_\perp; p_{||}, \mathbf{k}) - \mathbf{f}(\mathbf{p}_\perp - \mathbf{q}_\perp; p_{||}, \mathbf{k})\Big]$$

with $\delta E = k^0 + E_p - E_{p+k}$

Photon Radiation Rate

$$\frac{d\Gamma_\gamma}{d^3k} = \frac{d_F q^2 \alpha_{EM}}{4\pi^2 k} \int_{-\infty}^{\infty} \frac{dp_{||}}{2\pi} \int \frac{d\mathbf{p}_\perp}{(2\pi)^2} \left| \mathcal{J}_{p_{||} \leftarrow p_{||}+k} \right|^2 \times \mathrm{Re}\left\{ 2\mathbf{p}_\perp \cdot \mathbf{f}(\mathbf{p}_\perp; p_{||}, \mathbf{k}) \theta(p_{||}) \right\}$$

with

$$\left| \mathcal{J}_{p_{||} \leftarrow p_{||}+k} \right|^2 = \begin{cases} \frac{n_b(k+p_{||})[1+n_b(p_{||})]}{2p_{||}(p_{||}+k)} & \text{scalars} \\ \frac{n_f(k+p_{||})[1-n_b(p_{||})]}{2[p_{||}(p_{||}+k)]^2} \left[p_{||}^2 + (p_{||}+k)^2 \right] & \text{fermions} \end{cases}$$

Generalize to Gluon Radiation

SD Equation for Gluon Radiation

Must take care of:

- Gluon momentum $\mathbf{k}$ can change now.
- Color factors.
- Must keep track of quarks *and* gluons.

$$
\begin{aligned}
2\mathbf{h} &= i\delta E(\mathbf{h},p,k)\mathbf{F}(\mathbf{h}) + g^2 \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2} C(\mathbf{q}_\perp) \times \\
&\times \Big\{ (C_s - C_A/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h}-k\,\mathbf{q}_\perp)] \\
&\quad +(C_A/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h}+p\,\mathbf{q}_\perp)] \\
&\quad +(C_A/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h}-(p-k)\,\mathbf{q}_\perp)] \Big\}, \\
\delta E(\mathbf{h},p,k) &= \frac{\mathbf{h}^2}{2pk(p-k)} + \frac{m_k^2}{2k} + \frac{m_{p-k}^2}{2(p-k)} - \frac{m_p^2}{2p}\,.
\end{aligned}
$$

Gluon − Cont.

Here m^2 are the medium induced thermal masses, equal to $m_D^2/2$ for a gluon and $C_f g_{\mathsf{s}}^2 T^2/4 = g_{\mathsf{s}}^2 T^2/3$ for a quark. For the case of $g \longrightarrow qq$, the $(C_s - C_{\mathsf{A}}/2)$ term is the one with $\mathbf{F}(\mathbf{h} - p\,\mathbf{q}_\perp)$ rather than $\mathbf{F}(\mathbf{h} - k\,\mathbf{q}_\perp)$.

Gluon Radiation Rate

$$\frac{d\Gamma_g(p,k)}{dkdt} = \frac{C_s g_{\mathsf{s}}^2}{16\pi p^7}\frac{1}{1\pm e^{-k/T}}\frac{1}{1\pm e^{-(p-k)/T}} \times$$
$$\times \left\{ \begin{array}{cc} \frac{1+(1-x)^2}{x^3(1-x)^2} & q \to qg \\ N_{\mathsf{f}}\frac{x^2+(1-x)^2}{x^2(1-x)^2} & g \to qq \\ \frac{1+x^4+(1-x)^4}{x^3(1-x)^3} & g \to gg \end{array} \right\}$$
$$\times \int \frac{d^2\mathbf{h}}{(2\pi)^2} 2\mathbf{h} \cdot \mathrm{Re}\,\mathbf{F}(\mathbf{h},p,k)\,,$$

where $x \equiv k/p$ is the momentum fraction in the gluon (or either quark, for the case $g \to qq$).

$\mathbf{h} \equiv \mathbf{p} \times \mathbf{k}$: 2-D vector. $O(gT^2)$

Time evolution equation

$$\frac{dP_{q\bar{q}}(p)}{dt} = \int_k P_{q\bar{q}}(p+k)\frac{d\Gamma^q_{qg}(p+k,k)}{dkdt} - P_{q\bar{q}}(p)\frac{d\Gamma^q_{qg}(p,k)}{dkdt} + 2P_g(p+k)\frac{d\Gamma^g_{q\bar{q}}(p+k,k)}{dkdt},$$

$$\frac{dP_g(p)}{dt} = \int_k P_{q\bar{q}}(p+k)\frac{d\Gamma^q_{qg}(p+k,p)}{dpdt} + P_g(p+k)\frac{d\Gamma^g_{gg}(p+k,k)}{dkdt} - P_g(p)\left(\frac{d\Gamma^g_{q\bar{q}}(p,k)}{dkdt} + \frac{d\Gamma^g_{gg}(p,k)}{dkdt}\Theta(2k-p)\right),$$

- k integrals range: $(-\infty, \infty)$.
- $k < 0$: Absorption of thermal gluons.
- $k > p$: annihilation against and antiquark of energy $(k - p)$.
- $\Theta(2k - p)$: To prevent double counting of final states.

Relationship to BDMPS

(only to the equation, but not the solution)

(c.f. Eq.(20) in BDMS NPB **531**, 403 (1998).)

$$\frac{\partial}{\partial t} f(U,V,t) = \frac{i(U-xV)^2\mu^2}{2x(1-x)p} f(U,V,t)$$
$$+ \frac{\rho\sigma}{C_F} \int d^2Q\, V(Q^2) \Big[\frac{N_c}{2} f(U-Q,V-Q,t) - \frac{1}{2N_c} f(U,V-Q,t)$$
$$- \frac{N_c}{2} f(U,V,t) - \frac{C_F}{2} f(U,V,t) - \frac{C_F}{2} f(U,V,t) + \frac{N_c}{2} f(U-Q,V,t) \Big]$$

Can identify: $\int_{t_1}^{\infty} dt'\, f(U,V,t) \leftrightarrow \mathbf{F}(p,k$

Except: BDMPS use (i) $V(Q^2) = 1/(Q^2+m_D^2)^2$ instead of HTL result $1/[Q^2(Q^2+m_D^2)]$. (ii) $\delta E = \mathbf{h}^2/2pk(p-k)$ (mass terms are missing)

Relationship with BDMPS

(solutions)

- BDMPS typically solve the equation and calculate $\frac{dI}{d\omega}$ in a large h approximation, valid for large $p/T, k/T$ but unreliable for $k \leq 10T$.
- Time (medium) evolution: Probability to lose energy p

$$P(p) = \sum_{n=0}^{\infty} \frac{1}{n!} \left[\prod_{i=1}^{n} \int_0^{\infty} d\omega_i, \frac{dI(\omega_i)}{d\omega} \right] \delta(p - \sum_{i=1}^{n} \omega_i) \exp\left(- \int_0^{\infty} d\omega \frac{dI}{d\omega} \right)$$

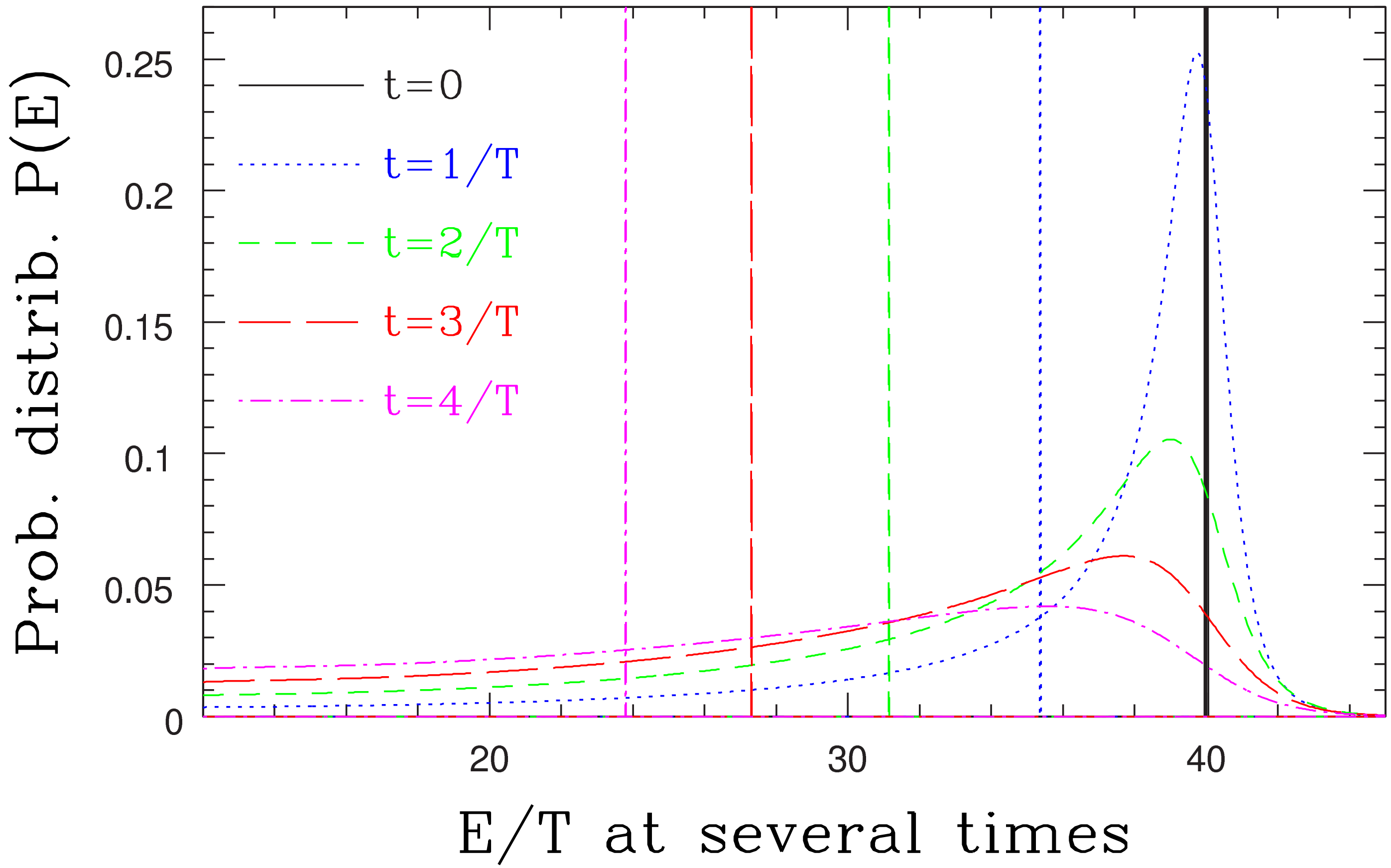
- This is the solution of

$$\frac{dP}{dt} = \int d\omega \Gamma_{BDMPS}(\omega) P(p+\omega, t) - P(p,t) \int d\omega \Gamma_{BDMPS}(\omega) \quad (1)$$

We do (only displaying one component), with $-\infty < \omega < \infty$.

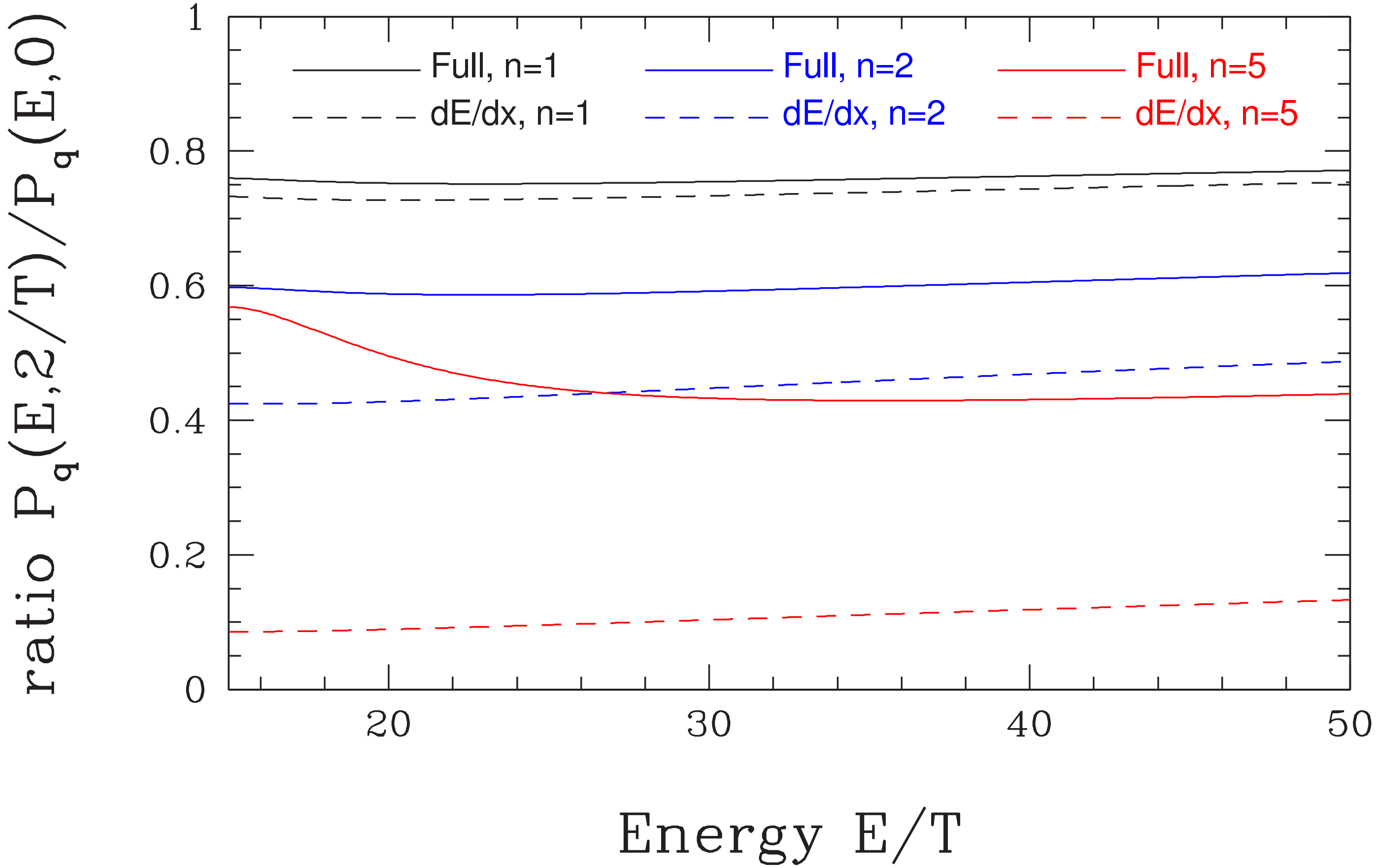
$$\frac{dP}{dt} = \int d\omega \Gamma(p+\omega, \omega) P(p+\omega, t) - P(p,t) \int d\omega \Gamma(p, \omega) \quad (2)$$

Results – Evolution



Lesson: Using just dE/dx is dangerous.

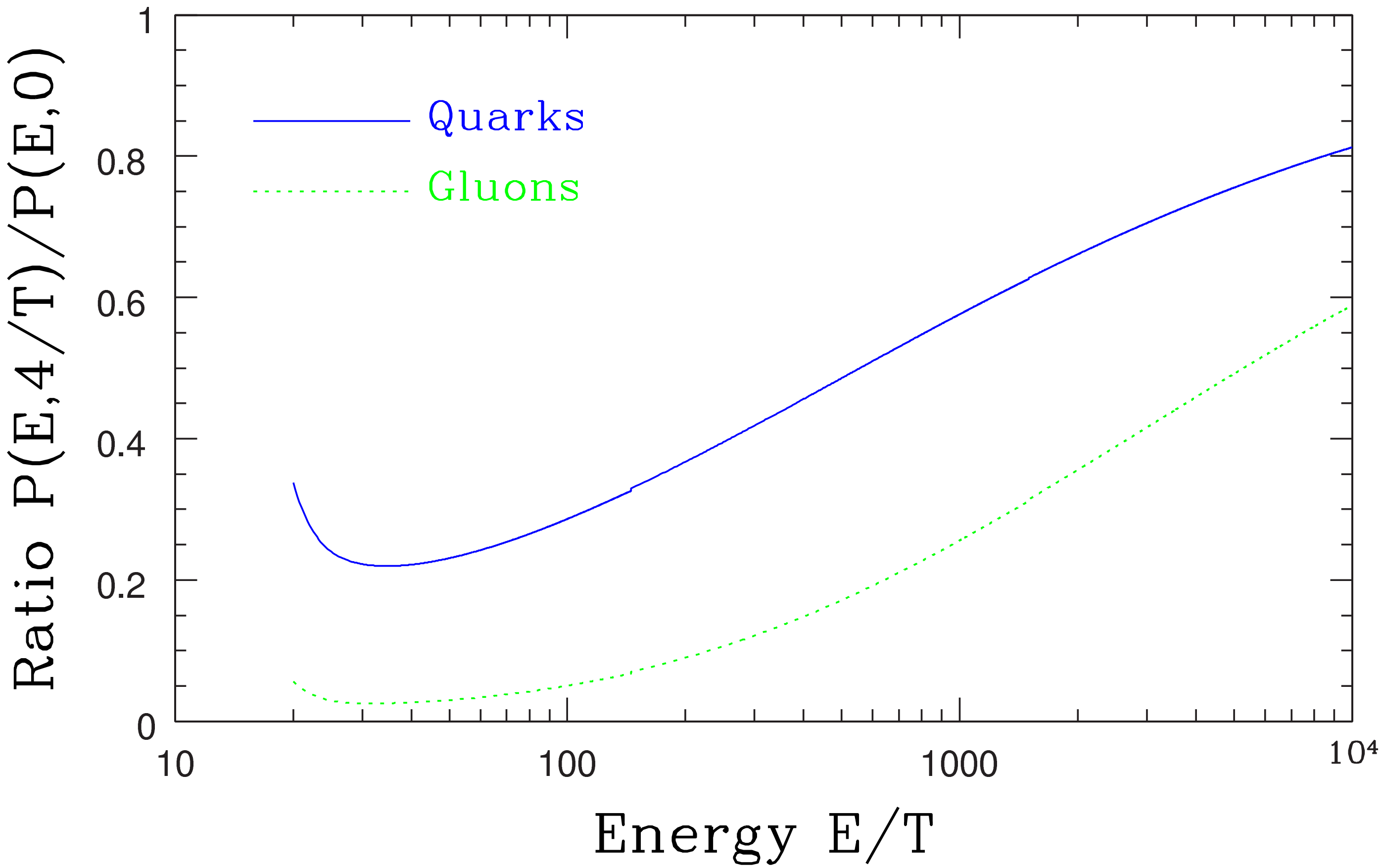
Parton distribution ratios



This is for illustration only. Using $P(p,0) = 1/(p^2 + p_0^2)^n$.

Lesson: Using just dE/dx is dangerous!

Parton distribution ratios



This is for illustration only.

Extreme LPM limit is reached, but only at very high energies.

Understanding the ratio

Use BDMPS expression for the quenching factor for $1/p^n$ with large n but with the energy range extended to $\omega < 0$:

$$R_{AA}(p) \approx \exp\left(-(1 - e^{-\omega n/p}) \int_{-\infty}^{\infty} d\omega \int_0^t dt' \, \Gamma(p, \omega, t)\right) \qquad (3)$$

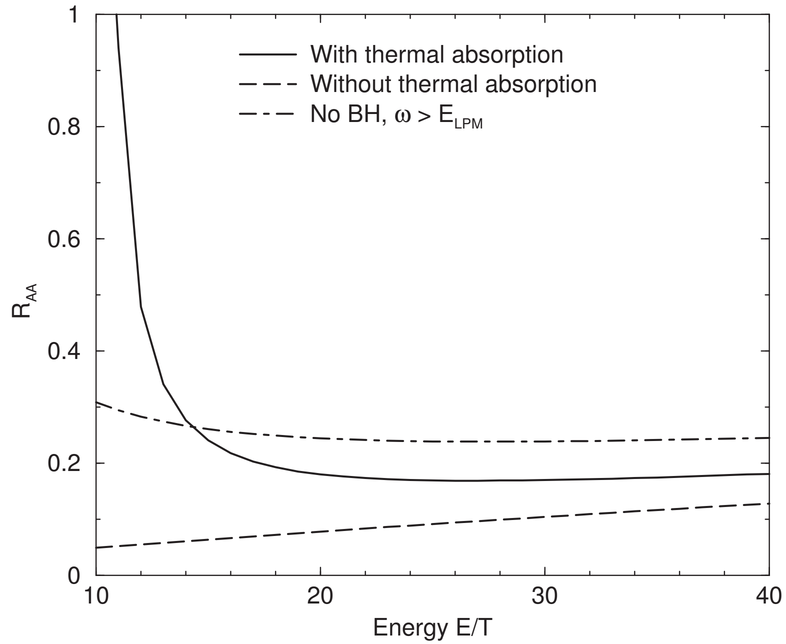
For Γ, use simple estimates

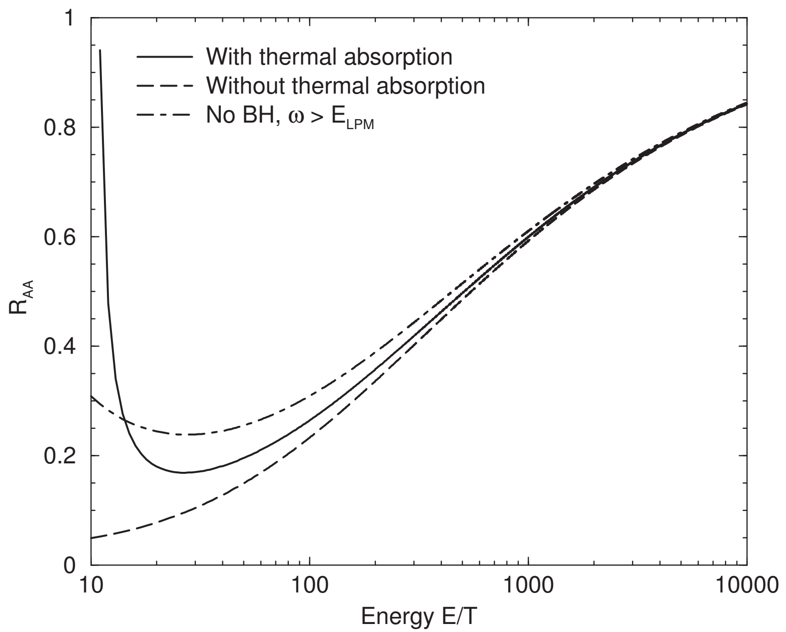
$$\omega \frac{dI}{d\omega dt} \approx \frac{\alpha}{\pi} \frac{N_c}{\lambda} \quad \text{for} \quad 0 < \omega < \lambda\mu^2$$

$$\omega \frac{dI}{d\omega dt} \approx \frac{\alpha}{\pi} N_c \sqrt{\frac{\mu^2}{\lambda\omega}} \quad \text{for} \quad \lambda\mu^2 < \omega < \lambda\mu^2 (L/\lambda)^2$$

$$\frac{dI}{d\omega dt} \approx \frac{\alpha}{\pi|\omega|} \frac{N_c}{\lambda} e^{-|\omega|/T} \quad \text{for} \quad \omega < 0$$

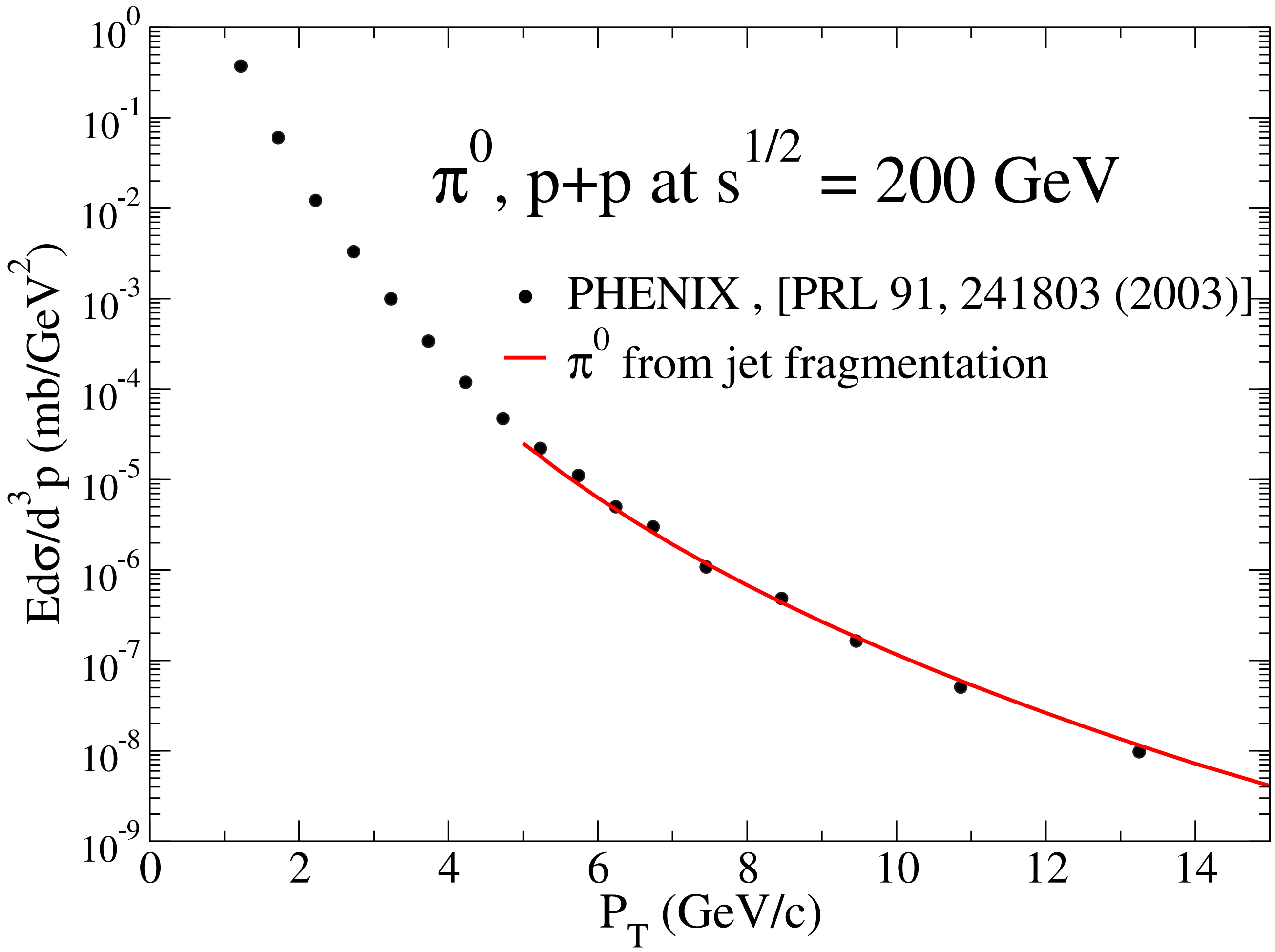
Understanding the ratio – Cont.





- Features are roughly produced.
- Flat ratio due to energy-loss *and* thermal absorption
- BH part of the energy-loss is important.

Baseline calculation



Using P.Aurenche et al.'s program.

Pion Production

$$\frac{dN_{AA}}{dyd^2\mathbf{p}_T} = \frac{\langle N_{\rm coll}\rangle}{\sigma_{in}} \sum_{a,b,c,d} \int dx_a dx_b \, g_A(x_a,Q) g_A(x_b,Q) \times K_{\rm jet} \frac{d\sigma_{a+b\to c+d}}{dt} \frac{\tilde{D}_{\pi^0/c}(z,Q)}{\pi z}$$

with

$$\tilde{D}_{\pi^0/c}(z,Q) = \int d^2 r_\perp \, \mathcal{P}(\mathbf{r}_\perp) \tilde{D}_{\pi^0/c}(z,Q,\mathbf{r}_\perp,\mathbf{n})$$

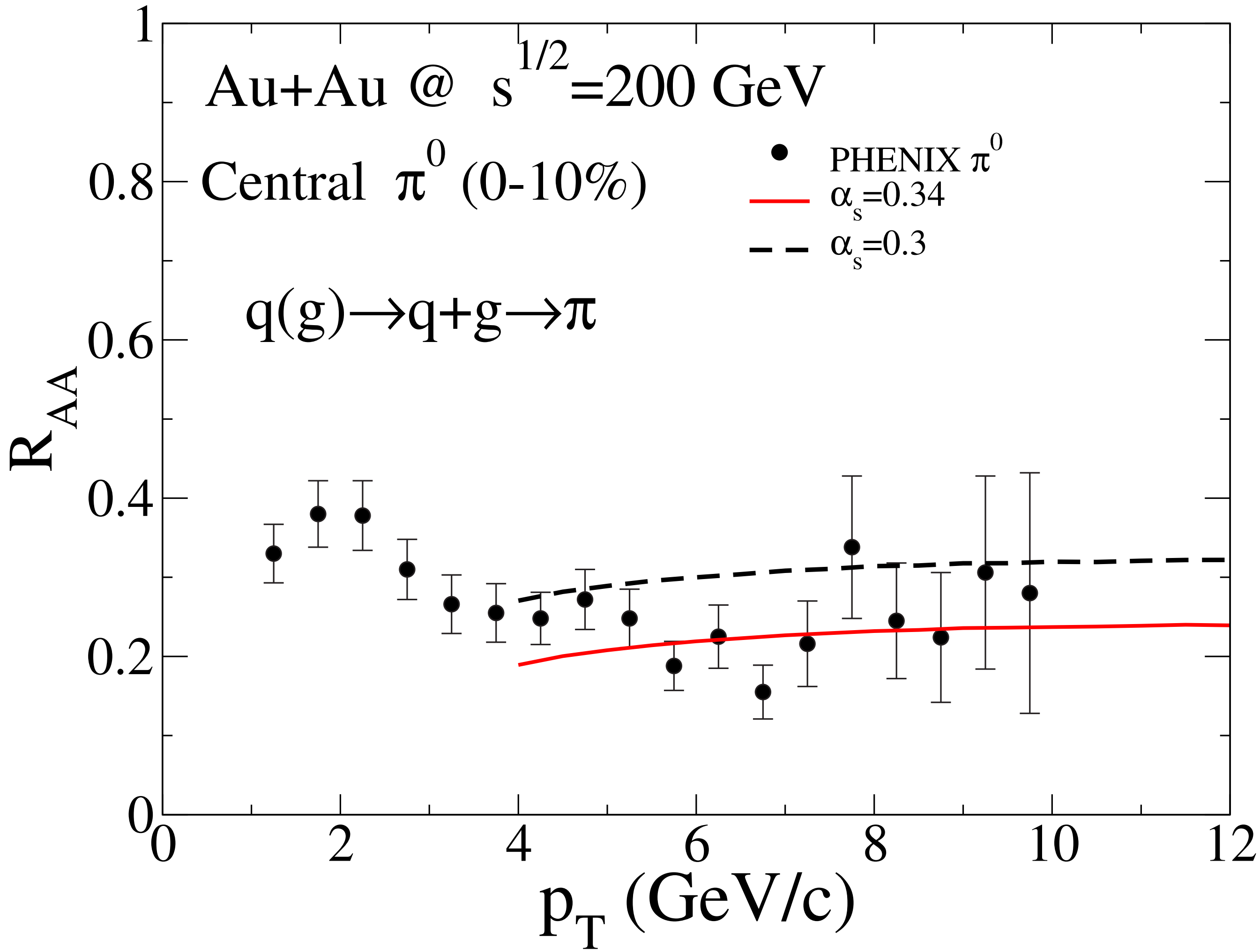
and

$$\tilde{D}_{\pi^0/c}(z,Q,\mathbf{r}_\perp,\mathbf{n}) = \int dp_f \frac{z'}{z} \Big(P_{qq/c}(p_f;p_i;\Delta t)\, D_{\pi^0/c}(z',Q) + P_{g/c}(p_f;p_i;\Delta t)\, D_{\pi^0/c}(z',Q) \Big)$$

with $z = p_T/p_i$ and $z' = p_T/p_f$.

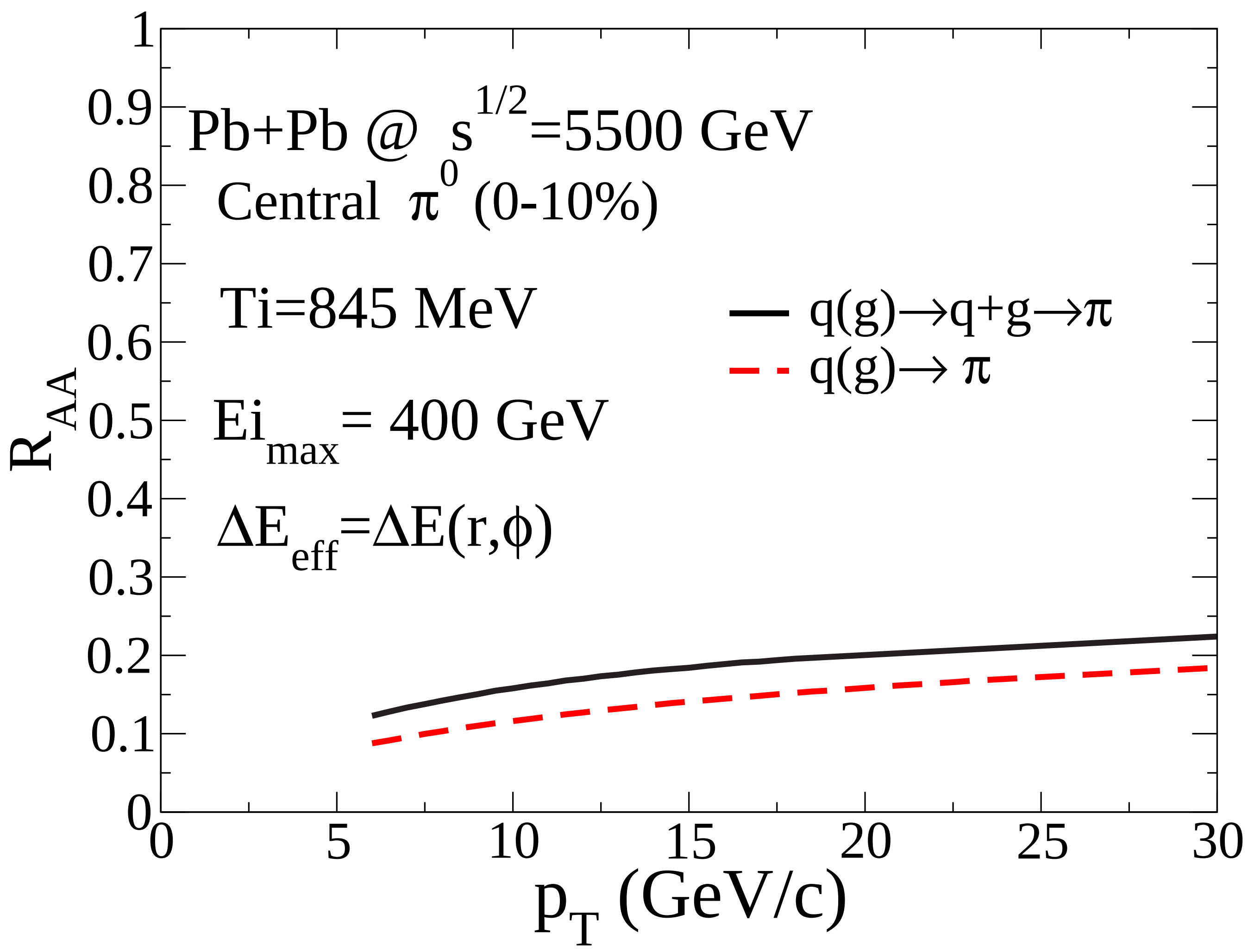
Δt determined by the location of the production $\mathbf{r}$ and the direction of the jet $\mathbf{n}$.

Nuclear Modification Factor

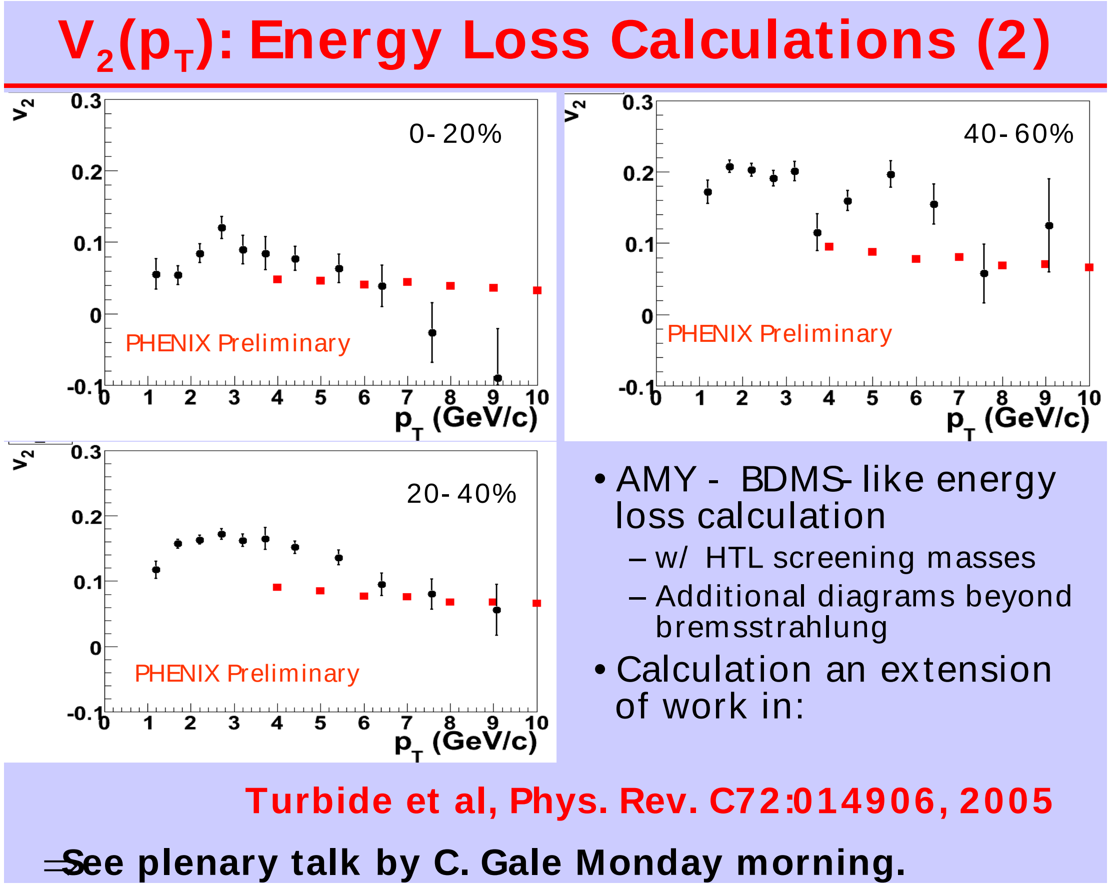


1-D expansion included.

Nuclear Modification Factor (LHC)



High p_T v_2



Brian Cole's QM05 plenary session slide. Mostly geometry.

γ from jets and QGP

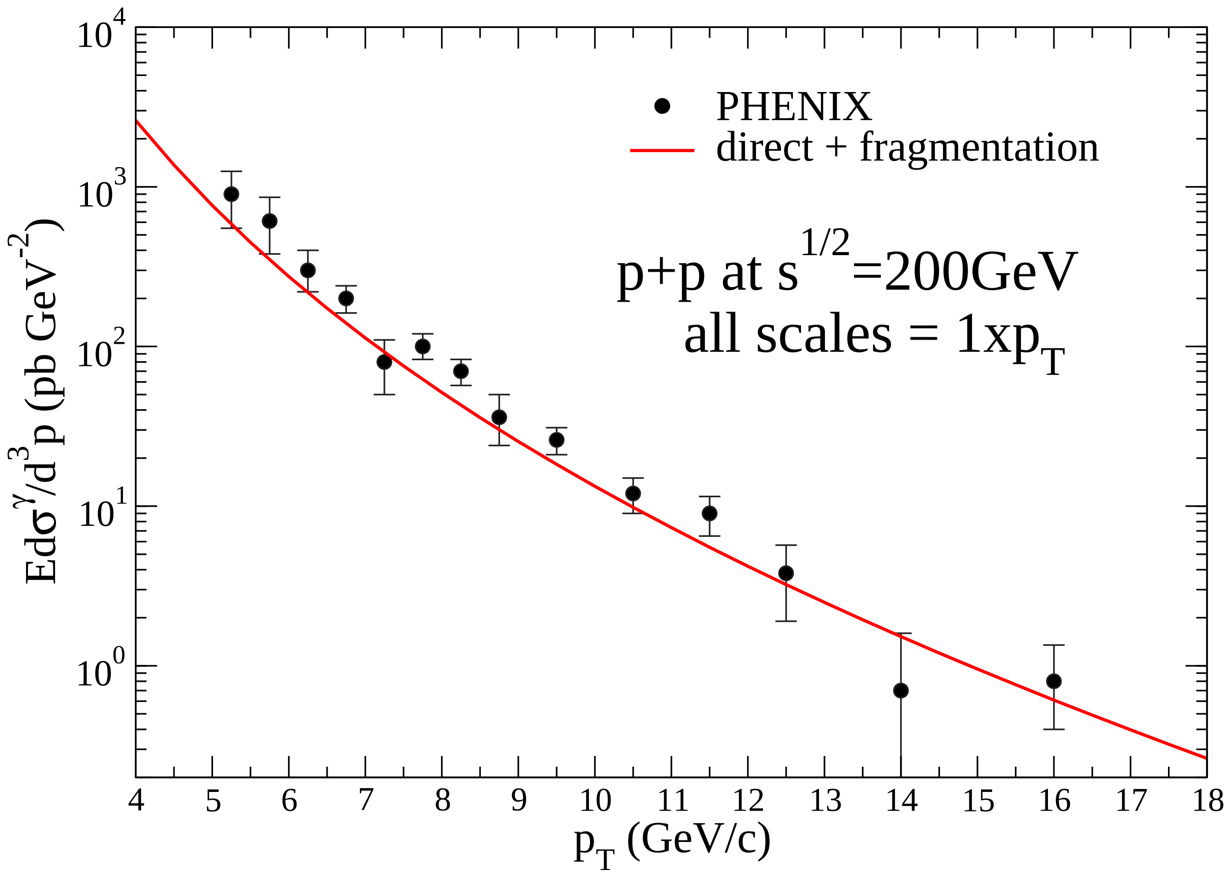
Simon Turbide's Ph.D. Thesis work (w/ Charles Gale).

Ref: Turbide, Gale, Jeon and Moore, PRC 72, 014906, 2005.

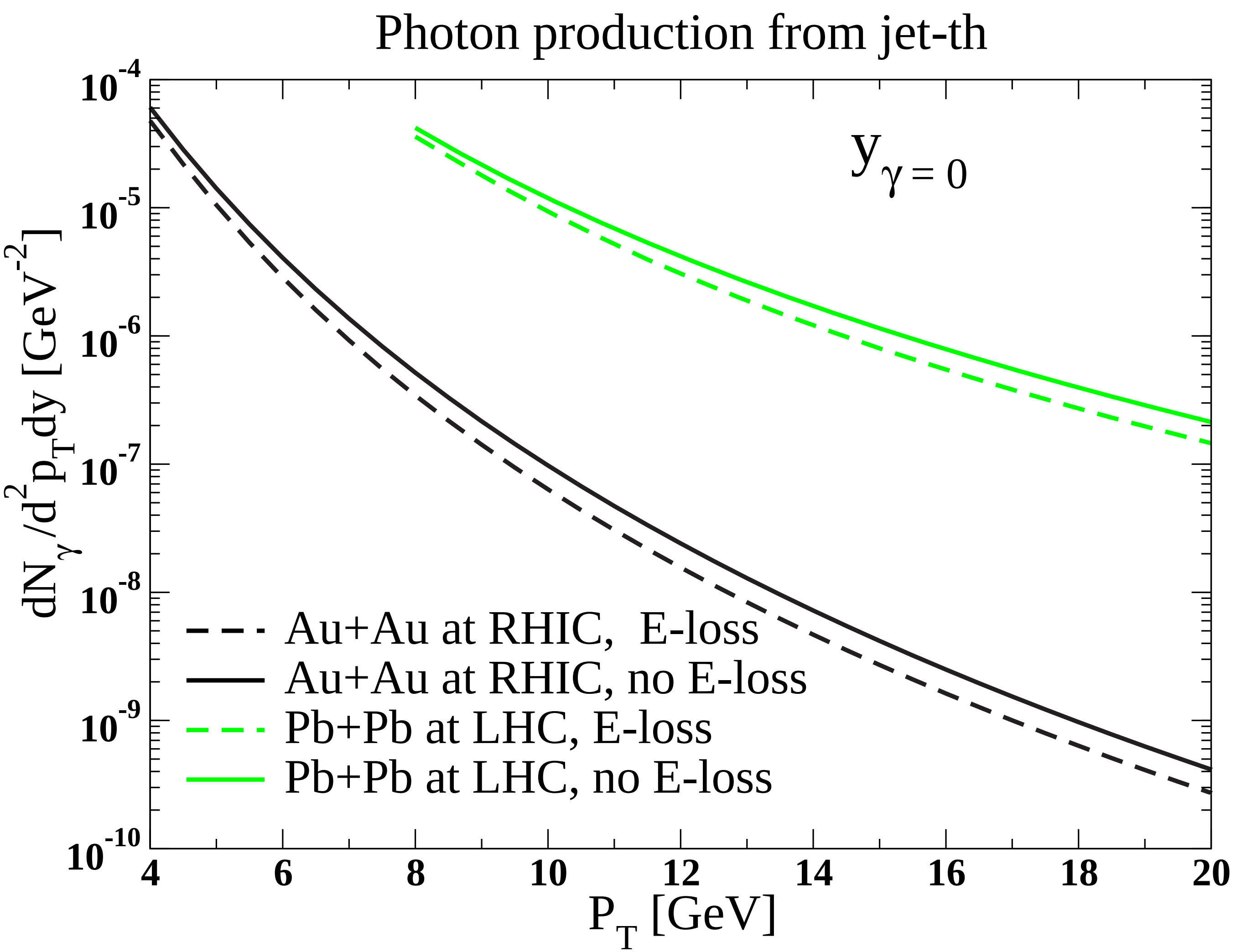
Photon sources:

- direct photons
- jet bremsstrahlung
- jet + thermal $q\bar{q} \to g\gamma$ and $gq \to \gamma q$
- thermal + thermal $q\bar{q} \to g\gamma$ and $gq \to \gamma q$
- jet fragmentation

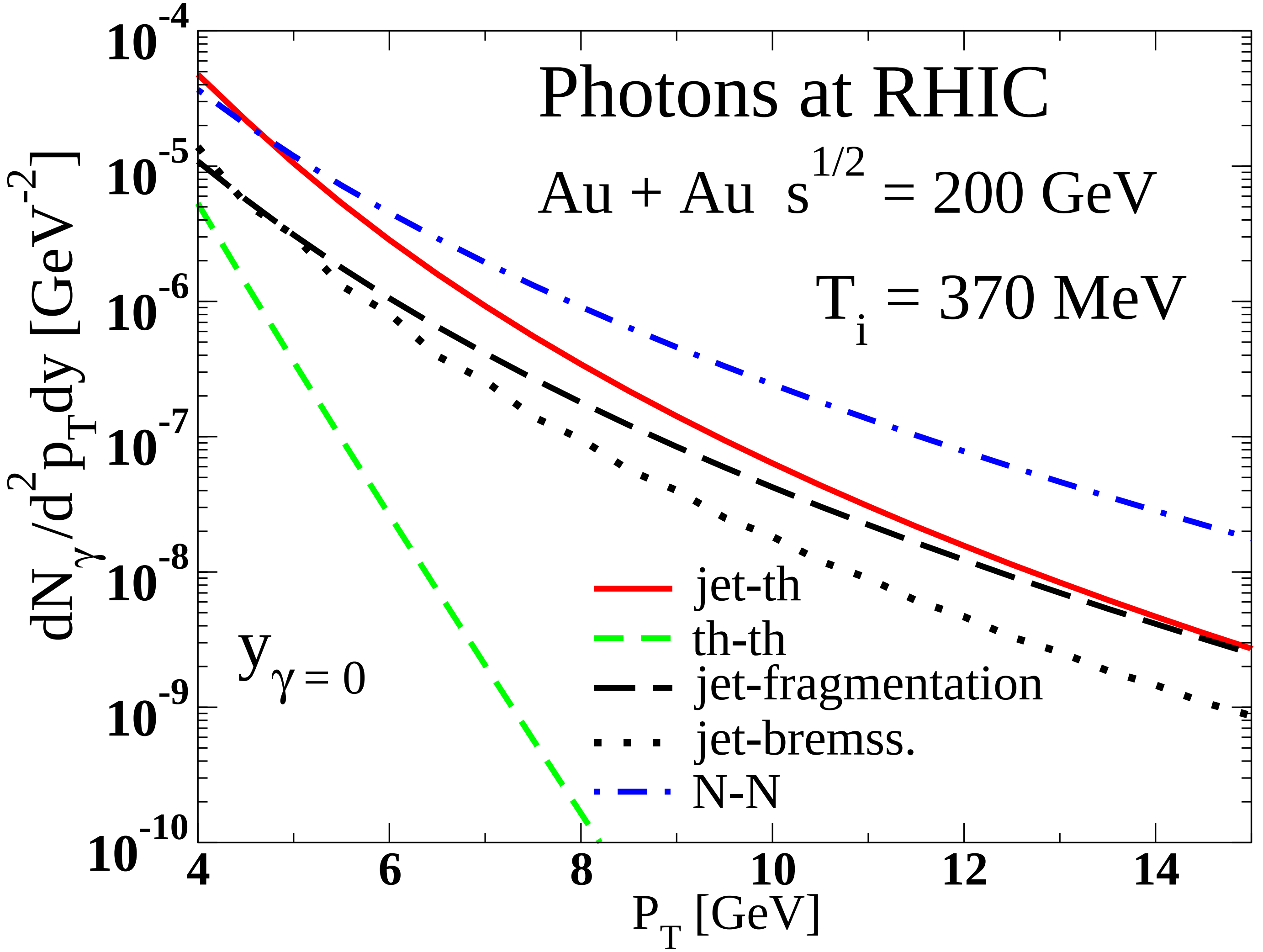
γ − Baseline



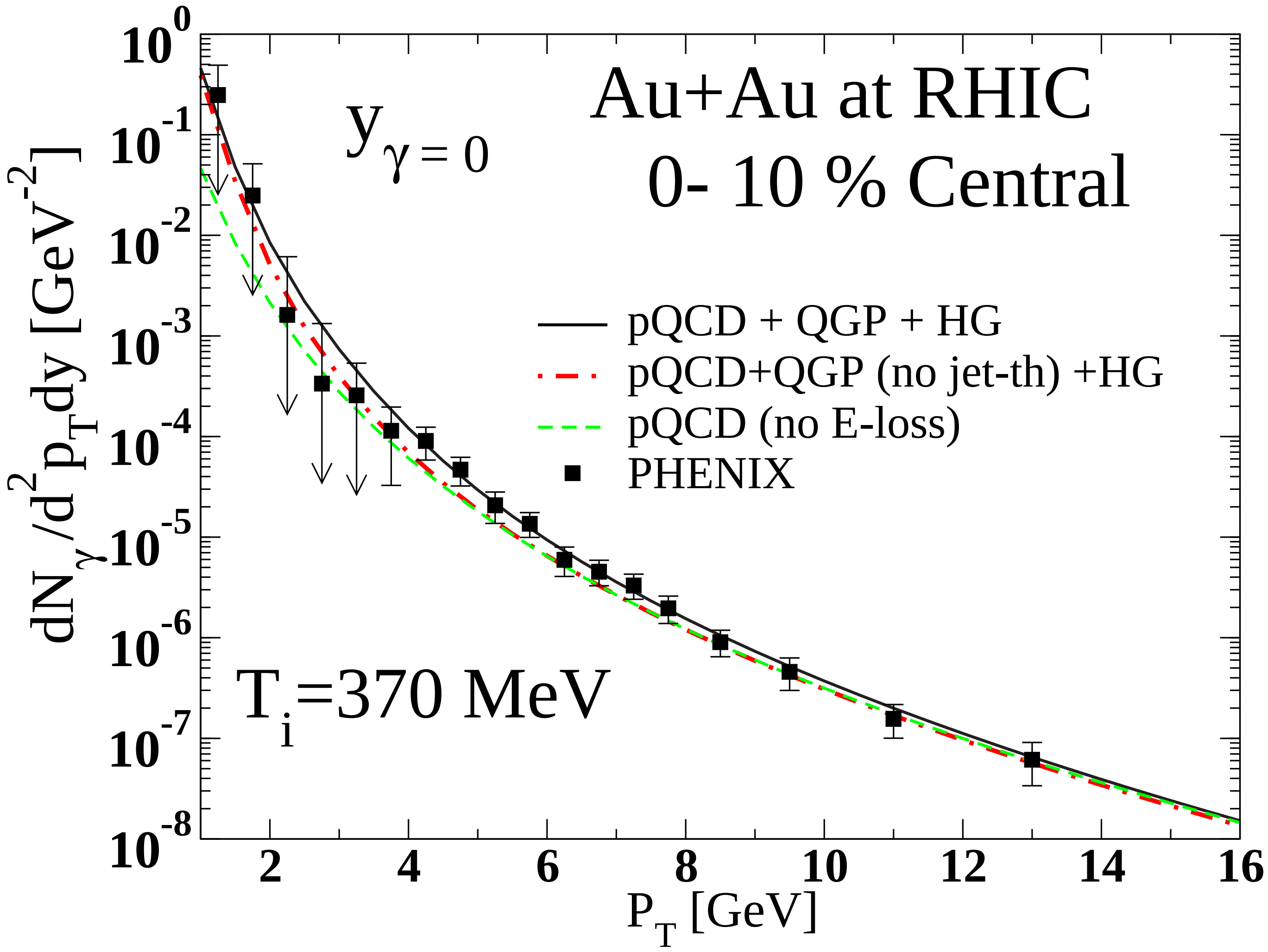
γ – Effect of parton energy loss



γ − Composition

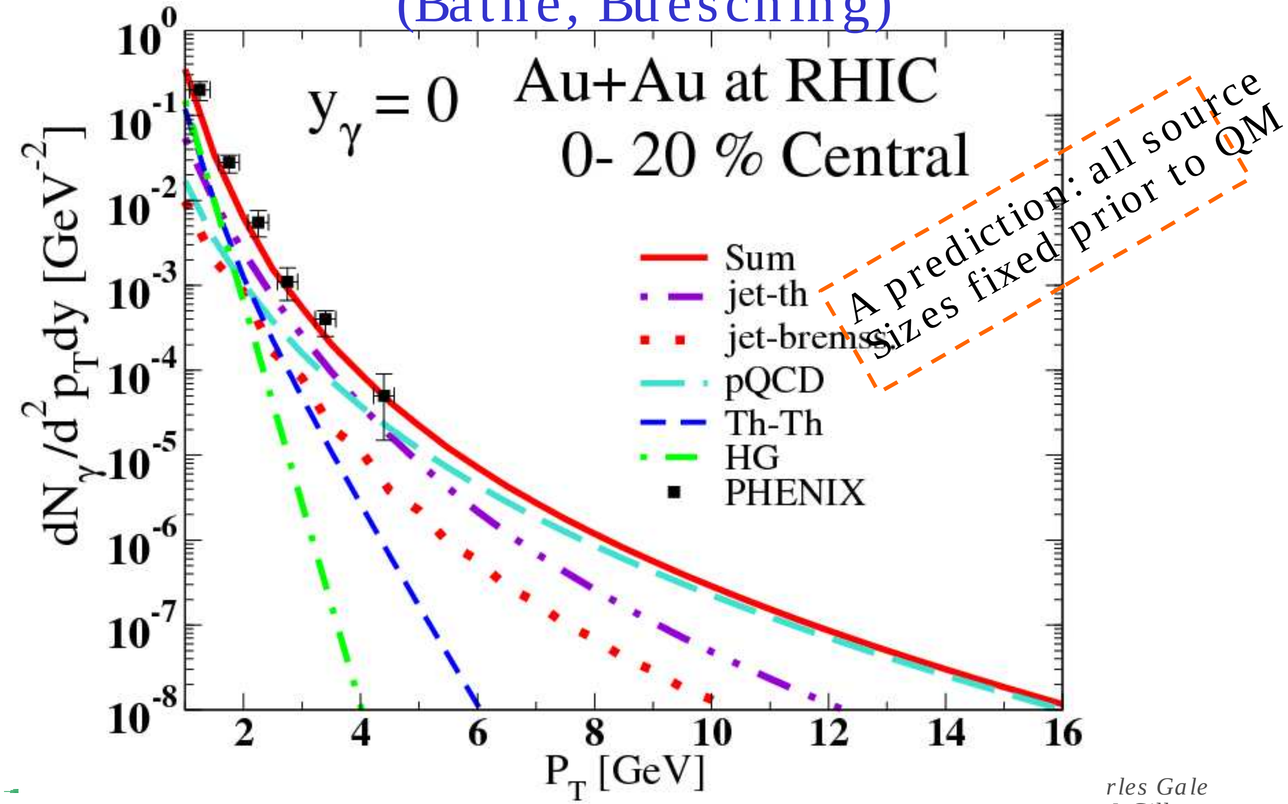


γ – PHENIX pre-QM05



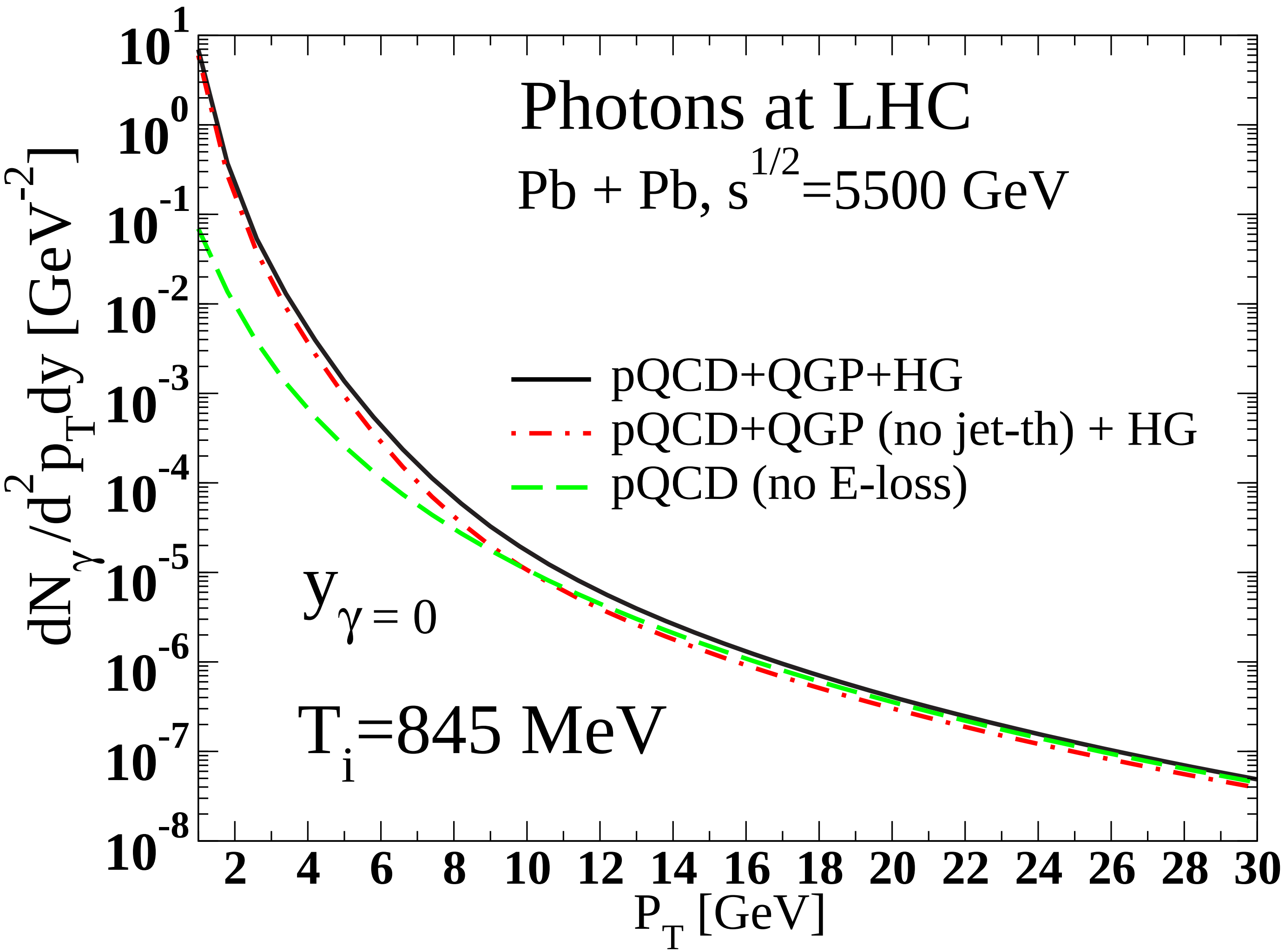
γ – PHENIX QM05

New (preliminary) PHENIX Data
(Bathe, Buesching)

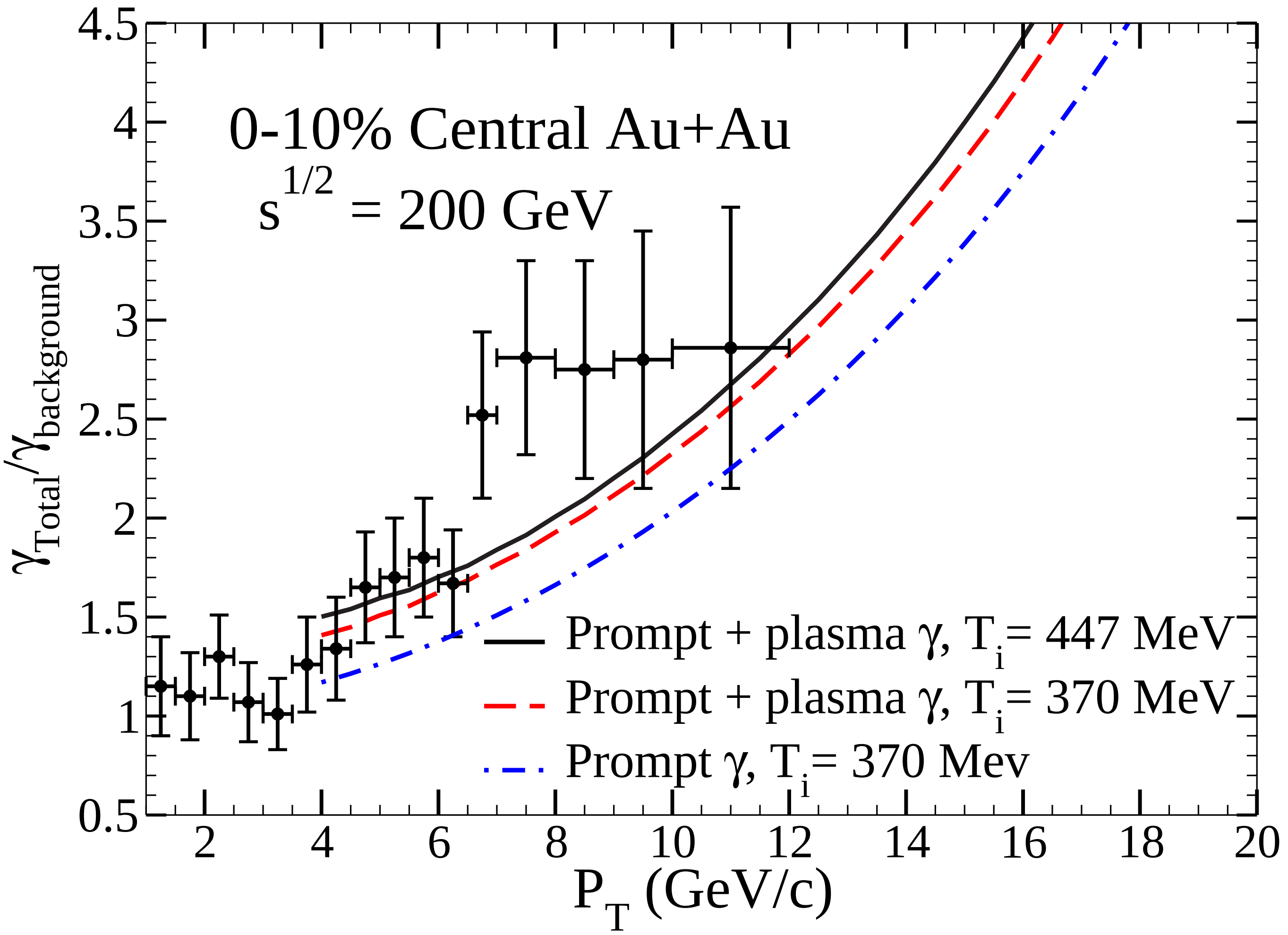


rles Gale
McGill

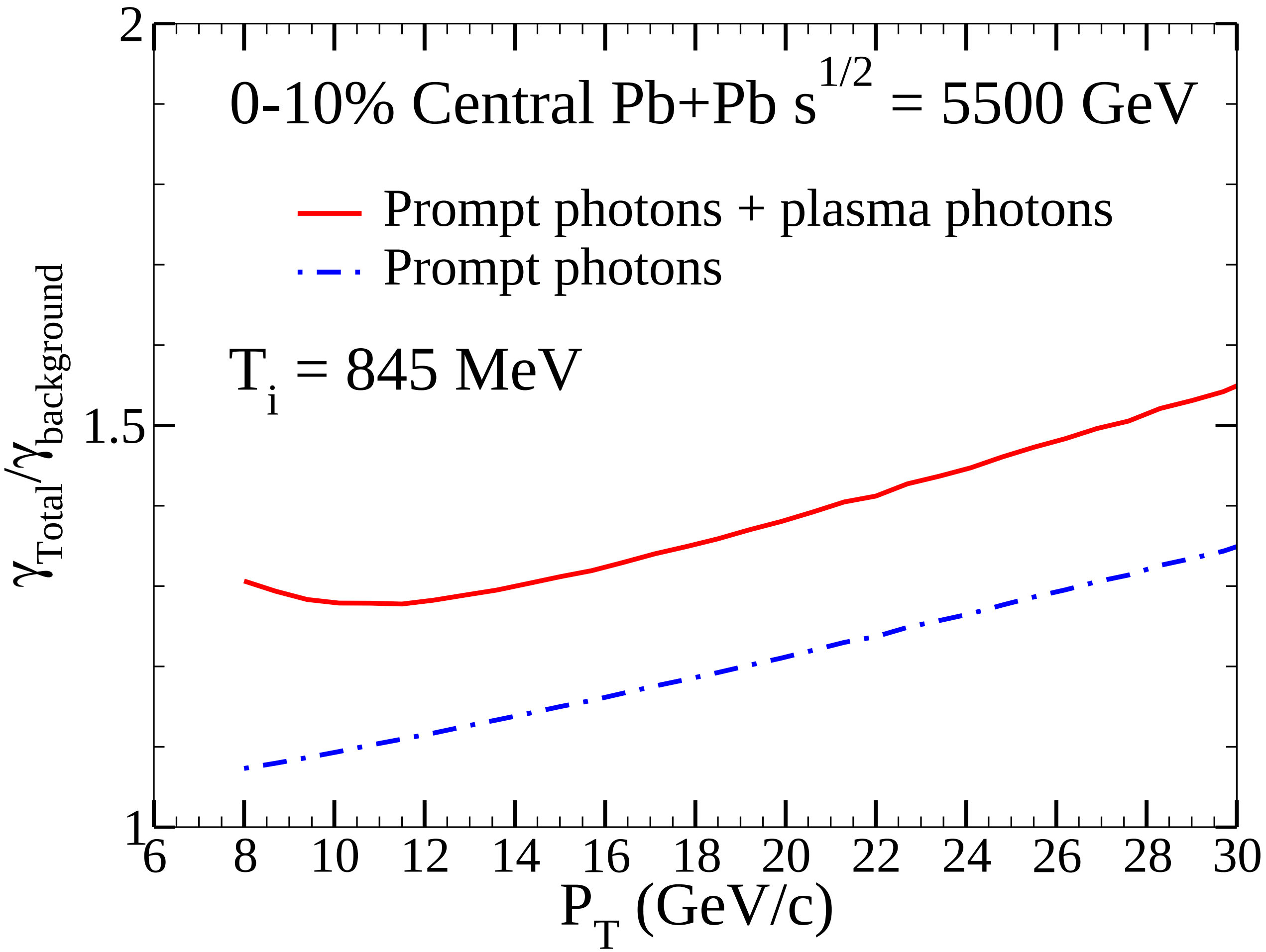
γ – LHC prediction



γ/γ Ratio – PHENIX



γ/γ Ratio – LHC prediction



Conclusions and Caveats

The Good

- Calculated and used the full leading order Hot QCD radiation rates for both gluons and photons.
- Important to use the full momentum distribution at any given time, not just dE/dx.
- Geometry and 1-D expansion included.
- Good description of existing data – pions and photons.
- For photons, jet-thermal interaction is crucial.
- LHC predictions – Should be better since pQCD should work better there.

The Bad

- Calculations consistent in the $g \ll 1$ limit for momenta $T < p$. Yet for quantitative calculations, we needed $\alpha_s \approx 1/3$ or $g \approx 2$! So in reality, one must sum all diagrams, not just pinching part of the ladder diagrams!
  - At this leading order, α_s is an overall factor. So one might hope that the structure of the solution is OK.
  - Right now, this is best we can do with perturbative calculation.
- $Q = p_T$?
  - Factor of 2 either way doesn't change results too much.
  - But really it should be something like μp_T (or may even be $Q_0^2 + \Delta k_T^2$).

The Ugly (Ducklings)

- Do better job with the medium evolution? – Need 3-D hydro code. Working on it.

- What about jet correlations? – Need to keep track of the evolution of the joint probability function of two jet energies. Single particle distribution was hard enough!

- Most energy-loss calculations these days do get R_{AA} right. Is there an experimental way to distinguish? – Maybe. Photon bremsstrahlung due to acceleration should be able to distinguish few hard radiations and many soft radiations. How to fish that out of all others is another matter. Maybe some corner of momentum space? Will work on it.