## Space-time evolution of hadronization in heavy-ion collisions at RHIC

# Arata Hayashigaki

(Goethe Univ. Germany)

#### collaboration with

Boris Kopeliovich (Santa Maria Univ. Chile) Jan Nemchik (IEP-SAV Slovakia)

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Goethe Univ. A. Hayashigaki



Slide 2

- **A.H.6** Let's start with s-t evolution of high energy heavy-ion collision described by Bjorken model. Here the head-on collison of two equal nuclei in the CM frame can be considered on this t-z plane, where t is a clock on the lab. frame and z is collisional axis. So now we assume the translational invariance in the transverse direction. First, the projectile nucleus A comes from z=-infinity wiht a velocity close to the speed of light and the target nucleus B comes from +infinity similarly. So these nulei have a Lorentz contraction in the z-direction like thin disks. Then at t=0, z=0 they meet and collide, then they run away each other on this light-cone. Arata Hayashigaki, 10/4/2005
- A.H.7 In a standard scenario, thus deposited energy in this hot/dense matter afer collision is carried away by parton production, and this leads to equilibration, hadrodynamical QGP, hadronization of parton and finally hadron freeze out. Their regions are naively classified by each proper time line in the frame of matter. Here tau\_0 means thermalization time at which plasma is produced. If this plasma formation time is comparable to the particle production time, at which particles begin to be produced out of the field spontaneously, this could be in the range of 0.4(Schwinger particle production mechanism) to 1.2 fm(QED model, Lund model), naively.

#### Arata Hayashigaki, 10/4/2005

**A.H.8** If we consider high-pt energetic parton production at mid-rapidity region, then in this Bjorken's model transverse flow is neglected and matter is at rest at z=0, of course, then tau equals normal t.

I think this energetic parton obey different s-t evolution from these whole background particles.

I mean, especially in production time discussed later.

So then, this would be a good probe of the matter.

Arata Hayashigaki, 10/4/2005

## Outline of strategy

1. Construct parton fragmentation  $D_{h/a}(z_h, Q^2)$ coherence time, vacuum EL, ... distribution function  $W(t_p)$  in pQCD model  $D_{h/q}$  vs.  $W(t_p)$ ,  $< t_p(z_h, Q^2) >$ 2. (pre-)hadron evolution ( $t_f$ ) after color-bremsstr. light-cone Green function method 3. Numerics in Au+Au  $\rightarrow \pi^0$ +X at RHIC(y=0) high-p<sub>T</sub> spectrum of  $R_{\Delta\Delta}$ 4. Summary

### **Parton fragmentation**



$$\sum_{h} \int_{0}^{1} dz_{h} z_{h} D_{h/q}(z_{h}, Q^{2}) = 1 \quad \text{: momentum conservation}$$
$$\int_{0}^{1} dz_{h} D_{h/q}(z_{h}, Q^{2}) = \langle n_{h} \rangle \quad \text{: average multiplicity for each h}$$

If we focuses on only one hadron containing an energetic parent parton,

$$\int_0^1 dz_h D_{h/q}(z_h, Q^2) = 1$$

#### **BKK, KKP, Kretzer parametrizations:**

$$D_{h/q}(\boldsymbol{z}_h, \boldsymbol{Q}^2) \propto \boldsymbol{z}_h^{\alpha(\boldsymbol{Q}^2)} (1 - \boldsymbol{z}_h)^{\beta(\boldsymbol{Q}^2)} (1 + \gamma(\boldsymbol{Q}^2) / \boldsymbol{z}_h)$$

**Define S-T pattern of hadronization** 

$$D_{h/q}(z_h,Q^2) = \int_0^\infty dt \ W_q(t,z_h,Q^2)$$

Dynamically two separate stages for  $W_a$ 







#### *high-p<sub>T</sub>* hadron productions at RHIC:

"Less" important parton energy loss even in hot/dense matter?! "More" important attenuation of compact pre-hadron?!

 $< r_T >_{q\bar{q}} \leq r_D = 0.3 \sim 0.8 \, fm$ 

$$\boldsymbol{D}_{h/q}^{mat}(\boldsymbol{z}_h, \boldsymbol{Q}^2) = \boldsymbol{D}_{h/q}^{vac}(\boldsymbol{z}_h, \boldsymbol{Q}^2) + \Delta \boldsymbol{D}_{h/q}(\boldsymbol{z}_h, \boldsymbol{Q}^2)$$

#### Additional effect to S-T pattern by matter

 $\Delta \boldsymbol{k}_{\star}$ Multiple scattering with matter Absorption of colorless qqbar dipole by matter Medium-induced energy loss hadron suppression  $\Delta E_{vac} \Rightarrow \Delta E_{vac} + \Delta E_{ind}$ hadron suppression

 $\mathbf{k}_{t}$ -broadening of a parton

 $Q^2 \Rightarrow Q^2 + \Delta k_t^2$ 

Leading hadron ( $z_h \ge 0.5$ )  $D_{h/q}^{mat}(z_h, Q^2) < D_{h/q}^{vac}(z_h, Q^2)$ 

#### **Radiative energy loss of a parton**



#### Vacuum energy loss in pQCD model





#### Quark distribution function $W_{\alpha}$ in pQCD model



#### <u>Gluon distribution function $W_{\alpha}$ in pQCD model</u>



$$W_{g}(t; \boldsymbol{z}_{h}, \boldsymbol{\nu}, \boldsymbol{Q}^{2})$$

$$\propto \int d\alpha \ dk_{T}^{2} \ dy \ dz \ \frac{dn_{g}}{d\alpha \ dk_{T}^{2}} \ \frac{d}{dt} \left(1 - e^{-t/t_{c}}\right) \delta\left(\boldsymbol{z}_{h} - \frac{\boldsymbol{E}_{g}(t)}{\boldsymbol{\nu}} \left\{\boldsymbol{z} + \alpha(\boldsymbol{y} - \boldsymbol{z})\right\}\right)$$

$$\times \int d\beta \ dl_{T}^{2} \ \delta\left(\boldsymbol{I}_{T}^{2} - \frac{(\boldsymbol{y} + \boldsymbol{z})^{2}}{4} \ \boldsymbol{k}_{T}^{2}\right) \delta\left(\beta - \frac{\alpha \boldsymbol{y}}{\alpha \boldsymbol{y} + (1 - \alpha)\boldsymbol{z}}\right) \left|\Psi_{h}(\beta, \boldsymbol{I}_{T}^{2})\right|^{2}$$

$$\times \exp\left[-\frac{\gamma}{2\boldsymbol{\nu}} \left(\frac{\boldsymbol{z}_{h}}{1 - \boldsymbol{z}_{h}}\right)^{\kappa} (\boldsymbol{Q}^{2} - \lambda^{2}) t\right]$$

$$3$$

Normalization of distribution functions  $W_{q,q}$ 

$$D_{q,g}(\boldsymbol{z}_h, \boldsymbol{v}, \boldsymbol{Q}^2) = \int_0^\infty dt \ W_{q,g}(t; \boldsymbol{z}_h, \boldsymbol{v}, \boldsymbol{Q}^2)$$

$$\int_0^1 dz_h \int_0^\infty dt W_{q,g}(t; z_h, v, Q^2) = \langle n_h \rangle \implies 1$$

$$D_{q,g}(z_h, v, Q^2) = \frac{\int_0^\infty dt \ W_{q,g}(t; z_h, v, Q^2)}{\int_0^1 dz_h \int_0^\infty dt \ W_{q,g}(t; z_h, v, Q^2)}$$

 $D_{q,g}^{\pi^0}(z_h, Q^2)$  : BKK,KKP parametrizations

#### VS.

$$D_{q,g}^{\pi^{0}}(z_{h},\nu,Q^{2}=\nu^{2}) < r_{\pi}^{2} >_{em} = 0.44 \text{ fm}^{2}$$
$$a_{0} = 1/12$$









 $D_g(z_{h^\prime}Q^2)$ 

$$t \sim t_{p} \propto \frac{(1-z_{h})\nu}{Q^{2}} \xrightarrow[Q^{2} \sim p_{T}]{Q^{2} \sim p_{T}}} \frac{1-z_{h}}{p_{T}}$$

$$< t_{p}(p_{T}) > = \frac{\int_{0}^{\infty} dt_{p} \int_{0}^{1} dz_{h} t_{p} W_{q,g}(t_{p}; z_{h}, p_{T})}{\int_{0}^{\infty} dt_{p} \int_{0}^{1} dz_{h} W_{q,g}(t_{p}; z_{h}, p_{T})}$$

$$< t_{p}(\boldsymbol{z}_{h},\boldsymbol{p}_{T}) > = \frac{\int_{0}^{\infty} dt_{p} t_{p} W_{q,g}(t_{p};\boldsymbol{z}_{h},\boldsymbol{p}_{T})}{\int_{0}^{\infty} dt_{p} W_{q,g}(t_{p};\boldsymbol{z}_{h},\boldsymbol{p}_{T})}$$





#### **Dipole propagation during formation time t**<sub>f</sub>



Two-dimensional LC Schroedinger eq. :

$$i\frac{d}{dt_2}G_{q\bar{q}}(t_2,\vec{r}_2;t_1,\vec{r}_1 \mid \rho_{matter}(t))$$

$$= \left[\frac{m_q^2 - \Delta_{\vec{r}_2}}{2E_h\alpha(1-\alpha)} + V_{q\bar{q}}(t_2,\vec{r}_2,\alpha)\right]G_{q\bar{q}}(t_2,\vec{r}_2;t_1,\vec{r}_1 \mid \rho_{matter}(t))$$

$$V_{q\bar{q}}(t,\vec{r},\alpha) = \left[\frac{a(\alpha)^4}{2E_h\alpha(1-\alpha)}\vec{r}^2\right]_{vac} + i\left[-\frac{1}{2}\rho_{matter}(t) C_1(E_h)\vec{r}^2\right]_{matter}$$

dipole approx.

**Boundary condition:** 

$$\boldsymbol{G}_{q\bar{q}}(\boldsymbol{t}_{2},\boldsymbol{\vec{r}}_{2};\boldsymbol{t}_{1},\boldsymbol{\vec{r}}_{1} \mid \boldsymbol{\rho}_{matter}(\boldsymbol{t})) \mid_{\boldsymbol{t}_{2}=\boldsymbol{t}_{1}} = \delta^{2}(\boldsymbol{\vec{r}}_{2}-\boldsymbol{\vec{r}}_{1})$$

Solution in constant  $\rho$ :

$$\begin{aligned} \mathbf{G}_{q\bar{q}}\left(t_{2},\bar{r}_{2};t_{1},\bar{r}_{1}\mid\rho_{matter}\right) \\ \propto \quad \frac{E_{h}\alpha(1-\alpha)\xi}{2\pi i\sin(\xi\Delta t)}\exp\left[i\frac{E_{h}\alpha(1-\alpha)\xi}{2\sin(\xi\Delta t)}\left\{(\bar{r}_{1}^{2}+\bar{r}_{2}^{2})\cos(\xi\Delta t)-2\bar{r}_{1}\cdot\bar{r}_{2}\right\}\right] \\ \Delta t = t_{2}-t_{1} \qquad \xi = \frac{\sqrt{a(\alpha)^{4}-iE_{h}\alpha(1-\alpha)C_{1}(E_{h})\rho_{matter}}}{E_{h}\alpha(1-\alpha)} \end{aligned}$$
For pion, 
$$a(\alpha)^{2} = v^{1.15} \times (0.112 \text{ GeV})^{2} + (1-v)^{1.15} \times (0.165 \text{ GeV})^{2}, \quad 0 < v < 1$$

$$C_{1}(E_{h}) = \frac{\sigma_{tot}^{\pi N}}{\langle r_{T}^{2} \rangle_{\pi}} = \frac{25 \text{ mb}}{3} \times 0.44 \text{ fm}^{2} \quad \sim 1.9$$

In varying  $\rho$ , use recursion formula for multi-step *t*.

Survival probability of pre-hadron in matter

$$Tr(t_{1},t_{2} | \rho_{matter}(t)) = \frac{\int d^{2}r_{1}d^{2}r_{2} \Psi_{h}^{*}(\vec{r}_{2}) G_{q\bar{q}}(t_{2},\vec{r}_{2};t_{1},\vec{r}_{1} | \rho_{matter}(t)) \Psi_{q\bar{q}}(\vec{r}_{1})}{\int d^{2}r \Psi_{h}^{*}(\vec{r}) \Psi_{q\bar{q}}(\vec{r})} \Big|^{2}$$

$$\Psi_{h}(\vec{r}_{2}) \propto \exp\left(-\frac{\vec{r}_{2}^{2}}{2\langle r_{T}^{2} \rangle_{h}}\right) \qquad \Psi_{q\bar{q}}(\vec{r}_{1}) \propto \exp\left(-\frac{\vec{r}_{1}^{2}}{2\langle r_{T}^{2}(t_{\rho}, v, Q^{2}) \rangle_{q\bar{q}}}\right)$$

$$\left\langle r_{T}^{2} \right\rangle_{\pi} \sim (1 \, \text{fm} \,)^{2}$$

$$\left\langle r_{T}^{2} (t_{p}; \nu, Q^{2}, z_{h}) \right\rangle_{q\bar{q}} = \frac{64}{9} \frac{\int_{\lambda^{2}}^{Q^{2}} dk_{T}^{2} \left[ dW_{q,g}^{*}(t_{p}; k_{T}^{2}, \nu, Q^{2}, z_{h}) / dk_{T}^{2} \right]}{\int_{\lambda^{2}}^{Q^{2}} dk_{T}^{2} \left[ dW_{q,g}^{*}(t_{p}; k_{T}^{2}, \nu, Q^{2}, z_{h}) / d\ln k_{T}^{2} \right]}$$

Iarge v → Smaller
One possible origin of color transparency



 $E_h \rightarrow \infty$  ("Frozen" limit)

$$G_{q\bar{q}}(t_{2},\vec{r}_{2};t_{1},\vec{r}_{1} | \rho_{matter}(t)) \propto \delta^{2}(\vec{r}_{2}-\vec{r}_{1}) \exp\left[-\frac{1}{2}C_{1}(E_{h})\vec{r}_{2}^{2}\int_{t_{1}}^{t_{2}}dt \rho_{matter}(t)\right]$$

#### Leading hadron production in AA collision



A.H.1 To get dilution of matter density due to expansion, we multiply this by a factor "". This is modeled to represent longitudinally Loretz-boost invariant expansion. At RHIC energy we use tau=1 fm as pointed in B.Muller's paper. C2 is an unknown factor to fit the data later.

Arata Hayashigaki, 10/2/2005

- **A.H.2** For high-pt hadrons yield, we take a binary NN collision profile like this, here inelastic NN Xsec about 40 mb. Arata Hayashigaki, 10/2/2005
- **A.H.3** For produced comoving medium density which is this green region, we assume this participant nucleon profile based on glauber model. Arata Hayashigaki, 10/2/2005

#### 2-to-2 hard parton scatterings in LO



**QCD factorization (** $p_T > 3 \text{ GeV}$ **)**: A.H.4

$$d\sigma = F_{a/A}(x_a, Q^2)dx_a F_{b/B}(x_a, Q^2)dx_b \frac{d\hat{\sigma}}{d\hat{t}}(ab \rightarrow cd)d\hat{t} D_{h/q,g}(z_h, Q^2)dz_h$$

LO pQCD: 
$$\frac{d\hat{\sigma}}{d\hat{t}}(ab \rightarrow cd) = \frac{\pi \alpha_s^2(Q^2)}{\hat{s}^2} |M(ab \rightarrow cd)|^2$$
  
SU<sub>f</sub>(2), g Feynman et al. ('78)

A.H.4 If we assum QCD collinear factorization at large pT, the single particle Xsec is given by this form. The last integral of zh is done easily due to delta function, because we have this momentum conservation among zh, xa and xb. Here is parton-parton differential Xsec, which is the sum of totally 8 independent leading order diagrams. For parton species, for now we consider u,d quark and gluon. Arata Hayashigaki, 10/2/2005

A.H.5 In the center of mass frame at the energy root s, we consider head-on collision of two equal nuclei. Each parton collides here with respective momentum fraction xa and xb, so F is nuclear parton distribution.

> In this point, new partons c, d are produced and parton c finally hadronize with pT momentum and fraction zh. If we consider only mid-rapidity region, zh is this function of xa and xb. We assum naively Q^2 evolution of D as pT^2. Arata Hayashigaki, 10/2/2005

$$\frac{d\sigma(AB \to hX)}{d^2 p_T dy} \bigg|_{y=0} = \sum_{a,b,c,d} \int_{x_a^{\min}}^1 dx_a \int_{x_b^{\min}}^1 dx_b \ F_{a/A}(x_a,Q^2) F_{b/B}(x_b,Q^2) \\ \times \frac{d\hat{\sigma}(ab \to cd)}{d\hat{t}} \frac{D_{h/c}(z_h,Q^2)}{\pi z_h}$$

$$\boldsymbol{x}_{a}^{\min} = \frac{\boldsymbol{p}_{T}}{\sqrt{s} - \boldsymbol{p}_{T}} \qquad \boldsymbol{x}_{b}^{\min} = \frac{\boldsymbol{p}_{T} \boldsymbol{x}_{a}}{\sqrt{s} \boldsymbol{x}_{a} - \boldsymbol{p}_{T}} \geq \frac{\boldsymbol{p}_{T}}{\sqrt{s}} \sim 0.01 \text{ at RHIC}$$

 $F_{A,B}(x)$ : EKS nuclear PDF ('99)  $F_N(x)$ : CTEQ5L nucleon PDF ('99)

**Explicit t-dependence:** 
$$D_{h/q,g}(z_h,Q^2) = \int_0^\infty dt_p W_{q,g}(t_p;z_h,Q^2)$$

$$\frac{d\sigma(AB \to hX)}{d^2 p_T dy}\bigg|_{y=0} = \int_0^\infty dt_p \left. \frac{d\sigma(AB \to hX; W_{q,g}(t_p))}{d^2 p_T dy dt_p} \right|_{y=0} \times Tr(t_1 = t_p, t_2 = \infty \mid \rho_{matter}(\vec{s}, \vec{b}, \vec{t}))$$

**Nuclear modification factor** 

$$\frac{d\sigma_{AB}^{\rho\rho}}{d^{2}p_{T}dyd^{2}b}\Big|_{y=0} = \int_{0}^{\infty} dss \int_{0}^{2\pi} d\theta T_{A}(\vec{s})T_{B}(\vec{s}-\vec{b}) \int_{0}^{2\pi} d\varphi$$

$$\times \int_{0}^{\infty} dt_{p} \frac{d\sigma(AB \rightarrow hX; W_{q,g}(t_{p}))}{d^{2}p_{T}dy dt_{p}}\Big|_{y=0}$$

$$\times Tr(t_{1} = t_{p}, t_{2} = \infty \mid \rho_{matter}(\vec{s}, \vec{b}, \vec{t}))$$

$$\rho_{matter} = 0$$

$$R_{AA}(p_{T},\vec{b}) = \frac{d\sigma_{AB}/d^{2}p_{T}dyd^{2}b|_{y=0}}{d\sigma_{pp}/d^{2}p_{T}dyd^{2}b|_{y=0}}$$

$$C_{1}(E_{h})\rho_{matter}(t,\vec{s},\vec{b}) = C_{1}(E_{h})\frac{C_{2}}{\tau + \tau_{0}}\frac{d^{2}N_{part}}{d^{2}s} \qquad \begin{array}{c} \text{dimensionless}\\ C = C_{1} \times C_{2}\end{array}$$

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#### Almost flat.

Due to small <t<sub>p</sub>>, small parton EL, dominant dipole attenuation.

Disregard Cronin effect.

Small effect from nuclear PDF (< 10 %).

$$\Delta E_{ind} = \frac{3}{8} \alpha_s(Q^2) \Delta k_t^2 t \qquad \sqrt{\langle r_T^2 \rangle_{q\bar{q}}} = 0.25 \text{ fm}$$
$$= \frac{3}{4} \alpha_s(Q^2) C_1(E_h) \rho_{matter} t$$

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$$\frac{\left. \boldsymbol{C}_{1} \times \boldsymbol{C}_{2} \right|_{200 \text{ GeV}}}{\left. \boldsymbol{C}_{1} \times \boldsymbol{C}_{2} \right|_{62.4 \text{ GeV}}} \sim 4.3$$

$$\frac{C_1|_{200 \text{ GeV}}}{C_1|_{62.4 \text{ GeV}}} \sim 4.3 \frac{dN/dy|_{62.4 \text{ GeV}}}{dN/dy|_{200 \text{ GeV}}} \sim 4.3 \frac{430}{650} = 2.8$$

## **Summary**

- 1. Distribution function  $W_{q,g}(t)$  based on a pQCD model.
- 2. Good agreement with BKK and KKP.
- 3. A shrinkage of W(t) with rising  $p_T$  like  $< t_p > \infty 1/p_T$  at variance with DIS, where  $< t_p > \infty \nu/Q^2$ .
- 4. At high  $p_T$ , earlier hadronization leads to a dominant (pre-)hadron absorption in matter.
- 5. Induced EL doesn't work well due to small  $t_p$ .
- 6. This explains well a flat  $R_{AA}$  from recent PHENIX data, but shows slight increase with  $p_T$  due to color transparency.
- Similar behavior (p<sub>T</sub> > 4 GeV) to (GLV) partonic EL scenario (X.Wang, I.Vitev's talks)