Atomic mass dependence of hadron production in DIS on nuclei

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Outline

Part I (The model):

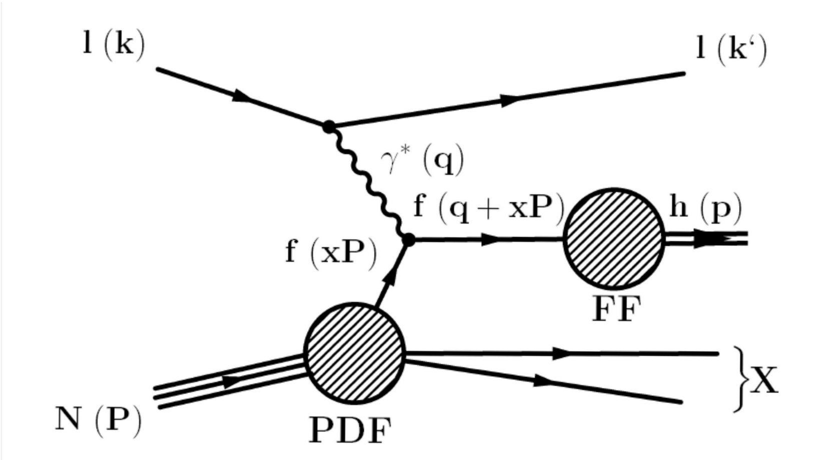
- Review: DIS
- Building blocks of the model:
  - Rescaling of PDF and FF
  - Absorption factor
- Comparison with HERMES data

Part II (A-dependence):

- Review: A dependence of different models
- Analytical investigation

  $\Rightarrow$A suitable observable
- Analysis of data with the new observable
- Conclusions

Review: Semi Inclusive deep inelastic scattering



Variable	Covariant	Lab. frame
Q^2	$-q^2$	$2Mx\nu$
ν	$\frac{q\,p}{\sqrt{P^2}}$	$E'-E$
x	$\frac{-q^2}{2Pq}$	$\frac{Q^2}{2M\nu}$
z	$\frac{p\,P}{q\,P}$	$\frac{E_h}{\nu}$

- Factorization theorem in QCD:

$$\left.\frac{d^2\sigma}{dx\,d\nu\,dz}\right|_{SIDIS} = \sum_f e_f^2 q_f(x,Q^2)\frac{d^2\sigma^{lq}}{dx\,d\nu}D_f^h(z,Q^2)$$

- Multiplicity:

$$M^h(z) = \frac{1}{N_A^{DIS}}\frac{dN_A^h(z)}{dz}$$

$$\frac{1}{N^{DIS}}\frac{dN^h(z)}{dz} = \frac{1}{\sigma^{lp}}\int_{\text{exp. cuts}} dx\,d\nu \sum_f e_f^2 q_f(x,Q^2)\frac{d\sigma^{lq}}{dx\,d\nu} \times D_f^h(z,Q^2)$$

$$\sigma^{lp} = \int_{\text{exp. cuts}} dx\,d\nu \sum_f e_f^2 q_f(x,\xi_A(Q^2)Q^2)\frac{d\sigma^{lq}}{dx\,d\nu}$$

Building Blocks of the model

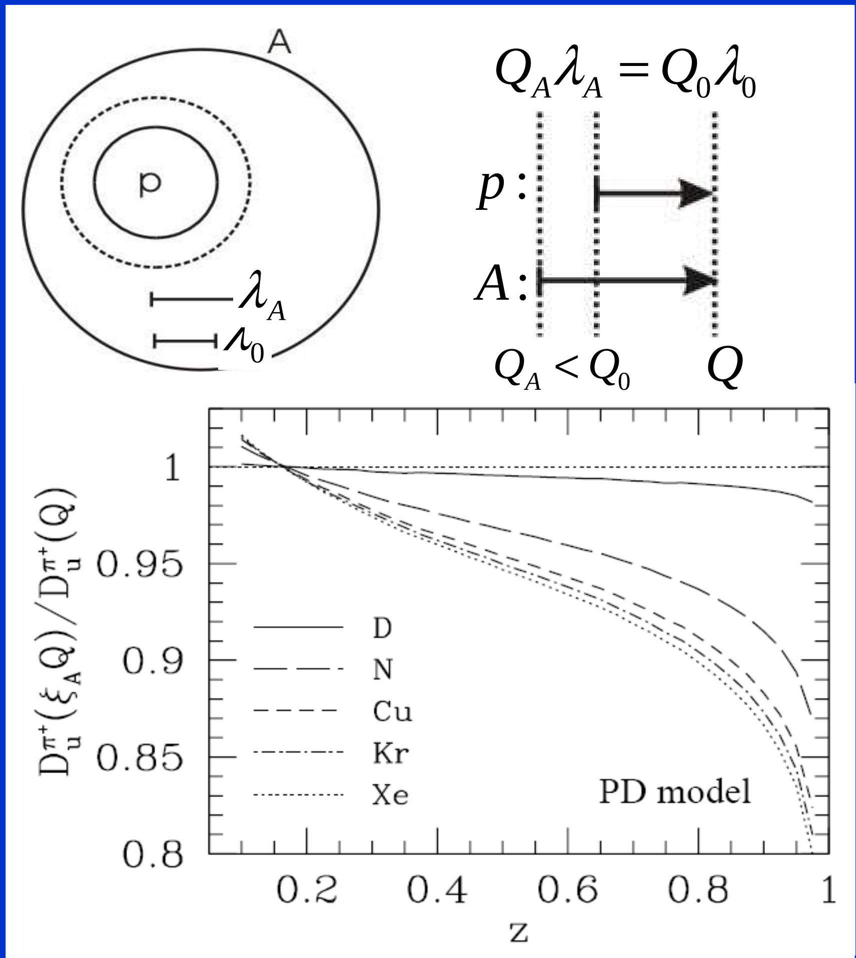
$$\frac{1}{N_A^{DIS}} \frac{dN_A^h(z)}{dz} = \frac{1}{\sigma^{lA}} \int_{\text{exp. cuts}} dx d\nu \sum_f e_f^2 q_f^A(x, \xi_A Q^2) \frac{d\sigma^{lq}}{dx d\nu}$$

$$\times D_f^h(z, \xi_A Q^2) N_A(z, \nu),$$

Rescaling of Parton Distribution, Rescaling of Fragmentation Function
Calculation of the mean formation times of the prehadron and hadron
Calculation of the Nuclear Absorption Factor N_A ,using formation times

Rescaling of PDF and FF

H.J. Pirner and O. Nachtmann Z. Phys. C21 (1984)



- Idea: Quarks in bound nucleons have access to a larger region in space $\lambda_A > \lambda_0$

$\Longrightarrow$ Smaller confinement scale

- Consequence:

$$\frac{1}{A} q_f^{N|A}(x, Q^2) = q_f^N(x, \xi_A(Q^2)Q^2)$$

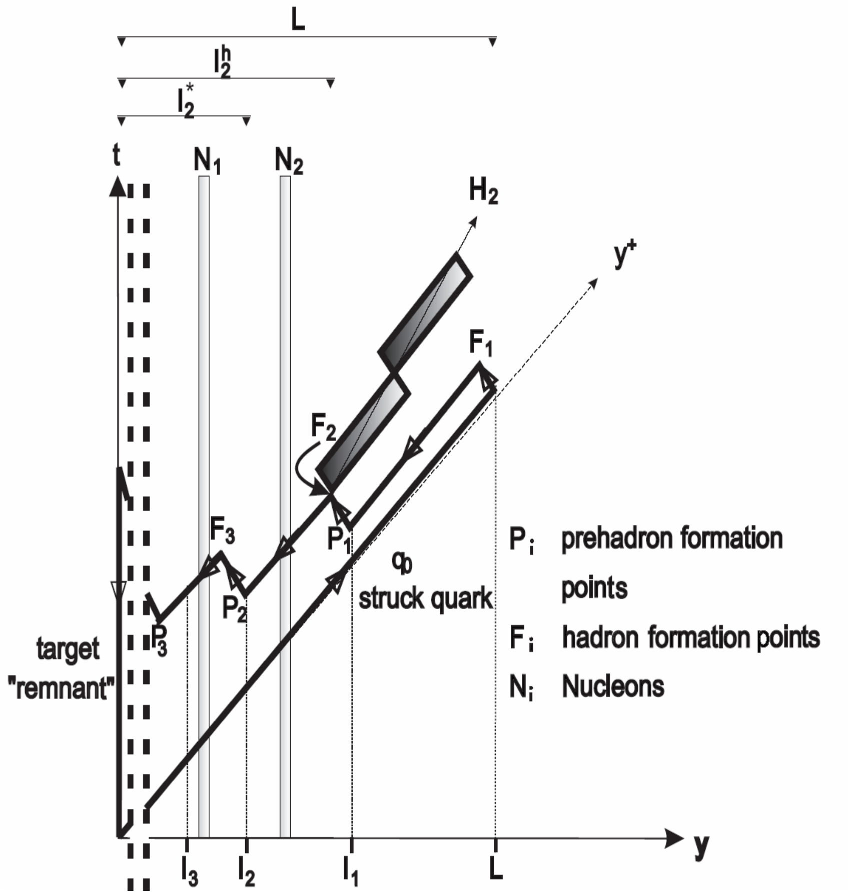
$$D_f^{h|A}(z, Q^2) = D_f^h(z, \xi_A(Q^2)Q^2)$$

$$\xi_A(Q^2) = \left(\frac{\lambda_A}{\lambda_0}\right)^{\frac{\bar{\alpha}_s}{\alpha_s(Q^2)}}$$

- Rescaling implies a longer DGLAP evolution (increased gluon shower)

The prehadron concept

schematic space time picture of hadronization



- prehadron: colorless object evolving into the observed hadron
- first rank hadron contains struck quark

  $\Rightarrow$ only hadrons which contain the struck quark are producible as first rank

Average prehadron formation lengths

computed in the LUND string fragmentation model

$\langle l_* \rangle$

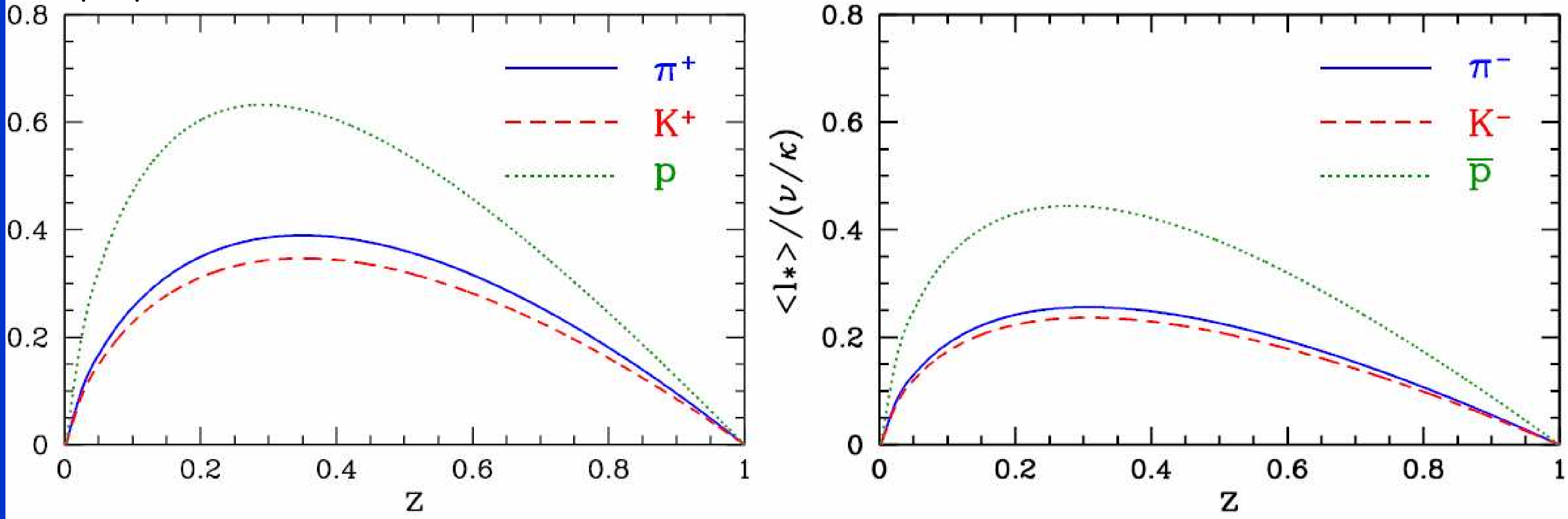


Fig. 3. Computed prehadron formation lengths when an up quark is struck by the virtual photon. *Left:* When a π^+, K^+ or p is observed, the corresponding prehadron can be created at rank $n \geq 1$. *Right:* When a π^-, K^- or $\bar{p}$ is observed, the corresponding prehadron can be created only at rank $n \geq 2$.

$$\langle l_h \rangle = \langle l_* \rangle + z\nu / \kappa$$

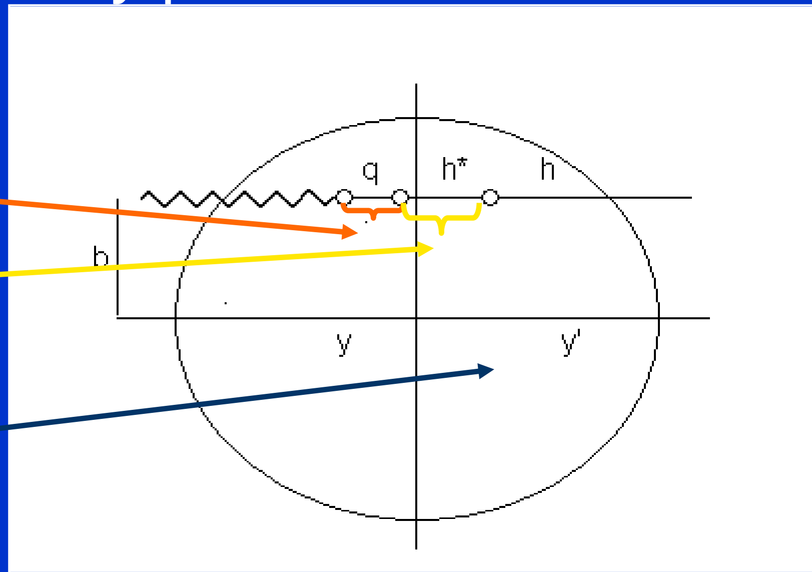
Absorption model

- Dominant contribution to observed particles:
  - Every inelastic interaction lowers the hadron energy
  - Fragmentation functions falls steeply for large z

  $\Longrightarrow$ Dominant contribution are hadrons which have not interacted
- Take into account only hadrons which have not interacted
- Consider hadronization as a decay process

  $\rightarrow$ three „timescales“:

1. $\langle l^* \rangle$
2. $\langle \Delta l \rangle = \langle l^h \rangle - \langle l^* \rangle$
3. $\lambda_{*,h}(y') = \dfrac{1}{\rho_A(y')\sigma_{*,h}}$



Absorption factor
evolution equations

- Decay equation for probability to find an intermediate state at y‘ if the initial interaction took place at y

$$\frac{\partial P_q(y,y')}{\partial y'} = -\frac{P_q(y,y')}{\langle l^* \rangle} \quad , P_q(y, y' = y) = 1$$

$$\frac{\partial P_*(y,y')}{\partial y'} = \frac{P_q(y,y')}{\langle l^* \rangle} - \frac{P_*(y,y')}{\langle \Delta l \rangle} - \frac{P_*(y,y')}{\lambda_*(y')} \quad , P_*(y, y' = y) = 0$$

$$\frac{\partial P_h(y,y')}{\partial y'} = \frac{P_*(y,y')}{\langle \Delta l \rangle} - \frac{P_h(y,y')}{\lambda_h(y')} \quad , P_h(y, y' = y) = 0$$

Take out from evo.

Absorption factor solution

probability that the prehadron has not decayed up to y'

$$P_q(y, y') = e^{-\frac{y'-y}{\langle l^* \rangle}}$$

$$P_*(y, y') = \int_y^{y'} dx \frac{e^{-\frac{x-y}{\langle l^* \rangle}}}{\langle l^* \rangle} e^{-\frac{y'-x}{\langle \Delta l \rangle}} e^{-\sigma_* \int_x^{y'} ds A \rho_A(s)}$$

$$P_h(y, y') = \int_y^{y'} dx' \int_y^{x'} dx \frac{e^{-\frac{x-y}{\langle l^* \rangle}}}{\langle l^* \rangle} e^{-\sigma_* \int_x^{x'} ds A \rho_A(s)} \frac{e^{-\frac{x'-x}{\langle \Delta l \rangle}}}{\langle \Delta l \rangle} e^{-\sigma_h \int_{x'}^{y'} ds A \rho_A(s)}$$

Integration over all prehadron formation points

Absorption factor

- Probability to find a hadron outside of the nucleus which has not interacted:

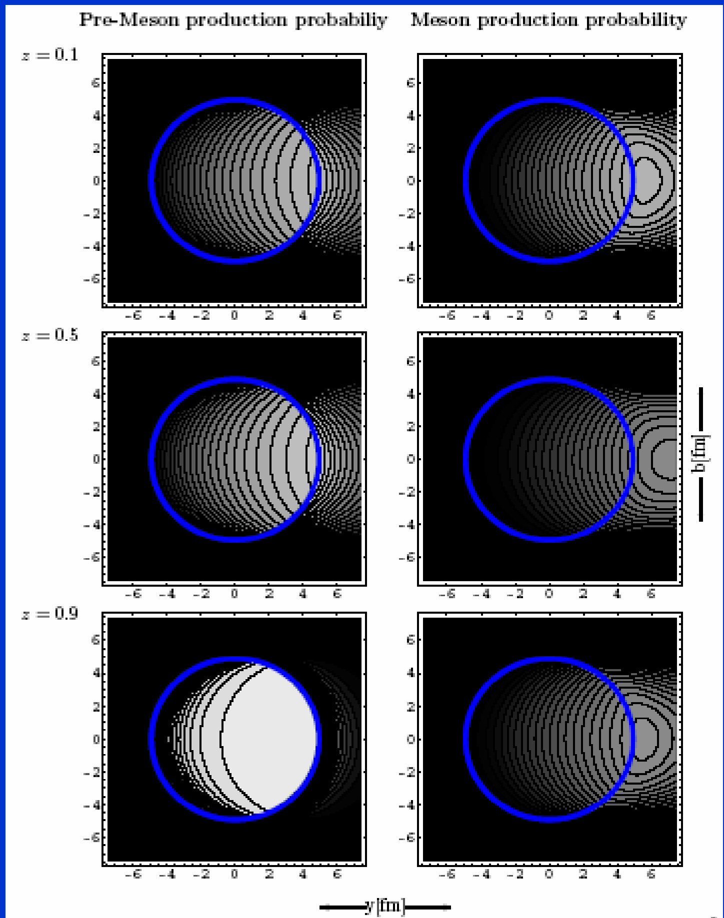
$$N_A = \lim_{y' \to \infty} \int d^2b \int_{-\infty}^{\infty} dy \rho_A(b,y) P_h(y',y)$$

$$= \int d^2b \int_{-\infty}^{\infty} dy \rho_A(b,y) \int_y^{\infty} dx' \int_y^{x'} dx \frac{e^{-\frac{x-y}{\langle l^* \rangle}}}{\langle l^* \rangle} e^{-\sigma_* \int_x^{x'} ds A \rho_A(s)}$$

$$\times \frac{e^{-\frac{x'-x}{\langle \Delta l \rangle}}}{\langle \Delta l \rangle} e^{-\sigma_h \int_{x'}^{\infty} ds A \rho_A(s)}$$

Prehadron and Hadron-Production probabilities

(at HERMES energies for Kr target)



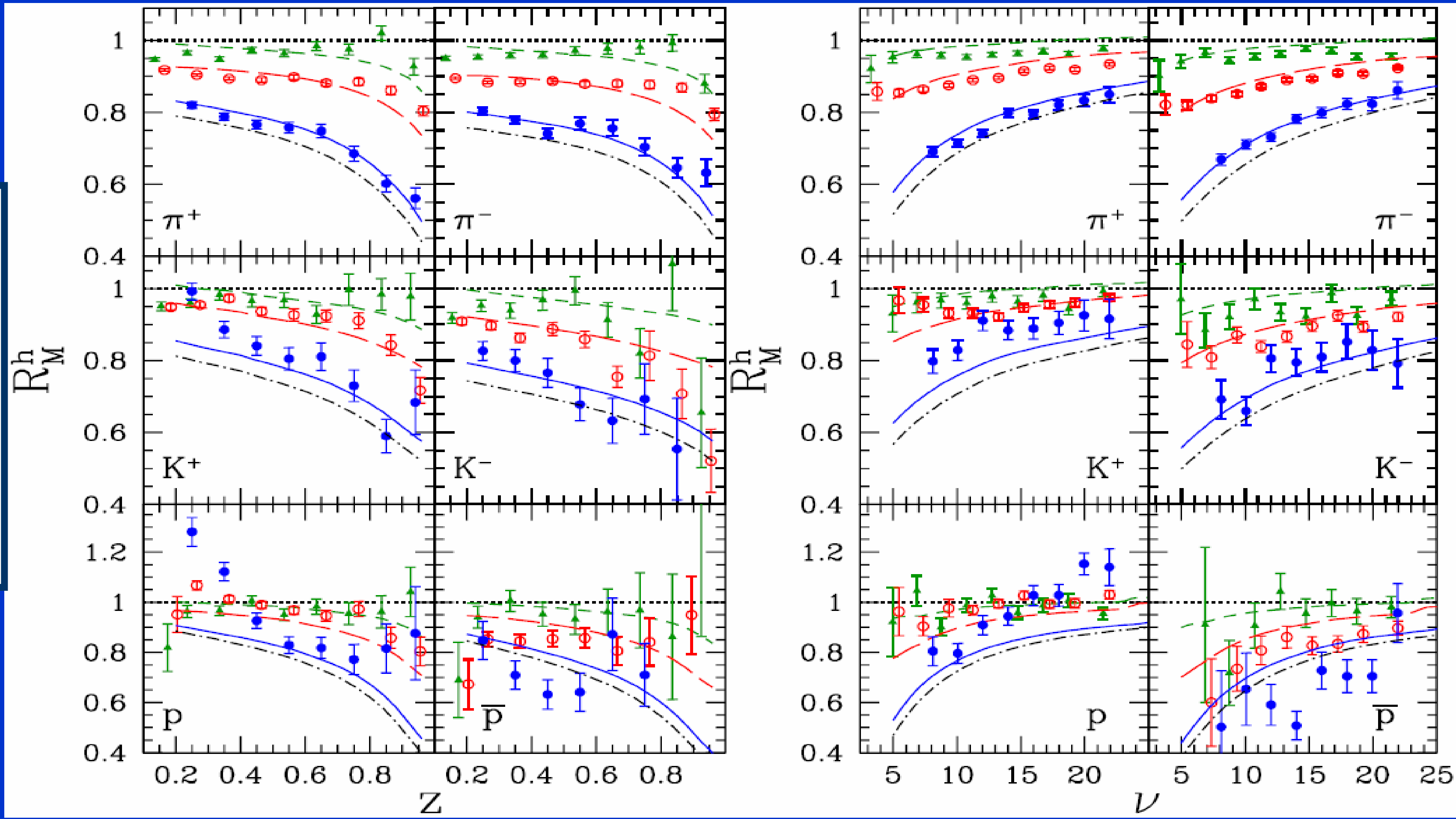
- Hadrons are mostly produced outside of the nucleus
  ⇒Attenuation dominated by pre-hadron absorption

Comparison with HERMES data

HERMES Coll., A. Airapetian et al. Phys. Lett B 577 (2003) [**],
G. Elbakian 11th International workshop on DIS [***]

$$\sigma_* = f * \sigma_h$$

$^{4}_{2}He$ [***]
$^{20}_{10}Ne$ [***]
$^{84}_{36}Kr$ [**]
$^{131}_{54}Xe$



A dependence
Review

- Absorption models:
  attenuation ∝ in medium path length ⇒
  $1 - R_M = c\,A^{1/3}$

- Energy loss models:
  attenuation ∝ in medium path length squared ⇒
  $1 - R_M = c\,A^{2/3}$

- But!!! ….

A dependence
analytical investigation

In order to obtain analytical results, make the following restrictions:

- Attenuation is dominated by pre-hadron absorption

  $\Rightarrow$ A prehadron which survives will yield a hadron with high probability

  $\Rightarrow$ Neglect hadrons in the evolution

- Consider nucleus as a hard sphere
- Neglect attenuation in Deuterium $\Rightarrow R_M \approx N_A$

A dependence
theoretical investigation

- Evolution equations without hadrons:

$$\frac{\partial P_q(y,y')}{\partial y'} = -\frac{P_q(y,y')}{\langle l^* \rangle} \qquad , P_q(y, y' = y) = 1$$

$$\frac{\partial P_*(y,y')}{\partial y'} = \frac{P_q(y,y')}{\langle l^* \rangle} - \frac{P_*(y,y')}{\langle \Delta l \rangle} - \frac{P_*(y,y')}{\lambda_*(y')} \qquad , P_*(y, y' = y) = 0$$

$$\frac{\partial P_h(y,y')}{\partial y'} = \frac{P_*(y,y')}{\langle \Delta l \rangle} - \frac{P_h(y,y')}{\lambda_h(y')} \qquad , P_h(y, y' = y) = 0$$

- Attenuation (hard sphere nucleus/neglecting D):

$$1 - R_M = 1 - \frac{\pi \rho_0}{A} \int_0^{R^2} db^2 \int_{-R(b)}^{R(b)} dy \int_y^{\infty} dx \frac{e^{-\frac{x-y}{\langle l^* \rangle}}}{\langle l^* \rangle} e^{-\rho_0 \sigma_* \int_x^{\infty} ds \Theta(R(b) - |s|)}$$

$$= \frac{\pi \rho_0}{2A} \langle l^* \rangle^3 \int_0^{2R/\langle l^* \rangle} dt\, t \int_0^t dr \int_0^r du\, e^{-u} \left[1 - e^{\frac{\langle l^* \rangle}{\lambda_0}(u-r)} \right]$$

A dependence
theoretical investigation

- Expansion in powers of u:

$$a = \langle l^* \rangle / \lambda_0 \qquad b = 2R/\langle l^* \rangle \qquad R = r_0 A^{1/3}$$

$$1 - R_M = \frac{1}{10} a b^2 - \frac{1}{48}(1+a) a b^3 + \frac{1}{280}(1 + a + a^2) a b^4 + \mathcal{O}[b^5]$$

$\Rightarrow$ leading order term $\propto A^{2/3}$

!!! contrary to common expectation !!!

- Expansion is not good for large values of a

and b $\Rightarrow$ perform a fit to the innermost integral

$$\int_0^r du e^{-u} \left[1 - e^{a(u-r)}\right] = \frac{1 - e^{-ar} - a\left(1 - e^{-r}\right)}{1 - a} \approx 1 - e^{-war^2} \qquad w = 0.19$$

A dependence
theoretical investigation

- This improves the convergence over the whole z range:

$$
\begin{aligned}
1-R_M &= \tfrac{1}{5}wab^2 - \tfrac{3}{70}(wab^2)^2 + \mathcal{O}[(wab^2)^3] \\
&= c_1 A^{2/3} + c_2 A^{4/3} + \mathcal{O}[A^2] \; .
\end{aligned}
$$

- Small value of w $\Rightarrow$ rapid convergence

z	$\langle l^h(z)\rangle$ [fm]	c_1	c_2	$\bar{A}$
.25	10.15	0.0095	-0.000096	980
.45	11.72	0.0103	-0.000114	860
.65	12.34	0.0142	-0.000217	530
.85	11.98	0.0314	-0.001059	160

- The series converges very quickly $\Rightarrow 1-R_M = c\,A^{\alpha}$

A dependence
theoretical investigation

Results from theory discussion:

- attenuation is proportional to $A^{2/3}$ in leading order
- higher order terms become important for large *A* and *z*
- *a lot of information on absorption dynamics is contained in c*
- *the strong dependence of c on z needs to be taken into account when analyzing data*

A dependence
a suitable observable

Requests for a suitable observable:

- fit to $c(z)\,A^{\alpha(z)}$ is suitable with c and α as free parameters
- take into account the correlations between c and α

  A small increase of the exponent α can be compensated by a small decrease of c

$\Rightarrow$ ***Perform a $c\,A^{\alpha}$ power law fit and present the result as confidence ellipses***

A dependence
a suitable observable

Requests for a suitable observable:

- fit to $c(z)\,A^{\alpha(z)}$ is suitable with c and α as free parameters
- take into account the correlations between c and α

  A small increase of the exponent α can be compensated by a small decrease of c

$$\chi^2(c,\alpha) = \sum_i \frac{1}{\sigma_i^2}\left[(1-R_M)(A_i) - cA_i^\alpha\right]$$

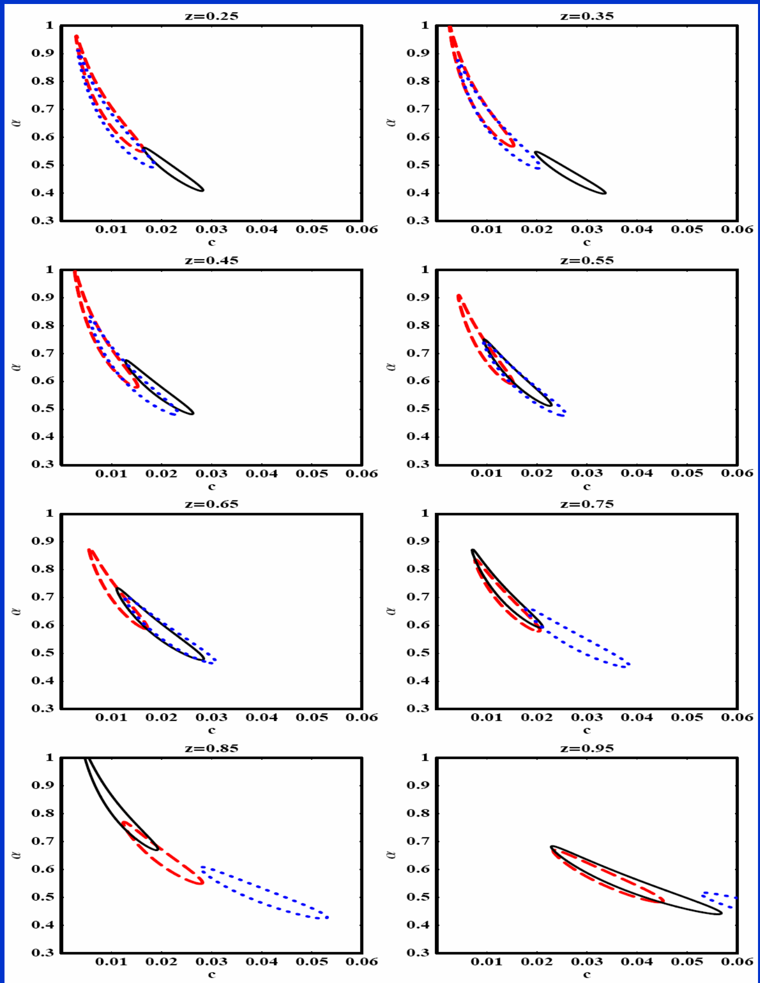
$$\chi^2_{\min} < \chi^2 < \chi^2_{\min} + \Delta\chi^2$$

$$\Delta\chi^2 = 4.61$$

A dependence
the proposed observable

exp. data
pure absorption model
abs. + rescaling model

He (N) Ne Kr



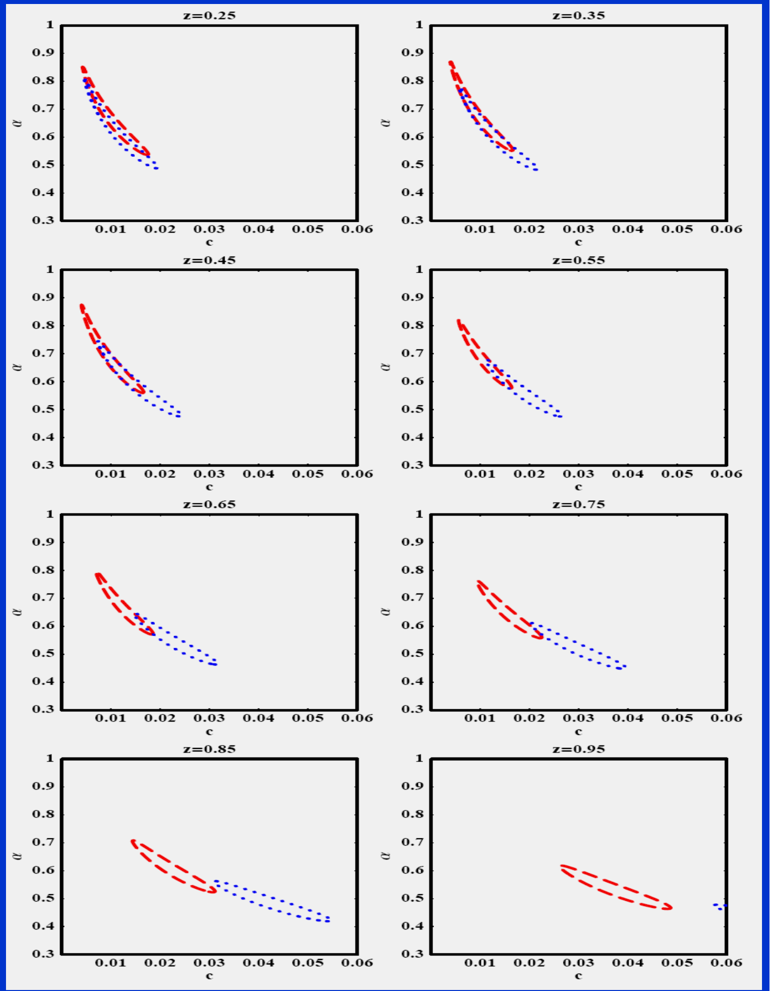
- Theory points originate from computation with realistic nuclear densities + 2 step hadronization process
- Theoretical uncertainty = 6% of attenuation
- N only included for z>= 0.55
  $\Longrightarrow$ two parameter fit to three „data“ points for z <0.55
- pure absorption model points follow the trend shown by the experimental data for z>=0.55
- For increasing z the agreement of the abs. + rescaling model decreases

$\Longrightarrow$ THIS SHOWS THE POWER OF THE PROPOSED ANALYSIS

A dependence
the proposed observable

exp. data
pure absorption model
abs. + rescaling model

He (N) Ne Kr Xe



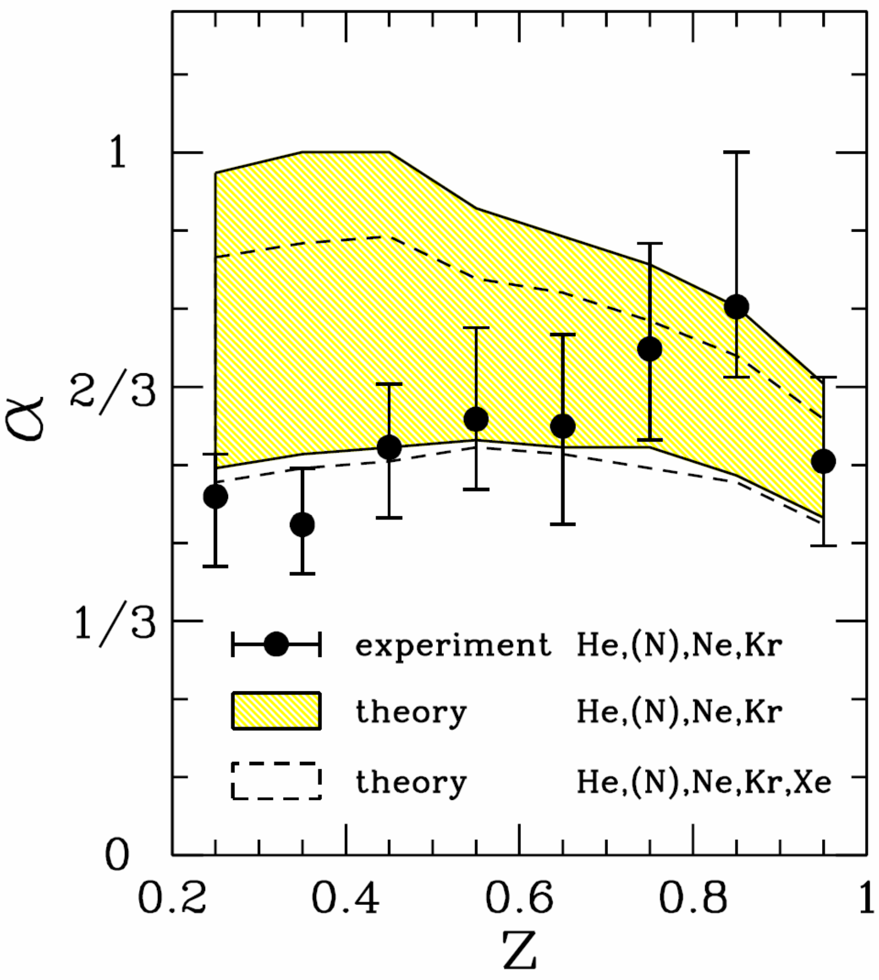
- To investigate the possible breaking of the power law:
  Include Xe data into the fitted data set
- Observed impact is rather small
- This is due to the relatively large error bars of the Xe attenuation making the weight small

A dependence
the proposed observable

Centroids of the confidence ellipsoids

	Experiment He (N) Ne Kr		Theory He (N) Ne Kr		Theory He (N) Ne Kr Xe	
z	$c\ [10^{-2}]$	α	$c\ [10^{-2}]$	α	$c\ [10^{-2}]$	α
.25	$2.1 \pm^{0.8}_{0.5}$	$0.51 \pm^{0.06}_{0.10}$	$0.7 \pm^{0.9}_{0.5}$	$0.75 \pm^{0.22}_{0.20}$	$0.9 \pm^{0.9}_{0.4}$	$0.70 \pm^{0.15}_{0.17}$
.35	$2.6 \pm^{0.8}_{0.6}$	$0.47 \pm^{0.08}_{0.07}$	$0.7 \pm^{0.9}_{0.4}$	$0.77 \pm^{0.23}_{0.20}$	$0.8 \pm^{0.9}_{0.4}$	$0.72 \pm^{0.15}_{0.17}$
.45	$1.9 \pm^{0.7}_{0.4}$	$0.58 \pm^{0.09}_{0.10}$	$0.7 \pm^{0.8}_{0.4}$	$0.78 \pm^{0.22}_{0.20}$	$0.8 \pm^{0.9}_{0.4}$	$0.73 \pm^{0.15}_{0.17}$
.55	$1.6 \pm^{0.7}_{0.6}$	$0.62 \pm^{0.13}_{0.10}$	$0.8 \pm^{0.7}_{0.4}$	$0.76 \pm^{0.16}_{0.17}$	$0.9 \pm^{0.7}_{0.4}$	$0.71 \pm^{0.11}_{0.13}$
.65	$1.8 \pm^{1.0}_{0.7}$	$0.61 \pm^{0.13}_{0.14}$	$1.0 \pm^{0.8}_{0.4}$	$0.74 \pm^{0.14}_{0.16}$	$1.1 \pm^{0.8}_{0.4}$	$0.70 \pm^{0.10}_{0.13}$
.75	$1.3 \pm^{0.8}_{0.6}$	$0.72 \pm^{0.15}_{0.13}$	$1.2 \pm^{0.9}_{0.5}$	$0.73 \pm^{0.11}_{0.15}$	$1.4 \pm^{0.9}_{0.4}$	$0.68 \pm^{0.08}_{0.13}$
.85	$1.2 \pm^{0.5}_{0.7}$	$0.78 \pm^{0.22}_{0.10}$	$1.7 \pm^{1.2}_{0.5}$	$0.69 \pm^{0.09}_{0.15}$	$1.9 \pm^{1.2}_{0.5}$	$0.65 \pm^{0.06}_{0.12}$
.95	$3.6 \pm^{2.1}_{1.3}$	$0.56 \pm^{0.12}_{0.12}$	$3.1 \pm^{1.5}_{0.8}$	$0.60 \pm^{0.07}_{0.12}$	$3.3 \pm^{1.6}_{0.7}$	$0.57 \pm^{0.05}_{0.10}$

A dependence
the proposed observable



Alpha versus z:

- Experimental data as well as absorption model compatible with $A^{2/3}$
- Xe shifts theory band to lower values of α
- Within error bars no definite statement about power-law breaking possible

Conclusions:

- Absorption model describes data-except p-production
- $\sigma_* = 2/3\sigma_h$
- The A-dependence of our absorption model shows a leading order $\propto A^{2/3}$ behavior contrary to common expectation
- Proposed analysis is a promising tool in order to distinguish several theoretical models
- Not only our absorption model shows the $A^{2/3}$ behavior (see hep-ph/0502072)