Retardation effect for heavy parton collisional energy loss in QGP.

- Motivation
- Collisional energy loss in stationnary regime
- Retardation effect; main results
- Radiation
- Conclusion & Perspectives

Pol-Bernard Gossiaux
work done with Stéphane Peigné & Thierry Gousset
SUBATECH (Nantes)
hep-ph/0509185

…collisional energy loss…

Recent papers advocating the possible role of this mechanism for partons of moderate energy:

Dutt Mazumder et al (Phys.Rev. D71 (2005) 094016) /
Mustafa et al (Phys.Rev. C72 (2005) 014905) /
Gay Ducati et al (hep-ph/0506241) /

But not the main stream

… heavy parton collisional energy loss in QGP.

Hot topic; Avoid some complications due to colinear divergences

Usual QGP: no quasi bound state, at rest, static, « confined » over some extansion L_{plasma}

… (Boring) heavy parton collisional energy loss in QGP.

Nearly everything is known; Braaten & Thoma (91)

$$\frac{dE_{coll}(E)}{dx}$$ in stationnary regime and then

$$\Delta E_{coll}(L) = \frac{dE_{coll}}{dx} \times L$$

stationnary ?

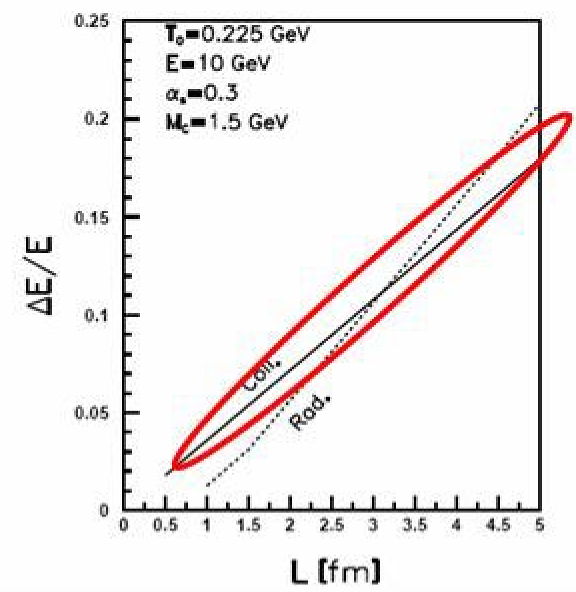
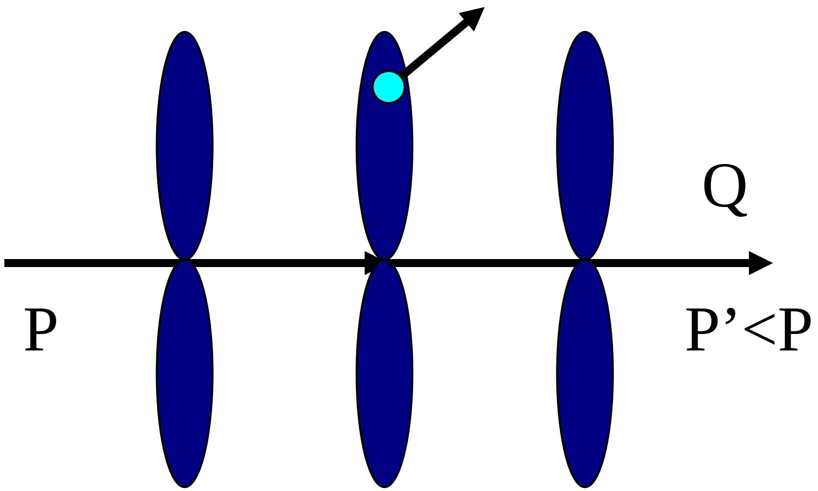


Figure taken from Mustafa et al (Phys.Rev. C72 (2005) 014905)

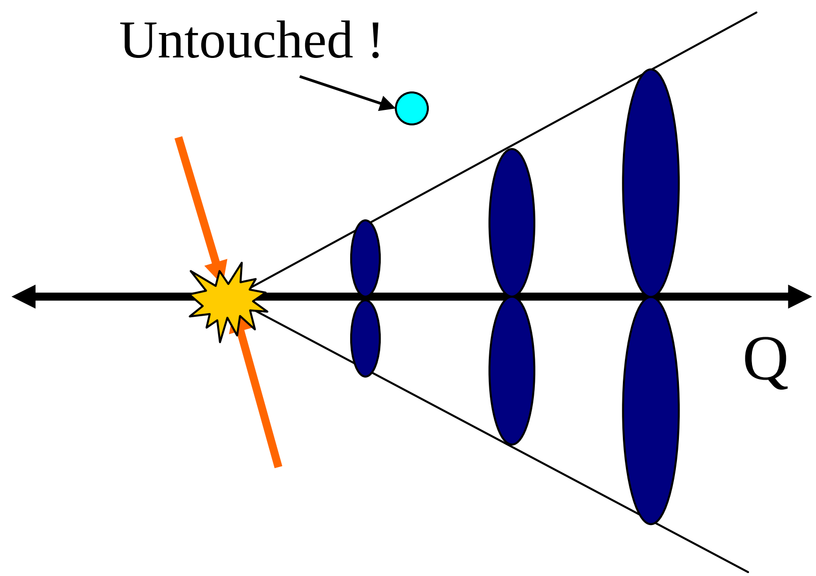
Retardation *effect* for heavy parton collisional energy loss in QGP.

What if the heavy quark is not produced at t = -∞ ?

World I



World II



Physically reasonable, but HOW BIG ?

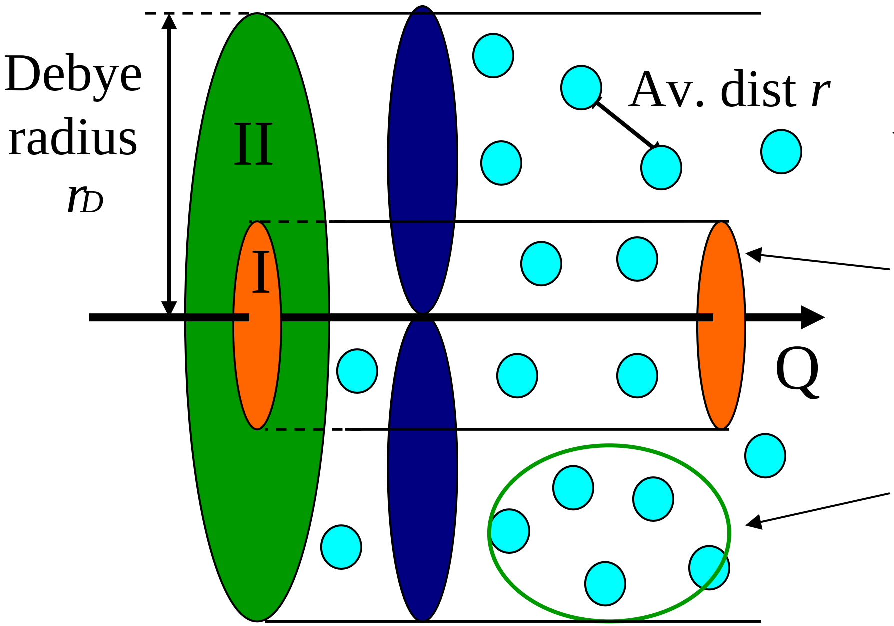
Retardation effect for heavy parton collisional energy loss in QGP.

- Motivation
- Collisional energy loss in stationnary regime
- Retardation effect; main results
- Radiation
- Conclusion & Perspectives

CEL in stationnary regime.

Assume pQCD regime at high temperature: g<<1 and ordering

$$\frac{1}{M} \ll r = \frac{1}{T} \ll \frac{1}{q^*} \ll r_D \approx \frac{1}{gT} \ll \lambda \approx \frac{1}{g^2 T}$$



Heavy quark probes the medium via virtual gluon of momentum k

Zone I: $k > q^*$: hard; close collisions; individual; incoherent.

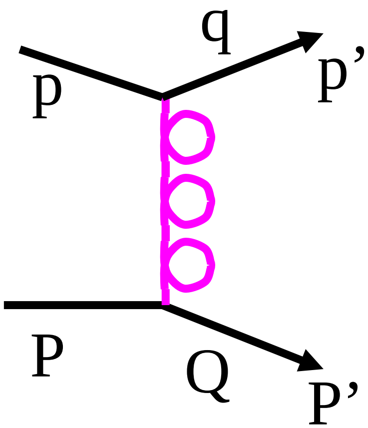
Zone II: $k < q^*$: soft; far collisions; collective; coherent; macroscopic.

CEL in stationnary regime: close collisions.

Bjorken (82) evaluated this contribution using kinetic theory.

Reaction rate Γ :

$$\Gamma = \sum_{\{P',p,p'\}} \left| \quad \right|^2 \times n(p)\,(1-n(p'))\,\delta(P+p-P'-q')$$



From rate to energy loss :

$$\frac{dE}{dx} = \int \frac{d\Gamma(E',E)}{dE'} \times \frac{E'-E}{v}\, dE' \;=\; \frac{2}{3}\,\alpha\, m_D^2 \ln\left(\frac{\sqrt{ET}}{q^*}\right) \quad \text{for } v \approx 1.$$

IR divergent

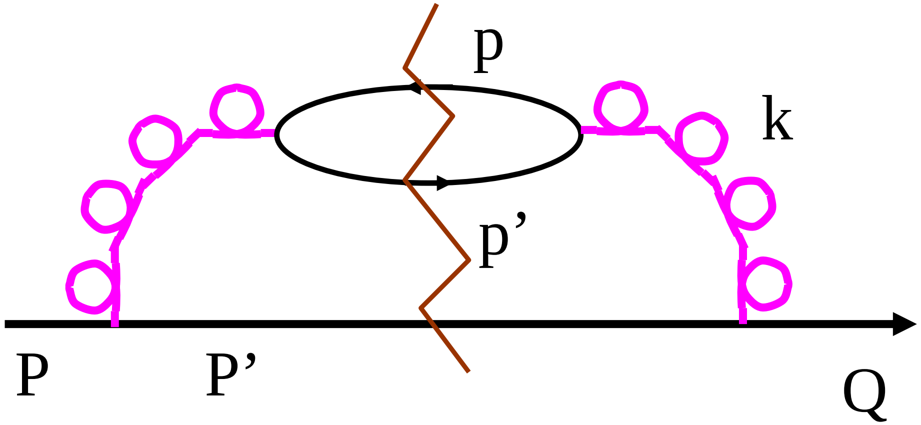
CEL in stationnary regime: far collisions.

Will embed most of the retardation effect.

First evaluated in stationnary regime by Braaten & Thoma (82)

following Weldon (83) : $\Gamma = -(1-n(E))/2E \times \mathrm{tr}[(P.\gamma+M)\,\mathrm{Im}(\Sigma)]$

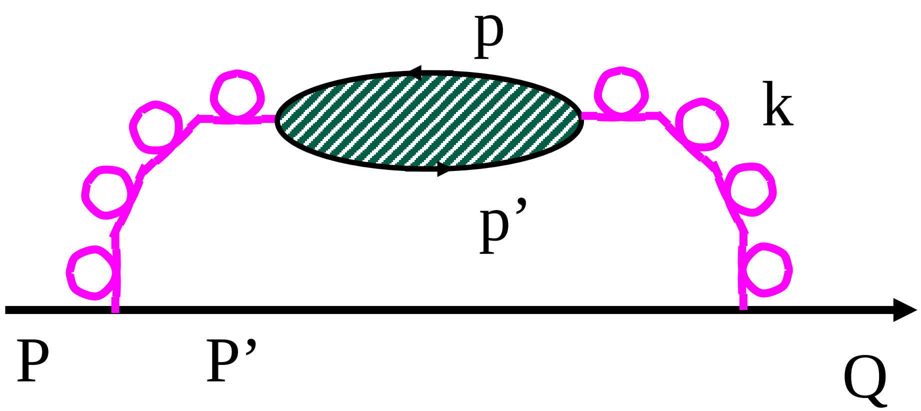
$\mathrm{Im}(\Sigma_{\mathrm{hard}})=$



Quite general relation : includes the soft collisions (k<q*) as well, provided one takes the full gluon propagator Π:

CEL in stationnary regime: far collisions (II).

$\Sigma =$



For k>q*: through kinetic expressions

For k<q*: evaluate the self energy Σ with HTL ressumation for Π (consistent with collective polarization)... Imaginary time formalism.

Legitimate as both contributions are separately gauge invariant.

CEL in stationnary regime: far collisions (III).

Results at the logarithmic accuracy:

0<v<1: | **v≈1:**

$$\frac{dE_{hard}}{dx}=\frac{2}{3}\alpha\, m_D^2\left[\frac{1}{v}-\frac{1-v^2}{v^2}\ln\left(\frac{1+v}{1-v}\right)\right]\ln\left(\frac{ET}{Mq^*}\right) \quad \not\longrightarrow \quad \frac{dE_{hard}}{dx}=\frac{2}{3}\alpha\, m_D^2\ln\left(\frac{\sqrt{ET}}{q^*}\right)$$

UV div →
$$\frac{dE_{soft}}{dx}=\frac{2}{3}\alpha\, m_D^2\left[\frac{1}{v}-\frac{1-v^2}{v^2}\ln\left(\frac{1+v}{1-v}\right)\right]\ln\left(\frac{q^*}{m_D/\sqrt{3}}\right) \quad \longrightarrow \quad \frac{dE_{soft}}{dx}=\frac{2}{3}\alpha\, m_D^2\ln\left(\frac{q^*}{m_D/\sqrt{3}}\right)$$

$$\frac{dE}{dx}=\frac{2}{3}\alpha\, m_D^2\left[\frac{1}{v}-\frac{1-v^2}{v^2}\ln\left(\frac{1+v}{1-v}\right)\right]\ln\left(\frac{ET/M}{m_D/\sqrt{3}}\right) \qquad \frac{dE}{dx}=\frac{2}{3}\alpha\, m_D^2\ln\left(\frac{\sqrt{ET}}{m_D/\sqrt{3}}\right)$$

Poor man's prescription: take $\frac{dE_{soft}}{dx}$ with some UV regulator k_{cut}:

$$k_{cut}=\min\left(\frac{ET}{M},\sqrt{ET}\right)$$

Method followed by Thoma & Gyulassy (91)

Far collisions: alternative formulation

Maxwell's Equations with appropriate dielectric functions (collective response of the medium in linear response approx.) lead to the same result for Energy loss.

Easy strategy: evaluate induced chromoelectric field (medium polarization E_{ind} and the Lorentz force of this field on the heavy parton / external current)

☺ Easy to generalize for non stationnary phenomena (heavy parton produced inside the QGP)… better suited to discuss physics.

☹ Misses some part of the hard dynamics… which plays little role for retardation ☺.

N.B.: L.R. ⇒ trivial color structure of dielectric functions.

Far collisions: alternative formulation (II)

Maxwell equations in Fourier space (ω,k) ⇒

$$\varepsilon_L \vec{E}_L + (\varepsilon_T - k^2/\omega^2)\, \vec{E}_T = \frac{4\pi}{i\omega}(\vec{j}_L + \vec{j}_T)$$

With ε_L and ε_T : dielectric functions of the plasma

$$\varepsilon_L(k,\omega) = 1 + \Pi_L(\omega/k)/k^2\ ;\ \varepsilon_T(k,\omega) = 1 - \Pi_T(\omega/k)/\omega^2$$

(Polarization Π taken in H.T.L approximation)

$\vec{j}_L$ ($\vec{j}_T$) : longitudinal (transverse) projection of the current on $\vec{k}$

$$\vec{E}_{ind}(\vec{k},\omega) = \frac{4\pi}{i}\left[\frac{k^2}{k^2+\Pi_L}\times\frac{\vec{j}_L}{\omega} + \frac{\omega}{\omega^2-k^2+\Pi_T}\times\vec{j}_T\right]_{\text{ind}} = 4i\pi\left[k^2\Delta_L\times\frac{\vec{j}_L}{\omega} + \omega\Delta_T \times\vec{j}_T\right]_{\text{ind}}$$

Substract the vacuum value

gluon propagator HTL

Far collisions: alternative formulation (III)

Fourier back and integrate the Lorentz force up to t=L/v:

$$-\Delta E(L) = i \sum_a \int \frac{d^3k}{4\pi^3} \int_{-\infty}^{+\infty} d\omega \left[\frac{k^2}{k^2+\Pi_L} \frac{\vec{j}_{L,a}(K)}{\omega} + \frac{\omega}{\omega^2-k^2+\Pi_T} \vec{j}_{T,a}(K) \right]_{\text{ind}} \cdot \int_0^{L/v} dt\, e^{-iK\cdot V\, t}\, q_a\, \vec{v}(t)$$

Special case of stationnary current: $j_a^\mu(t,\vec{x}) = q_a V^\mu \delta^3(\vec{x}-\vec{v}t)$ with $V^\mu=(1,\vec{v})$

= L/v

$$\Rightarrow \quad j_a^\mu(K) = 2\pi\, q_a V^\mu\, \delta(K\cdot V) = 2\pi\, q_a V^\mu\, \delta(\omega-\vec{k}\cdot\vec{v})$$

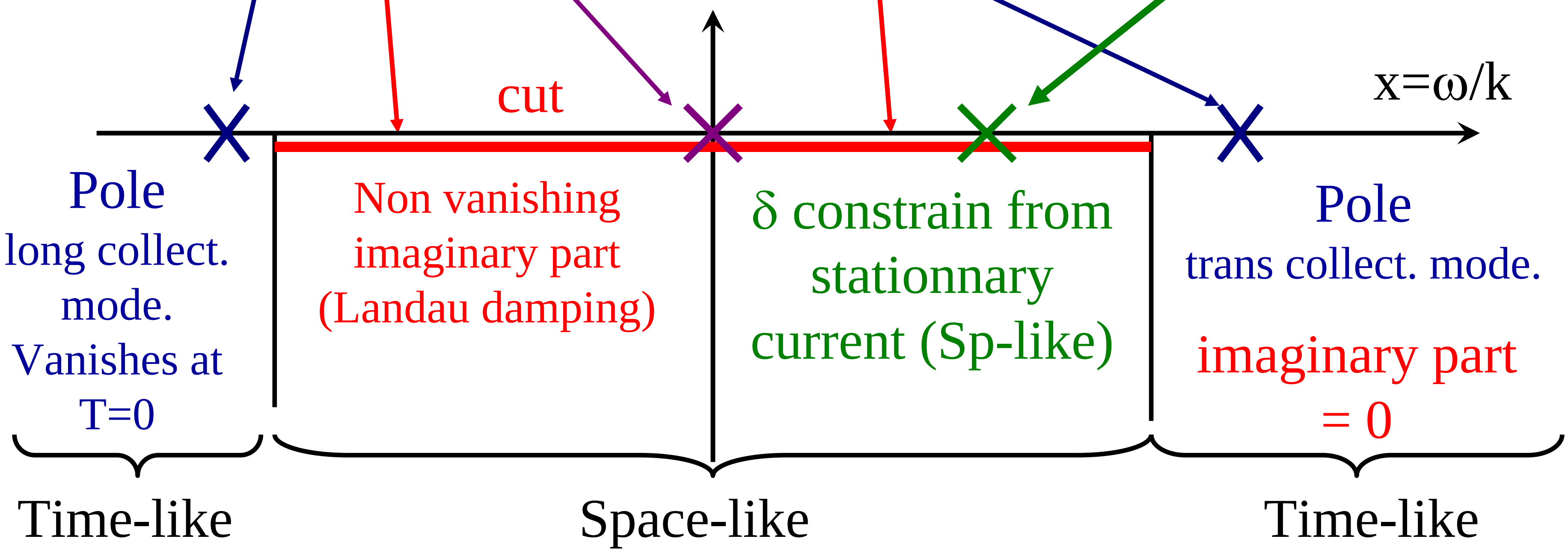
$$\frac{-\Delta E(L)}{C_R \alpha_S L} = \frac{i}{v} \int \frac{d^3k}{2\pi^2} \int_{-\infty}^{+\infty} d\omega \left[\frac{k^2}{k^2+\Pi_L} \frac{v_L^2(K)}{\omega} + \frac{\omega}{\omega^2-k^2+\Pi_T} v_T^2(K) \right]_{\text{ind}} \delta(\omega-\vec{k}\cdot\vec{v})$$

Only imaginary part contributes

N.B.: « induced » not really needed

Far collisions: alternative formulation (IV)

$$\left[\frac{k^2}{k^2+\Pi_L}\frac{v_L^2(K)}{\omega}+\frac{\omega}{\omega^2-k^2+\Pi_T}v_T^2(K)\right]_{\text{ind}}\delta(\omega-\vec{k}\cdot\vec{v})$$

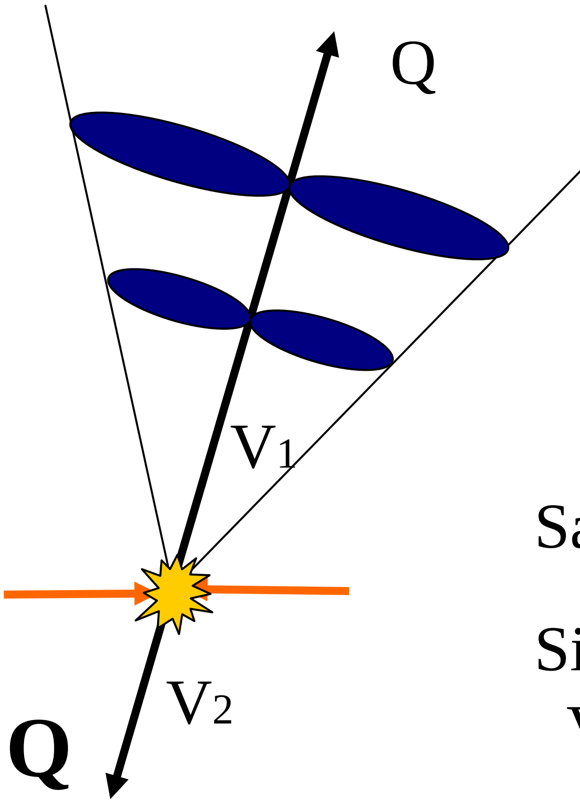


In stationnary regime: Only the Im(ε) contributes (ok); collective modes do not get excited; no Cherenkov radiation.

Retardation effect for heavy parton collisional energy loss in QGP.

- Motivation
- Collisional energy loss in stationnary regime
- Retardation effect; main results
- Radiation
- Conclusion & Perspectives

Retardation effect: model for partonic current



For partons produced at t=0

$$j_0^{\mu\, a}(K) = iq^a \left(\frac{V_1^\mu}{K.V_1 + i\eta} - \frac{V_2^\mu}{K.V_2 + i\eta} \right)$$

with $V_1 = (1, \vec{v}_1)$ and $V_2 = (1, \vec{v}_2)$

Satisfies current conservation $K \cdot V = 0$

Simplifying assumption: $v_2 = 0 \Rightarrow$
$V_2 = (1,0)$ and $V_1 = (1, \vec{v})$

Only **one** parton contributes to $\vec{j}$, but the system is in fact color neutral.

Retardation effect:generalization of ΔE

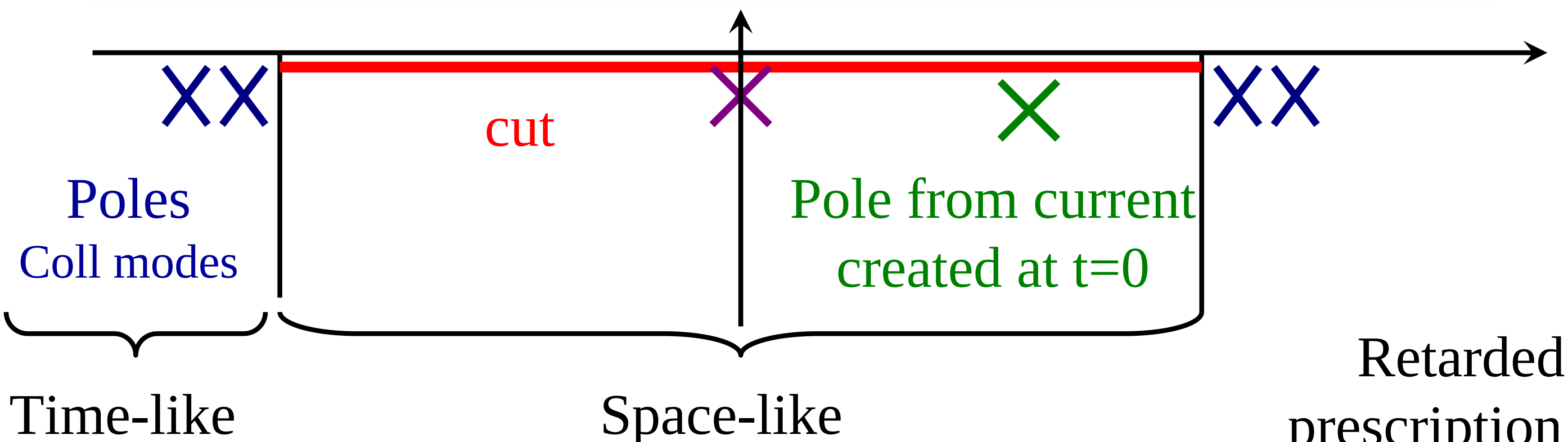
$$-\Delta E(L) = i\sum_{a}\int\frac{d^3k}{4\pi^3}\int_{-\infty}^{+\infty}d\omega\left[\frac{k^2}{k^2+\Pi_L}\,\frac{\vec{j}_{L,a}(K)}{\omega}+\frac{\omega}{\omega^2-k^2+\Pi_T}\,\vec{j}_{T,a}(K)\right]_{\rm ind}\cdot q_a\,\vec{v}\int_0^{L/v}dt\;e^{-iK\cdot V_1 t}$$

$K\cdot V=0$ but $K\cdot V_1\neq 0$

$$\frac{-\Delta E(L)}{C_R\alpha_s} = -iv^2\int\frac{d^3\vec{k}}{4\pi^3}\int_{-\infty}^{\infty}\frac{d\omega}{\omega}\left[k^2\cos^2\theta\,\Delta_L(\omega,k)+\omega^2\sin^2\theta\,\Delta_T(\omega,k)\right]_{\rm ind}$$
$$\times\left\{\frac{1-e^{-i(\omega-kv\cos\theta)L/v}}{(\omega-kv\cos\theta)(\omega-kv\cos\theta+i\eta)}\right\}.$$

complex →

← current



Retarded prescription !!!

Retardation effect:convergence and scales

Contour integration in lower half-(complex) plane :

$$\frac{-\Delta E(L)}{C_R \alpha_s} = \int \frac{d^3\vec{k}}{2\pi^2} \left\{ \frac{1-e^{ikL\cos\theta}}{k^2+m_D^2} + v^2 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi\omega} \left[k^2\cos^2\theta\, \rho_L + \omega^2 \sin^2\theta\, \rho_T \right] \right.$$
$$\left. \times\, \mathrm{P}\left(\frac{1}{\omega - kv\cos\theta}\right) \frac{1-e^{-i(\omega - kv\cos\theta)\, L/v}}{\omega - kv\cos\theta} \right\}_{\mathrm{ind}} .$$

With the spectral functions :

$$\rho_s(\omega,k) \equiv 2\,\mathrm{Im}\,\Delta_s(\omega+i\eta,k)$$
$$= 2\pi\,\mathrm{sgn}(\omega)\, z_s(k)\delta(\omega^2-\omega_s^2(k)) + \beta_s(\omega,k)\theta(k^2-\omega^2)$$

Residue

Magnitude of the cut

Retardation effect: convergence and scales

Contour integration in lower half-(complex) plane :

$$\frac{-\Delta E(L)}{C_R \alpha_s} = \int \frac{d^3\vec{k}}{2\pi^2} \left\{ \frac{1 - e^{ikL\cos\theta}}{k^2 + m_D^2} + v^2 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi\omega} \left[k^2 \cos^2\theta \, \rho_L + \omega^2 \sin^2\theta \, \rho_T \right] \times \mathrm{P}\left(\frac{1}{\omega - kv\cos\theta} \right) \frac{1 - e^{-i(\omega - kv\cos\theta)L/v}}{\omega - kv\cos\theta} \right\}_{\mathrm{ind}} .$$

$\rightarrow$ finite constant when L $\rightarrow \infty$ (due to the induced prescription)

Retardation effect: convergence and scales

Contour integration in lower half-(complex) plane :

$$\frac{-\Delta E(L)}{C_R \alpha_s} = \int \frac{d^3\vec{k}}{2\pi^2} \left\{ \frac{1-e^{ikL\cos\theta}}{k^2+m_D^2} + v^2 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi\omega} \left[k^2\cos^2\theta\,\rho_L + \omega^2 \sin^2\theta\,\rho_T\right] \right.$$
$$\left. \times\, \mathrm{P}\left(\frac{1}{\omega - kv\cos\theta}\right) \frac{1-e^{-i(\omega-kv\cos\theta)\,L/v}}{\omega - kv\cos\theta} \right\}_{\mathrm{ind}} .$$

"$\rightarrow$" $\frac{\pi L}{v}\delta(\omega - kv\cos\theta)$ when L $\rightarrow \infty \Rightarrow$ add and remove :

$$\frac{-\Delta E(L)}{C_R \alpha_s} = \int_{k<k_{cut}} \frac{d^3\vec{k}}{2\pi^2} \left\{ \frac{1-e^{ikL\cos\theta}}{k^2+m_D^2} + v^2 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi\omega} \left[k^2\cos^2\theta\,\rho_L + \omega^2 \sin^2\theta\,\rho_T\right] \right.$$
$$\left. \times \left[\left(2\frac{\sin^2\left((\omega - kv\cos\theta)L/(2v)\right)}{(\omega - kv\cos\theta)^2} - \frac{\pi L}{v}\delta(\omega - kv\cos\theta) \right) + \frac{\pi L}{v}\delta(\omega - kv\cos\theta) \right] \right\}_{\mathrm{ind}} .$$

Brings a 1/k additional factor instead of L $\Rightarrow$ UV convergent !

Exactly the stationnary E loss log divergent in UV

Retardation effect: convergence and scales (II)

Conclusion: $-\Delta E(L) = dE_{stat}/dx * L + B(L)$

With: B(L) UV safe (as anticipated) and $\to$ cst when L $\to \infty$.

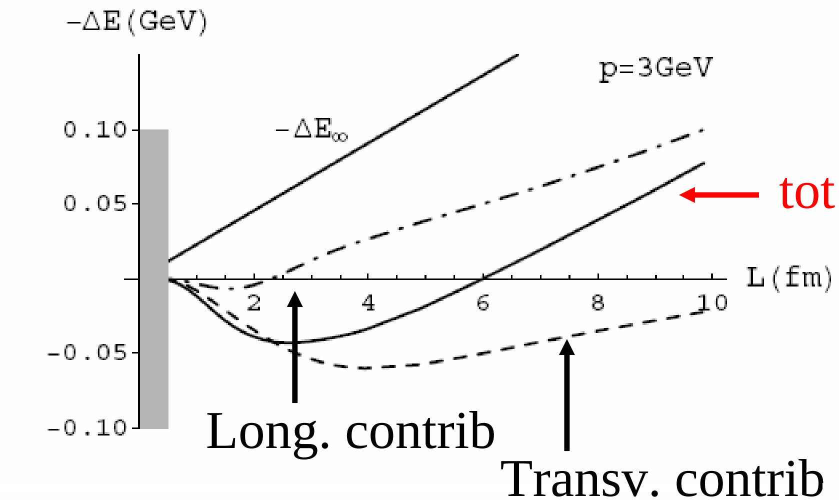
For small L, the hard scale « 1/L » would lead to k_{typ} in B(L) above k_{cut} $\Rightarrow$ Condition on L: must be typically larger then $1/m_D$.

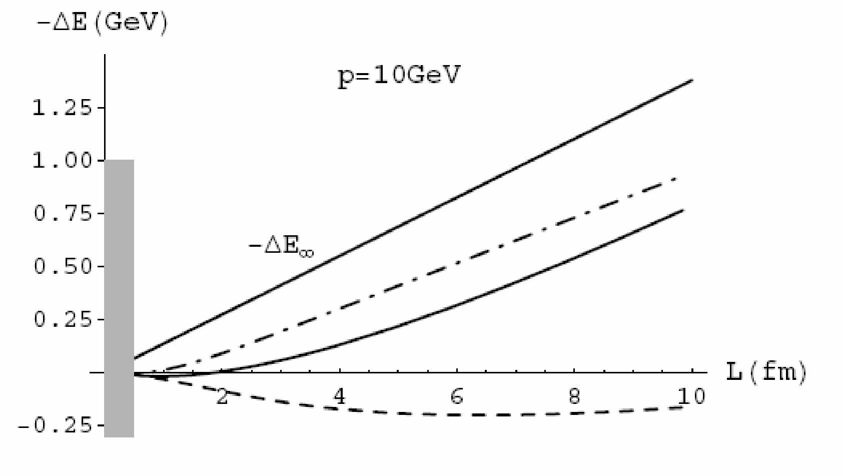
Behaviour at small L has been nevertheless studied, using *sum rules*; interesting cancelations were discovered.

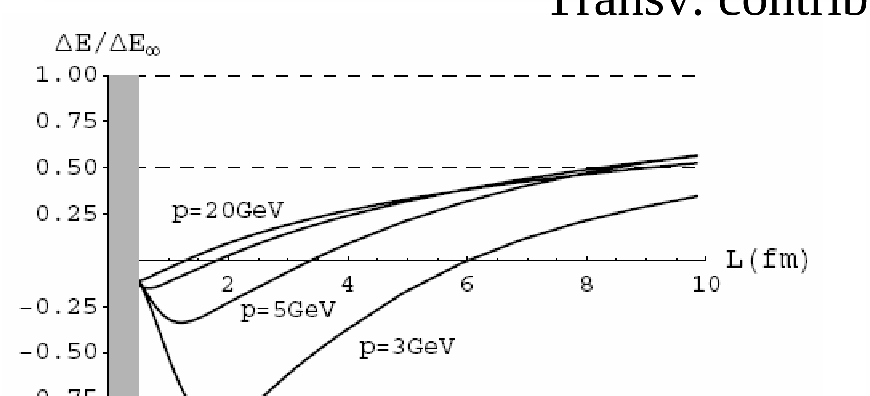
$$-\Delta E(L) \underset{L \ll k_{cut}^{-1}}{=} -\frac{C_R \alpha_s m_D^2 k_{cut}^3}{108\,\pi v^2} L^4$$

Retardation effect: Results

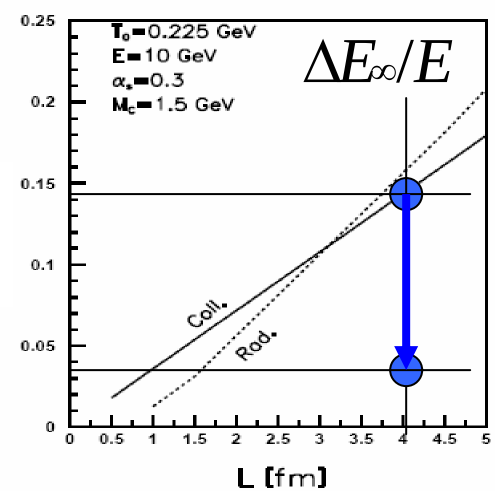
M = 1.5 GeV; α_s = 0.2; n_f = 2; T=0.25 GeV; m_D=0.46 GeV







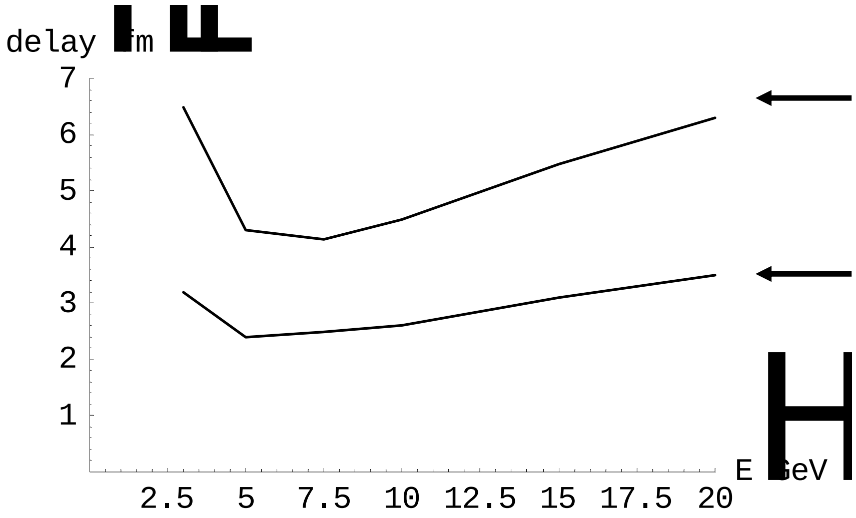
Negative ΔE(L) !!!



Large reduction of CEL due to retardation effect (Trento Tirami…)

Retardation effect: Results (II)

$$-\Delta E_{asymp}(L,E,\ldots) = dE_{stat}/dx * (L\text{-delay}(E,T))$$



T=0.25 GeV

T=0.5 GeV

M = 1.5 GeV; α_s = 0.2; n_f = 2

Delay roughly α 1/T, but not of the order of r_D !!!

Retardation effect: Results (III)

Might have some phenomenological implications :

Jet absorption and corona effect at RHIC. Extracting collision geometry from experimental data

V.S. Pantuev

University at Stony Brook, Stony Brook, NY 11794-3800

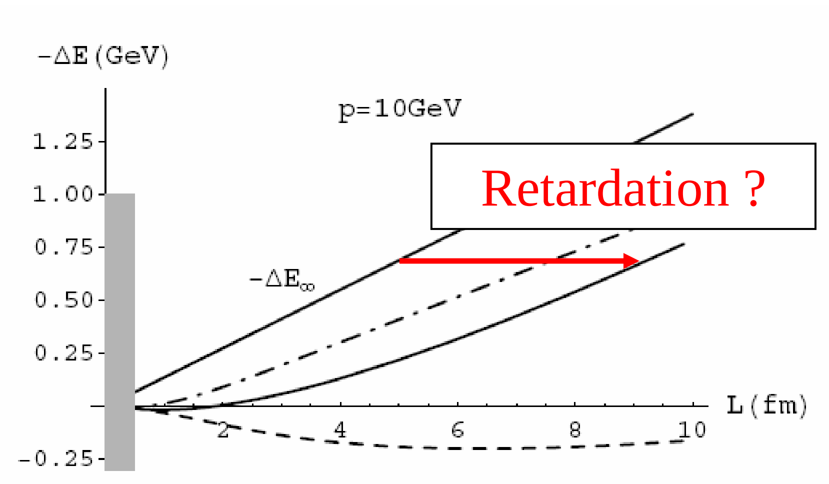
Abstract

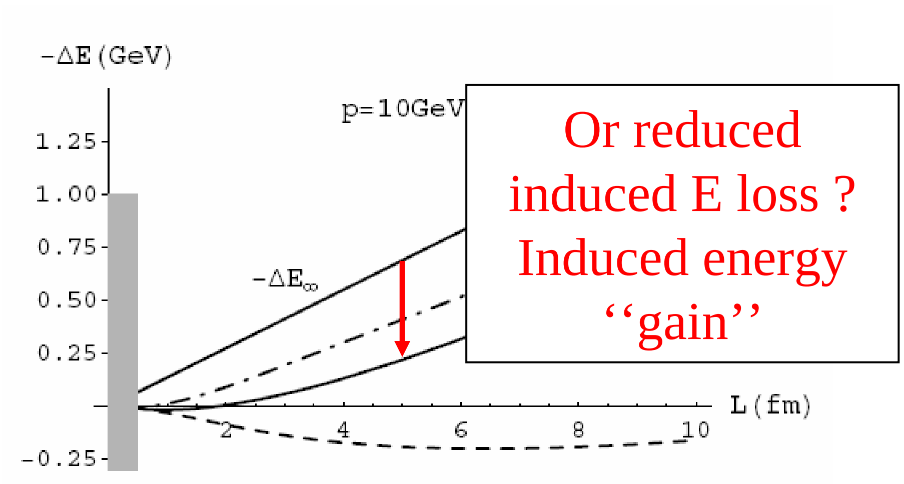
We propose a simple model based on Monte Carlo simulation of nucleus-nucleus collisions using a Glauber approach to explain experimental data on the angular dependence of the nuclear modification factor R_{AA} at high transverse momentum in the reaction plane. The model has one free parameter $L \simeq 2$ fm to describe the the thickness of the corona area and was ajusted to fit the experimental data on AuAu collisions at centrality 50-60%. The model nicely describes the R_{AA} dependence for all centrality classes. We extract the second Fourier component amplitude, v_2, for high pt particle azimuthal distribution and found v_2 should be at the level of 11-12% purely from the geometry of the collision with particle absorption in the core. We give a prediction for R_{AA} in Cu+Cu collisions at 200 GeV. Our physical interpretation of the parameter L is that it's actually the formation time $T = L/c \simeq 2$ fm/c.

Retardation effect for heavy parton collisional energy loss in QGP.

- Motivation
- Collisional energy loss in stationnary regime
- Retardation effect; main results
- Radiation
- Conclusion & Perspectives

Retardation effect… and other related mechanisms.





$$\frac{-\Delta E(L)}{C_R\alpha_s} = \int \frac{d^3\vec{k}}{2\pi^2}\left\{\frac{1-e^{ikL\cos\theta}}{k^2+m_D^2} + v^2\int_{-\infty}^{\infty}\frac{d\omega}{2\pi\omega}\left[k^2\cos^2\theta\,\rho_L+\omega^2\sin^2\theta\,\rho_T\right] \times \mathrm{P}\left(\frac{1}{\omega-kv\cos\theta}\right)\frac{1-e^{-i(\omega-kv\cos\theta)\,L/v}}{\omega-kv\cos\theta}\right\}_{\mathrm{ind}} .$$

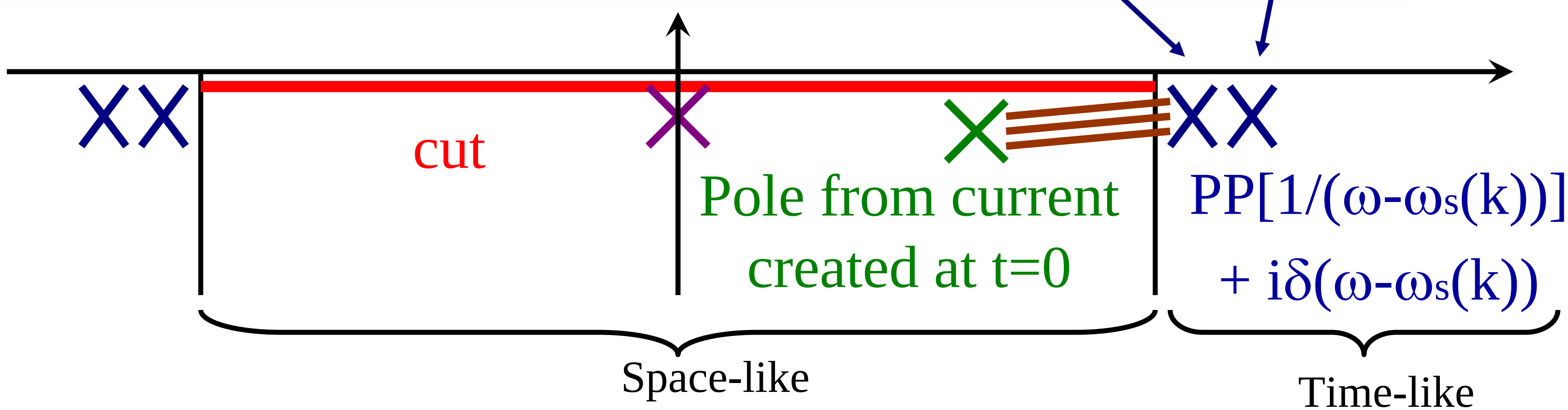
$\rightarrow$ finite negative constant when L $\rightarrow \infty$ (dipole separation easier in QGP than in vacuum)

Retardation effect… and other related mechanisms.

- If one just considers the force acting on the heavy parton: irrelevant question (It takes some time to reach its fully value, that’s all).
- Being less inclusive, one can identify another interesting mechanism...

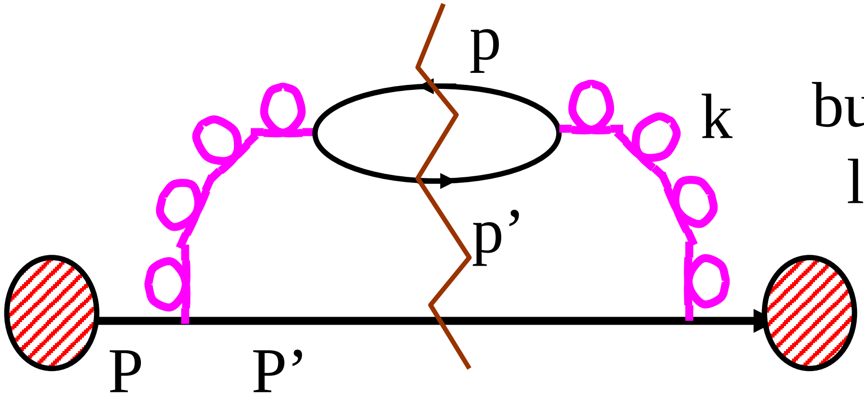
Retardation effect… and radiation.

$$\frac{-\Delta E(L)}{C_R\alpha_s} = -iv^2\int\frac{d^3\vec{k}}{4\pi^3}\int_{-\infty}^{\infty}\frac{d\omega}{\omega}\left[k^2\cos^2\theta\,\Delta_L(\omega,k)+\omega^2\sin^2\theta\,\Delta_T(\omega,k)\right]_{\rm ind} \times\left\{\frac{1-e^{-i(\omega-kv\cos\theta)\,L/v}}{(\omega-kv\cos\theta)(\omega-kv\cos\theta+i\eta)}\right\}.$$

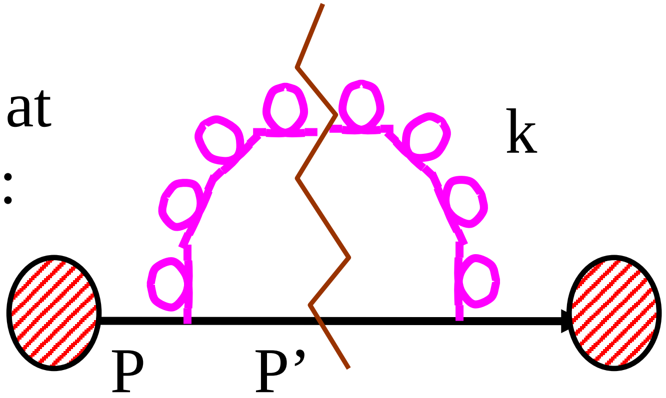


$\delta(\omega-\omega_s(k))$:
- Clear signature of some radiation (cf. identification of Cherenkov radiation in stationnary Collisional E loss)
- Does not scale like L $\Rightarrow$ Not Cherenkov (although emitted by the time dependent medium polarization), but ‘initial’ Brehmstrallung .
- ΔE incorporates the difference of energies radiated in the medium (neglecting radiative rescattering) and in the vacuum.

In-vacuum radiation



but also (even at lower order) :



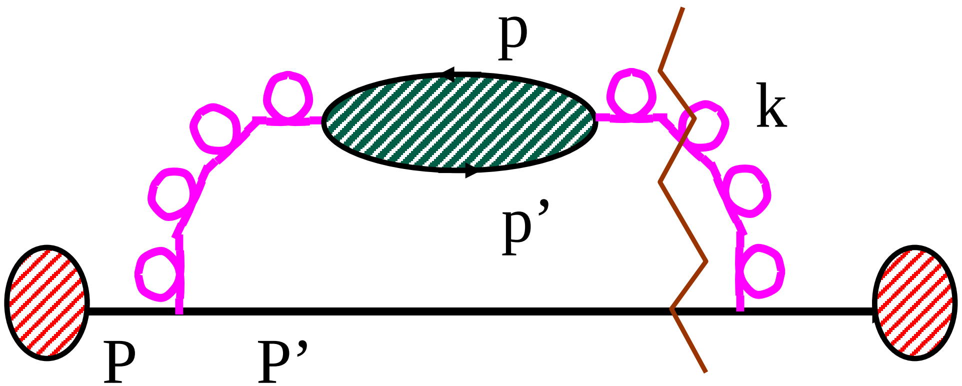
Radiative contribution W(L) to the energy loss :

$$\left.\frac{dW(L)}{dk\, d\cos\theta}\right|_{vac} = \frac{C_R\alpha_s}{\pi} \sin^2\theta \, \frac{\sin^2((k - kv\cos\theta)\, L/(2v))}{(\cos\theta - 1/v)^2} \quad \underset{L\to\infty}{\longrightarrow} \frac{C_R\alpha_s}{2\pi} \frac{v^2 \sin^2\theta}{(1 - v\cos\theta)^2}$$

<u>N.B.:</u> For v=1, this is just the Z.O.L. result of GLV for x<<1

''Radiates like crazy'', but what should matter is the difference with the ''In-medium'' radiation.

In-medium radiation



Radiation of collective modes, described at the level of the HTL ressumation

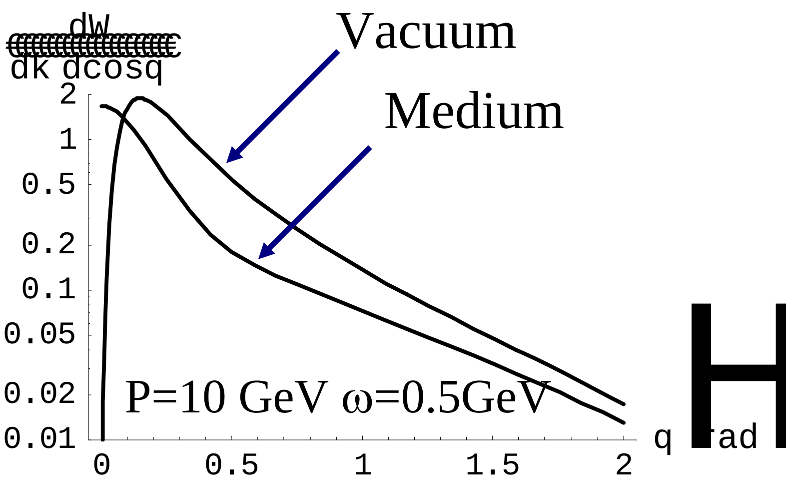
Not considered before, to my knowledge

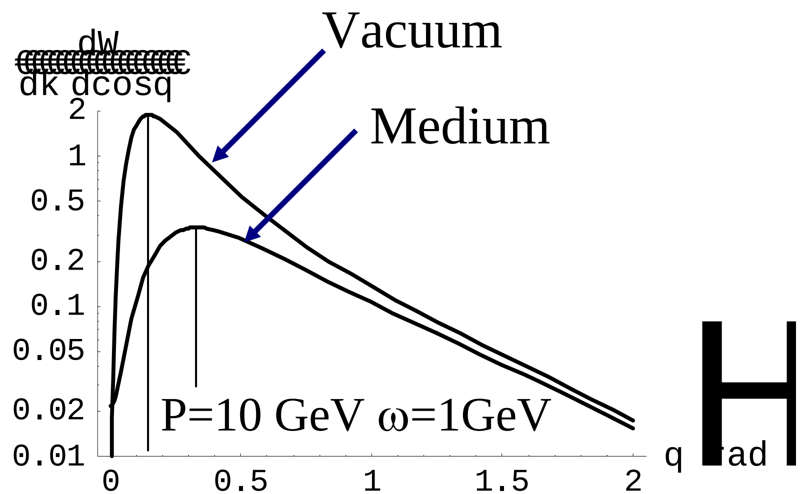
$$\frac{dW(L)}{dk\,d\cos\theta} = \frac{C_R\alpha_s}{\pi}\left\{ z_L(k)\frac{k^2}{\omega_L^2(k)}\cos^2\theta\,\frac{\sin^2((\omega_L(k)-kv\cos\theta)\,L/(2v))}{(\cos\theta-\omega_L(k)/(kv))^2} + z_T(k)\sin^2\theta\,\frac{\sin^2((\omega_T(k)-kv\cos\theta)\,L/(2v))}{(\cos\theta-\omega_T(k)/(kv))^2}\right\}$$

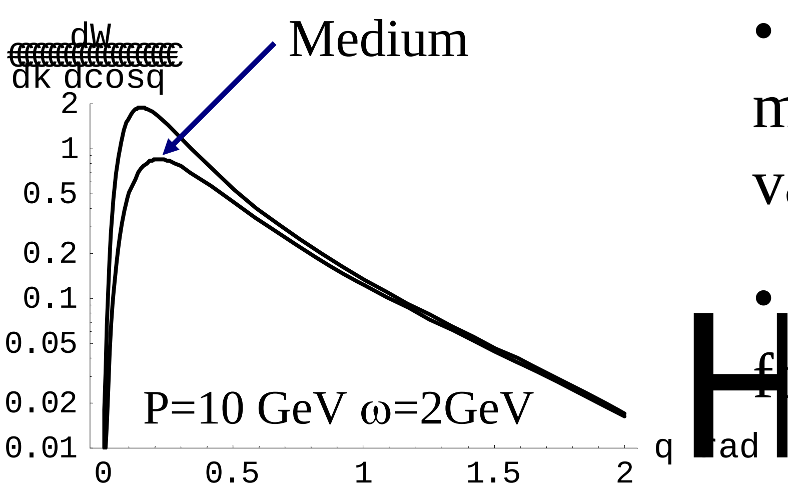
$$\xrightarrow[L\to\infty]{} \frac{C_R\alpha_s}{2\pi}\left\{ \underbrace{z_L(k)\frac{k^2}{\omega_L^2(k)}\,\frac{\cos^2\theta}{(\cos\theta-\omega_L(k)/(kv))^2}} + z_T(k)\,\frac{\sin^2\theta}{(\cos\theta-\omega_T(k)/(kv))^2}\right\}$$

Long: suppressed at large k

Vacuum vs Infinite medium



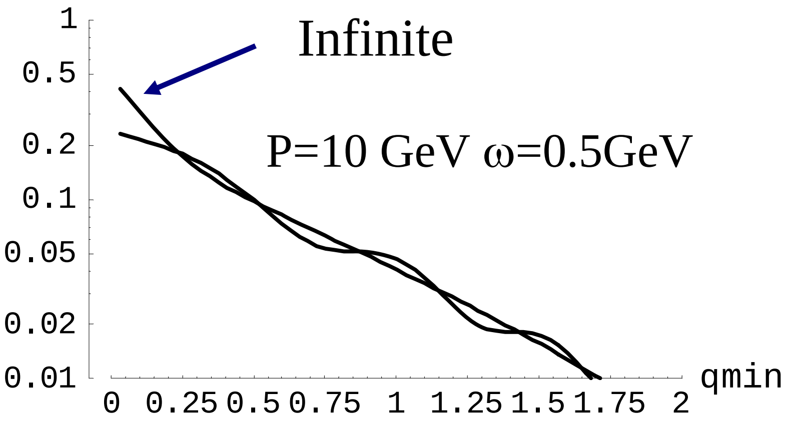


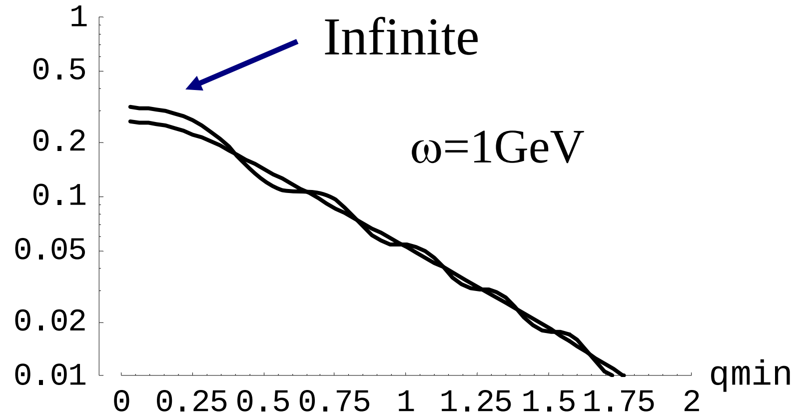


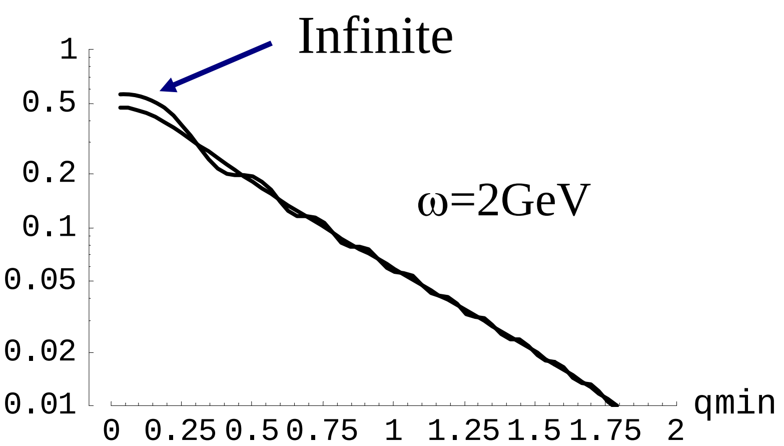
- In all cases, the θ–integrated In-medium radiation is weaker than in the vacuum (finite mass, residues < 1)

- Interplay of trans. and long. radiation for ω of the order of m_D

Infinite medium vs Finite (L=5fm) medium







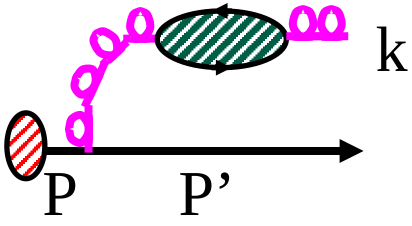
- Depletion at small angle
- Gluon formed:

$\sin^2((\omega_L(k) - kv\cos\theta)\, L/(2v))$ averages to ½

Provides a criteria for formation time.

- Possible consequences on the understanding of far away hadrons.

Conclusions & Perspectives.

1. Extansion of the formalism of Thoma and Guylassy in the case of partons produced inside the medium.
2. Collisional energy loss suppressed by large factor. Retardation time of the order of several fm/c.
3. In-vacuum gluon radiation is supressed by the medium. Radiation of collective mode is treated using the correct dispersion relation.
4. Crucial implications on the phenomenology of jet quenching.
5. Consequences on the radiative energy loss induced by rescatterings ?
6. Is 

the splitting function of some in-medium evolution equation ?