# Three-nucleon forces at low energy

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**RU**B

# Outline

- From QCD to nuclear physics

- Nuclear forces in chiral EFT

- Three-nucleon forces up to $N^3LO$

- Long-range part of three-nucleon forces up to $N^4LO$

- Summary & Outlook

# From QCD to nuclear physics
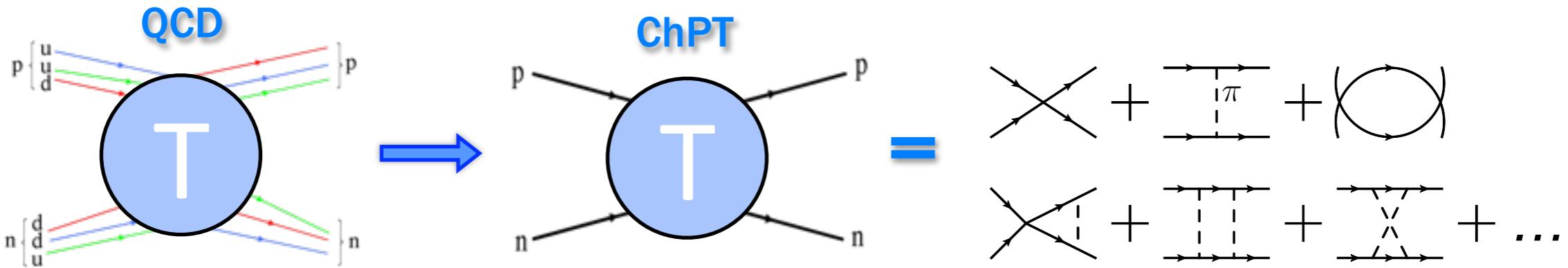


**NN interaction is strong**: resummations/nonperturbative methods needed

$1/m_N$ - expansion: nonrelativistic problem ($|\vec{p}_i| \sim M_\pi \ll m_N$) ⟹ the QM A-body problem

$$\left[\left(\sum_{i=1}^{A}\frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\textit{derived within ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$

Weinberg '91



contact interactions

multiple GB exchange (ChPT)
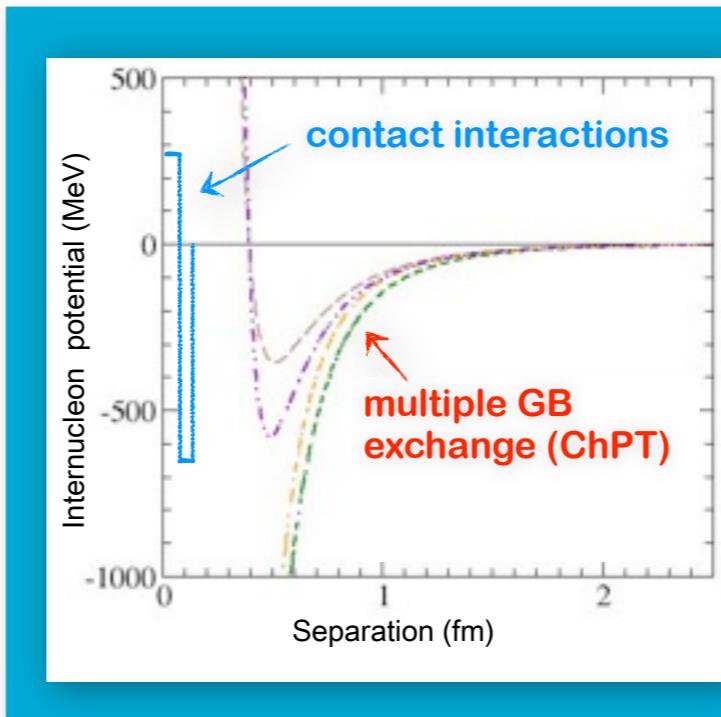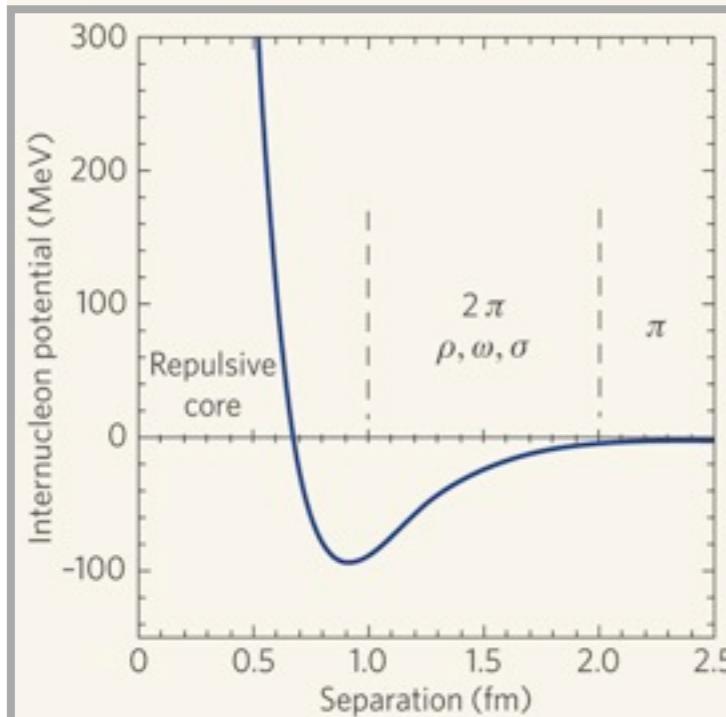
- unified description of $\pi\pi$, $\pi$N and NN
- consistent many-body forces and currents
- systematically improvable
- bridging different reactions (electroweak, $\pi$-prod., ...)
- precision physics with/from light nuclei

# Nucleon-nucleon force up to N³LO

*Ordonez et al. '94; Friar & Coon '94; Kaiser et al. '97; Epelbaum et al. '98,'03; Kaiser '99-'01; Higa et al. '03; ...*

**Chiral expansion for the 2N force:**

$$V_{2N} = V_{2N}^{(0)} + V_{2N}^{(2)} + V_{2N}^{(3)} + V_{2N}^{(4)} + ...$$
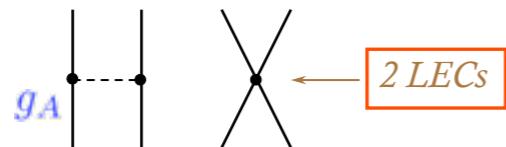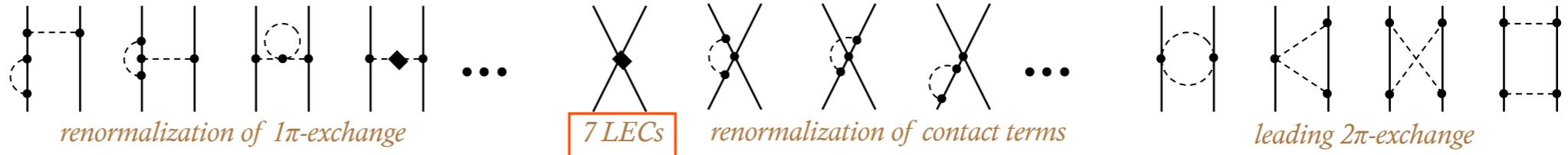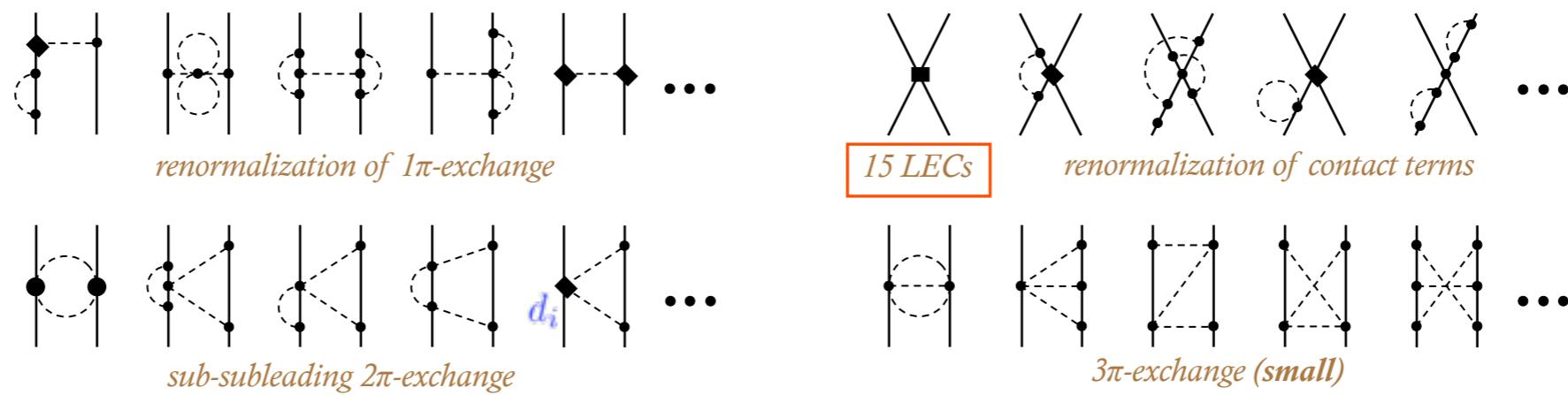


+ 1/m and isospin-breaking corrections…

# Neutron-proton phase shifts up to N³LO
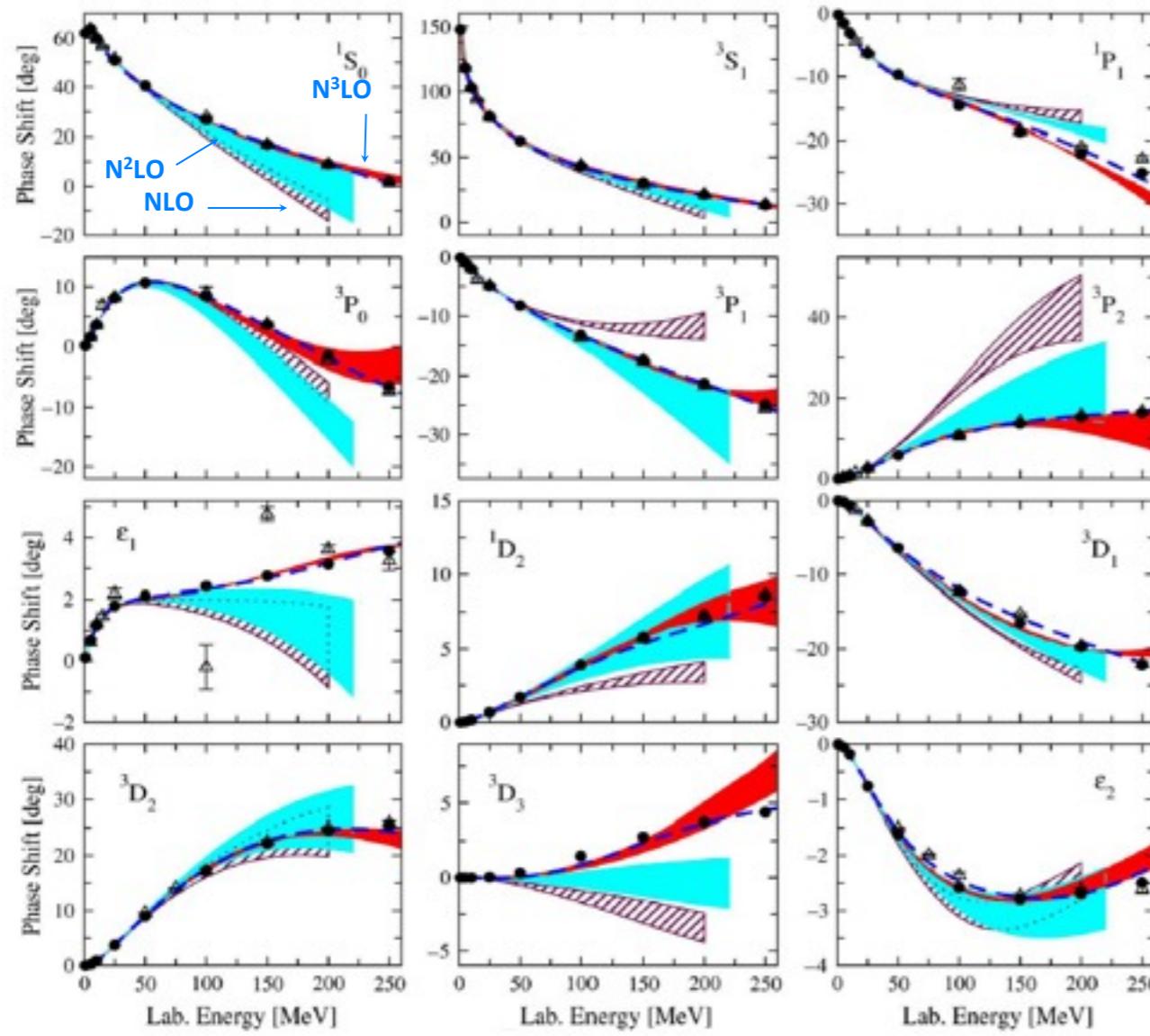
# np scattering at 50 MeV
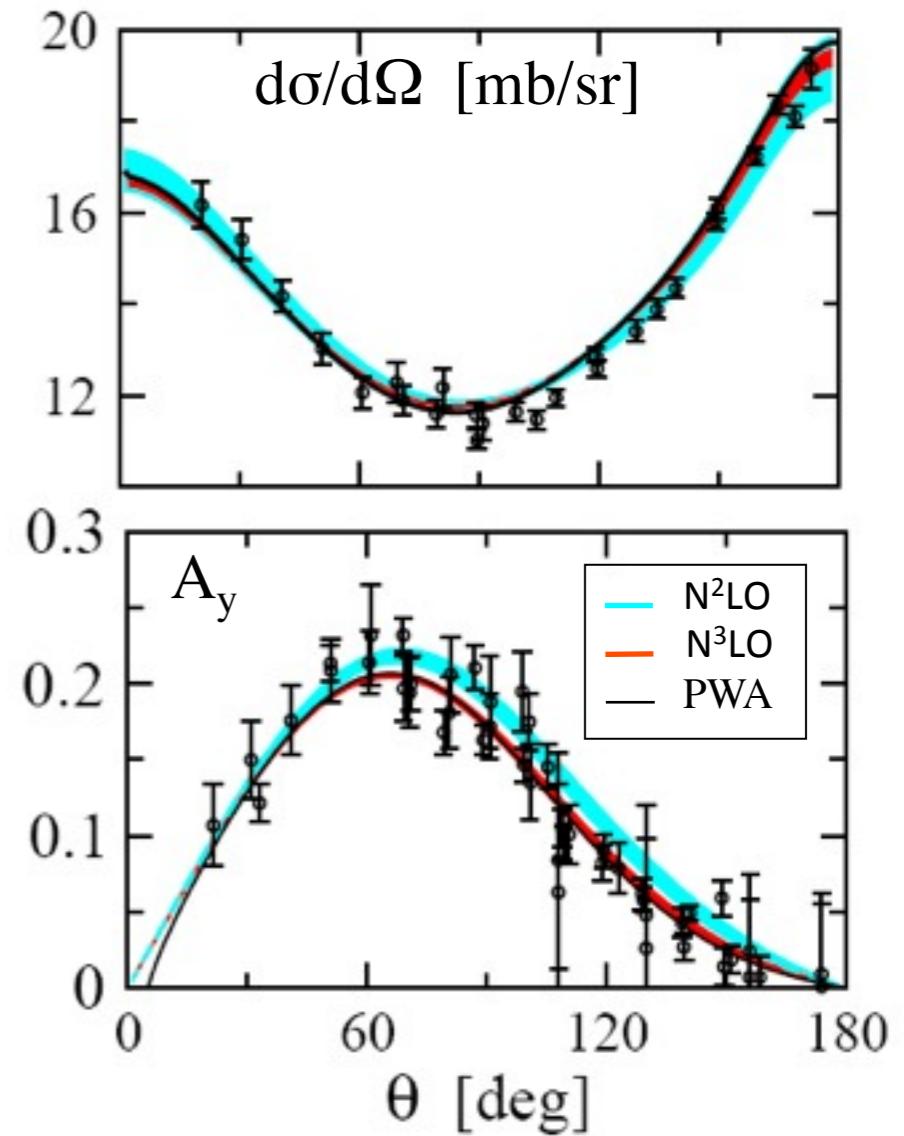


# Deuteron binding energy & asymptotic normalizations $A_s$ and $\eta_d$

|  | NLO | N²LO | N³LO | Exp |
|---|---|---|---|---|
| $E_d$ [MeV] | $-2.171\ldots-2.186$ | $-2.189\ldots-2.202$ | $-2.216\ldots-2.223$ | $-2.224575(9)$ |
| $A_S$ [fm$^{-1/2}$] | $0.868\ldots0.873$ | $0.874\ldots0.879$ | $0.882\ldots0.883$ | $0.8846(9)$ |
| $\eta_d$ | $0.0256\ldots0.0257$ | $0.0255\ldots0.0256$ | $0.0254\ldots0.0255$ | $0.0256(4)$ |

*Entem & Machleidt '03; Epelbaum, Glöckle & Meißner '05*

# Nuclear forces up to N³LO

## dimensional analysis counting



|  | Two-nucleon force | Three-nucleon force | Four-nucleon force |
|---|---|---|---|
| LO ($Q^0$) | | | |
| NLO ($Q^2$) | | | |
| N²LO ($Q^3$) | | | |
| N³LO ($Q^4$) | | | |

- converged
- accurate description of NN at least up to $E_{lab} \sim 200$ MeV

- not yet converged
- higher orders in progress
- impact on few- & many-N systems?

- converged ??
- presently out of reach for few- & many-N studies
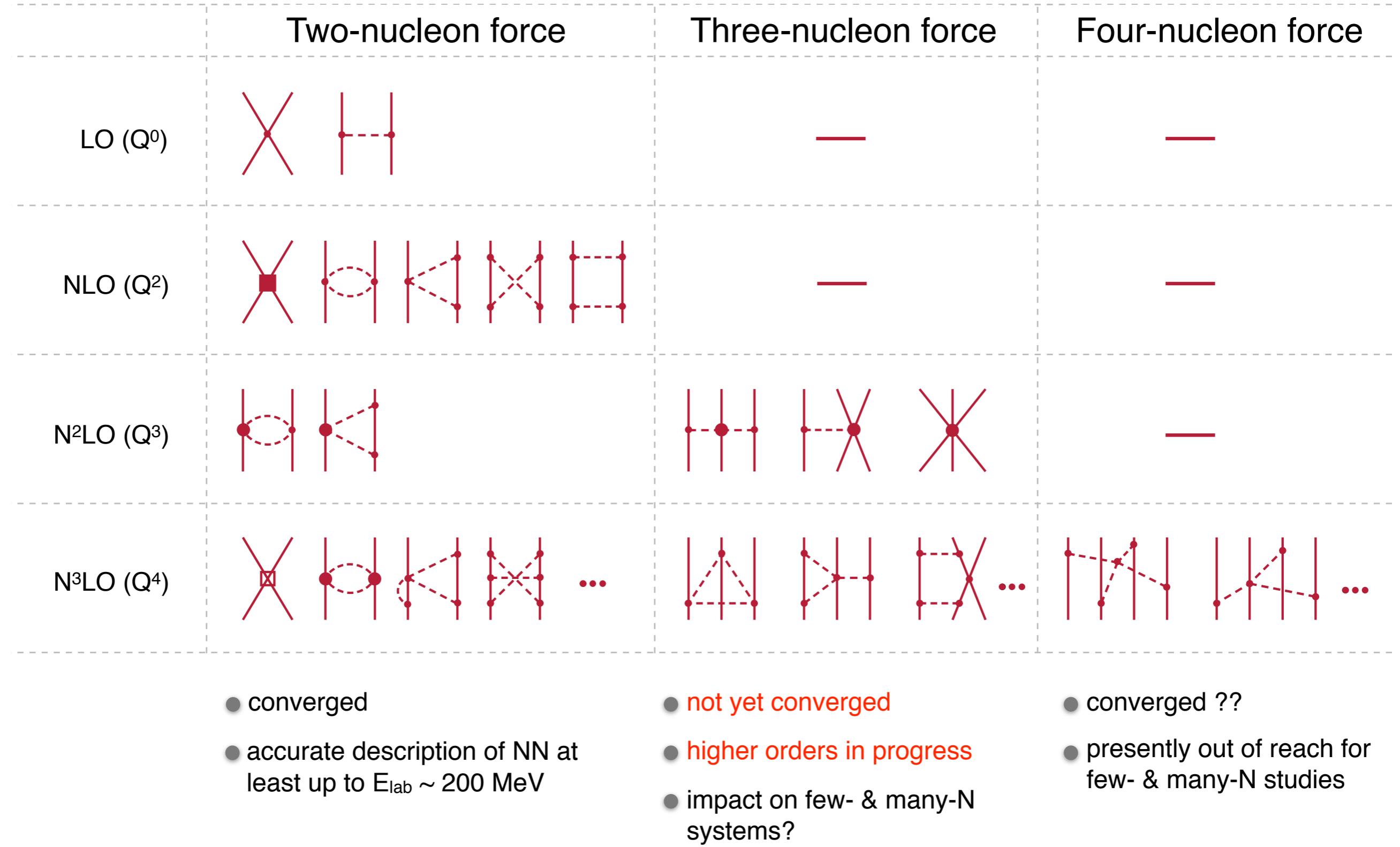
# Three-nucleon forces

- Three-nucleon forces in chiral EFT start to contribute at NNLO

    *(U. van Kolck ´94; Epelbaum et al. ´02; Nogga et al. ´05; Navratil et al. ´07)*
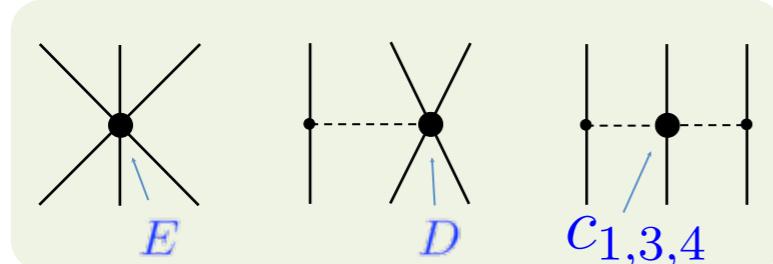


$c_{1,3,4}$ from the fit to $\pi N$-scattering data

$D, E$ from $^3\mathrm{H}, ^4\mathrm{He}, ^{10}\mathrm{B}$ binding energy + coherent $nd$ - scattering length

- LECs $D$ and $E$ incorporate short-range contr.
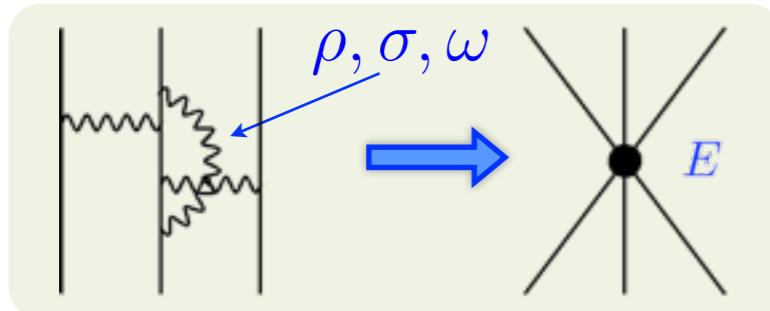


Resonance saturation interpretation of LECs
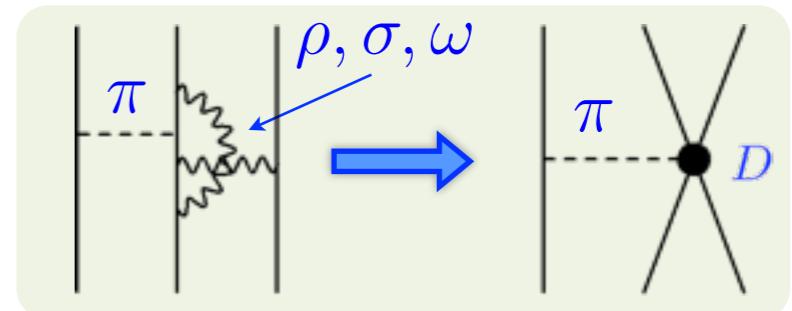
- Delta contributions encoded in LECs

    *(Bernard, Kaiser & Meißner '97)*
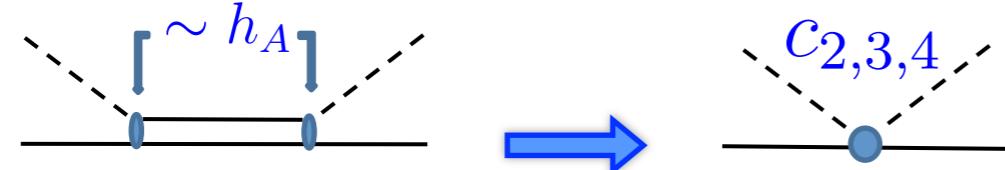


Delta-resonance saturation

$$c_3 = -2c_4 = c_3(\cancel{\Delta}) - \boxed{\frac{4h_A^2}{9\Delta}}$$

Enlargement due to Delta contribution

# nd elastic scattering

$E_{Lab} = 10\,\text{MeV}$

$E_{Lab} = 65\,\text{MeV}$

# nd break-up $[\text{mb}\,\text{MeV}^{-1}\text{sr}^{-2}]$

SST at $E_N = 12\,\text{MeV}$
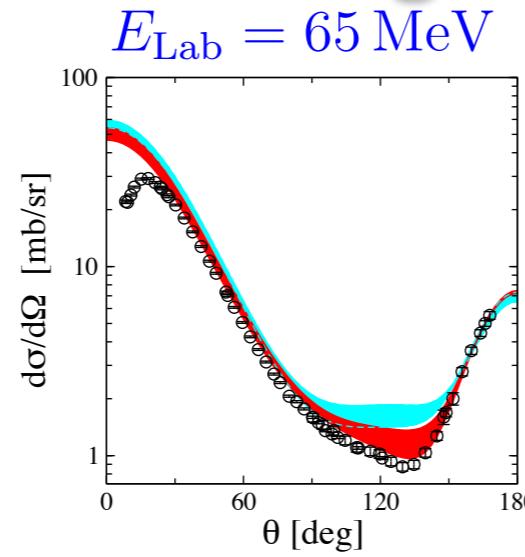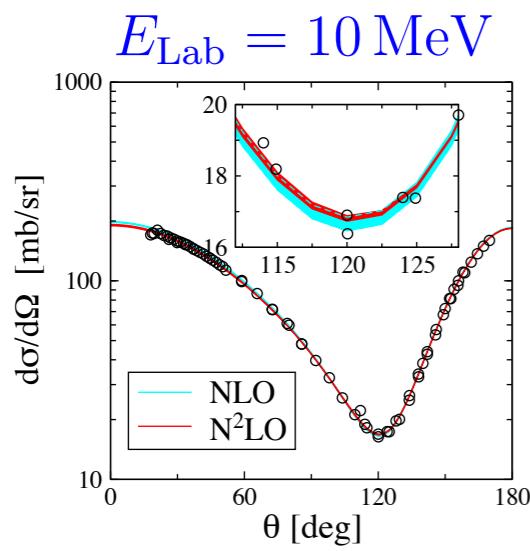
SCRE at $E_N = 19\,\text{MeV}$



# polarization transfer: $E_p^{Lab} = 22.7\,\text{MeV}$

$d(\vec{p}, \vec{p})d$

$d(\vec{p}, \vec{d})p$
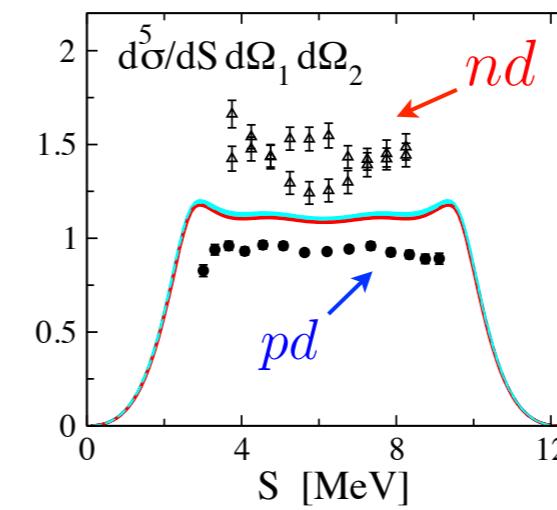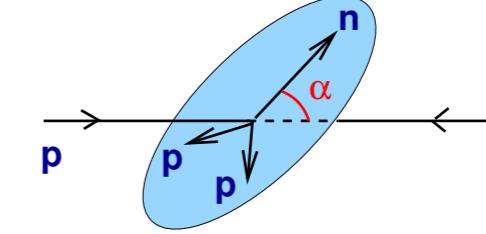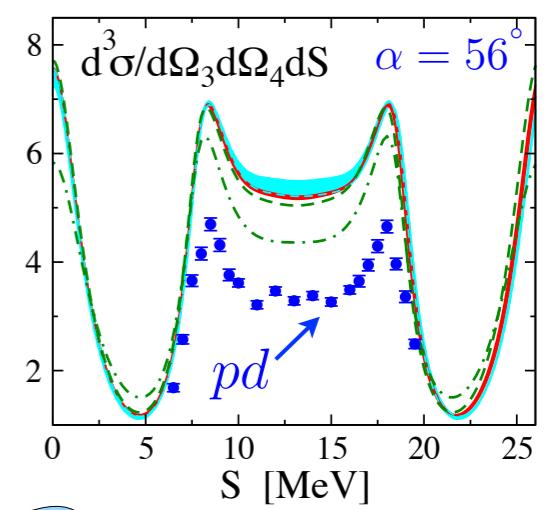


For references see recent reviews:

Epelbaum, Prog. Part Nucl. Phys. 57 (06) 654

Epelbaum, Hammer, Meißner, Rev. Mod. Phys. 81 (09) 1773

Entem, Machleidt, Phys. Rept. 503 (11) 1

Epelbaum, Meißner, arXiv:1201.2136,
       to appear in Ann. Rev. Nucl. Part. Sci.

Kalantar et al. Rep. Prog. Phys. 75 (12) 016301

- Generally good description of data. But some discrepancies arise. E.g. break-up observables for SCRE/SST configuration at low energy

- Hope for improvement at N3LO

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# Proton-³He elastic scattering

*Viviani, Girlanda, Kievsky, Marcucci, Rosati arXiv: 1004.1306*

p-³He differential cross section at low energies
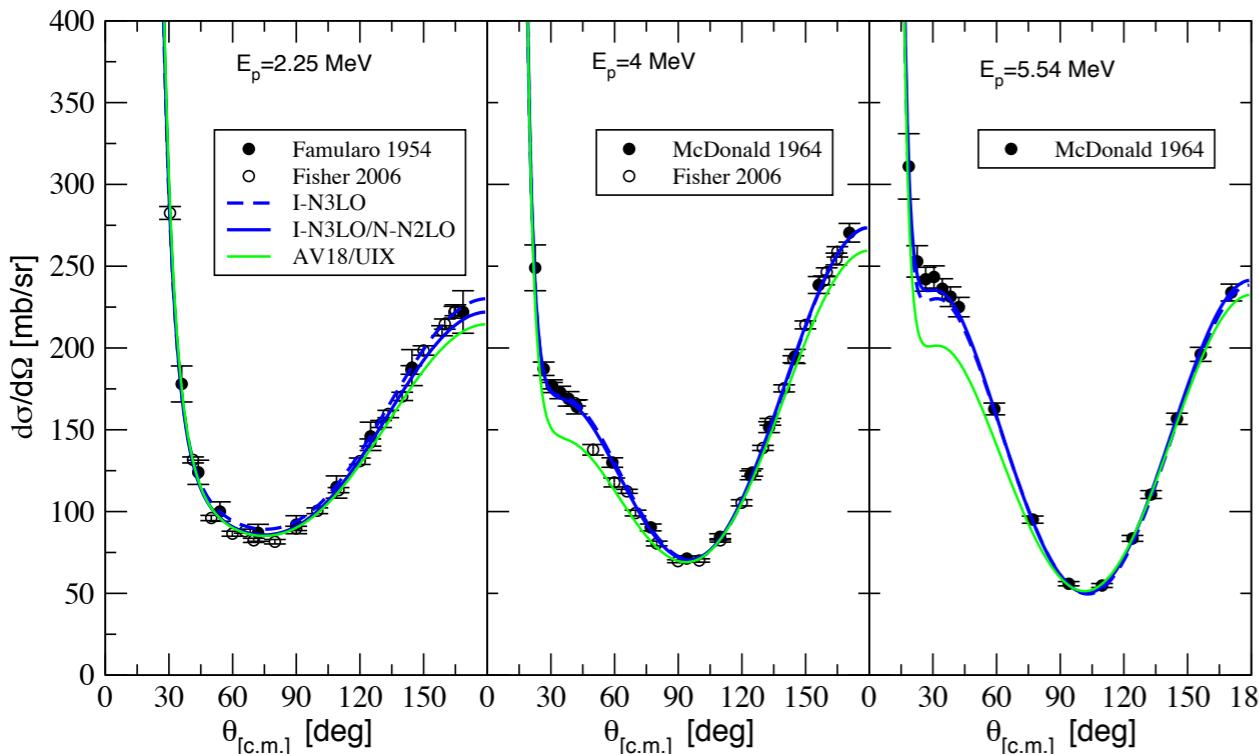
proton vector analyzing power $A_y$-puzzle
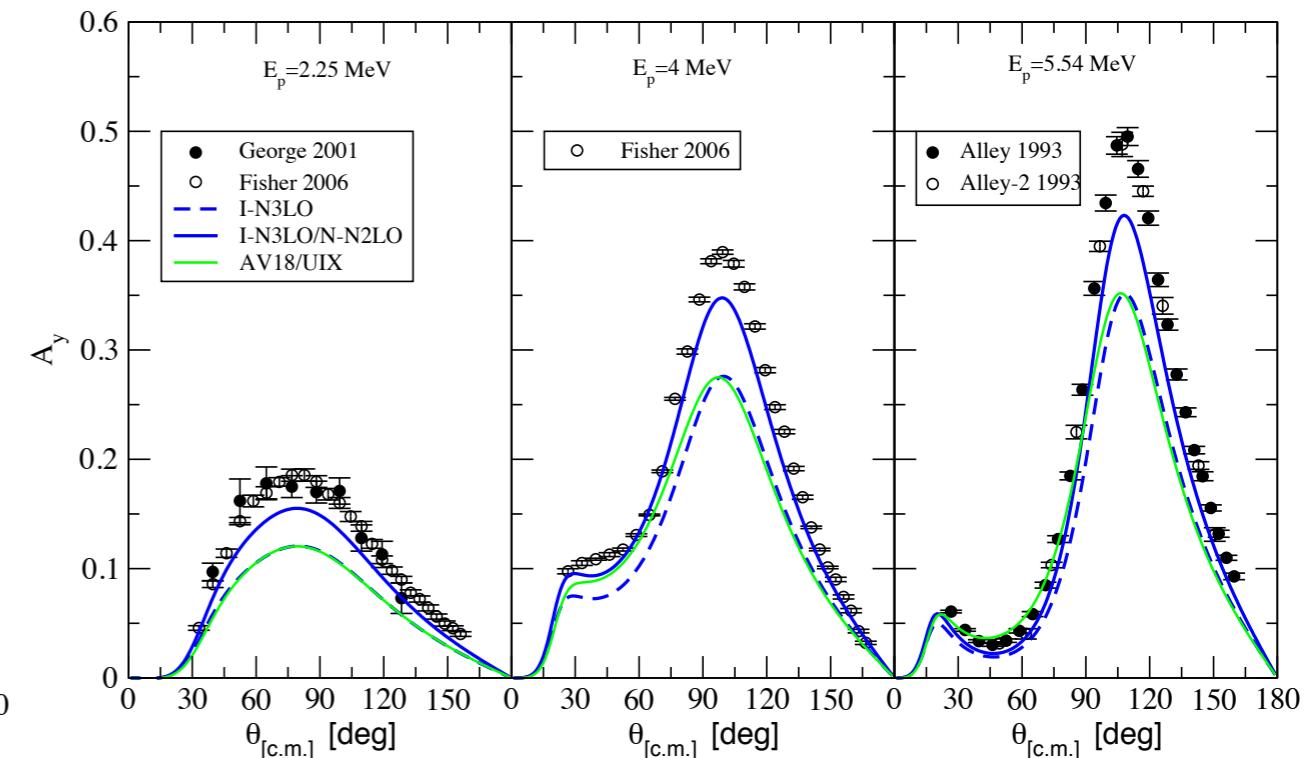


As in n-d scattering case $N^2LO$ 3NF's are not enough to resolve underprediction of $A_y$

Hope for improvement at higher orders

Many-body applications:

- EFT on the lattice *see talk by Dean Lee*

- No-Core-Shell-Model *see talk by Robert Roth*

Dispersive approach: *see talk by Ashot Gasparyan*

Further extensions/applications

- Electro-weak currents/few-body reactions:
  *see talks by Kölling, Myhrer, Girlanda, Pastore, Phillips*

- $\pi - d$ scattering: *see talk by Hoferichter*

# Three-nucleon forces

● Three-nucleon forces at N³LO

**Long range contributions**  *Bernard, Epelbaum, H.K., Meißner ´08; Ishikawa, Robilotta ´07*

● No additional free parameters

● Expressed in terms of $g_A, F_\pi, M_\pi$

● Rich isospin-spin-orbit structure

● $\Delta(1232)$-contr. are important

Large $c_i \sim$ 
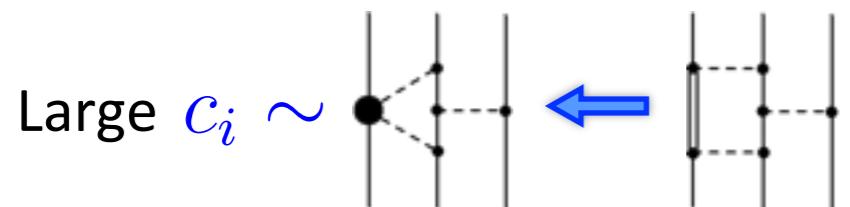


**Shorter range contributions**  *Bernard, Epelbaum, H.K., Meißner ´11*

● LECs needed for shorter range contr.
$$g_A, F_\pi, M_\pi, C_T$$

● Central NN contact interaction does not contribute

● Unique expressions in the static limit for a renormalizable 3NF
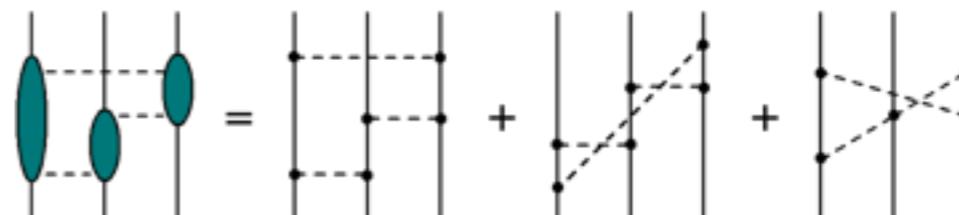
# Two-pion-exchange 3NF
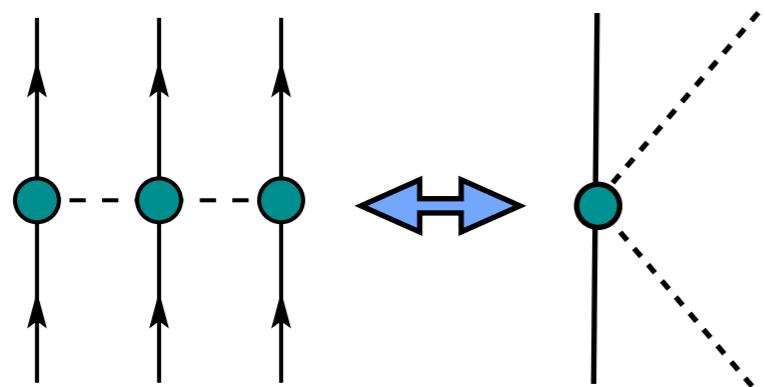


- Two-pion-exchange 3NF is connected to pion-nucleon scattering amplitude

  *Ishikawa, Robilotta ´07*

- The same linear combinations of LECs

- The same renormalization

$$V_{2\pi} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2]\,[q_3^2 + M_\pi^2]} \Big( \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big)$$

**NLO - contr.**



$\mathcal{L}_{\pi N}^{(1)}$
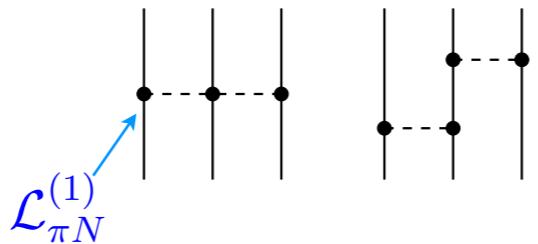
← yield vanishing 3NF contributions

**N²LO - contr.**



$\mathcal{L}_{\pi N}^{(2)}$
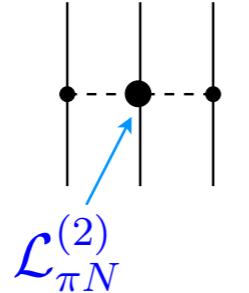
← first nonvanishing 3NF, encodes information about the Δ:

$$\mathcal{A}^{(3)}(q_2) = \frac{g_A^2}{8F_\pi^4}\Big( (2c_3 - 4c_1)M_\pi^2 + c_3 q_2^2 \Big)\,, \qquad \mathcal{B}^{(3)}(q_2) = \frac{g_A^2 c_4}{8F_\pi^4}$$

*van Kolck ´94*

# Two-pion-exchange 3NF

$$\mathcal{L}_{\pi N}^{(3)}$$

renormalization
nontrivial !
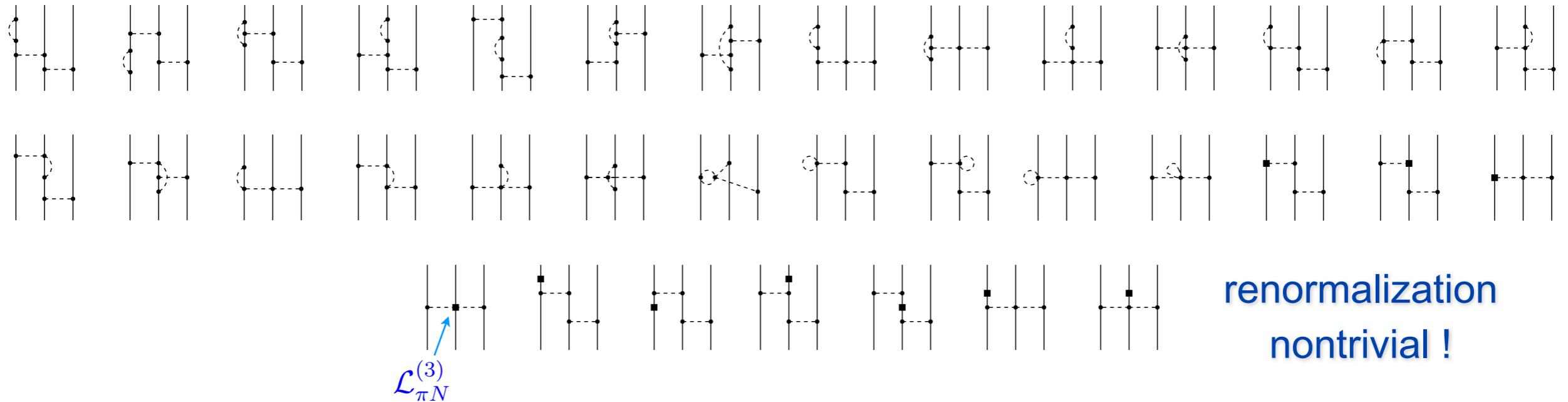
$$\mathcal{A}^{(4)}(q_2) = \frac{g_A^4}{256\pi F_\pi^6}\left[A(q_2)\left(2M_\pi^4 + 5M_\pi^2 q_2^2 + 2q_2^4\right) + \left(4g_A^2 + 1\right)M_\pi^3 + 2\left(g_A^2 + 1\right)M_\pi q_2^2\right],$$

$$\mathcal{B}^{(4)}(q_2) = -\frac{g_A^4}{256\pi F_\pi^6}\left[A(q_2)\left(4M_\pi^2 + q_2^2\right) + (2g_A^2 + 1)M_\pi\right]$$

*Ishikawa, Robilotta ´07,
Bernard, Epelbaum, HK, Meißner ´07*

- No unknown parameters at this order

- Everything is expressed in terms of loop function $A(q) = \frac{1}{2q}\arctan\frac{q}{2M_\pi}$

- Additional unitarity transformations required for proper renormalization

# Two-pion-exchange 3NF

**N⁴LO - contr. (subleading 1 loop)**  *Epelbaum, Gasparyan, H.K., ´12*
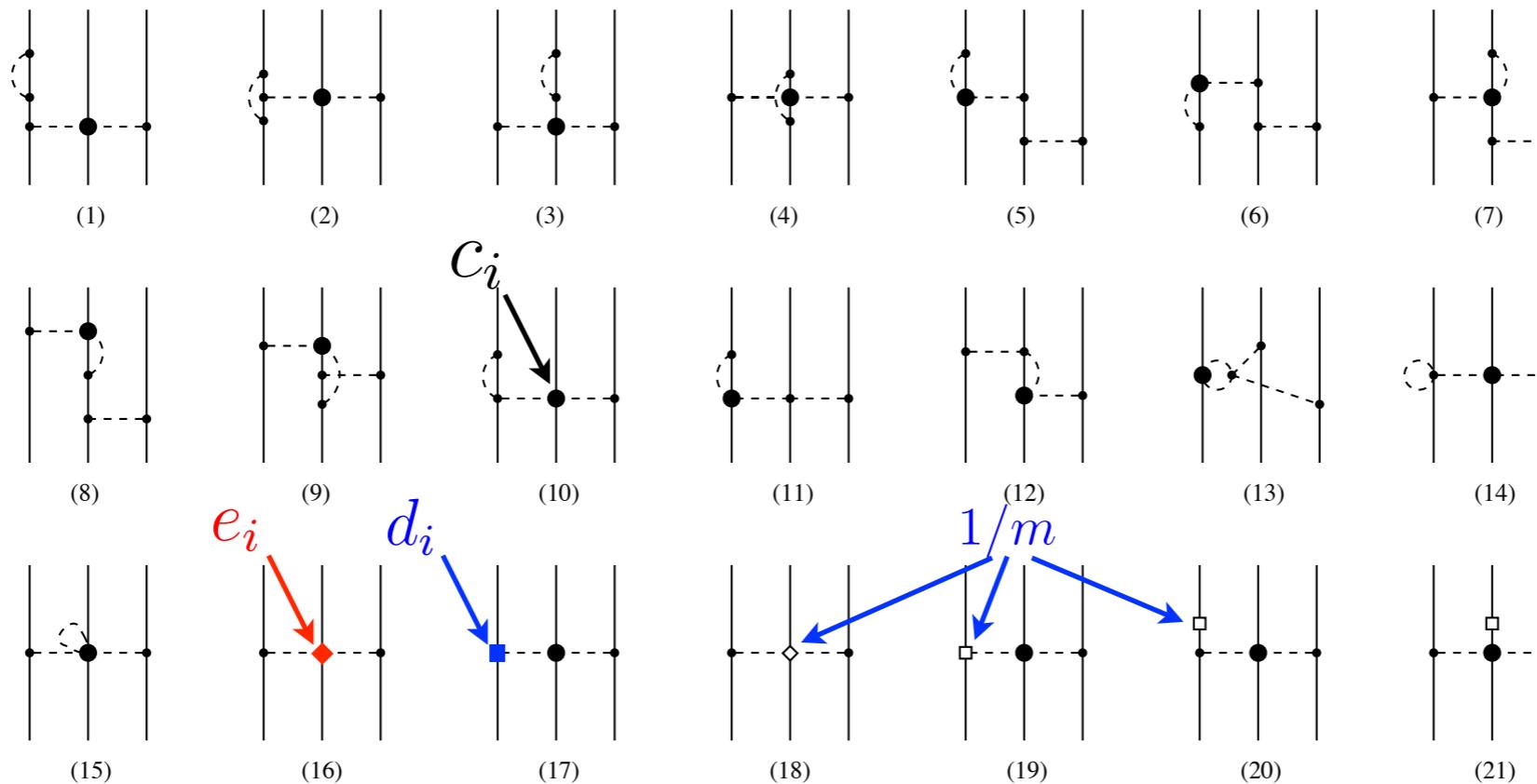


$c_i$'s LECs from $\mathcal{L}_{\pi N}^{(2)}$, $d_i$'s LECs from $\mathcal{L}_{\pi N}^{(3)}$, $e_i$'s LECs from $\mathcal{L}_{\pi N}^{(4)}$: fitted to $\pi N$ - scattering data

- Leading $\Delta$ - contributions are taken into account through $c_i$'s

- Vanishing $1/m$ - contributions at this order

# Two-pion-exchange 3NF at N⁴LO

$$\mathcal{A}^{(5)}(q_2) = \frac{g_A}{4608\pi^2 F_\pi^6}\Big[ M_\pi^2 q_2^2\big(F_\pi^2\big(2304\pi^2 g_A(4\bar{e}_{14} + 2\bar{e}_{19} - \bar{e}_{22} - \bar{e}_{36}) - 2304\pi^2 \bar{d}_{18}c_3\big)$$

$$+\ g_A(144c_1 - 53c_2 - 90c_3)\big) + M_\pi^4\big(F_\pi^2\big(4608\pi^2\bar{d}_{18}(2c_1 - c_3) + 4608\pi^2 g_A(2\bar{e}_{14} + 2\bar{e}_{19} - \bar{e}_{36} - 4\bar{e}_{38})\big)$$

$$+\ g_A\big(72\big(64\pi^2\bar{l}_3 + 1\big)c_1 - 24c_2 - 36c_3\big)\big) + q_2^4\big(2304\pi^2\bar{e}_{14}F_\pi^2 g_A - 2g_A(5c_2 + 18c_3)\big)\Big]$$

$$-\ \frac{g_A^2}{768\pi^2 F_\pi^6}L(q_2)\big(M_\pi^2 + 2q_2^2\big)\big(4M_\pi^2(6c_1 - c_2 - 3c_3) + q_2^2(-c_2 - 6c_3)\big)$$

$$\mathcal{B}^{(5)}(q_2) = -\frac{g_A}{2304\pi^2 F_\pi^6}\Big[ M_\pi^2\big(F_\pi^2\big(1152\pi^2\bar{d}_{18}c_4 - 1152\pi^2 g_A(2\bar{e}_{17} + 2\bar{e}_{21} - \bar{e}_{37})\big) + 108g_A^3 c_4 + 24g_A c_4\big)$$

$$+\ q_2^2\big(5g_A c_4 - 1152\pi^2\bar{e}_{17}F_\pi^2 g_A\big)\Big] + \frac{g_A^2 c_4}{384\pi^2 F_\pi^6}L(q_2)\big(4M_\pi^2 + q_2^2\big)$$

Some LECs can be absorbed by shifting $c_i$'s

$$c_1 \rightarrow c_1 - 2M_\pi^2\left(\bar{e}_{22} - 4\bar{e}_{38} - \frac{\bar{l}_3 c_1}{F_\pi^2}\right),$$

$$c_3 \rightarrow c_3 + 4M_\pi^2\left(2\bar{e}_{19} - \bar{e}_{22} - \bar{e}_{36} + 2\frac{\bar{l}_3 c_1}{F_\pi^2}\right),$$

$$c_4 \rightarrow c_4 + 4M_\pi^2(2\bar{e}_{21} - \bar{e}_{37}),$$

$$g_{\pi NN} = \frac{g_A\, m}{F_\pi}\left(1 - \frac{2M_\pi^2\,\bar{d}_{18}}{g_A}\right) \Longleftarrow \text{Violation of Goldberger-Treiman relation}$$

$$L(q) = \frac{\sqrt{q^2 + 4M_\pi^2}}{q}\log\frac{\sqrt{q^2 + 4M_\pi^2} + q}{2M_\pi}$$

🔵 No $d_i$ dependence of TPE-contr. besides $d_{18}$

🔵 Pion-nucleon scattering does strongly depend on $d_i$'s

# Pion-nucleon scattering

Heavy baryon calculation up to order $q^4$  *Fettes, Meißner Nucl. Phys. A676 (2000) 311*

1/m power counting used in FM work $\Longrightarrow$ $\dfrac{p}{m} \sim \dfrac{q}{\Lambda_\chi}$

Difference in Weinberg's power counting for NN $\Longrightarrow$ $\dfrac{p}{m} \sim \left(\dfrac{q}{\Lambda_\chi}\right)^2$

Refit of $d_i$ and $e_i$ LECs is needed

$$\pi^a(q_1) + N(p_1) \to \pi^b(q_2) + N(p_2)$$

$$T^{ba}_{\pi N} = \frac{E+m}{2m}\left(\delta^{ba}\left[g^+(\omega,t) + i\,\vec{\sigma}\cdot\vec{q}_2\times\vec{q}_1\,h^+(\omega,t)\right] + i\,\epsilon^{bac}\tau^c\left[g^-(\omega,t) + i\vec{\sigma}\cdot\vec{q}_2\times\vec{q}_1\,h^-(\omega,t)\right]\right)$$

CMS kinematics: $\omega = q_1^0 = q_2^0,\quad E = E_1 = E_2 = \sqrt{\vec{q}^{\,2}+m^2},\quad \vec{q}_1^{\,2} = \vec{q}_2^{\,2} = \vec{q}^{\,2},\quad t = (q_1 - q_2)^2$

Partial wave amplitudes: $f^\pm_{l\pm}(s) = \dfrac{E+m}{16\pi\sqrt{s}}\displaystyle\int_{-1}^{1} dz\left[g^\pm P_l(z) + \vec{q}^{\,2}h^\pm\left(P_{l\pm1}(z) - zP_l(z)\right)\right]$

In the isospin basis: $f^{1/2}_{l\pm} = f^+_{l\pm} + 2f^-_{l\pm},\quad f^{3/2}_{l\pm} = f^+_{l\pm} - f^-_{l\pm}$

Absence of inelasticity below the two-pion production threshold

$$\delta^I_{l\pm}(s) = \arctan\left(|\vec{q}|\mathcal{R}e\,f^I_{l\pm}(s)\right)$$

Dispersive analysis of pion-nucleon scattering: *see talks by Ditsche and Hoferichter*

# Two-pion-exchange at N⁴LO

Data fitted for $p_{Lab} < 150\,\text{MeV}$



KH
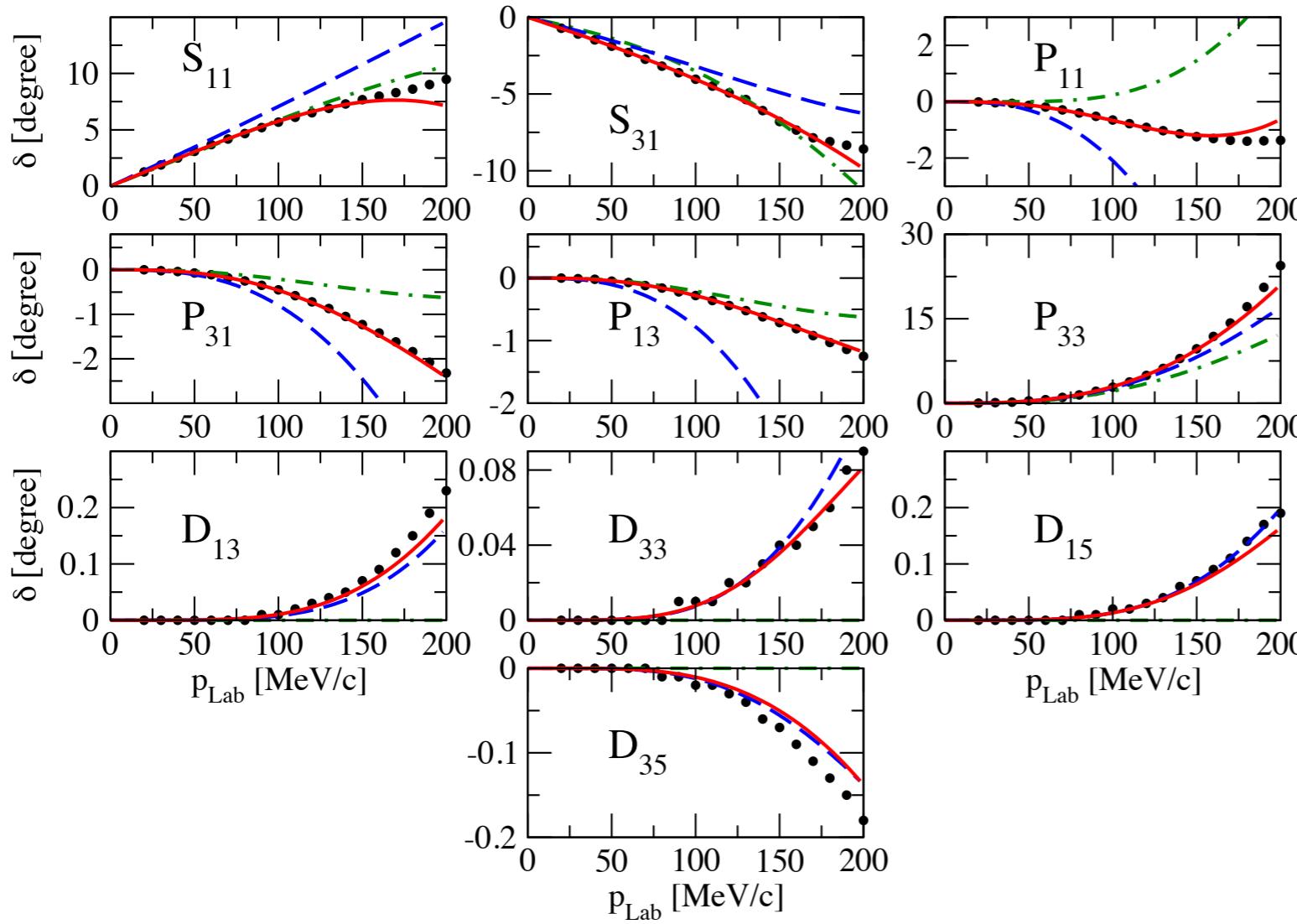
*Karlsruhe-Helsinki (KH) PWA: R. Koch Nucl. Phys. A 448 (1986) 707*

*Similar fit to George-Washington (GW) PWA: Arndt et al. Phys. Rev. C 74 (2006) 045205*
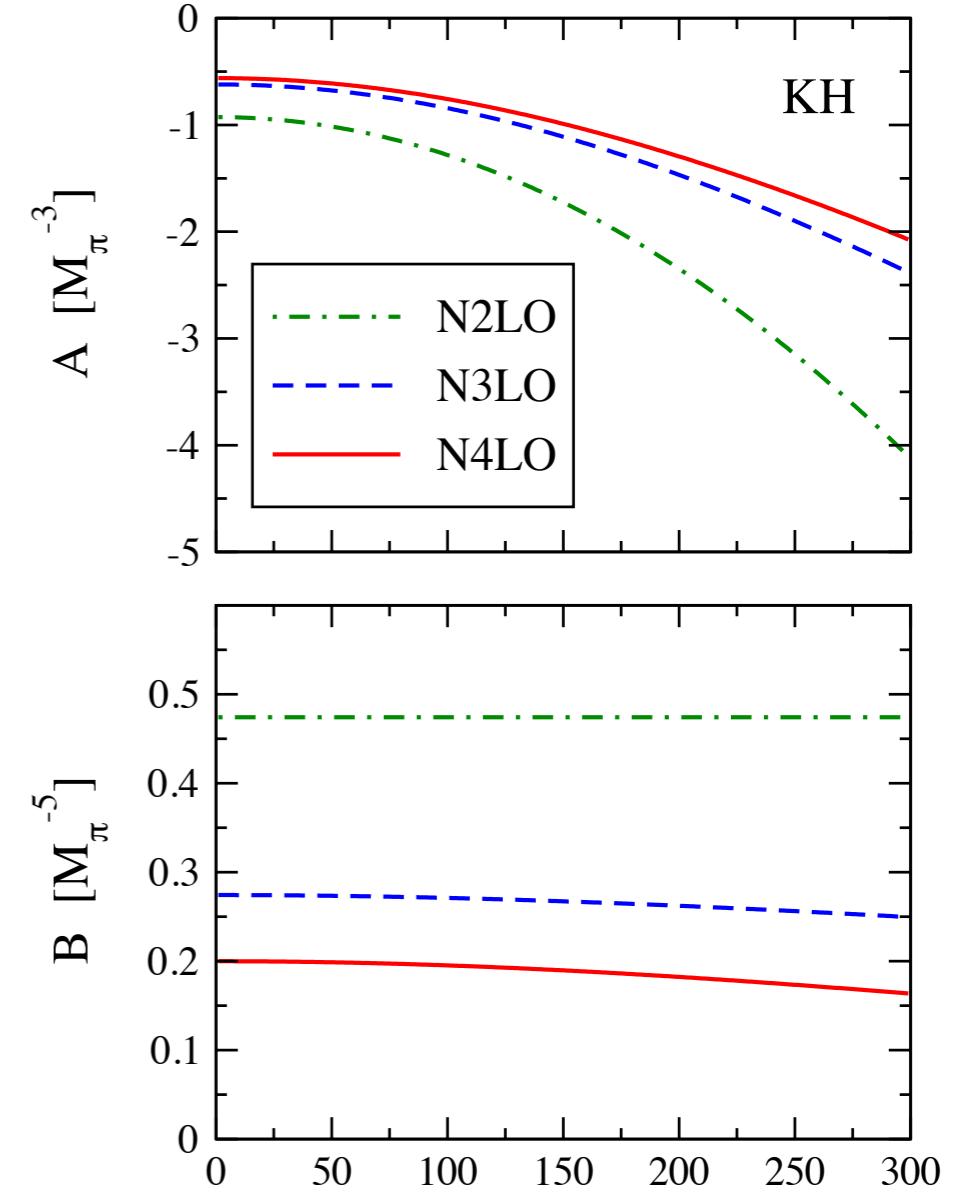
| | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $\bar{d}_1 + \bar{d}_2$ | $\bar{d}_3$ | $\bar{d}_5$ | $\bar{d}_{14} - \bar{d}_{15}$ | $\bar{e}_{14}$ | $\bar{e}_{15}$ | $\bar{e}_{16}$ | $\bar{e}_{17}$ | $\bar{e}_{18}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| GW-fit | −1.13 | 3.69 | −5.51 | 3.71 | 5.57 | −5.35 | 0.02 | −10.26 | 1.75 | −5.80 | 1.76 | −0.58 | 0.96 |
| KH-fit | −0.75 | 3.49 | −4.77 | 3.34 | 6.21 | −6.83 | 0.78 | −12.02 | 1.52 | −10.41 | 6.08 | −0.37 | 3.26 |

🔵 No dependence on $d_i$'s    🔵 $e_i$'s are of natural size    🔵 Good convergence of TPE 3NF

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# Working with N²LO 3NF
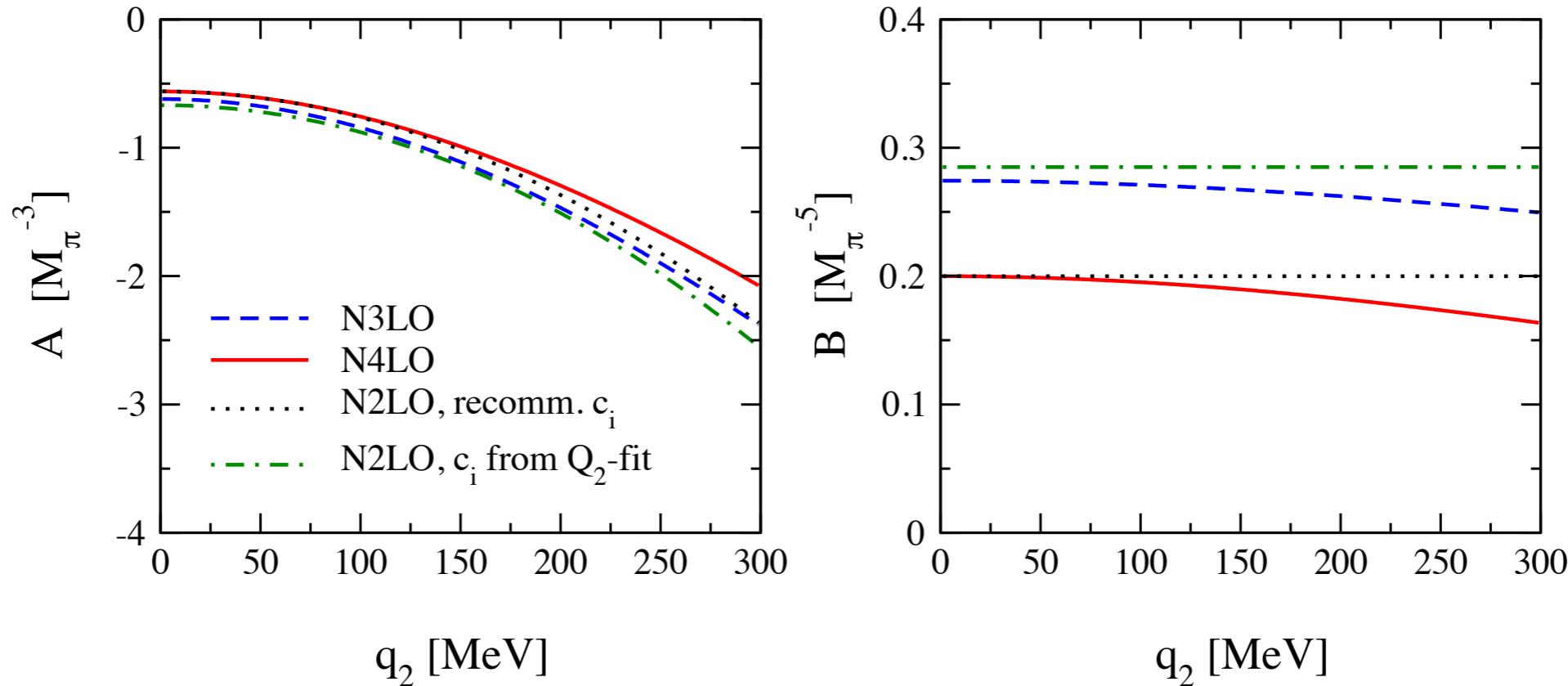


Recommended $c_i$'s by working with N²LO 3NF

$$c_1^{\text{KH}} = -0.37\,\text{GeV}^{-1}, \quad c_3^{\text{KH}} = -2.71\,\text{GeV}^{-1}, \quad c_4^{\text{KH}} = 1.41\,\text{GeV}^{-1},$$
$$c_1^{\text{GW}} = -0.73\,\text{GeV}^{-1}, \quad c_3^{\text{GW}} = -3.38\,\text{GeV}^{-1}, \quad c_4^{\text{GW}} = 1.69\,\text{GeV}^{-1}.$$

● With these parameters we get at $q_2 = 0$ the value and slope of N⁴LO result

● $c_i$'s fitted to pion-nucleon $Q^2$(KH-fit) lead to slightly different results for B-function

$$c_1 = -0.25\,\text{GeV}^{-1}, \quad c_2 = 2.02\,\text{GeV}^{-1}, \quad c_3 = -2.80\,\text{GeV}^{-1}, \quad c_4 = 2.01\,\text{GeV}^{-1}.$$

# Most general structure of a local 3NF

*Epelbaum, Gasparyan, H.K., in preparation*

Up to N⁴LO, the computed contributions are local ⟶ it is natural to switch to r-space.

A meaningful comparison requires a complete set of independent operators

| Generators $\mathcal{G}$ of 89 independent operators | $S$ | $A$ | $G_1$ | $G_2$ | $G_1(12)$ | $G_2(12)$ |
|---|---|---|---|---|---|---|
| $1$ | $\mathcal{O}_1$ | - | - | - | - | - |
| $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$ | $\mathcal{O}_2$ | - | $\mathcal{O}_3$ | $\mathcal{O}_4$ | - | - |
| $\vec{\sigma}_1\cdot\vec{\sigma}_3$ | $\mathcal{O}_5$ | - | $\mathcal{O}_6$ | $\mathcal{O}_7$ | - | - |
| $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\vec{\sigma}_1\cdot\vec{\sigma}_3$ | $\mathcal{O}_8$ | - | $\mathcal{O}_9$ | $\mathcal{O}_{10}$ | - | - |
| $\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\vec{\sigma}_1\cdot\vec{\sigma}_2$ | $\mathcal{O}_{11}$ | $\mathcal{O}_{12}$ | $\mathcal{O}_{13}$ | $\mathcal{O}_{14}$ | $\mathcal{O}_{15}$ | $\mathcal{O}_{16}$ |
| $\boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\vec{\sigma}_1\cdot(\vec{\sigma}_2\times\vec{\sigma}_3)$ | $\mathcal{O}_{17}$ | - | - | - | - | - |
| $\boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\vec{\sigma}_2\cdot(\vec{r}_{12}\times\vec{r}_{23})$ | $\mathcal{O}_{18}$ | - | $\mathcal{O}_{19}$ | $\mathcal{O}_{20}$ | - | - |
| $\vec{r}_{23}\cdot\vec{\sigma}_1\vec{r}_{23}\cdot\vec{\sigma}_3$ | $\mathcal{O}_{21}$ | $\mathcal{O}_{22}$ | $\mathcal{O}_{23}$ | $\mathcal{O}_{24}$ | $\mathcal{O}_{25}$ | $\mathcal{O}_{26}$ |
| $\vec{r}_{23}\cdot\vec{\sigma}_3\vec{r}_{12}\cdot\vec{\sigma}_1$ | $\mathcal{O}_{27}$ | - | $\mathcal{O}_{28}$ | $\mathcal{O}_{29}$ | - | - |
| $\vec{r}_{23}\cdot\vec{\sigma}_1\vec{r}_{12}\cdot\vec{\sigma}_3$ | $\mathcal{O}_{30}$ | - | $\mathcal{O}_{31}$ | $\mathcal{O}_{32}$ | - | - |
| $\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\vec{r}_{23}\cdot\vec{\sigma}_1\vec{r}_{23}\cdot\vec{\sigma}_2$ | $\mathcal{O}_{33}$ | $\mathcal{O}_{34}$ | $\mathcal{O}_{35}$ | $\mathcal{O}_{36}$ | $\mathcal{O}_{37}$ | $\mathcal{O}_{38}$ |
| $\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\vec{r}_{23}\cdot\vec{\sigma}_1\vec{r}_{12}\cdot\vec{\sigma}_2$ | $\mathcal{O}_{39}$ | $\mathcal{O}_{40}$ | $\mathcal{O}_{41}$ | $\mathcal{O}_{42}$ | $\mathcal{O}_{43}$ | $\mathcal{O}_{44}$ |
| $\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\vec{r}_{12}\cdot\vec{\sigma}_1\vec{r}_{23}\cdot\vec{\sigma}_2$ | $\mathcal{O}_{45}$ | $\mathcal{O}_{46}$ | $\mathcal{O}_{47}$ | $\mathcal{O}_{48}$ | $\mathcal{O}_{49}$ | $\mathcal{O}_{50}$ |
| $\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\vec{r}_{12}\cdot\vec{\sigma}_1\vec{r}_{12}\cdot\vec{\sigma}_2$ | $\mathcal{O}_{51}$ | $\mathcal{O}_{52}$ | $\mathcal{O}_{53}$ | $\mathcal{O}_{54}$ | $\mathcal{O}_{55}$ | $\mathcal{O}_{56}$ |
| $\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\vec{r}_{23}\cdot\vec{\sigma}_2\vec{r}_{23}\cdot\vec{\sigma}_3$ | $\mathcal{O}_{57}$ | - | $\mathcal{O}_{58}$ | $\mathcal{O}_{59}$ | - | - |
| $\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\vec{r}_{12}\cdot\vec{\sigma}_2\vec{r}_{12}\cdot\vec{\sigma}_3$ | $\mathcal{O}_{60}$ | $\mathcal{O}_{61}$ | $\mathcal{O}_{62}$ | $\mathcal{O}_{63}$ | $\mathcal{O}_{64}$ | $\mathcal{O}_{65}$ |
| $\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\vec{r}_{23}\cdot\vec{\sigma}_2\vec{r}_{12}\cdot\vec{\sigma}_3$ | $\mathcal{O}_{66}$ | - | $\mathcal{O}_{67}$ | $\mathcal{O}_{68}$ | - | - |
| $\boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\vec{\sigma}_1\cdot\vec{\sigma}_2\vec{\sigma}_3\cdot(\vec{r}_{12}\times\vec{r}_{23})$ | $\mathcal{O}_{69}$ | - | $\mathcal{O}_{70}$ | $\mathcal{O}_{71}$ | - | - |
| $\boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\vec{\sigma}_3\cdot\vec{r}_{23}\vec{r}_{23}\cdot(\vec{\sigma}_1\times\vec{\sigma}_2)$ | $\mathcal{O}_{72}$ | $\mathcal{O}_{73}$ | $\mathcal{O}_{74}$ | $\mathcal{O}_{75}$ | $\mathcal{O}_{76}$ | $\mathcal{O}_{77}$ |
| $\boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\vec{\sigma}_1\cdot\vec{r}_{23}\vec{\sigma}_2\cdot\vec{r}_{23}\vec{\sigma}_3\cdot(\vec{r}_{12}\times\vec{r}_{23})$ | $\mathcal{O}_{78}$ | $\mathcal{O}_{79}$ | $\mathcal{O}_{80}$ | $\mathcal{O}_{81}$ | $\mathcal{O}_{82}$ | $\mathcal{O}_{83}$ |
| $\boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\vec{\sigma}_1\cdot\vec{r}_{12}\vec{\sigma}_2\cdot\vec{r}_{12}\vec{\sigma}_3\cdot(\vec{r}_{12}\times\vec{r}_{23})$ | $\mathcal{O}_{84}$ | - | $\mathcal{O}_{85}$ | $\mathcal{O}_{86}$ | - | - |
| $\boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\vec{\sigma}_1\cdot\vec{r}_{23}\vec{\sigma}_2\cdot\vec{r}_{12}\vec{\sigma}_3\cdot(\vec{r}_{12}\times\vec{r}_{23})$ | $\mathcal{O}_{87}$ | - | $\mathcal{O}_{88}$ | $\mathcal{O}_{89}$ | - | - |

Most general, local 3NF involves 89 operators, can be generated (by permutations) from 22 structures:

$$V_{3N}^{\text{loc}} = \sum_{i=1}^{22}\mathcal{G}_i F_i(r_{12}, r_{23}, r_{31}) + 5\,\text{perm.}$$

The structures $\mathcal{O}_i$ are defined as:

$$S(\mathcal{G}) := \frac{1}{6}\sum_{P\in S_3} P\mathcal{G}$$

$$A(\mathcal{G}) := \frac{1}{6}\sum_{P\in S_3}(-1)^P P\mathcal{G}$$

$$G_1(\mathcal{G}) := \left[S_{13} - \frac{1}{2}(S_{23}S_{13} + S_{12}S_{13})\right](\mathcal{G})$$

$$G_2(\mathcal{G}) := \frac{\sqrt{3}}{2}\left[S_{23}S_{13} - S_{12}S_{13}\right](\mathcal{G})$$

# Two-pion-exchange up to N⁴LO

*Epelbaum, Gasparyan, H.K., in preparation*

Chiral expansion of TPE „structure functions" $F_i$ (in MeV) in the equilateral-triangle configuration
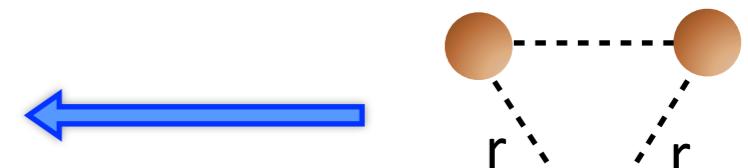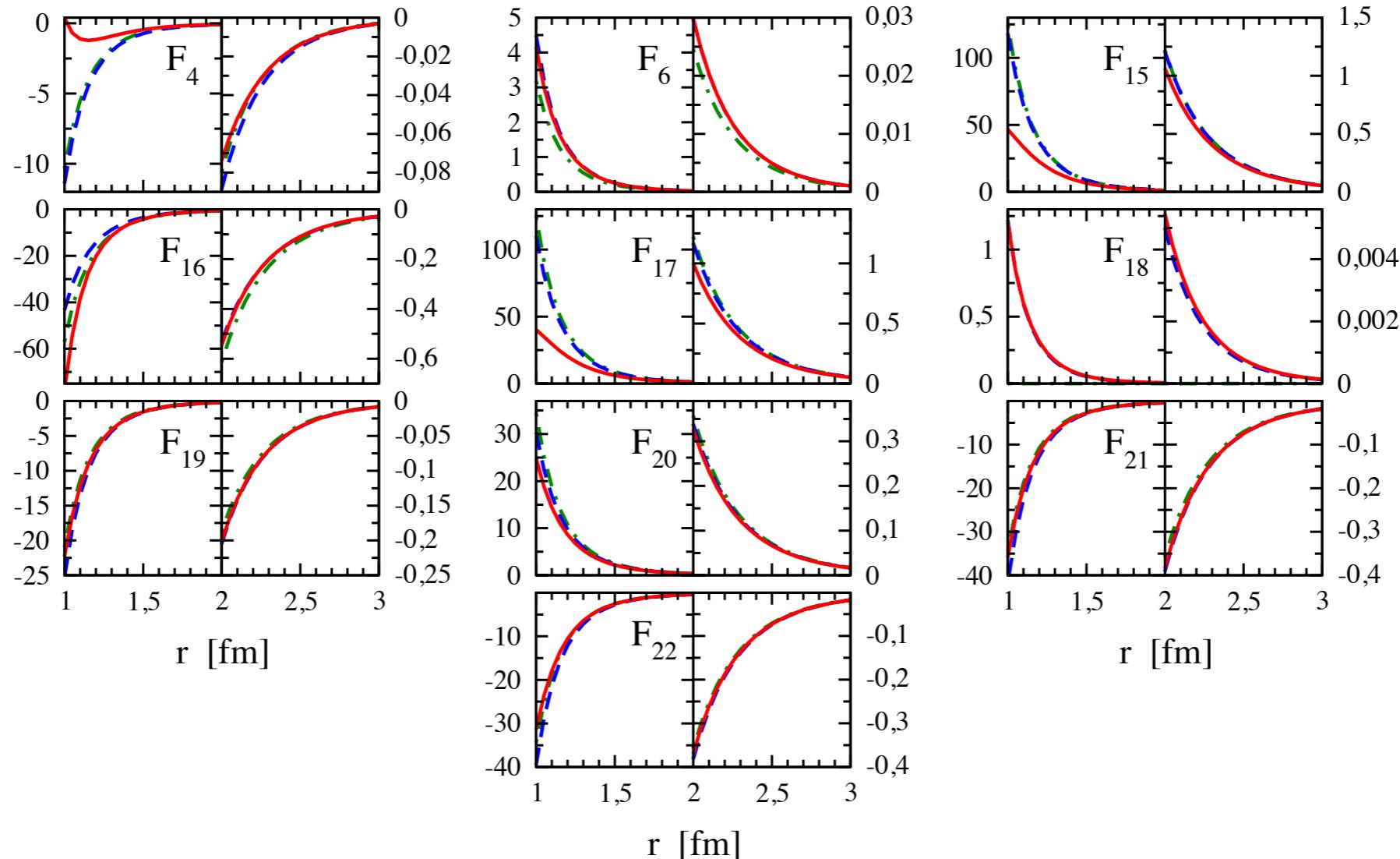


Excellent convergence of TPE-force at distance $r \geq 2\,\mathrm{fm}$

# Complete long-range 3NF up to N⁴LO

*Epelbaum, Gasparyan, H.K., in preparation*
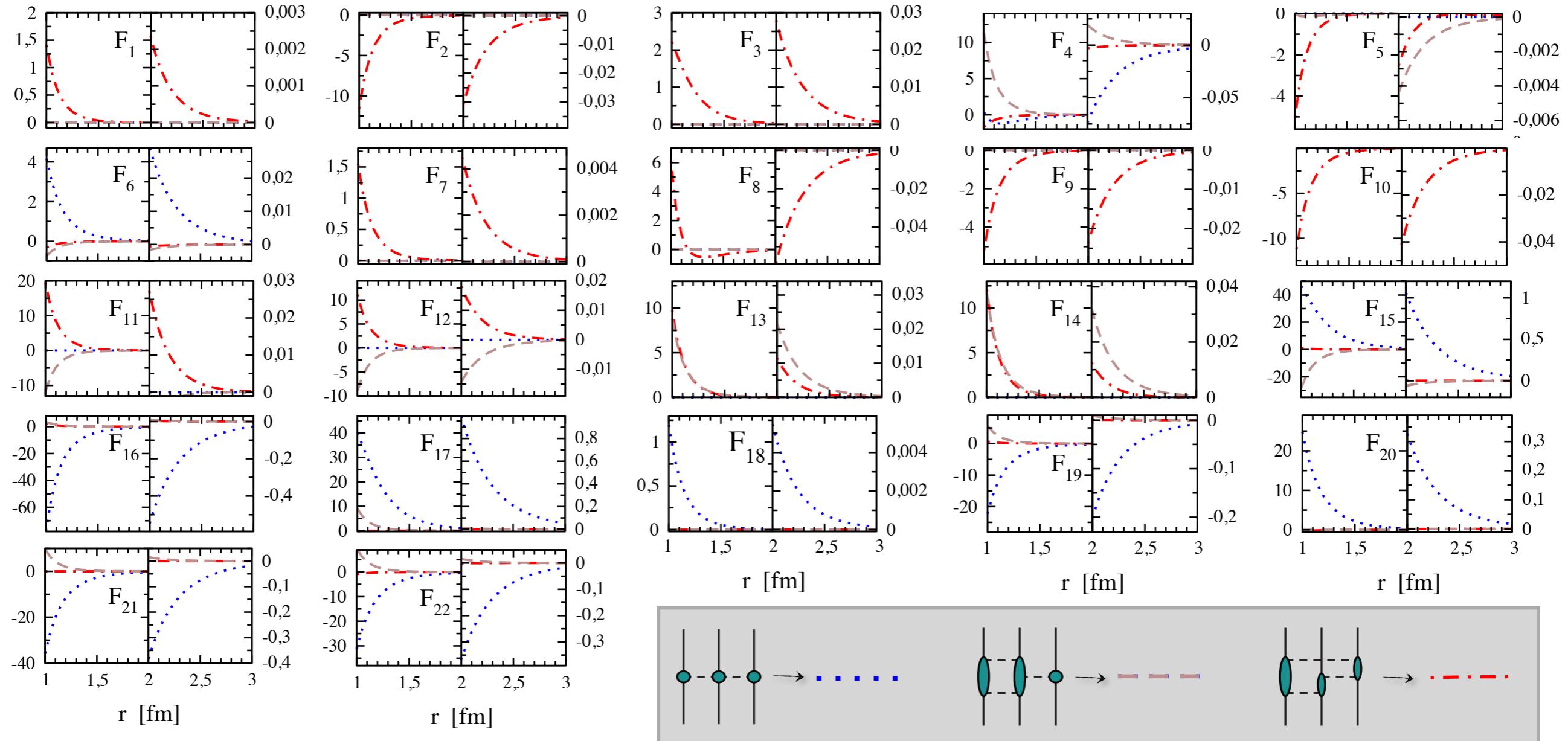


- 🔵 Predictions based entirely on chiral symmetry + input from $\pi N$, benchmarks for lattice-QCD

- 🔵 Implications for Nd, light nuclei & nuclear matter? *(work in progress …)*

- 🔵 $2\pi - 1\pi$ and ring-topology: already converged? ChPT with explicit $\Delta$´s *(work in progress …)*

# Partial wave decomposition

- Faddeev equation is solved in the partial wave basis

$$|p, q, \alpha\rangle \equiv |pq(ls)j(\lambda \tfrac{1}{2})I(jI)JM_J\rangle \, |(t\tfrac{1}{2})TM_T\rangle$$

- Too many terms for doing PWD by hand $\Longrightarrow$ Automatization

$$\underbrace{\langle p'q'\alpha'|V|pq\alpha\rangle}_{} = \int \underbrace{d\hat{p}' \, d\hat{q}' \, d\hat{p} \, d\hat{q}}_{} \sum_{m_l,\ldots} \Big(\text{CG coeffs.}\Big)\Big(Y_{l,m_l}(\hat{p}) \, Y_{l',m_l'}(\hat{p}') \ldots\Big)\underbrace{\langle m_{s_1}' m_{s_2}' m_{s_3}'|V|m_{s_1} m_{s_2} m_{s_3}\rangle}_{}$$

matrix ~ $10^5$ x $10^5$ | can be reduced to 5 dim. integral | depends on spin & isospin

- Ring-diagram-contr. expensive to calculate on the fly

| We prestore ring-contr. to 3nf's on a fine momentum grid | $\Longrightarrow$ | Numerical interpolation of ring terms |

Matrix-elements are so far calculated upto $j_{max}$=2 and J=5/2

Supercomputers used: JUGENE in FZ-Jülich and OSC in Ohio State University

- PWD matrix-elements can be used to produce matrix-elements in harmonic oscillator basis
  *see talk by Robert Roth*

Straightforward implementation of high order 3nf's in many-body calc.
within No-Core Shell Model
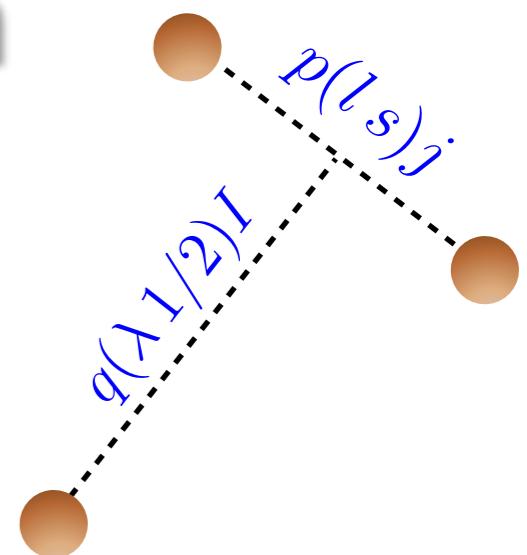
# Summary

- Chiral nuclear forces are analyzed up to $N^3LO$

- Long-range part of chiral three-nucleon forces is analyzed up to $N^4LO$

- In general there are 89 spin-isospin structures in local 3NF's built out of 22 + perm.

- Two-pion-exchange part dominates 3NF but does not fill all 22 structures

- With two-pion-one-pion-exchange and ring diagrams all 22 structures are filled

# Outlook

- Partial wave decomposition of $N^3LO$ three-nucleon forces

- Complete study of 3NF and 4NF up to $N^4LO$ with explicit delta-isobar

- Implementations in Nd, light nuclei & nuclear matter

# GW-Fit to pion-nucleon scattering

GW-PWA: Arndt et al. Phys. Rev. C 74 (2006) 045205
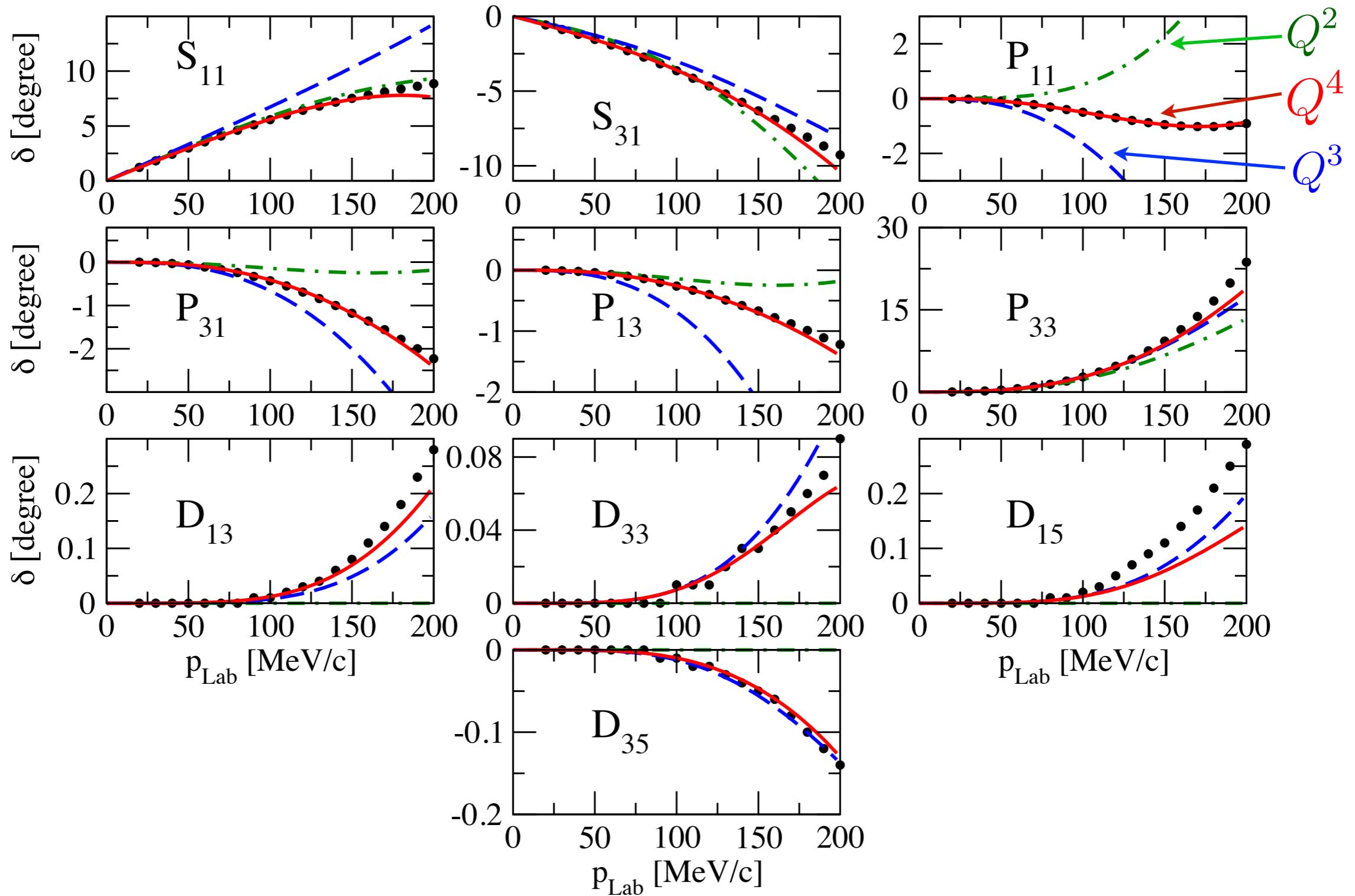


Data fitted for $p_{Lab} < 150\,\mathrm{MeV}$

# Nd elastic scattering

## Cross section & vector analyzing power

*E.pelbaum, PPNP 57 (2006) 654*
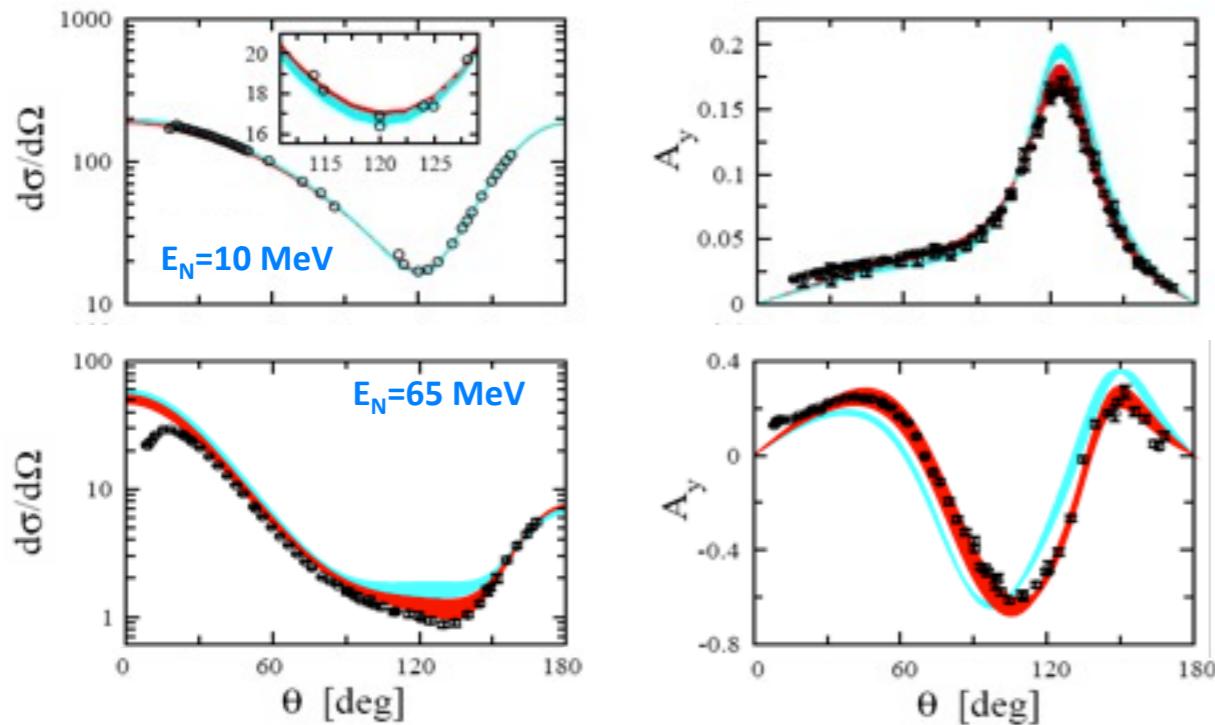


# Deuteron break-up

## SCRE configuration at $E_d$=19 MeV

*Ley et al., PRC 73 (2006) 064001*
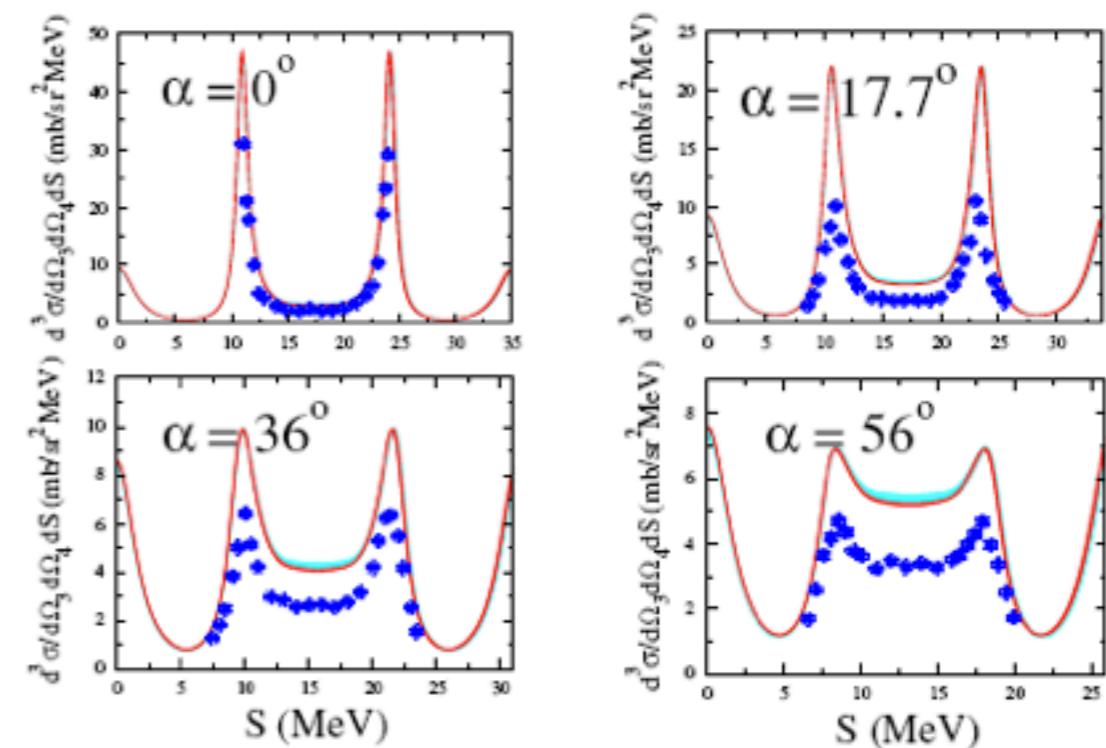


## Polarization transfer coefficients

*Witała et al., PRC 73 (2006) 044004*
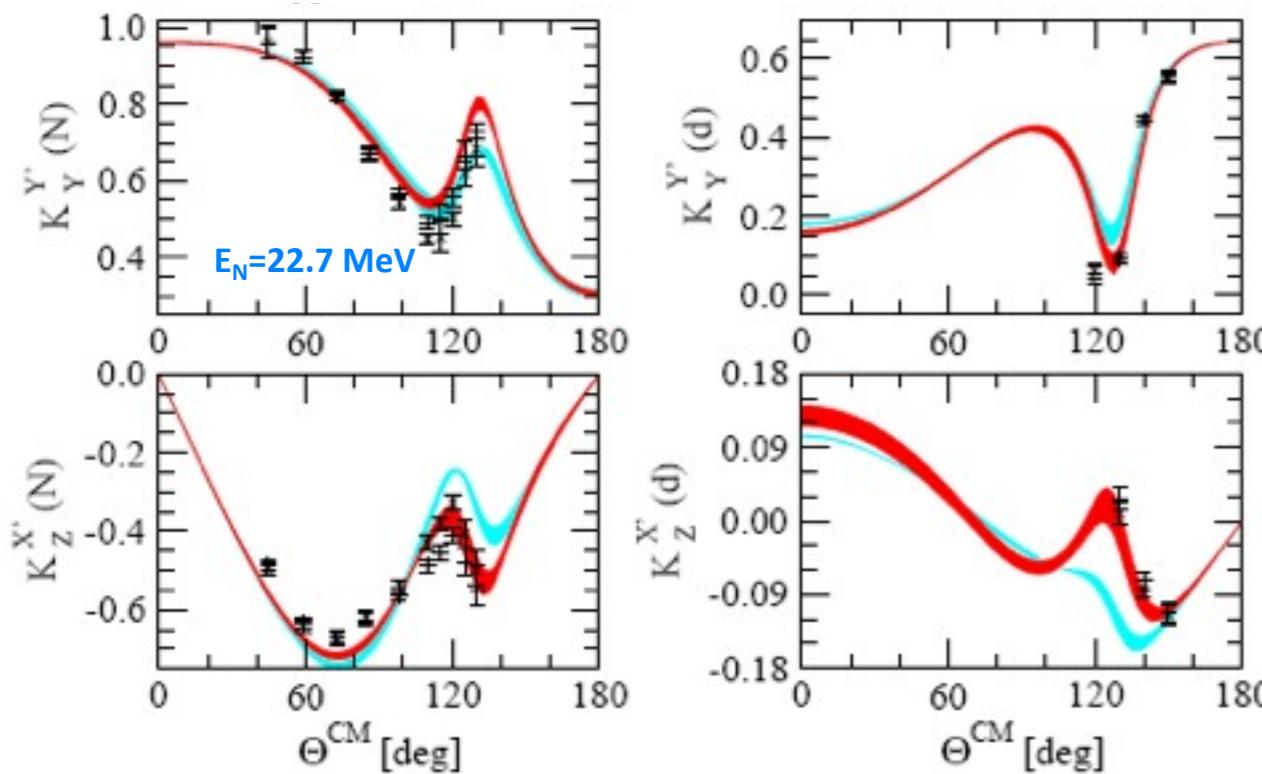


- Promising NNLO results for Nd elastic scattering

- Generally good description of break-up observables except for SCRE/SST break-up configuration at low energy

- Hope for improvement at $N^3LO$