Meson chiral perturbation theory meets lattice QCD

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(Special thanks to A. Sastre and to colleagues from BMWc and FLAG)

LQCD and χPT: a long history

- For many years χPT compensated LQCD's shortcomings
  - quenched χPT (qχPT) (Morel '87, Sharpe '90-'92, Bernard et al '92, . . .)

    $\longrightarrow$ limitations of quenched approximation

    $\longrightarrow$ (too) long extrapolations $m_{ud}^{\text{val}} \searrow m_{ud}^{\text{ph}}$
  - partially-quenched χPT (pqχPT) (Bernard et al '94, Sharpe et al '00, . . .)

    $\longrightarrow$ (too long) extrapolations $m_{ud}^{\text{val}}, m_{ud}^{\text{sea}} \searrow m_{ud}^{\text{ph}}$
  - χPT w/ Symanzik expansion in a^n (Bär et al '04, Sharpe et al '04, . . .)

    $\longrightarrow$ interplay of continuum and chiral limits
  - rooted staggered χPT (rSχPT) (Sharpe '94, Lee et al '99, Aubin et al '03, . . .)

    $\longrightarrow$ remedy staggered flavor excess
  - PGBs lightest dofs in QCD $\longrightarrow$ χPT gives finite-volume (FV) effects

LQCD and χPT: a long history

- Huge progress in LQCD simulations
  - now possible to vary m_{ud} (and m_s) $\searrow m_{ud}^{\rm ph}$

  $\Rightarrow$ test χPT

  $\Rightarrow$ determine LECs from first principles

  $\Rightarrow$ compute important observables for PGB phenomenology
  - Better than Nature:
    - m_{ud} and m_s freely tuned
    - Can tune valence and sea quark masses independently

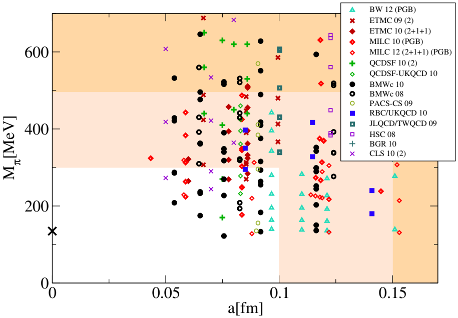
    $\Rightarrow$ LEC combinations that cannot be determined from phenomenology
    - Can further play with N_f, N_c, ...

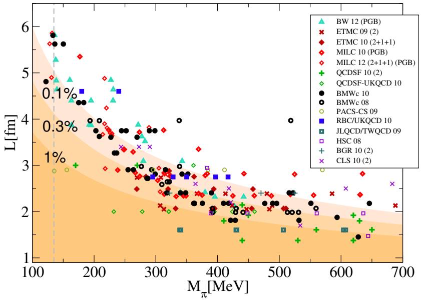
    $\Rightarrow$ try out different regimes of χPT (see Bijnens' review)

"Ask not what χPT can do for LQCD – ask what LQCD can do for χPT"

State of the art in 2012

Comparison of parameters reached by different LQCD collaborations





- Time of CD 09, typically $M_\pi \gtrsim 250\,\mathrm{MeV}$, $a \gtrsim 0.09\,\mathrm{fm}$ and $L \lesssim 3\,\mathrm{fm}$
- Today, $M_\pi \searrow 120\,\mathrm{MeV}$, $a \searrow 0.05\,\mathrm{fm}$ and $L \nearrow 6\,\mathrm{fm}$

$\Rightarrow$ can clearly make abundant contact with χPT

χPT meets LQCD on a torus

Lattice QCD (LQCD) calculations performed in finite periodic box $V = T \times L^3$ w/ $T \sim L$

For large V such that

$$p \sim \frac{2\pi}{L} \ll \Lambda_\chi \sim 4\pi F \Leftrightarrow FL \gg 1$$

and small m_q such that

$$M^2 = \frac{2m_q\Sigma}{F^2} \ll \Lambda_\chi^2$$

low energy physics described by usual chiral lagrangian (Gasser et al '88)

$$\mathcal{L} = \frac{F^2}{4}\,\mathrm{tr}\left\{\partial_\mu U^\dagger \partial_\mu U\right\} - \frac{\Sigma}{2}\left\{\mathcal{M}U^\dagger + \mathrm{cc}\right\} + O(p^4)$$

with $U = e^{i2\pi/F} \in SU(N_q)$ and $\mathcal{M} = mI$

Gaussian part of action

$$\Rightarrow \left\langle \frac{(\pi_p^a)^2}{F^2} \right\rangle \sim \frac{1}{F^2V}\frac{1}{p^2+M^2} \sim \frac{1}{(FL)^2}\frac{1}{(2\pi)^2n^2+(ML)^2}$$

and relative size of ML and 1 or $FL \gg 1$ determines different regimes of χPT

Regimes of χPT on a torus

$$\langle \frac{(\pi_p^a)^2}{F^2} \rangle \sim \frac{1}{F^2 V} \frac{1}{p^2+M^2} \sim \frac{1}{(FL)^2} \frac{1}{(2\pi)^2 n^2+(ML)^2}$$

(1) $ML \gg 1\ (2\pi)$ $\longrightarrow$ $\frac{|\pi_p^a|}{F} \sim \frac{1}{(FL)(ML)} \ll 1$

$\longrightarrow$ $U = 1 + i\frac{2\pi}{F} + \cdots$ & $\frac{1}{V}\sum_p \simeq \int \frac{d^4p}{(2\pi)^4}$

$\longrightarrow$ ∞**-volume p-expansion** w/ $p \sim M$, up to e^{-ML} corrections

(2) $ML \sim 1\ (2\pi)$ $\longrightarrow$ $\frac{|\pi_p^a|}{F} \sim \frac{1}{(FL)} \ll 1$

$\longrightarrow$ $U = 1 + i\frac{2\pi}{F} + \cdots$ but $\frac{1}{V}\sum_p \neq \int \frac{d^4p}{(2\pi)^4}$

$\longrightarrow$ **finite-volume p-expansion** w/ $p \sim M$

(3) $ML \lesssim 1/FL \Leftrightarrow m_q \Sigma V \lesssim 1 \Rightarrow ML \ll 1$

non-zero modes

$\longrightarrow \frac{|\pi_p^a|}{F} \sim \frac{1}{2\pi\sqrt{n^2}(FL)} \ll 1$

$\longrightarrow U = 1 + i\frac{2\pi}{F} + \cdots$

zero modes

$\longrightarrow \frac{|\pi_p^a|}{F} \sim \frac{1}{(FL)(ML)} \gtrsim 1$

$\longrightarrow U_0 \neq 1 + i\frac{2\pi}{F} + \cdots$

$\longrightarrow$ ϵ**-expansion** w/ $M/\Lambda_\chi \sim (p/\Lambda_\chi)^2 \sim \epsilon^2$ and U_0 treated non-perturbatively (Gasser et al '87)

Here focus on (1)

The flavors of LQCD and χPT

LQCD is QCD when $a \to 0$ and $V \to \infty$

$N_f{=}2$: $m_u^{\rm sea}{=}m_d^{\rm sea}$, $m_{s,\cdots,t}^{\rm sea}{\to}\infty$

- Not a valid approx. of QCD
- No systematic deviation yet observed

$N_f{=}2{+}1$: $m_u^{\rm sea}{=}m_d^{\rm sea}{\le}m_s^{\rm sea}$, $m_{c,\cdots,t}^{\rm sea}{\to}\infty$

- Approximates low-E QCD up to $\frac{1}{N_c}\left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^2$ corrections

$\to$ Good to % level

$N_f{=}2{+}1{+}1$: $m_u^{\rm sea}{=}m_d^{\rm sea}{\le}m_s^{\rm sea}{\le}m_c^{\rm sea}$, $m_{b,t}^{\rm sea} \to \infty$

- Approximates low-E QCD up to $\frac{1}{N_c}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2$ corrections

$\to$ Good to per mil level

χPT is a systematic expansion of QCD at low-E (see talk by Bijnens)

$N_f{=}2$: $m_u, m_d{\to}0$, $m_{s,\cdots,t}$ fixed

- Expansion in $\frac{p^2, M_\pi^2}{(\Lambda_\chi^{N_f=2})^2}$, $\frac{m_{ud}}{m_s}$
- Can be applied to $N_f{=}2$ LQCD results in χ-regime, but LECs **are not** QCD LECs
- When applied to $N_f{\ge}2{+}1$ LQCD results in χ-regime, LECs **are** QCD LECs, up to corrections on LHS

$N_f{=}3$: $m_u, m_d, m_s{\to}0$, $m_{c,\cdots,t}$ fixed

- Expansion in $\frac{p^2, M_{\pi,K,\eta}^2}{(\Lambda_\chi^{N_f=3})^2}$
- Cannot be applied to $N_f = 2$ LQCD results
- When applied to $N_f \ge 2{+}1$ LQCD results in χ-regime, fitted LECs **are** QCD LECs, up to corrections on LHS

F_π and M_π in $SU(2)$ χPT

At NNLO (Colangelo et al '01)

$$M_\pi^2 = M^2 \left\{ 1 - \frac{1}{2} x \ln \frac{\Lambda_3^2}{M^2} + \frac{17}{8} x^2 \left(\ln \frac{\Lambda_M^2}{M^2} \right)^2 + x^2 k_M + O(x^3) \right\}$$

$$F_\pi = F_2 \left\{ 1 + x \ln \frac{\Lambda_4^2}{M^2} - \frac{5}{4} x^2 \left(\ln \frac{\Lambda_F^2}{M^2} \right)^2 + x^2 k_F + O(x^3) \right\}$$

where $x = \frac{M^2}{(4\pi F_2)^2}$ and $M^2 = 2 m_{ud} B_2$ and

$$\ln \frac{\Lambda_M^2}{\bar{M}_\pi^2},\ \ln \frac{\Lambda_F^2}{\bar{M}_\pi^2} \leftrightarrow \frac{7\bar{\ell}_1 + 8\bar{\ell}_2}{15}$$

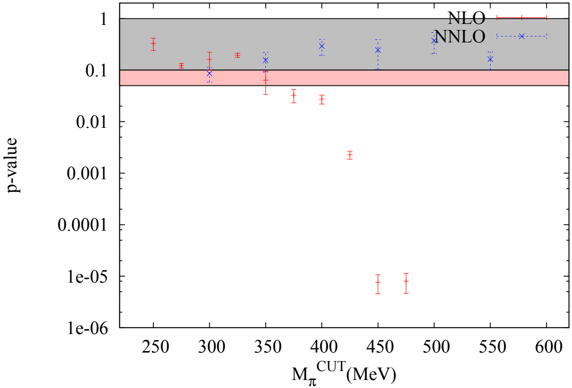
Illustration: (see also Scholz's talk) combined, correlated fits to m_{ud} dependence of N_f=2+1 BMWc '10 results from 47 simulations w/

$$M_\pi \searrow 120\,\mathrm{MeV}, \quad 5\ a\text{'s w/ } a: 0.116 \searrow 0.054\,\mathrm{fm} \quad \text{and} \quad L \nearrow 6\,\mathrm{fm}$$

Small terms in powers of $(M_{s\bar{s}}^2 - M_{s\bar{s}}^{\mathrm{ph},2})$ and of a, and FV corrections, as required

Reach of $SU(2)$ χPT? – Fit quality

PRELIMINARY

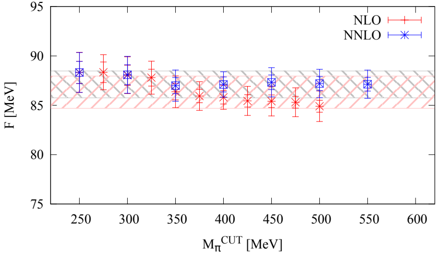


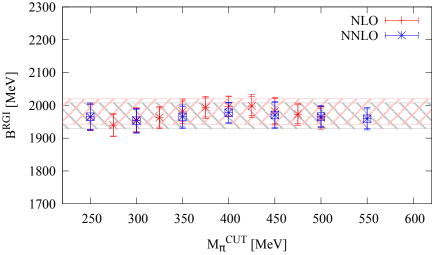
- NLO appears to work up to $M_\pi \lesssim 350-400\,\mathrm{MeV}$
- NNLO shows no sign of breaking down up to $M_\pi \sim 550\,\mathrm{MeV}$
- Loose priors imposed on NNLO terms
- Dependence of results on priors must be studied

Reach of $SU(2)$ χPT? – Stability of LO LECs

PRELIMINARY

Above ranges of validity for χPT only make sense if LECs' values independent of M_π^{cut} in range





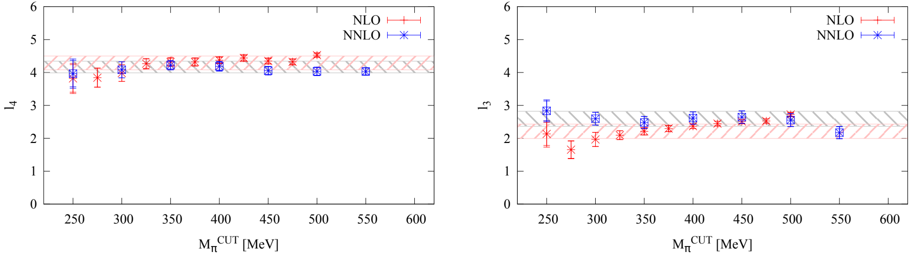
- Observed stability up to $M_\pi \lesssim 350 - 400\,\text{MeV}$ for NLO
- Up to $M_\pi \lesssim 550\,\text{MeV}$ for NNLO

$\rightarrow$ range given by fit quality confirmed

Reach of $SU(2)$ χPT? – Stability of NLO LECs

PRELIMINARY

Above ranges of validity for χPT only make sense if LECs' values independent of M_π^{cut} in range



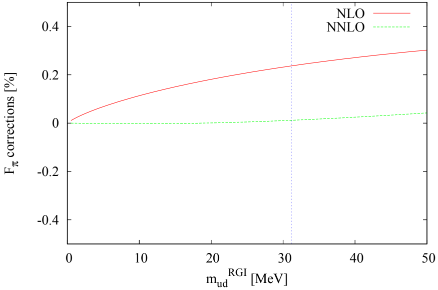
- Relative stability up to $M_\pi \lesssim 400\,\mathrm{MeV}$ for NLO $\longrightarrow$ fit quality range seems correct
- NNLO more stable over full range
- Have to push NNLO fits to 600 MeV to confirm

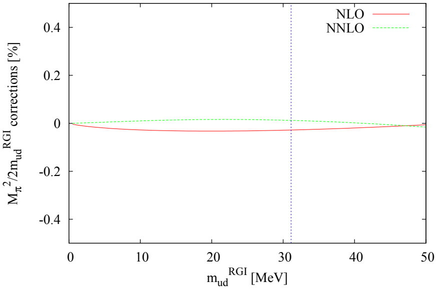
Reach of $SU(2)$ χPT? – Relative size of corrections

PRELIMINARY

Above ranges of validity only make sense if LO $\gg$ NLO $\gg$ NNLO in range

From typical fit





Coherent overall picture $\Rightarrow$ range of validity of:

- NLO $SU(2)$ χPT extends up to $M_\pi \lesssim 350\text{–}400\,\mathrm{MeV}$
- NNLO $SU(2)$ χPT extends up to $M_\pi \sim 500\,\mathrm{MeV}$

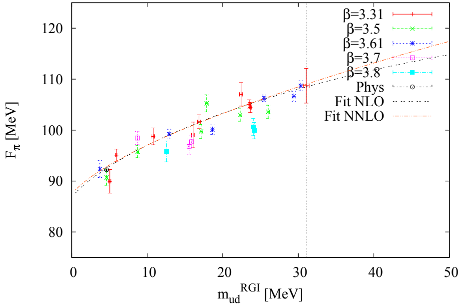
w/ BMWc's percent level errors. Caveat: χPT asymptotic expansion . . .

Typical fit

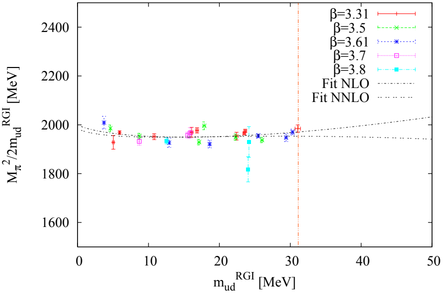
PRELIMINARY

$M_\pi \leq 350\,\mathrm{MeV}$

$(\chi^2/dof)_{\mathrm{NLO}} = 59./46 = 1.3$



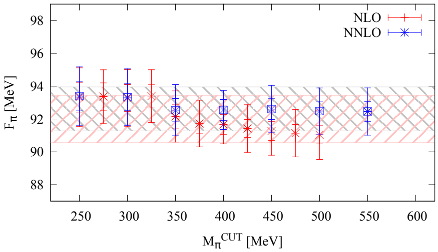
$(\chi^2/dof)_{\mathrm{NNLO}} = 50./43 = 1.2$

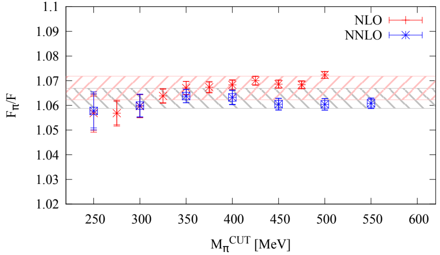


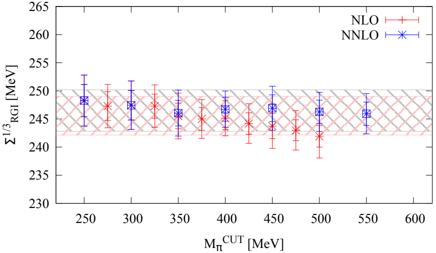
(Lattice data are corrected for m_s, a and FV effects using NNLO fit results)

Other $SU(2)$ observables

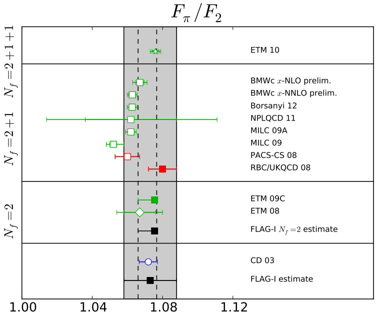
PRELIMINARY

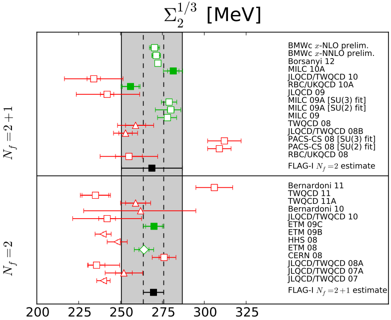






Comparison w/ FLAG LO compilation





(Σ_2 in $\overline{\mathrm{MS}}$ @ 2 GeV)

Very nice consistency of recent $N_f = 2+1$ results

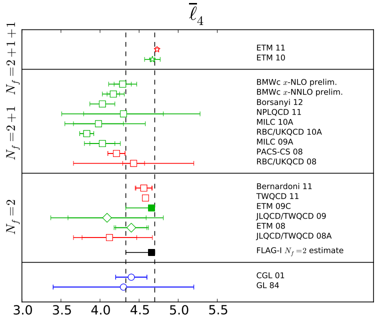
$$\frac{F_\pi}{F} = 1.073(15) \quad [1.4\%] \quad \text{(FLAG-I)}$$

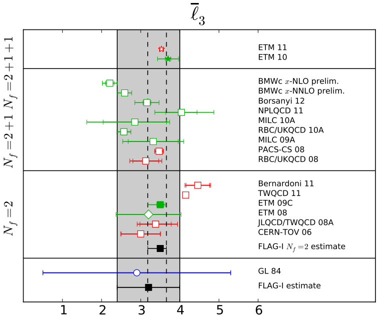
Aiming for error of $\sim$ 0.6%

$$\Sigma^{1/3} = 269(18)\,\mathrm{MeV} \quad [7\%] \quad \text{(FLAG-I)}$$

Aiming for error of $\sim$ 1.5%

Comparison w/ FLAG NLO compilation





- Poor agreement between $N_f = 2+1$ results
- $N_f = 2+1+1$ results for $\bar{\ell}_4$ are a bit off, but $M_\pi \geq 270\,\mathrm{MeV}$

- No $N_f = 2+1$ FLAG estimate for the moment
- Aiming for error $\lesssim 5\%$

- $N_f = 2+1$ FLAG-I estimate:

$$\bar{\ell}_3 = 3.2(8) \quad [25\%]$$

- Aiming for error $\lesssim 15\%$

Light quark masses: motivation

Very similar studies give m_{ud} and m_s *ab initio*

Need QED on lattice or phenomenological input to get m_u and m_d

- Fundamental parameters of the standard model
- Precise values $\rightarrow$ stability of matter, N-N scattering lengths, presence or absence of strong CP violation, etc.
- Couplings to the Higgs
- Information about BSM: theory of fermion masses must reproduce these values
- Nonperturbative (NP) computation is required
- Would be needle in a haystack problem if not for χSB

$\longrightarrow$ tremendous progress has been made in recent years

Fixing m_u, m_d, m_s

χSB $\rightarrow$ observables most sensitive to m_u, m_d, m_s are PGB masses:

$$M^2_{\pi^+} \sim m_{ud}, \qquad M^2_{K^+} \sim (m_s + m_u), \qquad M^2_{K^0} \sim (m_s + m_d)$$

NB:– π^0 avoided because of π^0-η mixing and quark-disconnected contributions
– Need 4th observable to fix a

1. LQCD simulation w/ $m_u^{\rm sea} \neq m_d^{\rm sea}$ and QED
   $\rightarrow$ tune $m_q^{\rm lat}$, $q = u, d, s$, so that $M_P^{\rm lat} = M_P^{\rm ph}$, $P = \pi^+, K^+, K^0$
2. $N_f \geq 2+1$ simulation w/ $m_u^{\rm sea} = m_d^{\rm sea}$ and no QED
   $\rightarrow$ tune $m_q^{\rm lat}$, $q = ud, s$, so that $M_P^{\rm lat} = \bar{M}_P$, $P = \pi, K$
   $\rightarrow$ $\bar{M}_{\pi,K}$ are PGB in isospin limit (with QED corrections subtracted)
   $\rightarrow$ need χPT and phenomenology (see FLAG '11)

$$\bar{M}_\pi = 134.8(3)\,\mathrm{MeV}\ [0.2\%], \qquad \bar{M}_K = 494.2(5)\,\mathrm{MeV}\ [0.1\%]$$

Not limiting factor

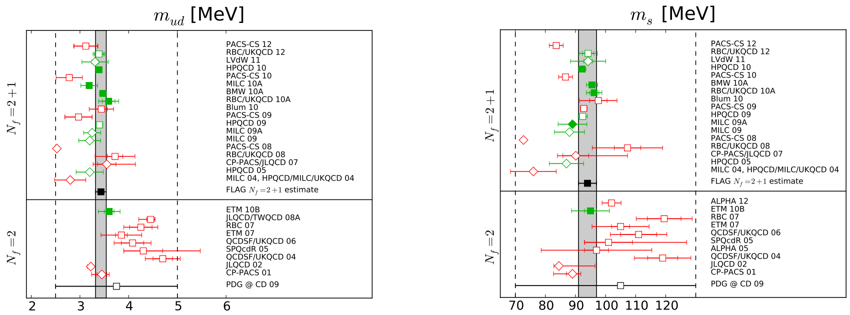
Light quark masses: FLAG tables – $N_f = 2 + 1$

Collab.	pub.	$m_{ud} \to m_{ud}^{ph}$	$a \to 0$	$V \to \infty$	renorm.	run.	m_{ud}	m_s
PACS-CS 12*	P	★	■	■	★	*a*	3.12(24)(8)	83.60(0.58)(2.23)
RBC/UKQCD 12	C	★	○	★	★	*c*	3.39(9)(4)(2)(7)	94.2(1.9)(1.0)(0.4)(2.1)
LVdW 11	C	○	★	★	○	–	3.31(7)(20)(17)	94.2(1.4)(3.2)(4.7)
PACS-CS 10	A	★	■	■	★	*a*	2.78(27)	86.7(2.3)
MILC 10A	C	○	★	★	○	–	3.19(4)(5)(16)	–
HPQCD 10	A	○	★	★	★	–	3.39(6)*	92.2(1.3)
BMW 10A, 10B^{+}	A	★	★	★	★	*b*	3.469(47)(48)	95.5(1.1)(1.5)
RBC/UKQCD 10A	A	○	○	★	★	*c*	3.59(13)(14)(8)	96.2(1.6)(0.2)(2.1)
Blum 10†	A	○	■	○	★	–	3.44(12)(22)	97.6(2.9)(5.5)
PACS-CS 09	A	★	■	■	★	*a*	2.97(28)(3)	92.75(58)(95)
HPQCD 09	A	○	★	★	★	–	3.40(7)	92.4(1.5)
MILC 09A	C	○	★	★	○	–	3.25 (1)(7)(16)(0)	89.0(0.2)(1.6)(4.5)(0.1)
MILC 09	A	○	★	★	○	–	3.2(0)(1)(2)(0)	88(0)(3)(4)(0)
PACS-CS 08	A	★	■	■	■	–	2.527(47)	72.72(78)
RBC/UKQCD 08	A	○	■	★	★	–	3.72(16)(33)(18)	107.3(4.4)(9.7)(4.9)
CP-PACS/ JLQCD 07	A	■	★	★	■	–	$3.55(19)(^{+56}_{-20})$	$90.1(4.3)(^{+16.7}_{-4.3})$
HPQCD 05	A	○	○	○	○	–	3.2(0)(2)(2)(0)‡	87(0)(4)(4)(0)‡
MILC 04, HPQCD/ MILC/UKQCD 04	A	○	○	○	■	–	2.8(0)(1)(3)(0)	76(0)(3)(7)(0)

Light quark masses: FLAG tables – $N_f=2$

Collab.	pub.	$m_{ud}\to m_{ud}^{ph}$	$a\to 0$	$V\to\infty$	renorm.	run.	m_{ud}	m_s
ETM 10B	A	○	★	○	★	*a*	3.6(1)(2)	95(2)(6)
JLQCD/TWQCD 08A	A	○	■	■	★	–	$4.452(81)(38)\left(^{+0}_{-227}\right)$	–
RBC 07†	A	■	■	★	★	–	4.25(23)(26)	119.5(5.6)(7.4)
ETM 07	A	○	■	○	★	–	3.85(12)(40)	105(3)(9)
QCDSF/ UKQCD 06	A	■	★	■	★	–	4.08(23)(19)(23)	111(6)(4)(6)
SPQcdR 05	A	■	○	○	★	–	$4.3(4)\left(^{+1.1}_{-0.0}\right)$	$101(8)\left(^{+25}_{-0}\right)$
ALPHA 05	A	■	○	★	★	*b*		$97(4)(18)^{\S}$
QCDSF/ UKQCD 04	A	■	★	■	★	–	4.7(2)(3)	119(5)(8)
JLQCD 02	A	■	■	○	■	–	$3.223\left(^{+46}_{-69}\right)$	$84.5\left(^{+12.0}_{-1.7}\right)$
CP-PACS 01	A	■	■	★	■	–	$3.45(10)\left(^{+11}_{-18}\right)$	$89(2)\left(^{+2}_{-6}\right)^{\star}$

m_{ud} and m_s @ 2 GeV in $\overline{\mathrm{MS}}$ – FLAG compilation



$m_{ud} = 2.5 \div 5.0\,\mathrm{MeV}$ [33%] PDG @ CD 09
$\rightarrow 3.43(11)\,\mathrm{MeV}$ [3%] FLAG

$m_s = 105^{+25}_{-35}\,\mathrm{MeV}$ [29%] PDG @ CD 09
$\rightarrow 94(3)\,\mathrm{MeV}$ [3%] FLAG

$\frac{m_s}{m_{ud}} = 25 \div 30$ [9%] PDG @ CD 09 $\longrightarrow$ 27.4(4) [1.5%] FLAG

. . . and FLAG is being conservative too . . .

Accuracies reached in calculations $\lesssim 2\%$ $\longrightarrow$ worry about $(\Lambda_{\mathrm{QCD}}/m_c)^2$, $\alpha_s(m_c)$ [α, $(m_u - m_d)/\Lambda_{\mathrm{QCD}}$] corrections

Individual m_u and m_d

$N_f \geq 2+1$ calculations are performed w/ $m_u = m_d$ and no QED

$\Rightarrow$ need χPT and phenomenology
$\Rightarrow$ leave *ab initio* realm

Use χPT (Dashen '67, Gasser et al '84, '85) and pheno. to subtract EM effects (FLAG '11)

$$\hat{M}_{K^+} = 491.2(7)\,\mathrm{MeV}$$
$$\hat{M}_{K^0} = 497.2(4)\,\mathrm{MeV}$$

- Compute partially quenched $M_{K^+}^{\rm val}$ & $M_{K^0}^{\rm val}$ w/ $m_u^{\rm val} \neq m_d^{\rm val}$
- Tune $m_u^{\rm val}$ & $m_d^{\rm val}$ so that $M_P^{\rm val} = \hat{M}_P$, $P = K^+, K^0$
- Correct up to $O\left(\frac{m_u - m_d}{M_{\rm QCD}}\right)^2$ sea effects

Use dispersive analysis of $\eta \to 3\pi$ (Anisovich et al '96, Ditsche et al '09) (see talk by Lanz)

$$A(s,t,u) \propto -\frac{1}{Q^2}$$

$$Q^2 \equiv \frac{m_s^2 - m_{ud}^2}{m_d^2 - m_u^2}$$

☛ Precise m_{ud} and $m_s/m_{ud} \Rightarrow$

$$m_{u/d} = m_{ud}\left\{1 \mp \frac{1}{4Q^2}\left[\left(\frac{m_s}{m_{ud}}\right)^2 - 1\right]\right\}$$

- Conservative $Q = 22.3(8)$ (Leutwyler '09)

Individual m_u and m_d

Assumes Dashen thm violations (FLAG '11)

$$(\Delta^\gamma_{K^+} - \Delta^\gamma_{K^0}) - (\Delta^\gamma_{\pi^+} - \Delta^\gamma_{\pi^0}) \equiv \epsilon \Delta_\pi$$

$$\epsilon = 0.7(5)\ [71\%]$$

$$\Delta^\gamma_P \equiv M_P^2 - \hat{M}_P^2, \quad \Delta_\pi = M^2_{\pi^+} - M^2_{\pi^0}$$

from Q^2 including error from unknown NNLO $SU(3)$ corrections

Turning around $Q^2 \leftrightarrow \epsilon$ gives (FLAG '11)

$$\Rightarrow Q^2 \overset{\mathrm{NLO}}{=} 497(94)\ [19\%]$$

Systematic error driven by

$$\hat{M}^2_{K^0} - \hat{M}^2_{K^+} = 6.0(8)\ [13\%]$$

$$\Rightarrow \delta(m_d - m_u) \sim 13\%$$

Relation to LH method (Gasser et al '85)

$$Q^2 = \frac{\hat{M}_K^2}{\hat{M}_\pi^2} \frac{\hat{M}_K^2 - M_\pi^2}{\hat{M}^2_{K^0} - \hat{M}^2_{K^+}} \left[1 + O\left(\frac{m_s}{\Lambda_\chi^{N_f=3}}\right)^2\right]$$

$$\hat{M}_P^2 \equiv \frac{\hat{M}^2_{P^+} + \hat{M}^2_{P^0}}{2}, \quad P = \pi, K$$

Using EM and smaller corrections (FLAG '11)

$$\Rightarrow \epsilon = 0.70(28) + O\left(\frac{m_s}{\Lambda_\chi^{N_f=3}}\right)^2 \quad [40\%]$$

Systematic error driven by

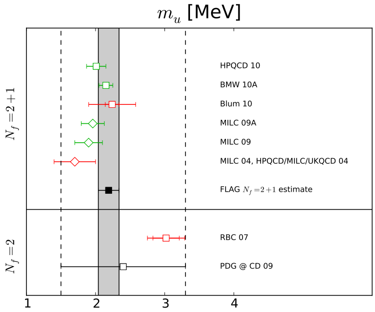
$$Q^2 = 497(36)\ [7\%]$$

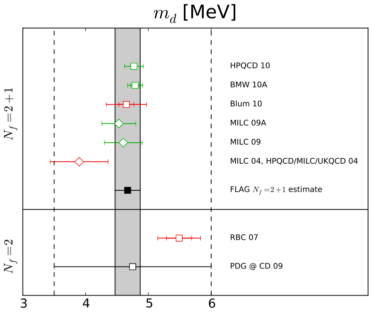
$$\Rightarrow \delta(m_d - m_u) \sim 7\%$$

Error from RHS smaller since ϵ obtained from Q^2 up to NNLO $SU(3)$ corrections

$\rightarrow$ will change w/ improved QCD+QED calculations (see Izubuchi's review)

m_u and m_d @ 2 GeV in $\overline{\mathrm{MS}}$ – FLAG compilation





$m_u = 1.5 \div 3.3\,\mathrm{MeV}$ [38%] PDG @ CD 09
$\rightarrow 2.19(15)\,\mathrm{MeV}$ [7%] FLAG

$m_d = 3.5 \div 6.0\,\mathrm{MeV}$ [26%] PDG @ CD 09
$\rightarrow 4.67(20)\,\mathrm{MeV}$ [4%] FLAG

$$\frac{m_u}{m_d} = 0.35 \div 0.6 \quad [26\%] \quad \text{PDG @ CD 09} \quad \longrightarrow \quad 0.47(4) \quad [9\%] \quad \text{FLAG}$$

Improvement less spectacular because EM corrections are estimated w/ phenomenology

Further improvement requires QCD+QED calculations (see Izubuchi's talk)

Conclusion

- LQCD and χPT have a long history
- Freedom to vary all of QCD parameters

  $\rightarrow$ new combinations of LECs
  $\rightarrow$ other regimes of χPT

- Tremendous recent advances in LQCD

  $\rightarrow$ LQCD finally paying back debt to χPT
  $\rightarrow$ explore the range of validity of χPT
  $\rightarrow$ precise determinations of LECs

Future:

- many more systematic analyses around and below M_π^{ph}

  $\rightarrow$ more and more precise LECs
  $\rightarrow$ NNLO LECs
  $\rightarrow$ high precision results for F_K/F_π, $f_+(0)$, $K \rightarrow \pi\pi$, etc.

- inclusion of QED and isospin breaking effects
- systematic inclusion of sea charm

Conclusion

LQCD and meson χPT meet in many places:

- Future already here in Newport News – talks by:
  - Chris Sachrajda: "Non-leptonic and rare Kaon decays in Lattice QCD"
  - Taku Izubuchi: "Isospin breaking studies from lattice QCD + QED"
  - E. Scholz: "Determination of SU(2) ChPT LECs from 2+1 flavor staggered lattice simulations"
  - A. Deuzeman: "Light meson physics from 2+1+1 flavours of twisted mass Wilson fermions"
  - C. Bernard: "Electromagnetic contributions to pseudoscalar masses"
  - W. Lee: "Recent progress in staggered ChPT"
  - P. Fritzsch: "The Lambda parameter and strange quark mass in two-flavour QCD"
  - M. Golterman: "Two-pion excited state contribution to the axial vector and pseudoscalar correlators"
  - A. Shindler: "Corrections to the Banks-Casher relation with Wilson quarks"
- See also FLAG review (EPJC 71 (2011))