Pion-mass dispersion relations in the baryon sector

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Outline

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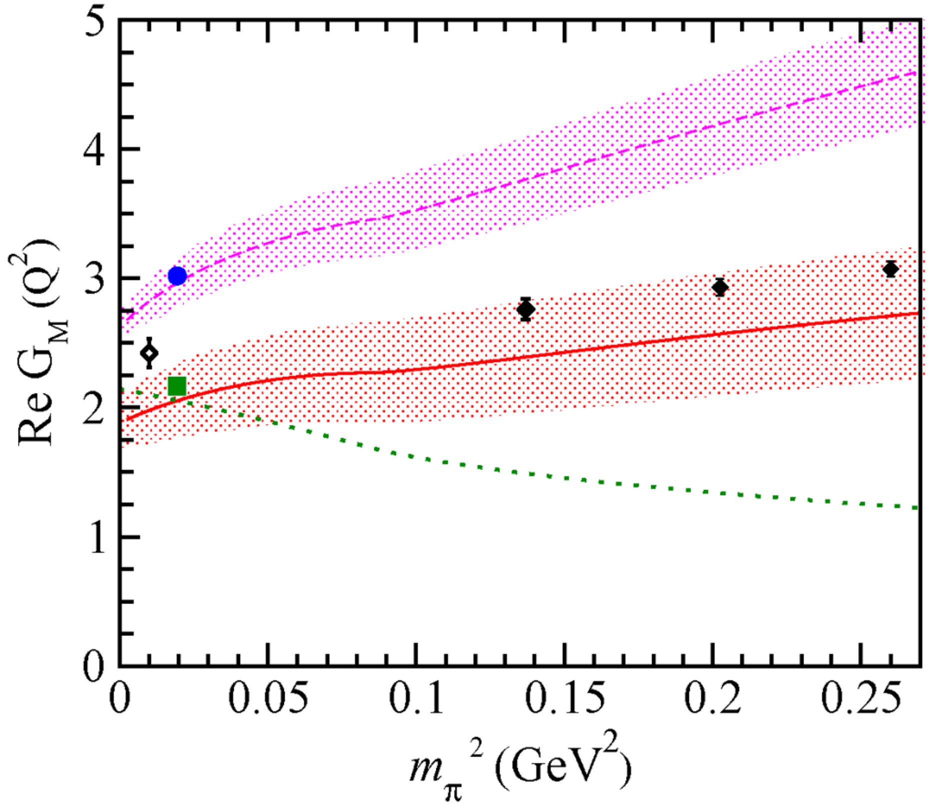
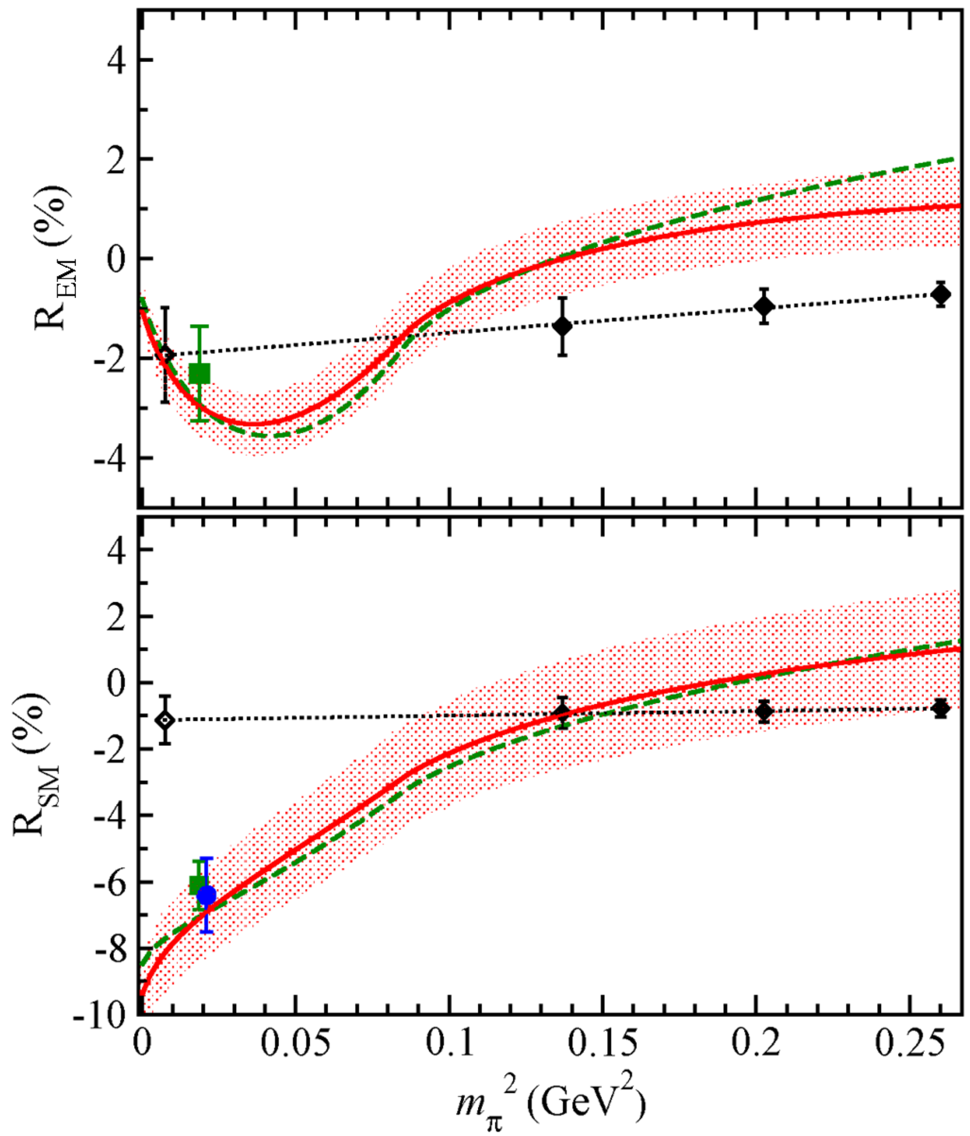
γNΔ form factors: chiral behavior at low Q

$$R_{EM} = -\frac{G_E^*}{G_M^*}, \qquad R_{SM} = -\frac{Q_+Q_-}{4M_\Delta^2}\frac{G_C^*}{G_M^*}.$$

W=1.232 GeV, Q² = 0.1 GeV²





data points : MAMI, MIT-Bates,
lattice QCD [Nicosia group]

[V.P. & Vanderhaeghen, PRL 95 (2005); PRD 73 (2006)]

Example - Nucleon Mass

[Gasser, Sainio & Svarc, NPB (1988); ...]

$$\mathcal{L} = \sum_k \mathcal{L}^{(k)}, \qquad k = \#\text{ of pion derivatives and masses}$$

$$\mathcal{L}^{(1)}_{\pi N} = \bar{N}(i\not{D} - \mathring{M}_N + \mathring{g}_A\, a_\mu \gamma^\mu \gamma_5) N$$

$$= \bar{N}\left(i\not{\partial} - \mathring{M}_N + \frac{\mathring{g}_A}{2 f_\pi}(\partial_\mu \pi)\gamma^\mu \gamma_5\right) N + O(\pi^2)$$

$$\mathcal{L}^{(2)}_{\pi N} = 4\,\mathring{c}_{1N}\, m_\pi^2\, \bar{N} N + \ldots$$

Power-counting: p^n

$$n = \sum_k k V_k + 4L - 2N_\pi - N_N$$

V_k	# of vertices from $\mathcal{L}^{(k)}$
L	# of Loops
N_π	# of internal pions
N_N	# of internal nucleons

LECs

$k = 1,\ V_k = 2,\ L = 1,\ N_\pi = 1,\ N_N = 1$

$$M_N = \mathring{M}_N - 4\,\mathring{c}_{1N}\, m_\pi^2 -$$  + ...

$O(p^3)$

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But, actually 

$$= \frac{3g_A^2}{2(4\pi f_\pi)^2}\left\{ -M_N^3\, L + M_N\,(1-L)\, m_\pi^2 - m_\pi^3\left(\sqrt{1 - m_\pi^2/4M_N^2}\ \arccos\frac{m_\pi}{2M_N} + \frac{m_\pi^4}{4M_N}\ \ln\frac{m_\pi^2}{M_N^2}\right)\right\}$$

where $L = \frac{1}{\epsilon} + \ldots$ contains the UV-divergence, removed by MS-bar: $L = 0$

remaining m_π^2 "complicates life a lot" [GSS88]. Violation of power counting?!

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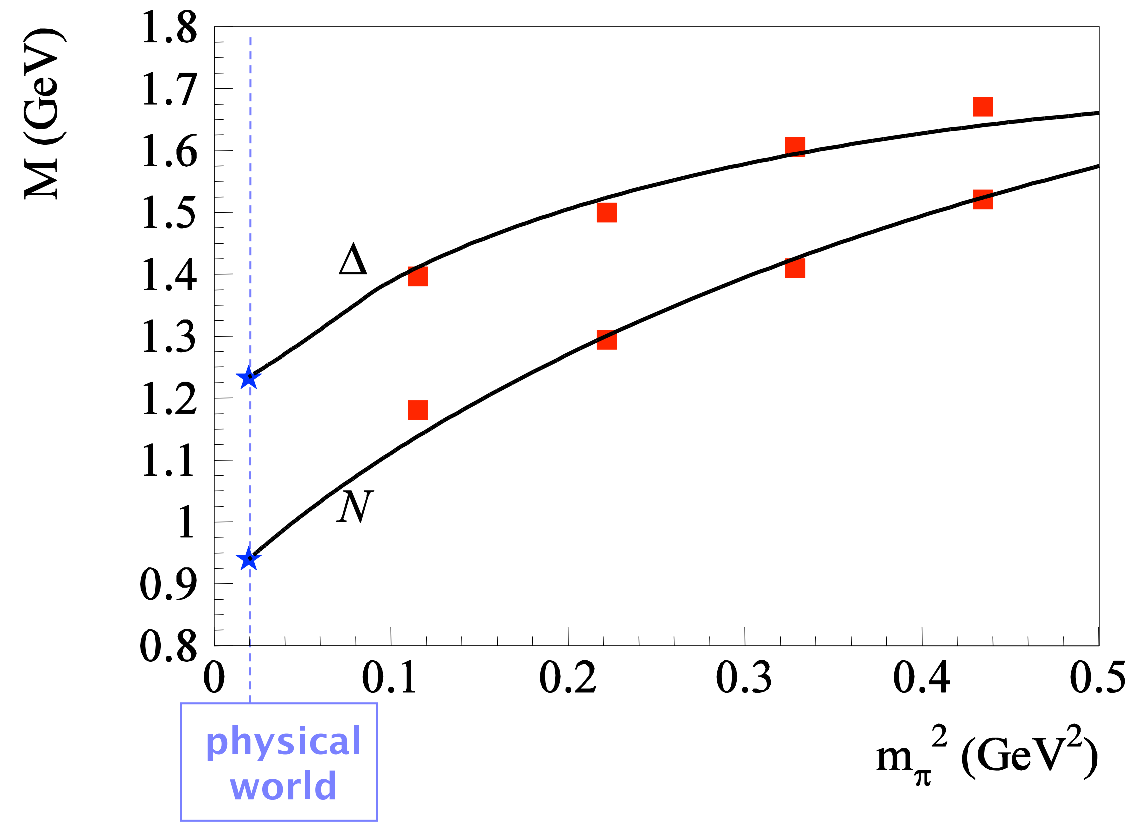

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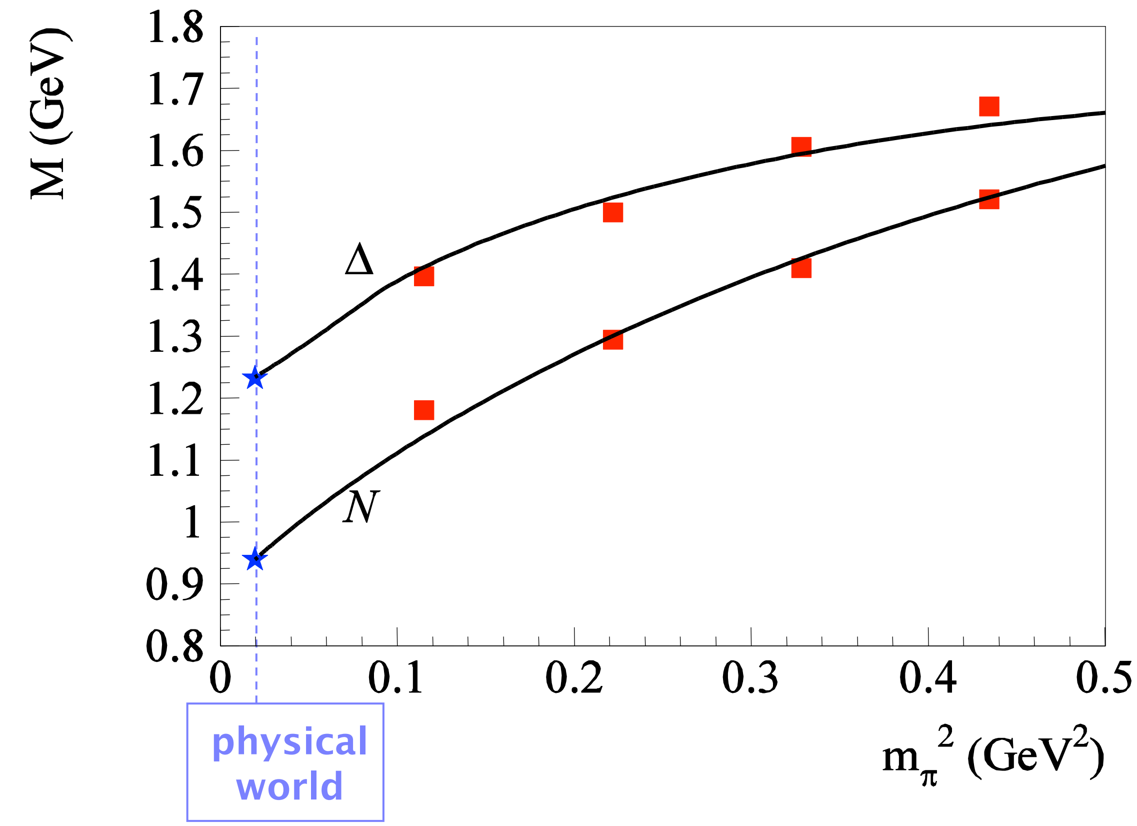


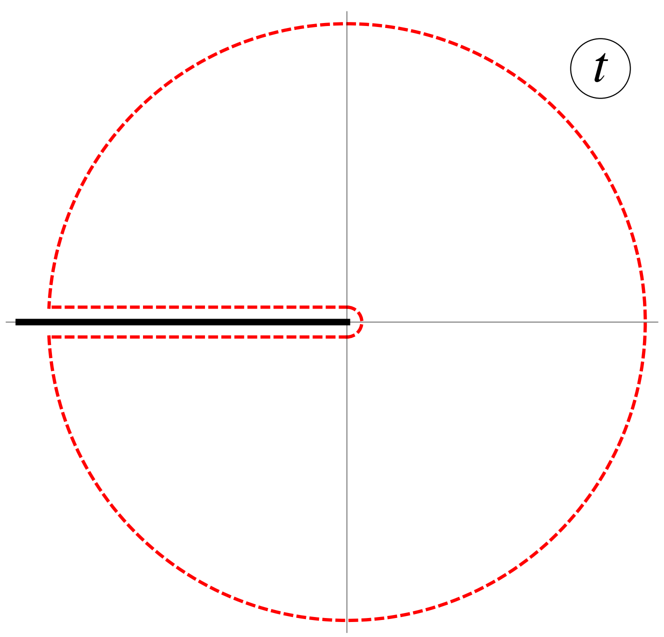
[V.P. & Vanderhaeghen, PLB (2006)]

Presently nucleon mass is computed up to p^6 in HB-ChPT [Birse & McGovern (2006), Schindler et al (2007)], and to p^4 in BChPT.

Analyticity in pion-mass squared

[Ledwig, V.P. & Vanderhaeghen, PLB (2010)]

$t = m_\pi^2$



$$f(m_\pi^2) = -\frac{1}{\pi}\int_{-\infty}^{0} \mathrm{d}t \, \frac{\mathrm{Im}\, f(t)}{t - m_\pi^2 + i0^+}$$

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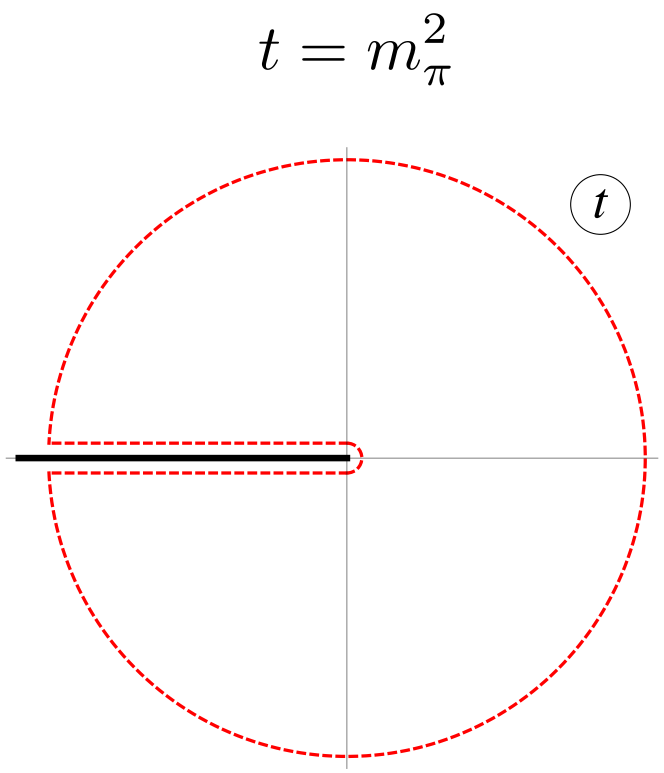

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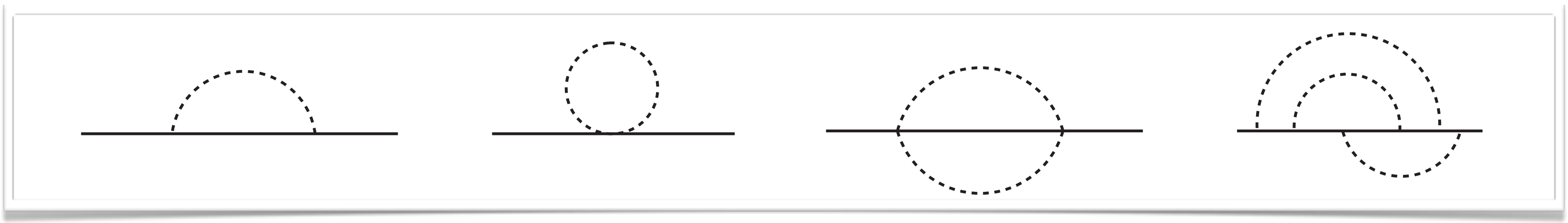

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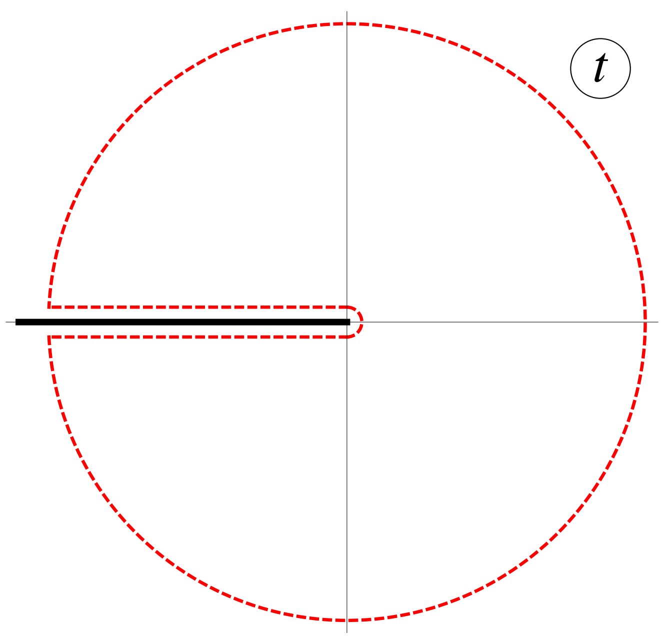

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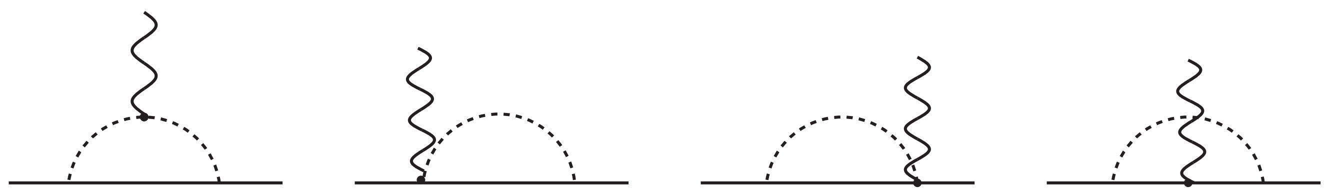
Verified for nucleon mass, a.m.m., polarizabilities at order p^3.

Example: magnetic moment



$$\mathrm{Im}\,\kappa_p^{(p^3\ \mathrm{loop})}(t) = \frac{g_A^2 M_N^2}{(4\pi f_\pi)^2}\frac{\pi}{2\lambda}\left(\tfrac{1}{2}\tau+\lambda\right)^2\left(1-\tfrac{3}{2}(\tfrac{1}{2}\tau+\lambda)\right)\theta(-t)\,,$$

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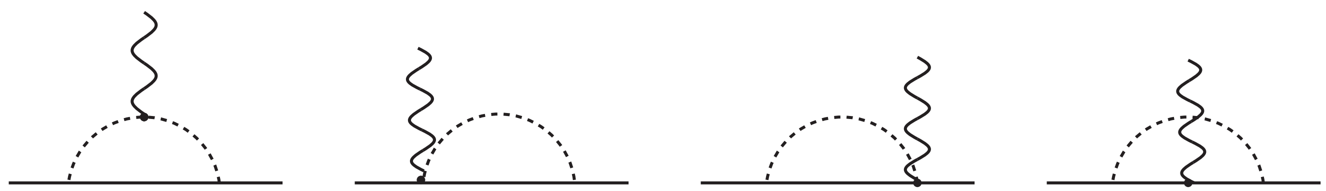

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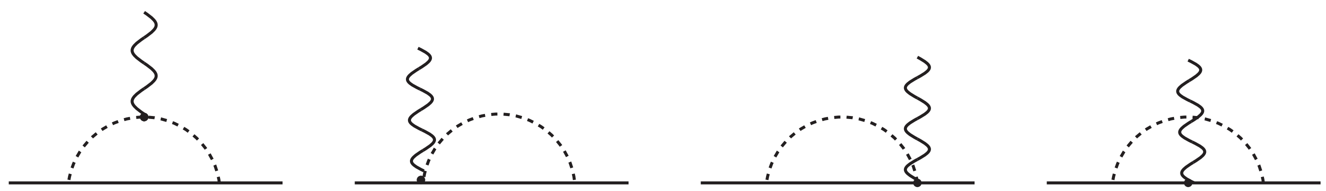

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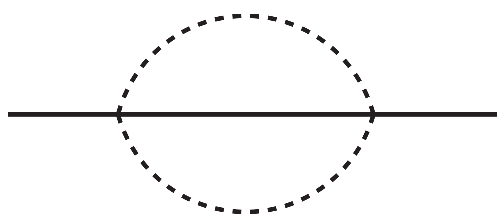
direct calculation of the loops results in the same r.h.s!

c.f.: Holstein, V.P. & Vanderhaeghen, PLB (2004), PRD (2005);
Bethe & de Hoffman, *Mesons and Fields*, Vol. 2 (MIT Press, 1970)

Example: two-loop selfenergy (sunset diagram)



$$J_{\text{sunset}}(m^2, M^2) = \pi^{-d} \int \mathrm{d}^d k_1 \int \mathrm{d}^d k_2 \frac{1}{(k_1^2 - m^2)(k_2^2 - m^2)[(p - k_1 - k_2)^2 - M^2]}.$$

defining the dimensionless: $\tilde{J}(t) = \frac{M^{2(2\varepsilon-1)}}{\Gamma^2(1+\varepsilon)} J_{\text{sunset}}(m^2, M^2)\,, \qquad t = m^2/M^2$

and # of dimensions: $d = 4 - 2\varepsilon,$

$$t(t-1)\frac{\mathrm{d}^2\tilde{J}(t)}{\mathrm{d}t^2} + \left[\frac{1}{2} - 2\varepsilon + \left(-\frac{3}{2} + 4\varepsilon\right)t\right]\frac{\mathrm{d}\tilde{J}(t)}{\mathrm{d}t} + \frac{1}{2}(1-2\varepsilon)(2-3\varepsilon)\tilde{J}(t) = \frac{1}{2\varepsilon^2}\left(t^{1-2\varepsilon} - 2t^{-\varepsilon}\right).$$

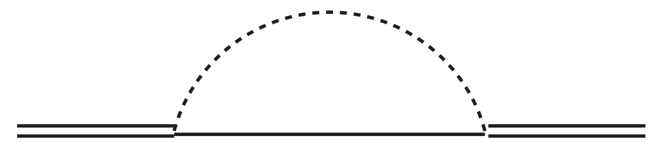
Since for real t the equation is linear with real coefficients we deduce that the solution develops an imaginary part when the inhomogeneous term (the r.h.s.) develops an imaginary part, i.e., for $t < 0$.

The solution for the imaginary part is of the form

$$\operatorname{Im}\tilde{J}(t) = \theta(-t)\,\pi\left[-\frac{2t}{\varepsilon} + t\left(-7 + (2+t)\ln(-t)\right) - (1-t)^2\ln(1-t) + O(\varepsilon)\right],$$

which agrees with F. A. Berends, A. I. Davydychev and N. I. Ussyukina, Phys. Lett. B **426**, 95 (1998).

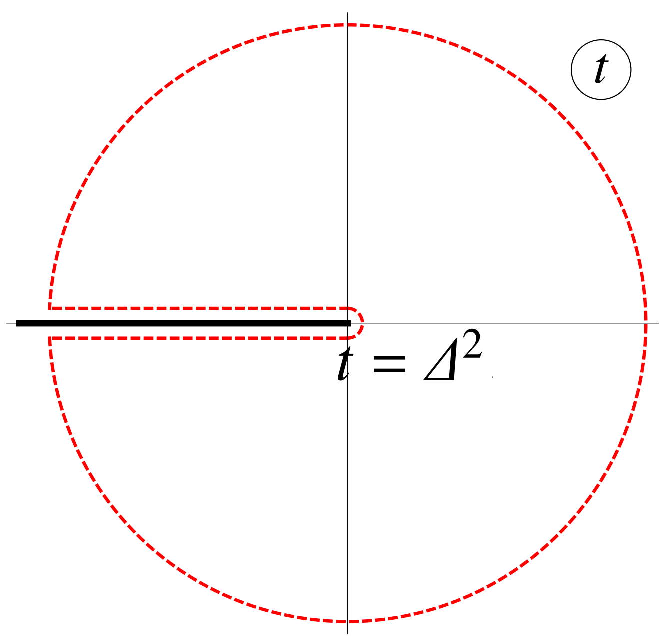
Example: Delta(1232) mass and width



$$\mathrm{Im}\,\Sigma_{\Delta}^{(\pi N\,\mathrm{loop})}(t) = \pi \frac{M_\Delta}{2}\left(\frac{h_A M_\Delta}{8\pi f_\pi}\right)^2 \times \begin{cases} \frac{1}{3}(\alpha + r)\Big[-2\lambda^3 + (1-\alpha)(\tau - 2\lambda^2)\Big] + \frac{1}{4}\tau^2, & t < 0 \\ -\frac{4}{3}(\alpha + r)\lambda^3, & 0 \le t \le \varDelta^2 \\ 0, & t > \varDelta^2 . \end{cases}$$

$\varDelta = M_\Delta - M_N$, the Delta-nucleon mass difference.

$r = M_N/M_\Delta$, $\tau = t/M_\Delta^2$, $\alpha = \frac{1}{2}(1 + r^2 - \tau)$, and $\lambda^2 = \alpha^2 - r^2$.



$$\mathrm{Re}\,\Sigma_{\Delta}^{(\pi N\,\mathrm{loop})}(m_\pi^2) = -\frac{1}{\pi}\int_{-\infty}^{\varDelta^2} \mathrm{d}t' \, \frac{\mathrm{Im}\,\Sigma_{\Delta}^{(\pi N\,\mathrm{loop})}(t')}{t' - m_\pi^2}\left(\frac{m_\pi^2}{t'}\right)^2$$

Chiral corrections to Delta's mass and width are defined unambiguously this way, independent of field redefinitions, etc.

Insights to the (bad) convergence of the HB expansion

Becher & Leutwyler (1999)

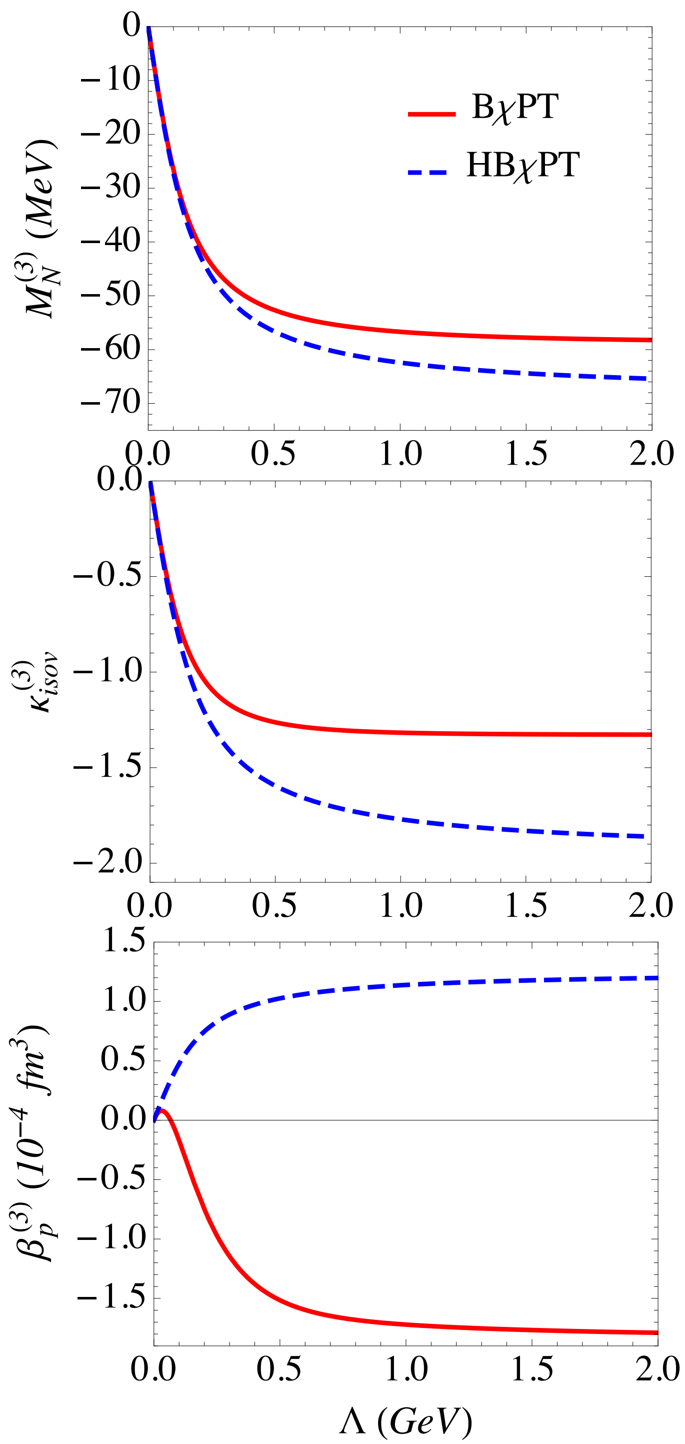
[“Infrared Regularization” violates analyticity, cf. Becher & Leutwyler (1999).]

J.Hall & V.P. (2012) arXiv:1203.0724

$$f(m_\pi^2;\Lambda^2) = -\frac{1}{\pi}\int\limits_{-\Lambda^2}^{0} \mathrm{d}t\,\frac{\mathrm{Im}\,f(t)}{t-m_\pi^2}\left(\frac{m_\pi^2}{t}\right)^n$$

where Λ interpreted as momentum cutoff

Cutoff dependence in HB- and B-ChPT



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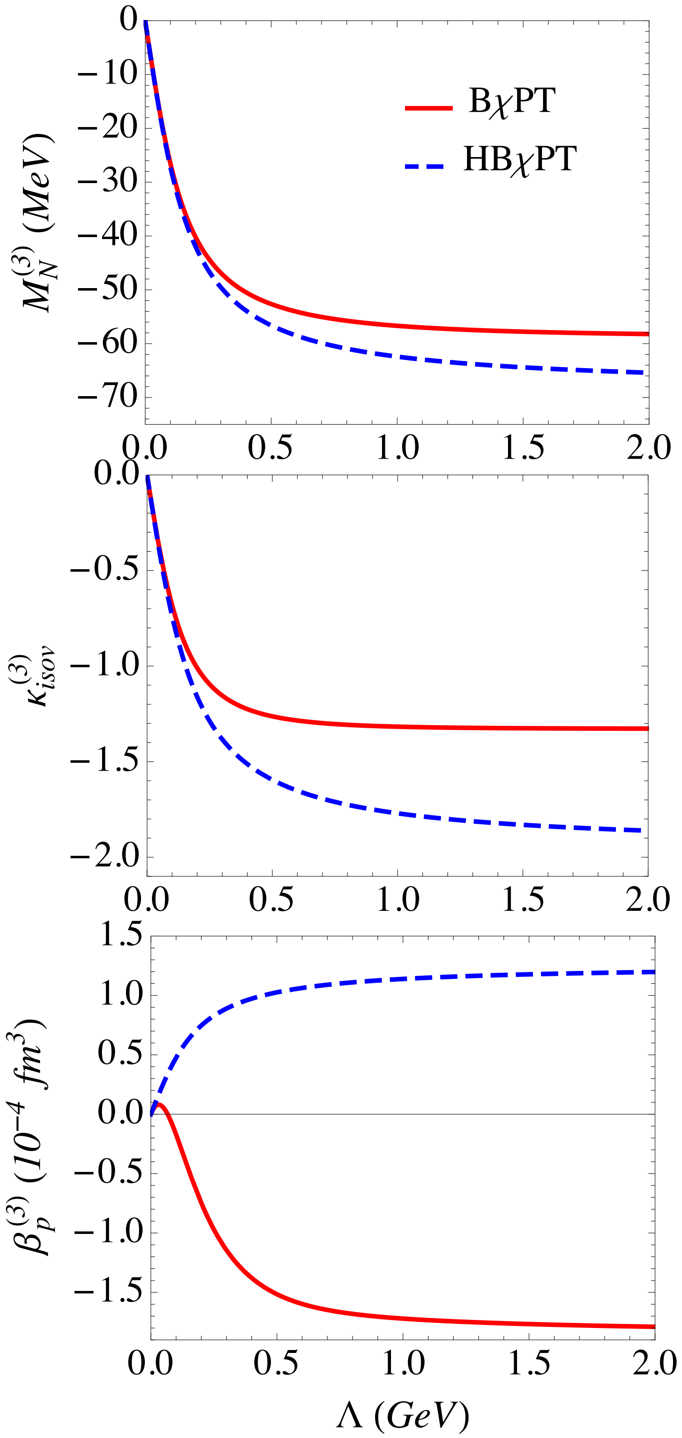

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$$\beta_M \sim \frac{1}{m_\pi}$$

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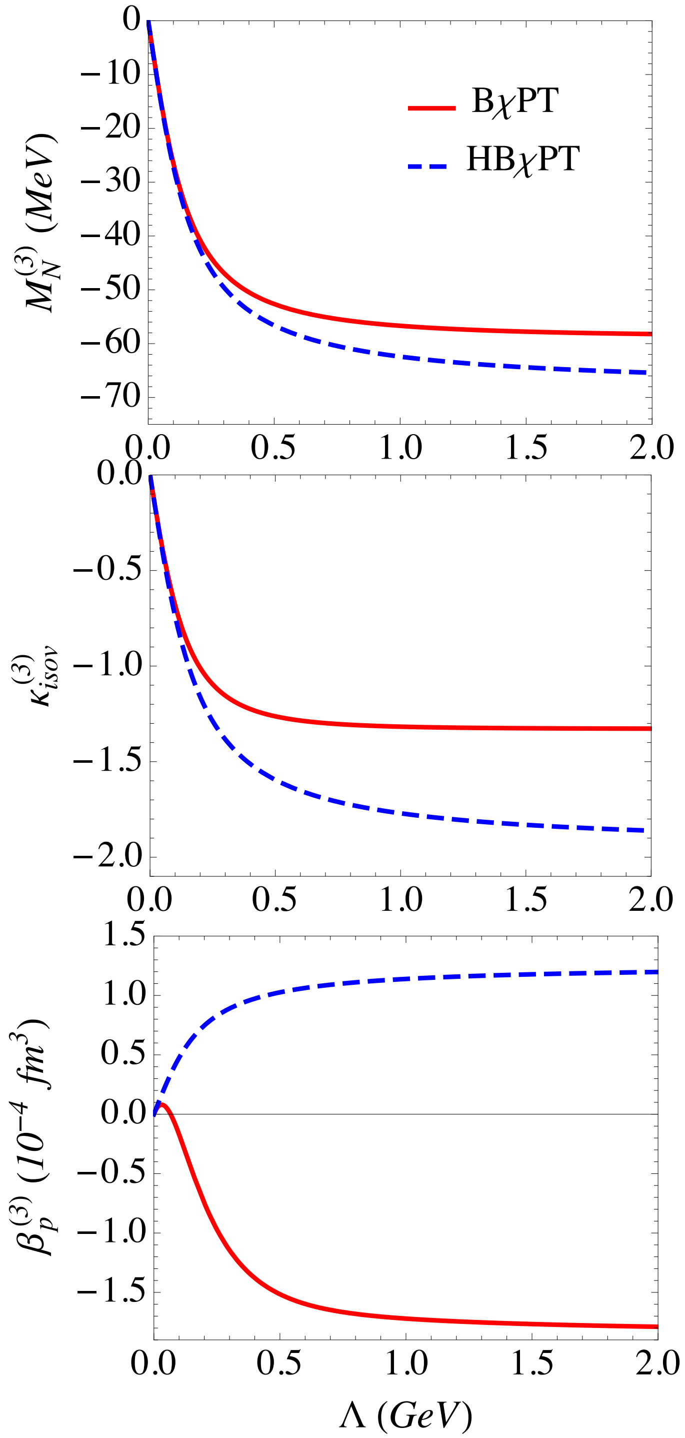

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Heavy-Baryon expansion fails for quantities where

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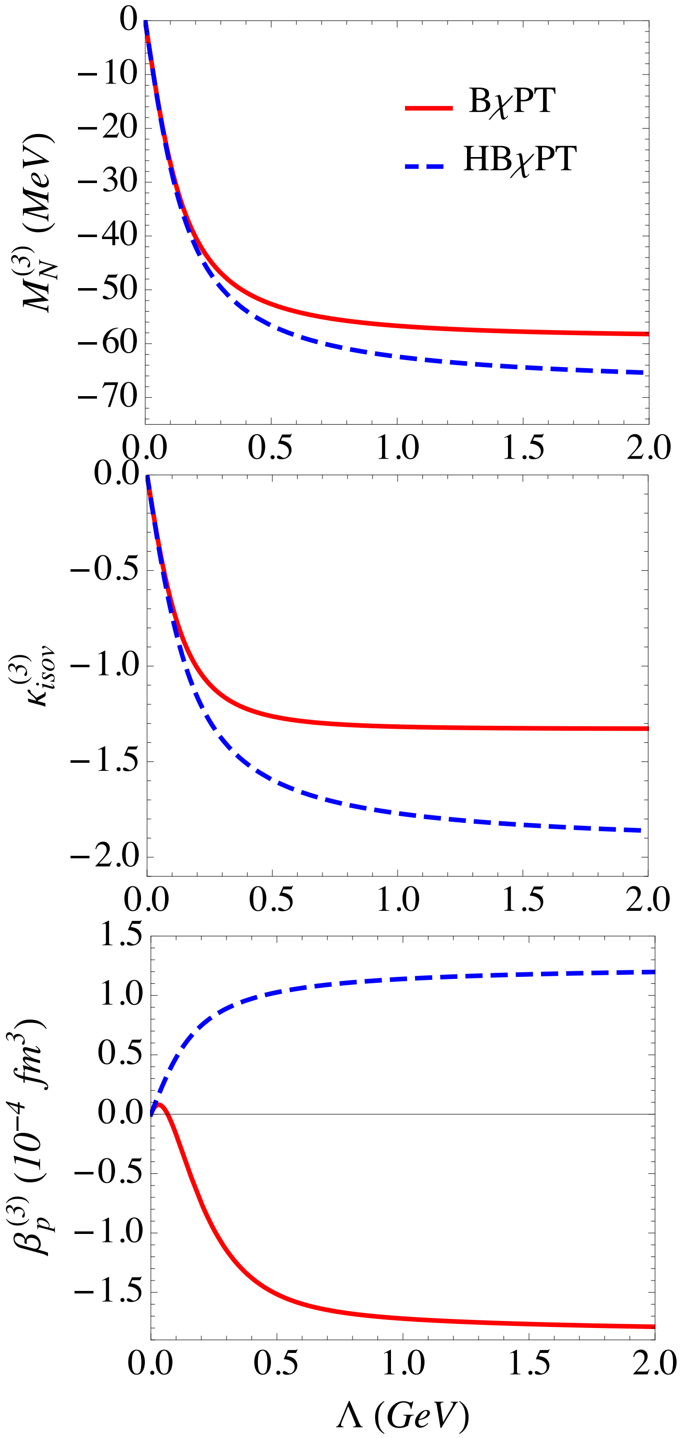

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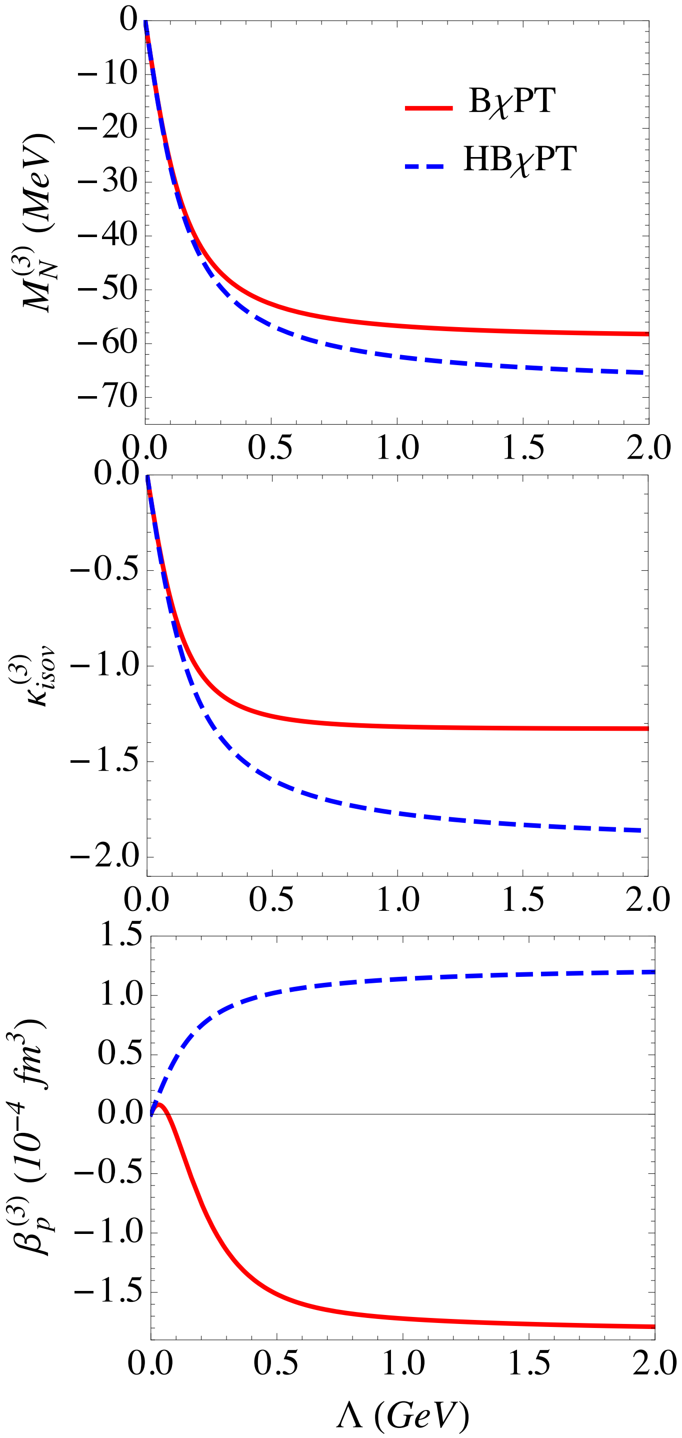
Cutoff dependence in HB- and B-ChPT



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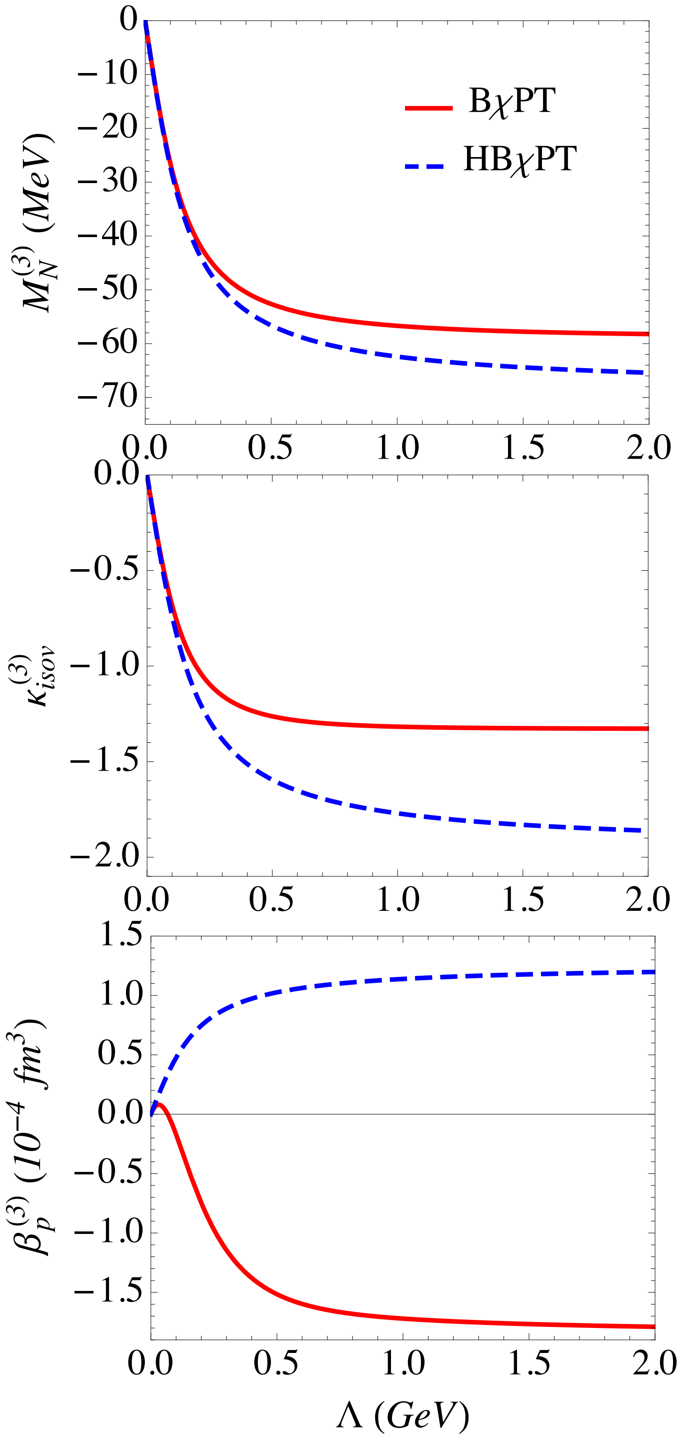

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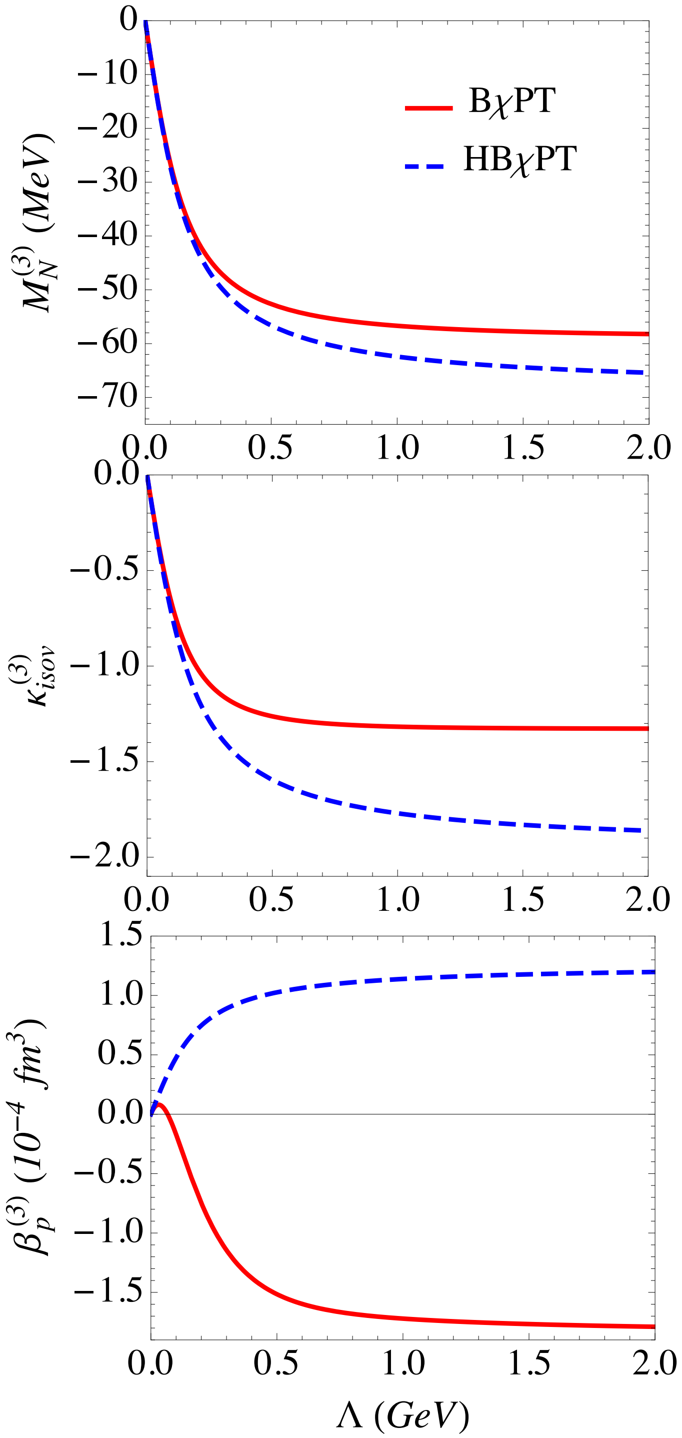

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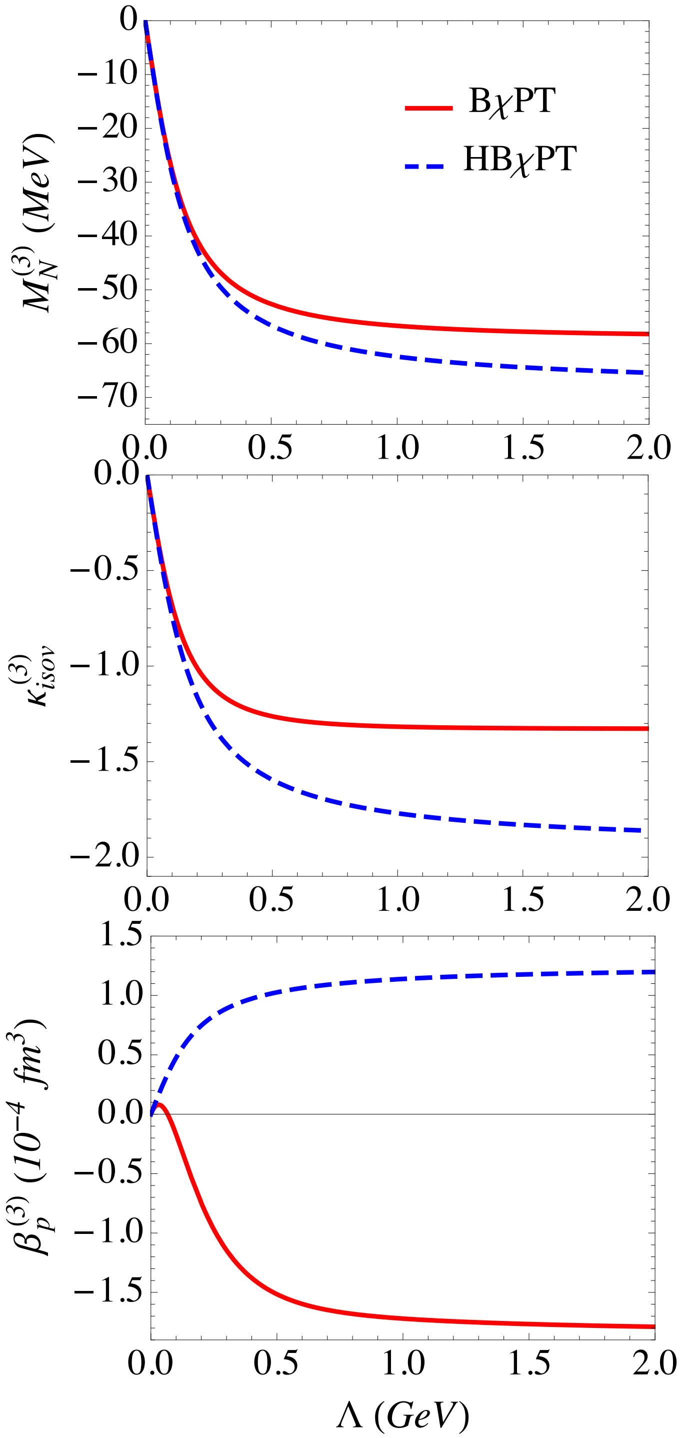

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Polarizabilities: a CONFLICT with PDG

	BχPT (HBχPT)			PDG
	$\mathcal{O}(p^3)$	$\mathcal{O}(p^3)+\mathcal{O}(p^4/\Delta)$	$\mathcal{O}(p^4)$ est.	[45]
$\alpha^{(p)}$	6.8 (12.2)	10.8 (20.8)	± 0.7	12.0 ± 0.6
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TABLE I: Predictions of baryon χPT for electric (α) and magnetic (β) polarizabilities of the proton in units of $10^{-4}\,\mathrm{fm}^3$, compared with the Particle Data Group summary of experimental values.

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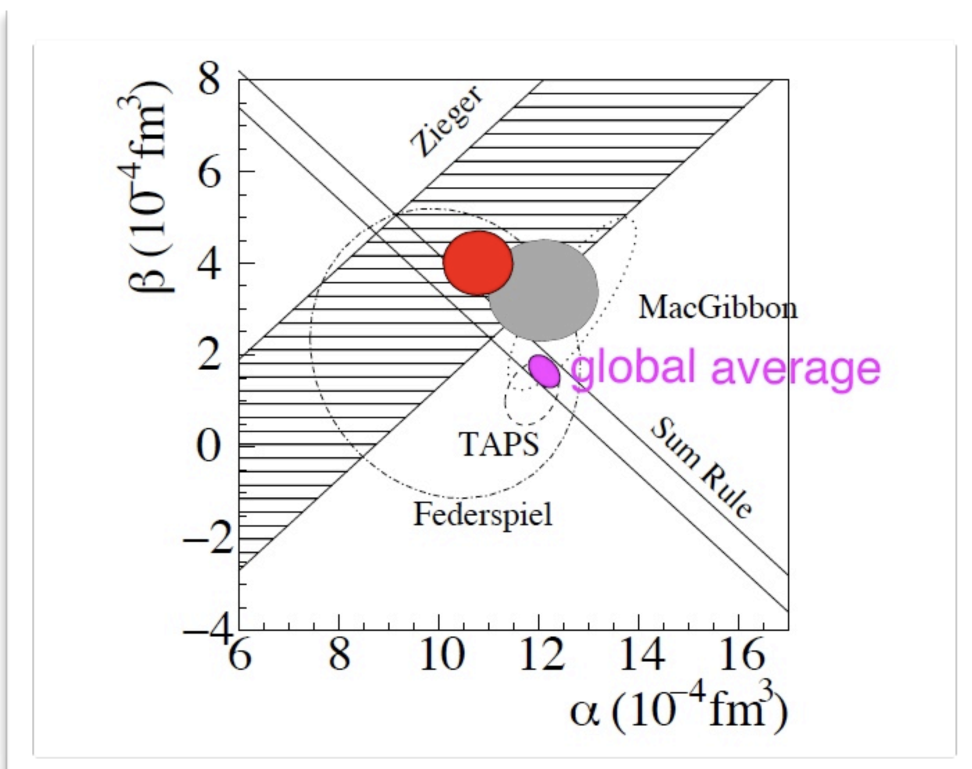

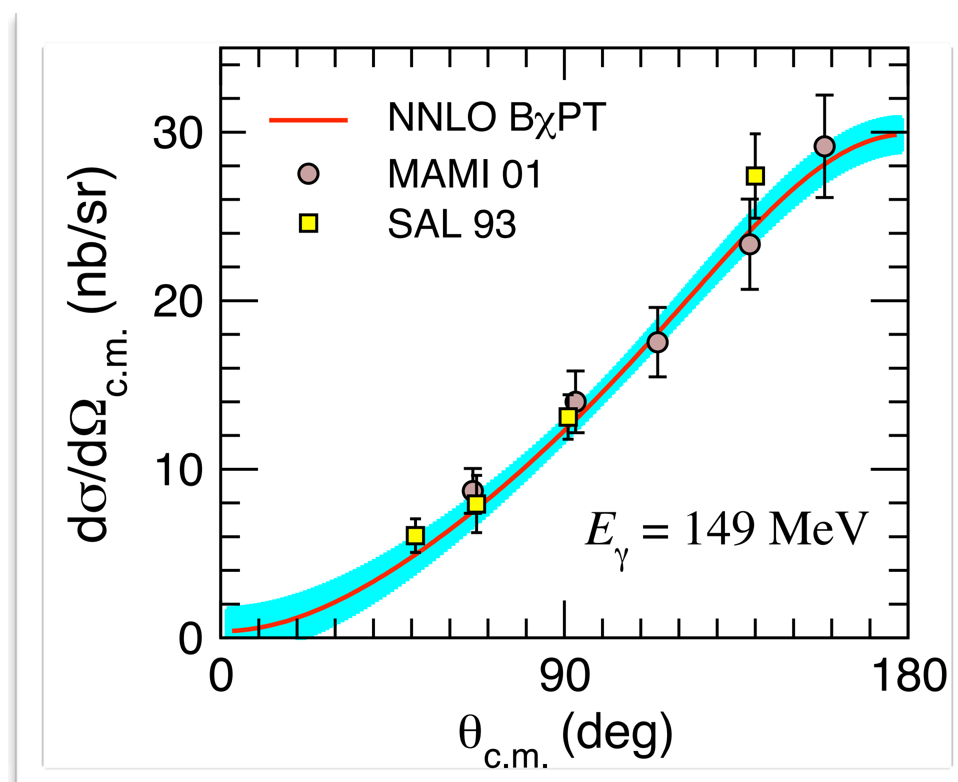

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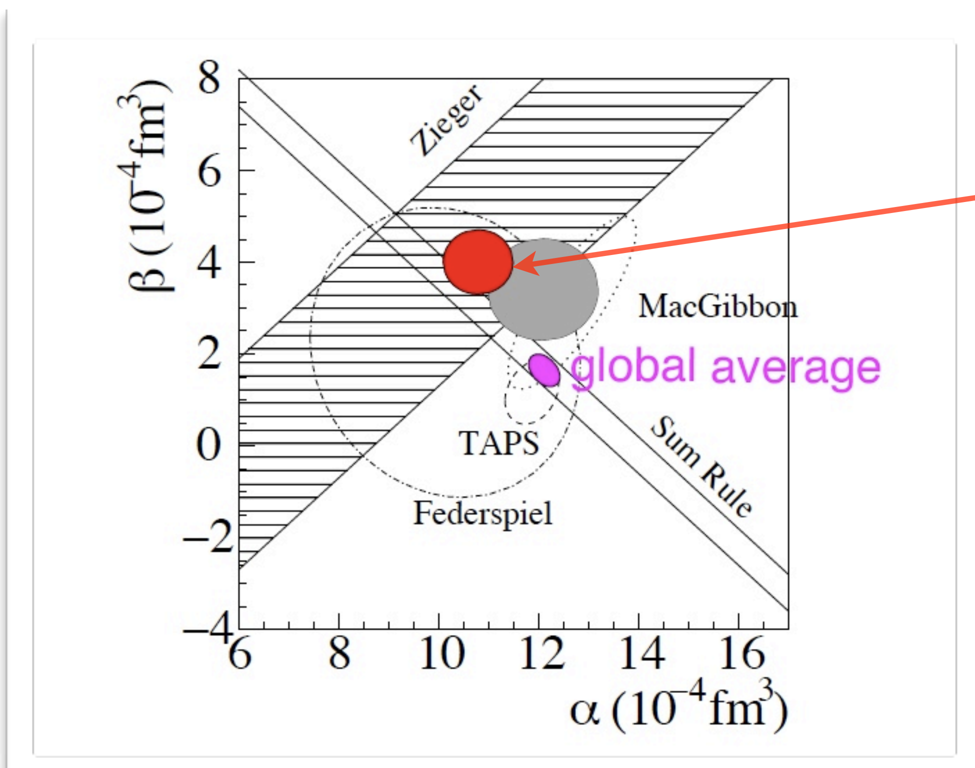

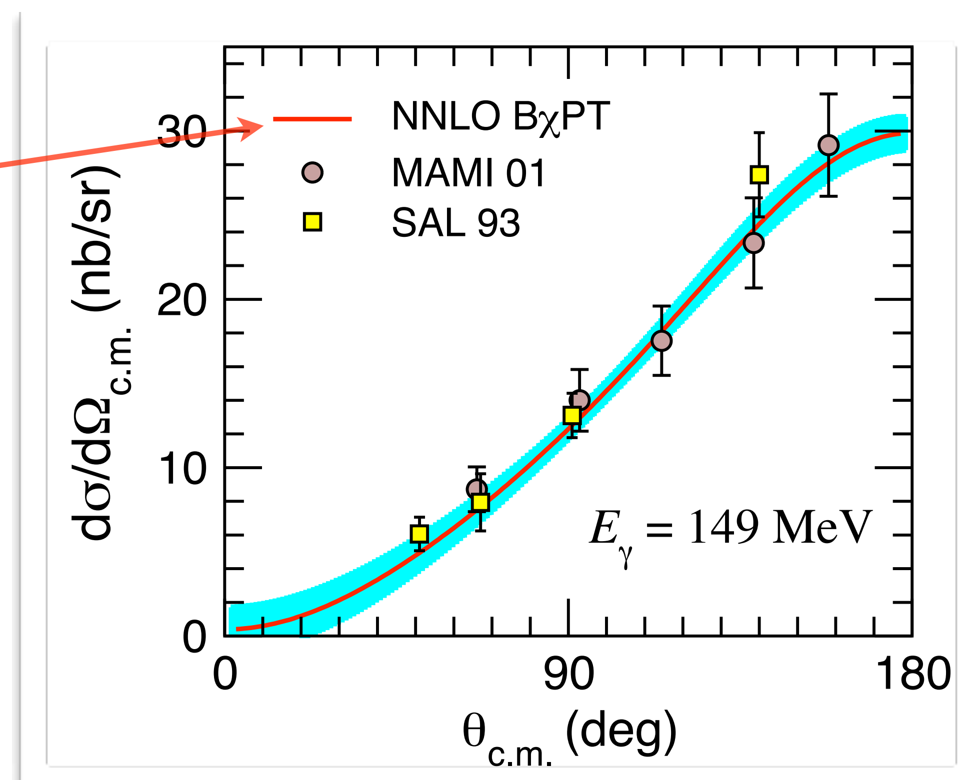

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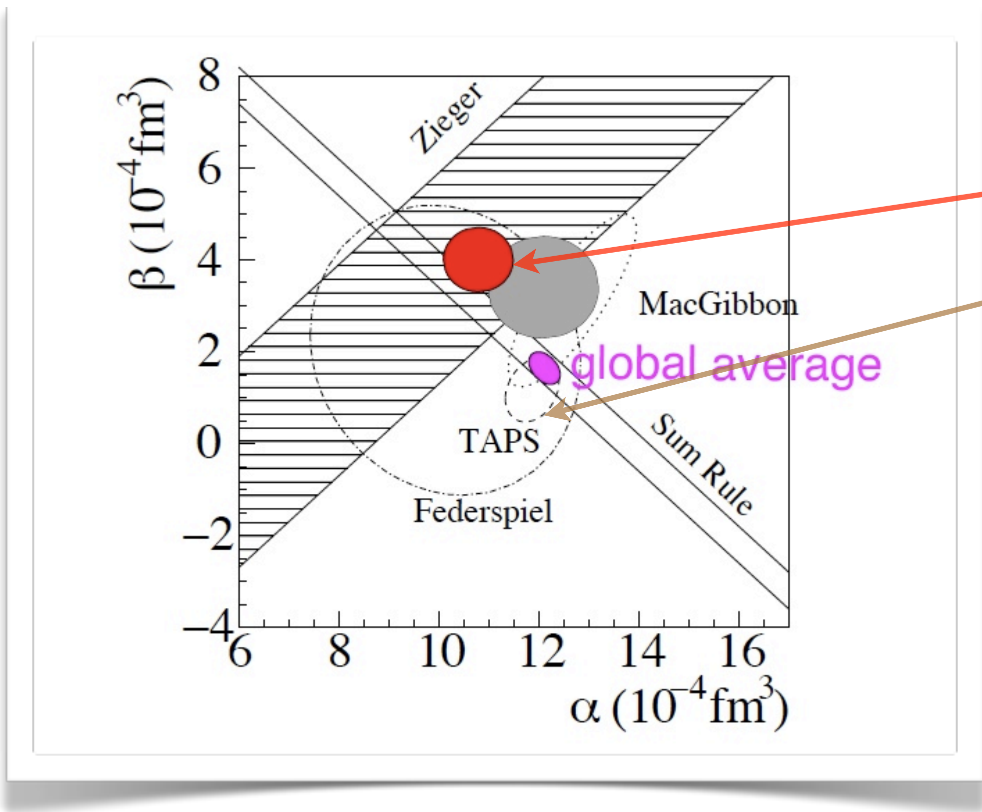

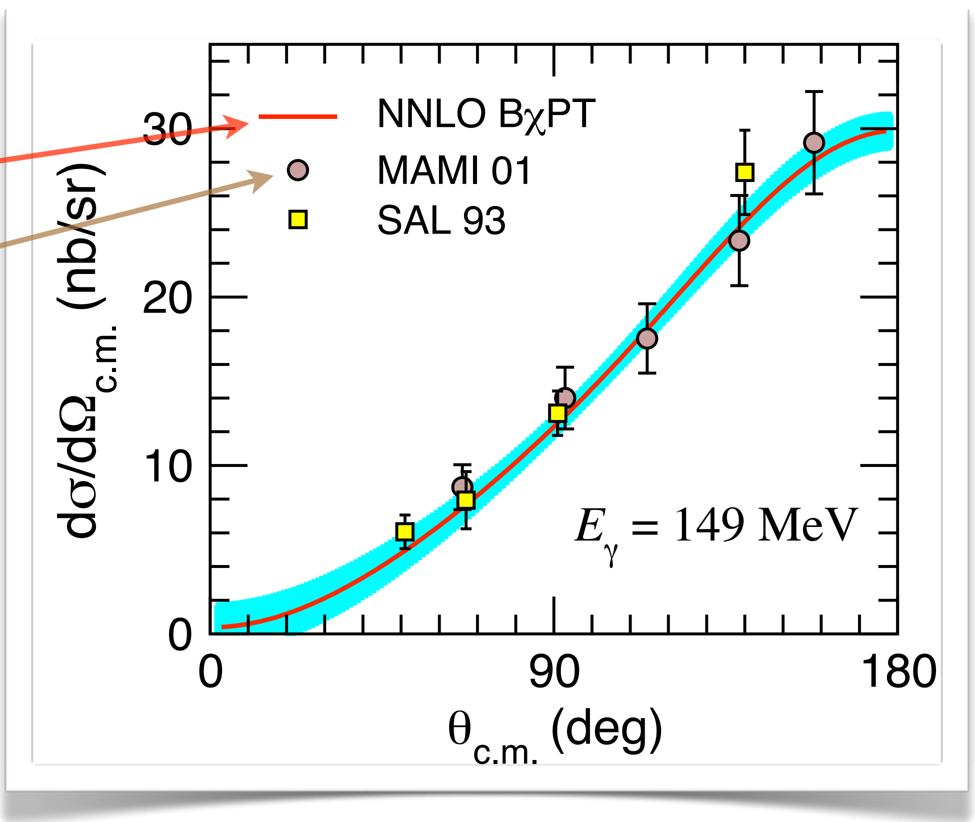

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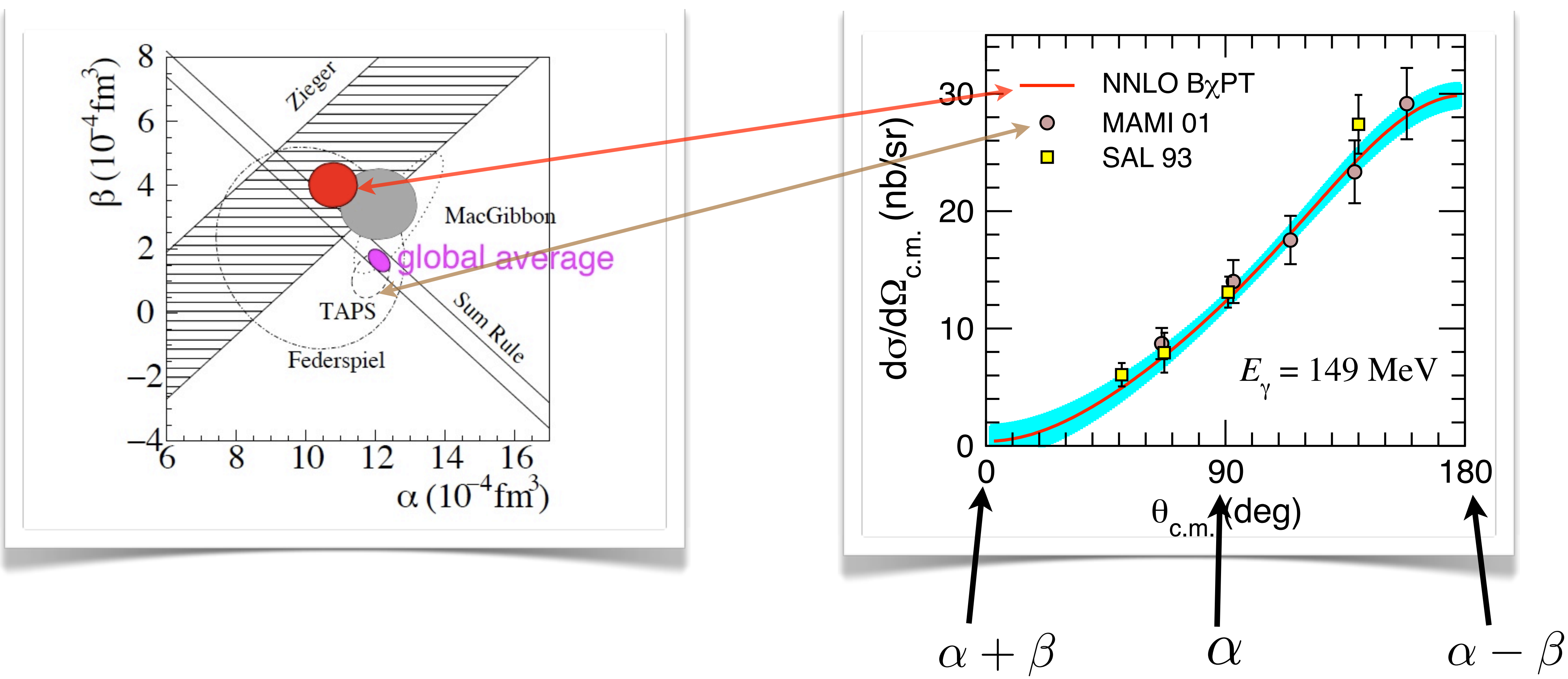

Summary

$$f(m_\pi^2) = -\frac{1}{\pi}\int_{-\infty}^{0} \mathrm{d}t \, \frac{\mathrm{Im}\, f(t)}{t - m_\pi^2 + i0^+}$$

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- insights from the FRR: quantities which expansion begins with inverse powers of pion mass converge badly (unnaturally) in HB-ChPT,

e.g. polarizabilities, NN effective range parameters.