Dispersive analysis of $\omega/\phi \to 3\pi$ decays and the $\omega/\phi \to \pi^0\gamma^*$ transition form factors

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with B. Kubis and F. Niecknig, EPJC **72**, 2014 (2012); arXiv:1206.3098 [hep-ph]

Outline

Motivation

Dispersive framework for $\omega/\phi \to 3\pi$

Dispersive framework for the $\omega/\phi \to \pi^0\gamma^*$ transition form factor

Conclusions and outlook

Why dispersive analyses?

- advent of high-statistics experiments allows for accurate measurements of decay amplitudes
  BES-III, WASA-at-COSY, MAMI-B/-C, CLAS@JLAB, CMD, KLOE, ELSA
  $\Rightarrow$ need to match this accuracy on the theoretical side
- final-state interactions in hadronic three-body decays play essential role in precision amplitude analyses
- perturbative approaches (ChPT, NREFT,...): implement final-state-interactions up to a certain order in a small power-counting parameter
- goal of dispersion relations: resum effects of hadronic rescattering to all orders $\Rightarrow$ precise implementation of final-state interactions, allows extension to higher energies
- high-accuracy parametrizations of phase-shifts required $\Rightarrow$ now available in some cases ($\pi\pi$, πK,...)

Physics case

$\omega/\phi \to 3\pi$:

- most simple imaginable system with physical relevance
  $\Rightarrow$ P-wave interactions only (neglecting F- and higher waves)
  $\Rightarrow$ ideal testing ground for the approach
- large existent (ϕ: KLOE/CMD-2) and upcoming (ω: WASA, CLAS) data base
- $\phi \to 3\pi$: study crossed-channel effects on resonances in the decay region

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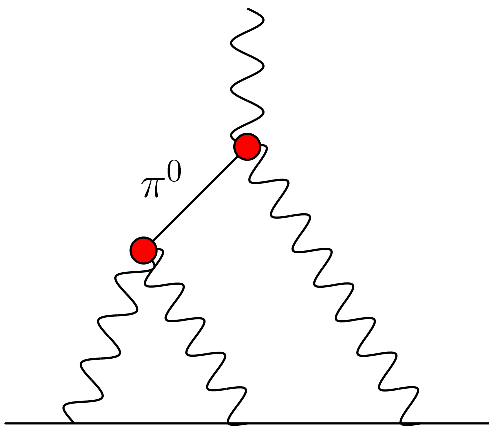
$\omega/\phi \to \pi^0\gamma^*$ transition form factors:

- can help to constrain pseudoscalar pole terms (π^0, η, η') in hadronic light-by-light
- strength determined by decay $\pi^0 \to \gamma^*\gamma^*$: doubly-virtual form factor $F_{\pi^0\gamma^*\gamma^*}(M_{\pi^0}^2, q_1^2, q_2^2)$
- for fixed isoscalar photon virtuality: can extract $F_{\pi^0\gamma^*\gamma^*}(M_{\pi^0}^2, q_1^2, M_\omega^2)$ from $\omega \to \pi^0\ell^+\ell^-$



The framework: fundamentals

- was applied in $\eta \to 3\pi$ decays before, but also $\eta' \to \eta\pi\pi, K_{\ell 4}, \ldots$ possible   Anisovich, Leutwyler '98; Lanz; Stoffer (see talks at CD12 on tuesday)

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- integral equations based on fundamental principles: unitarity, analyticity and crossing symmetry
- assume elastic $\pi\pi$ rescattering
- decay amplitude can be decomposed according to:

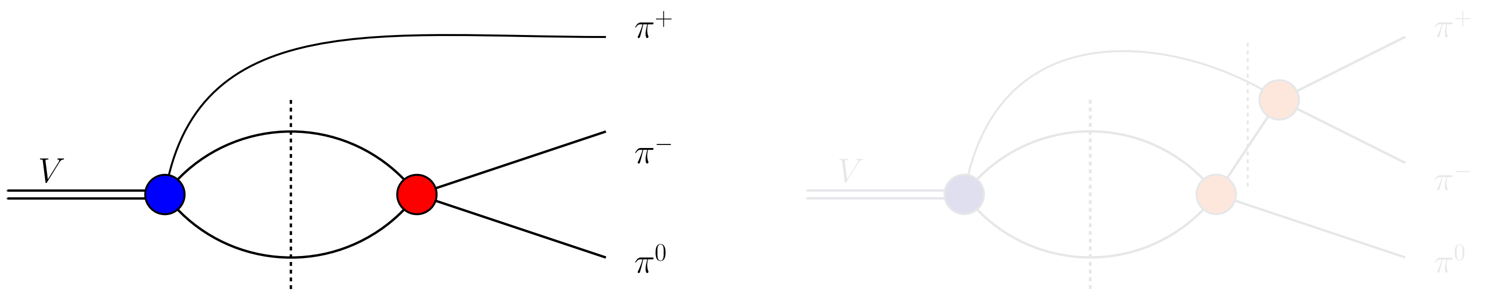
$$\omega/\phi \to 3\pi : \mathcal{F}(s,t,u) = \mathcal{F}(s) + \mathcal{F}(t) + \mathcal{F}(u)$$

$$s + t + u = M_{\omega/\phi}^2 + 3M_\pi^2 \doteq 3s_0$$

▷ $\mathcal{F}(s)$ functions of one variable with only a right-hand cut

▷ decomposition exact only if $l \geq 3$ partial waves are real

From unitarity to integral equations



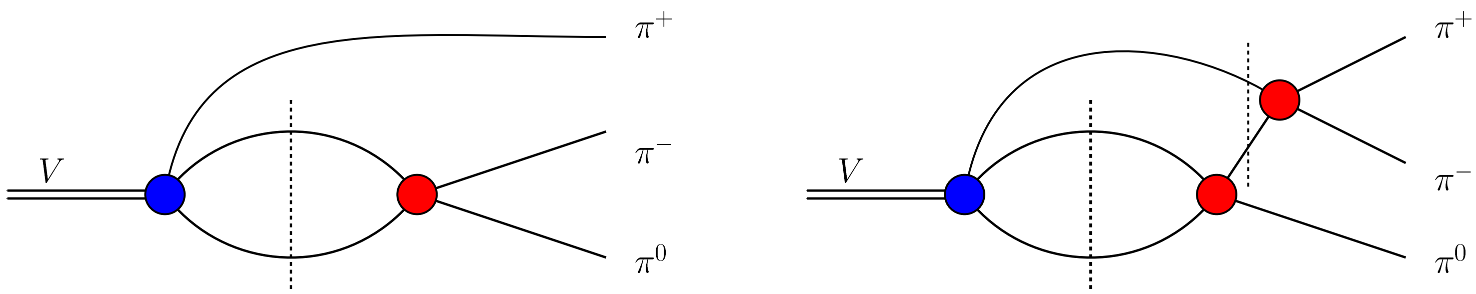
- from unitarity:

$$\text{disc}\,\mathcal{F}(s) = 2i\,\theta(s-4\,M_\pi^2)\{\mathcal{F}(s)+\hat{\mathcal{F}}(s)\}\,\sin\,\delta_1^1(s)\,e^{-i\delta_1^1(s)}$$

- simple ⇒ solved by an Omnès function $\Omega(s)$ Omnès '58

$$\mathcal{F}(s) = P(s)\Omega(s)\,,\quad \Omega(s) = \exp\Big\{\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds'\frac{\delta_1^1(s')}{s'(s'-s)}\Big\}$$

From unitarity to integral equations



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- unitarity relation gets more complicated for three-particle final states
- crossed-channel scattering between s-, t-, and u-channel
  $\Rightarrow$ inhomogeneities $\hat{\mathcal{F}}(s)$: angular integration over $\mathcal{F}(s)$
- correct analytic continuation necessitates path deformation of the angular integral

Integral equations

- Solution to:

$$\text{disc}\,\mathcal{F}(s) = 2i\,\theta(s-4\,M_\pi^2)\{\mathcal{F}(s)+\hat{\mathcal{F}}(s)\}\,\sin\,\delta_1^1(s)\,e^{-i\delta_1^1(s)}$$

$$\mathcal{F}(s) = \Omega(s)\Big\{a + \frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\sin\,\delta_1^1(s')\hat{\mathcal{F}}(s')}{|\Omega(s')|(s'-s-i\epsilon)}\Big\}$$

Niecknig, Kubis, SPS '12
Anisovich, Leutwyler '98

Khuri,Treiman '60; Aitchison, Pasquier '66

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Niecknig, Kubis, SPS ’12
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- only one subtraction constant in this system
  - ▷ dynamics (Dalitz plot) do not depend on the specific choice of this subtraction constant!
  - ▷ a matched to reproduce the $\omega/\phi \to 3\pi$ partial width
- $\delta_1^1(s)$ from phenomenological analyses (Roy equations)

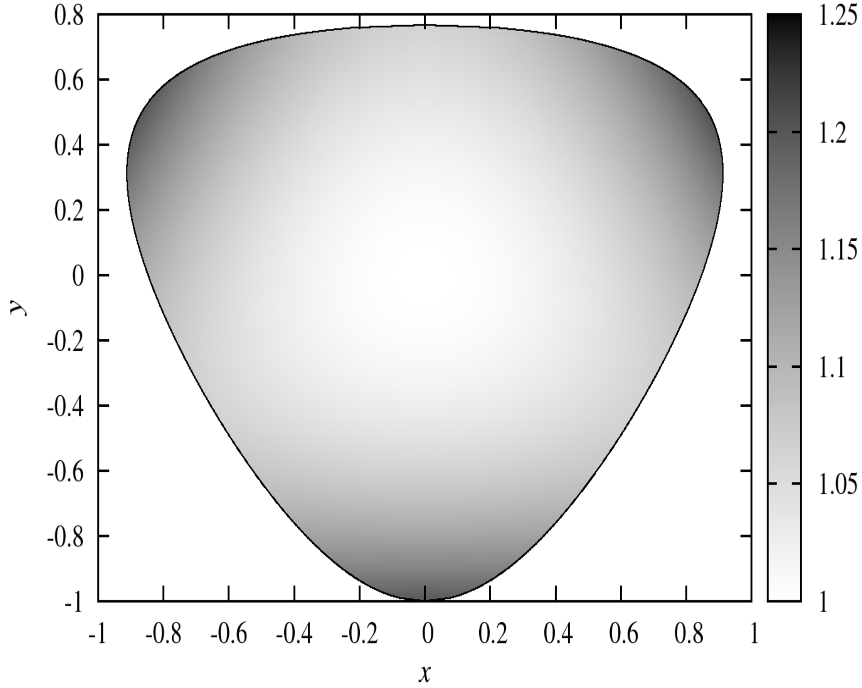
Caprini et al. (in preparation), García-Martín et al. ’11

- solve these equations by an iterative numerical procedure

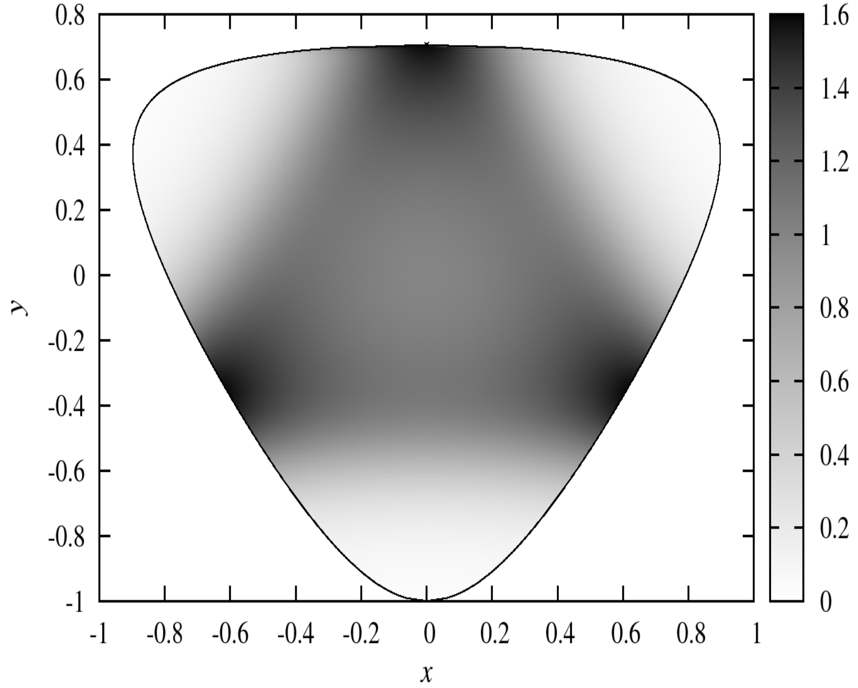
$\omega/\phi \to 3\pi$ Dalitz plot

- normalized Dalitz plot ($y = \frac{3(s_0 - s)}{2\,M_V(M_V - 3M_\pi)}$, $x = \frac{\sqrt{3}(t-u)}{2\,M_V(M_V - 3M_\pi)}$):

$\omega \to 3\pi$:



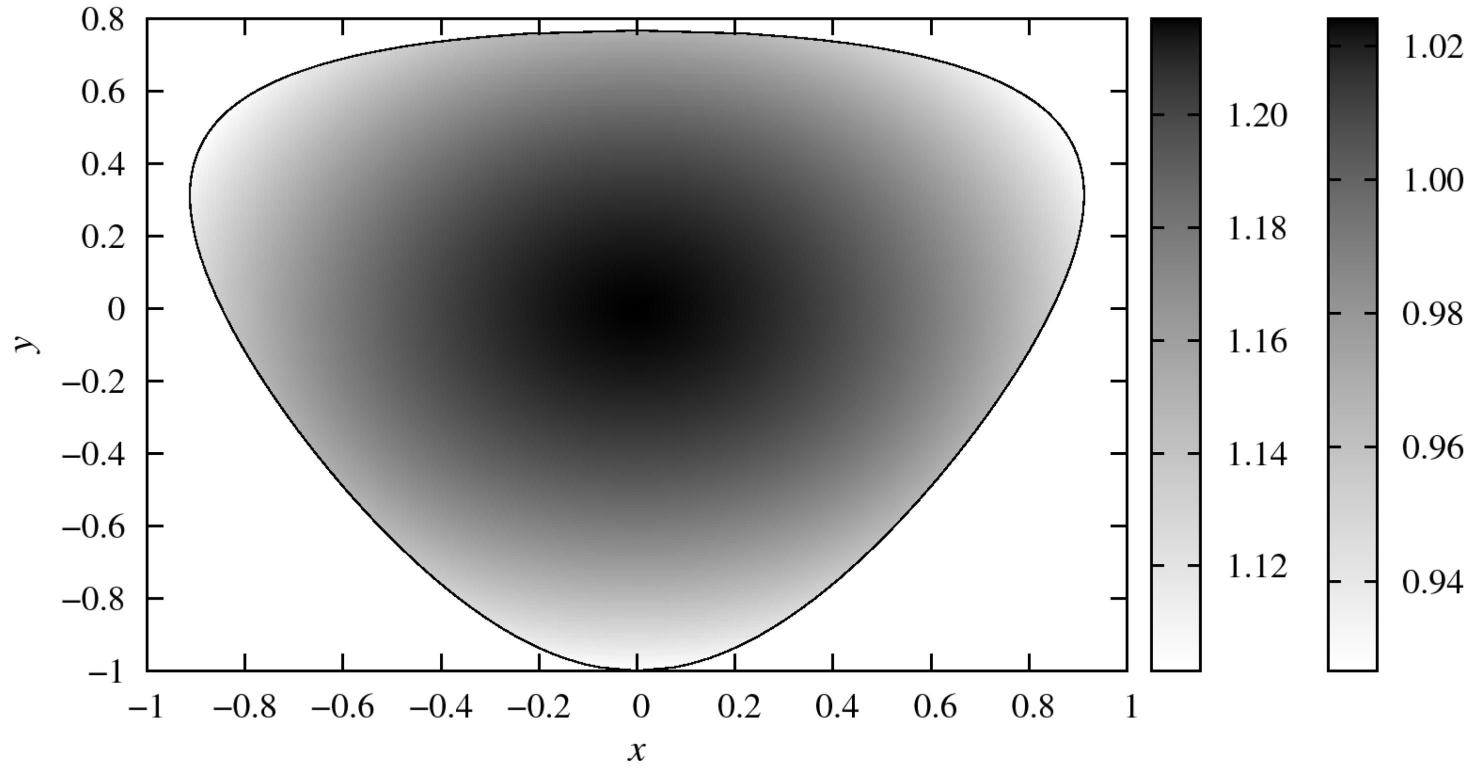
$\phi \to 3\pi$:



- normalized Dalitz plot is independent of the subtraction constant!
- ω Dalitz plot is relatively smooth
- ϕ Dalitz plot clearly shows ρ resonance bands
- so what are the effects of crossed-channel rescattering?

Crossed-channel effects in $\omega \to 3\pi$

- shown is $|\mathcal{F}_{\text{full}}(s,t,u)|^2/|\mathcal{F}_{\hat{\mathcal{F}}=0}|^2$



- left scale: a fixed to decay rate before iteration
  - partial width increased by about 16%
- right scale: a fixed to decay rate before and after iteration
  - significant part of changes absorbed in overall normalization

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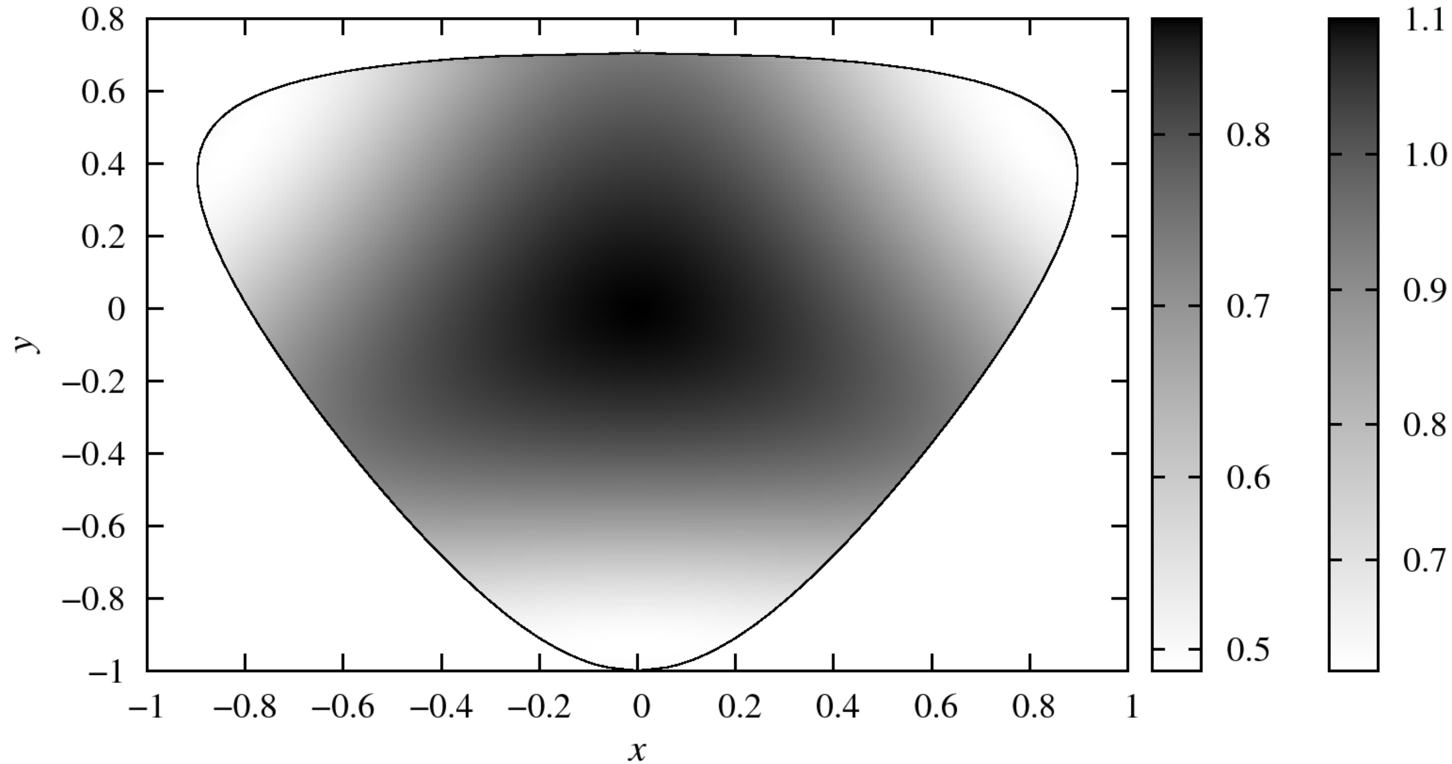
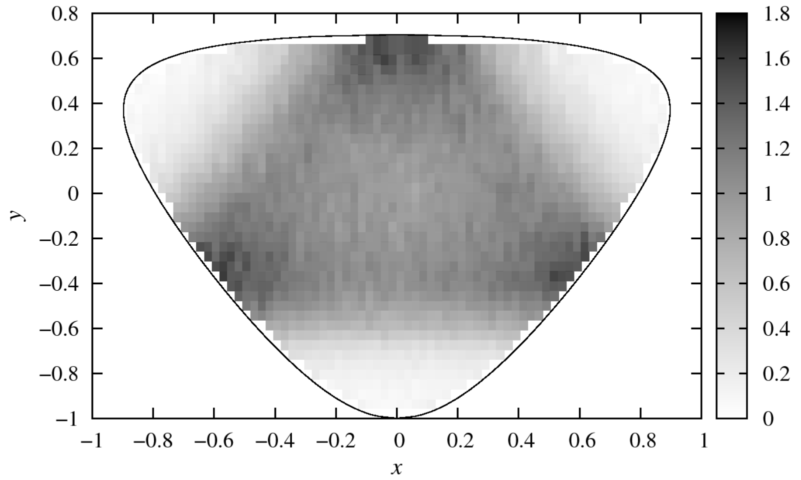
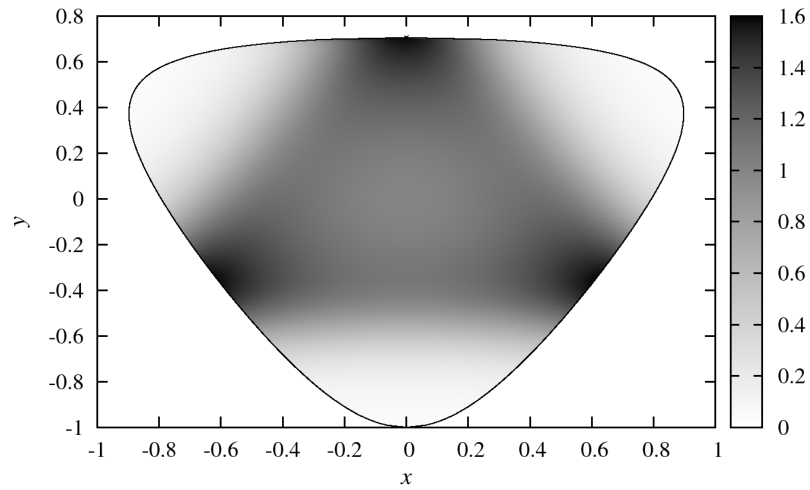


- left scale: a fixed to decay rate before iteration
  - ▷ partial width decreased by about 20%
- right scale: a fixed to decay rate before and after iteration
  - ▷ significant part of changes absorbed in overall normalisation
  - ▷ ρ bands relatively unaffected

Comparison with experiment

Compare to experimental $\phi \to 3\pi$ data:





"Fit" results KLOE:

	$\hat{\mathcal{F}} = 0$	full, once-subtracted
χ^2/ndof	$1.71 \ldots 2.06$	$1.17 \ldots 1.50$

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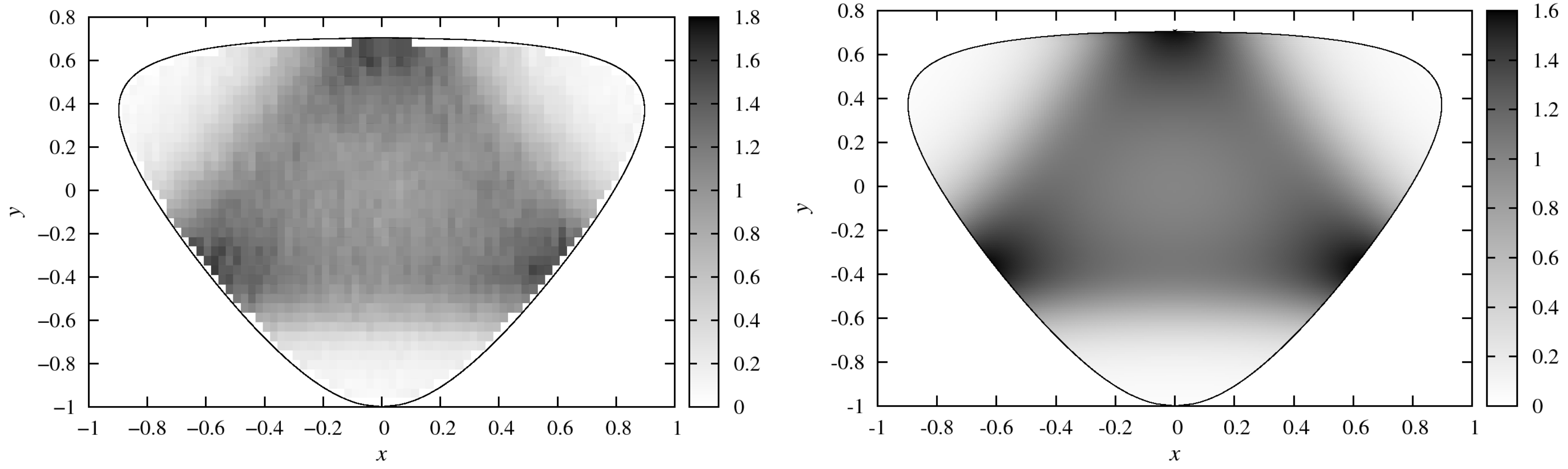

"Fit" results KLOE:

	$\hat{\mathcal{F}} = 0$	full, once-subtracted
χ^2/ndof	$1.71 \ldots 2.06$	$1.17 \ldots 1.50$

- looks ok – but certainly not perfect
- add additional subtraction $\Rightarrow$ suppress contributions from higher energies

Two subtractions

- twice-subtracted dispersion relation

$$\mathcal{F}(s) = \Omega(s)\left\{a + b\,s + \frac{s^2}{\pi}\int_{4M_\pi^2}^{\infty} \frac{ds'}{s'^2}\frac{\sin\,\delta_1^1(s')\hat{\mathcal{F}}(s')}{|\Omega(s')|(s'-s-i\epsilon)}\right\}$$

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	KLOE		CMD-2	
	Bern	Madrid-Cracow	Bern	Madrid-Cracow
χ^2/ndof	1.02	1.03	0.96	0.94
$\lvert b\rvert \times \mathrm{GeV}^2$	0.97 ± 0.03	0.94 ± 0.03	$0.97^{+0.16}_{-0.13}$	$0.95^{+0.15}_{-0.12}$
$\arg\, b$	0.52 ± 0.03	0.42 ± 0.03	0.00 ± 0.16	-0.18 ± 0.18

- perfect fits for both data sets $\Rightarrow$ representation respects unitarity, analyticity, and crossing symmetry
- apparent disagreement between KLOE and CMD-2 $\Rightarrow$ systematics?

Prediction of the $\omega \to 3\pi$ Dalitz plot parameters

- $\omega \to 3\pi$ Dalitz plot smooth $\Rightarrow$ polynomial parameterisation

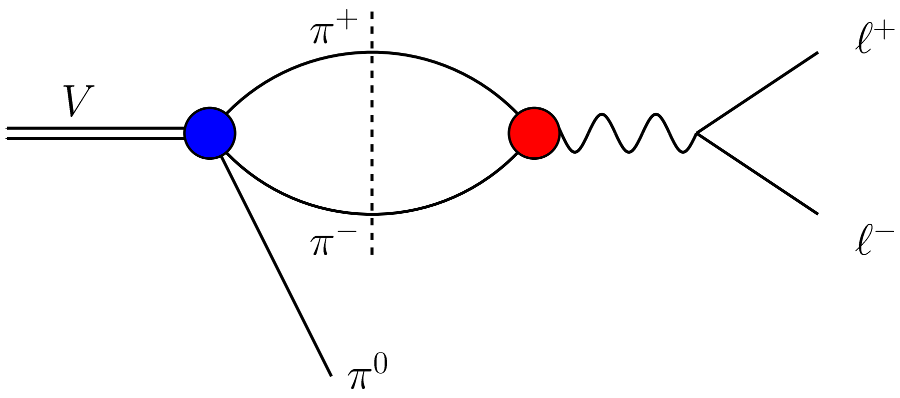
$$|\mathcal{F}_{\rm pol}(z,\phi)|^2 = |\mathcal{N}|^2\Big\{1 + 2\alpha z + 2\beta z^{3/2}\sin 3\phi + 2\gamma z^2 + 2\delta z^{5/2}\sin 3\phi\Big\}$$

$\alpha \times 10^3$	$\beta \times 10^3$	$\gamma \times 10^3$	$\delta \times 10^3$
$84\ldots96$	—	—	—
$74\ldots84$	$24\ldots28$	—	—
$73\ldots81$	$24\ldots28$	$3\ldots6$	—
$74\ldots83$	$21\ldots24$	$0\ldots2$	$7\ldots8$

- two Dalitz plot parameters sufficient at 1% accuracy
- compare $\eta \to 3\pi^0$ (same 3-fold symmetry):
  $\alpha = (-31.7 \pm 1.6) \times 10^{-3}$ PDG average
  $\beta \simeq -4 \times 10^{-3}$ $\gamma \simeq +1 \times 10^{-3}$ SPS,Kubis,Ditsche '11

Transition form factor: unitarity implications

$V \to \pi^0\gamma^*$ transition form factor is dominated by $\pi\pi$ intermediate states (γ^* is an isovector $\Rightarrow$ no 3π intermediate states):



$\Rightarrow$ discontinuity of the TFF from unitarity

$$\text{disc}\, f_{V\pi^0}(s) = \frac{i q_{\pi\pi}^3(s)}{6\pi\sqrt{s}} f_1(s) F_\pi^{V*}(s)\,, \qquad q_{\pi\pi}(s) = \sqrt{\frac{s}{4} - M_\pi^2}$$

Köpp '74

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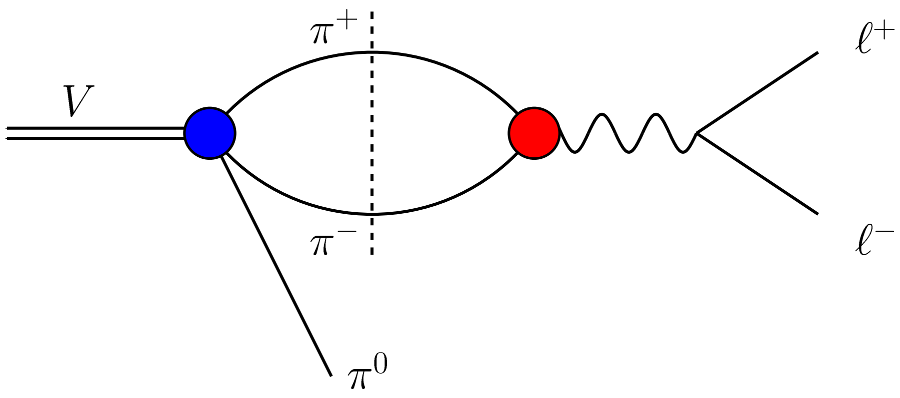


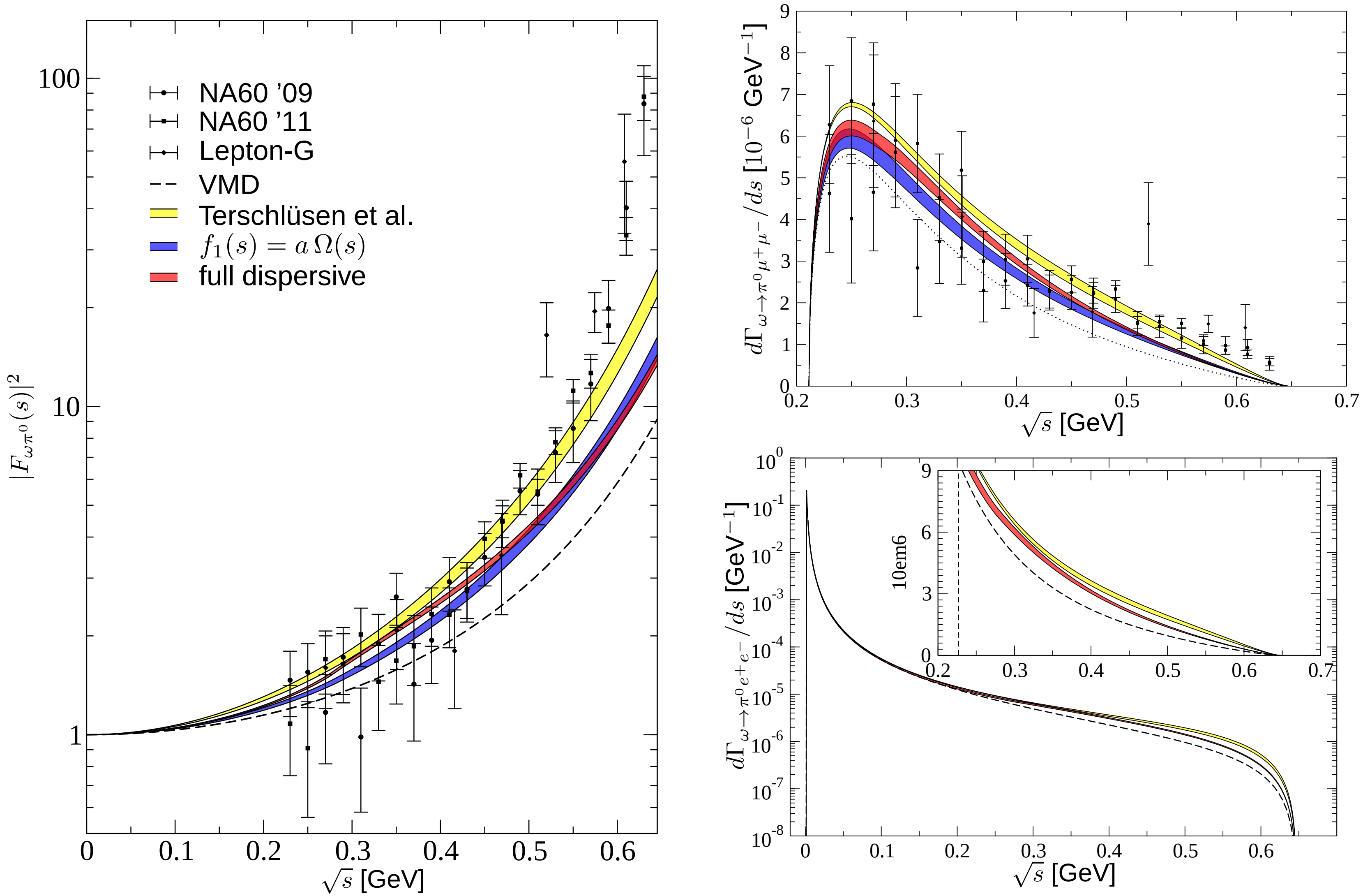
$\Rightarrow$ discontinuity of the TFF from unitarity

$$\Rightarrow f_{V\pi^0}(s) = f_{V\pi^0}(0) + \frac{s}{12\pi^2}\int_{4M_\pi^2}^{\infty} ds' \frac{q_{\pi\pi}^3(s') F_\pi^{V*}(s') f_1(s')}{s'^{3/2}(s'-s)}$$

Köpp '74

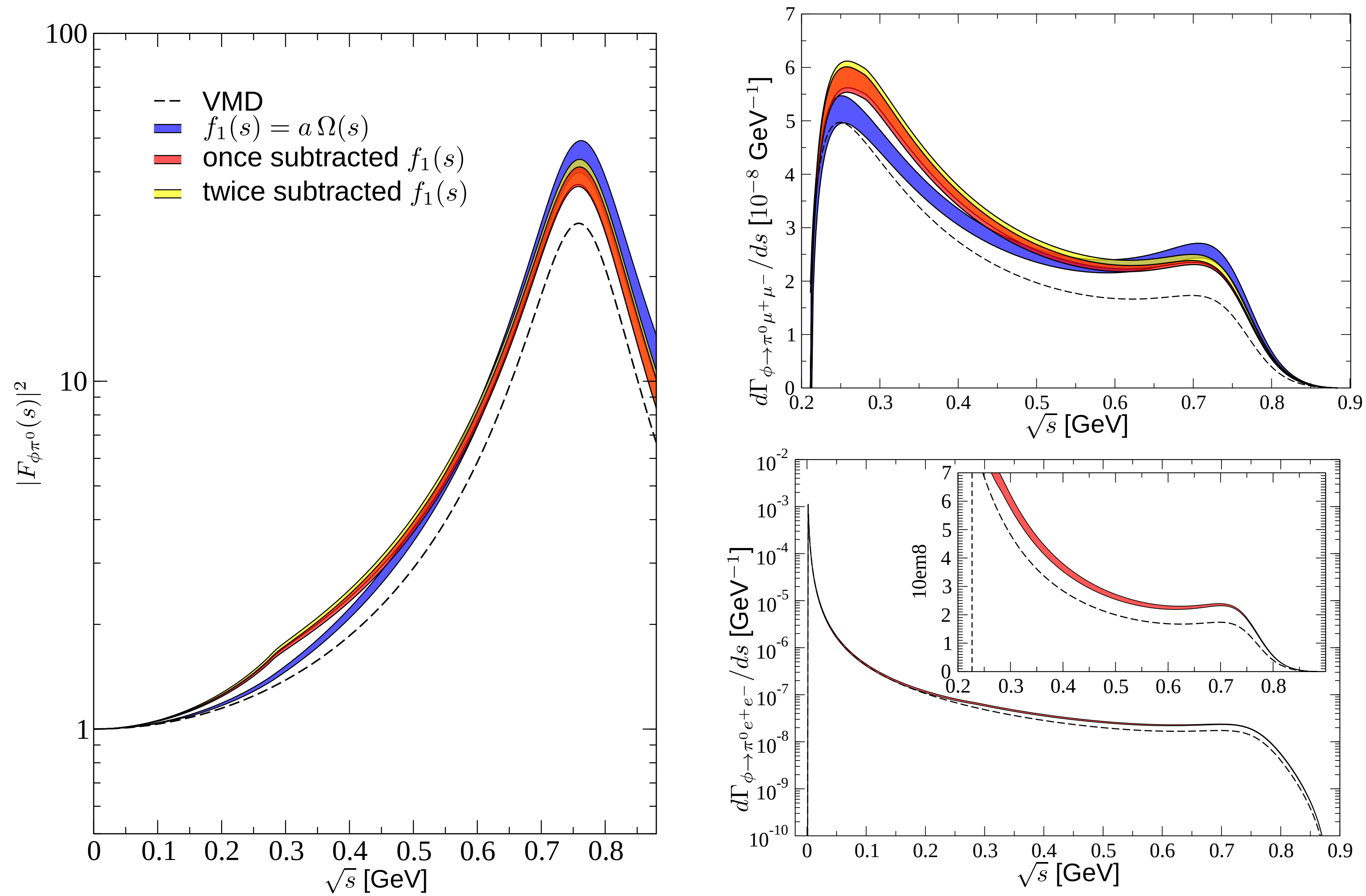
- $F_\pi^{V*}(s)$ pion vector form factor
- $f_1(s)$ $l = 1$ partial-wave amplitude for $V \to 3\pi$
- determine $f_{V\pi^0}(0)$ from $\Gamma_{V\to\pi^0\gamma}$

Numerical results: $\omega \to \pi^0\gamma^*$



- unable to account for steep rise $\Rightarrow$ pole structure???
- partial-wave amplitude not backed up by $\omega \to 3\pi$ experiment

Numerical results: $\phi \to \pi^0\gamma^*$



- measurement extremely helpful $\Rightarrow$ investigate "pole structure"
- partial-wave amplitude backed up by experiment

Conclusions and outlook

Conclusions...

- dispersion relations are a strong tool to study hadronic interactions
- based on fundamental principles of unitarity, analyticity and crossing symmetry
- application in $\phi \to 3\pi$ produces promising results
  $\Rightarrow$ perfect fit data with two subtractions
- steep rise in $\omega \to \pi^0\gamma^*$ transition form factor cannot be explained in our framework
- measurement of $\phi \to \pi^0\gamma^*$ should give insights!

...and Outlook

- short term: $\eta' \to \eta\pi\pi$ and $D^+ \to K^-\pi^+\pi^+$ in progress

Spares

The inhomogeneities $\hat{\mathcal{F}}(s)$

$$\hat{\mathcal{F}}(s) = \frac{3}{\kappa(s)} \int_{s_-(s)}^{s_+(s)} ds' \left[1 - \left(\frac{2s' - 3s_0 + s}{\kappa(s)}\right)^2\right] \mathcal{F}(s')$$

$$s_\pm(s) = \frac{1}{2}(3s_0 - s \pm \kappa(s))$$

$$\kappa(s) = \sqrt{\frac{s - 4\,M_\pi^2}{s}} \times \sqrt{(s - (M_V + M_\pi)^2)(s - (M_V - M_\pi)^2)}$$

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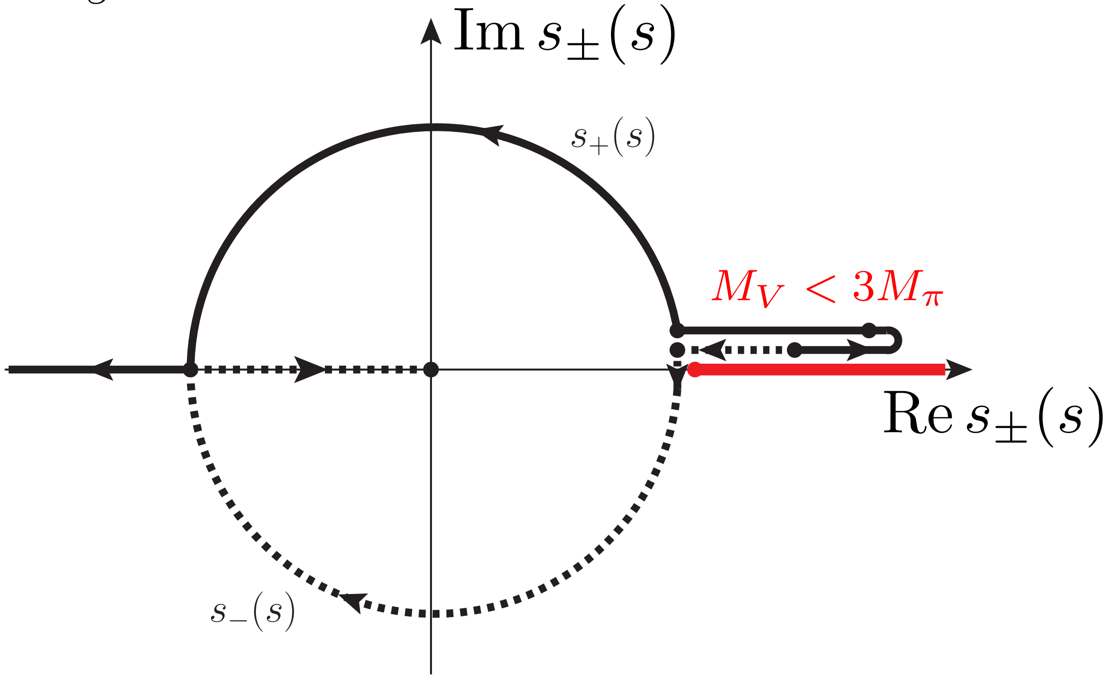

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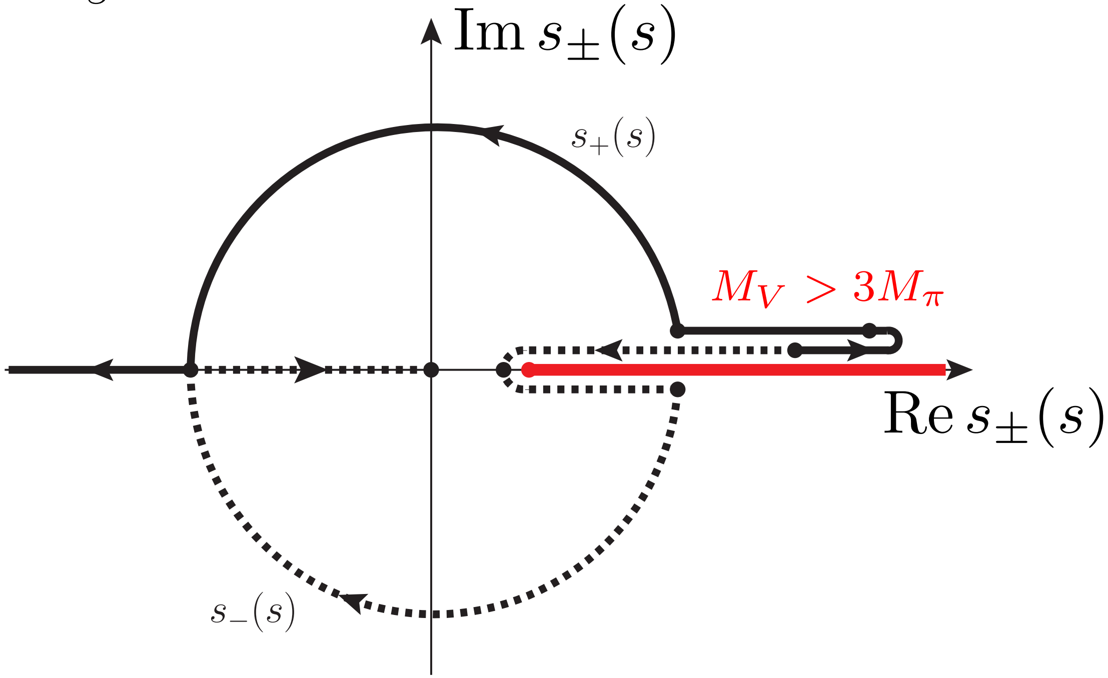
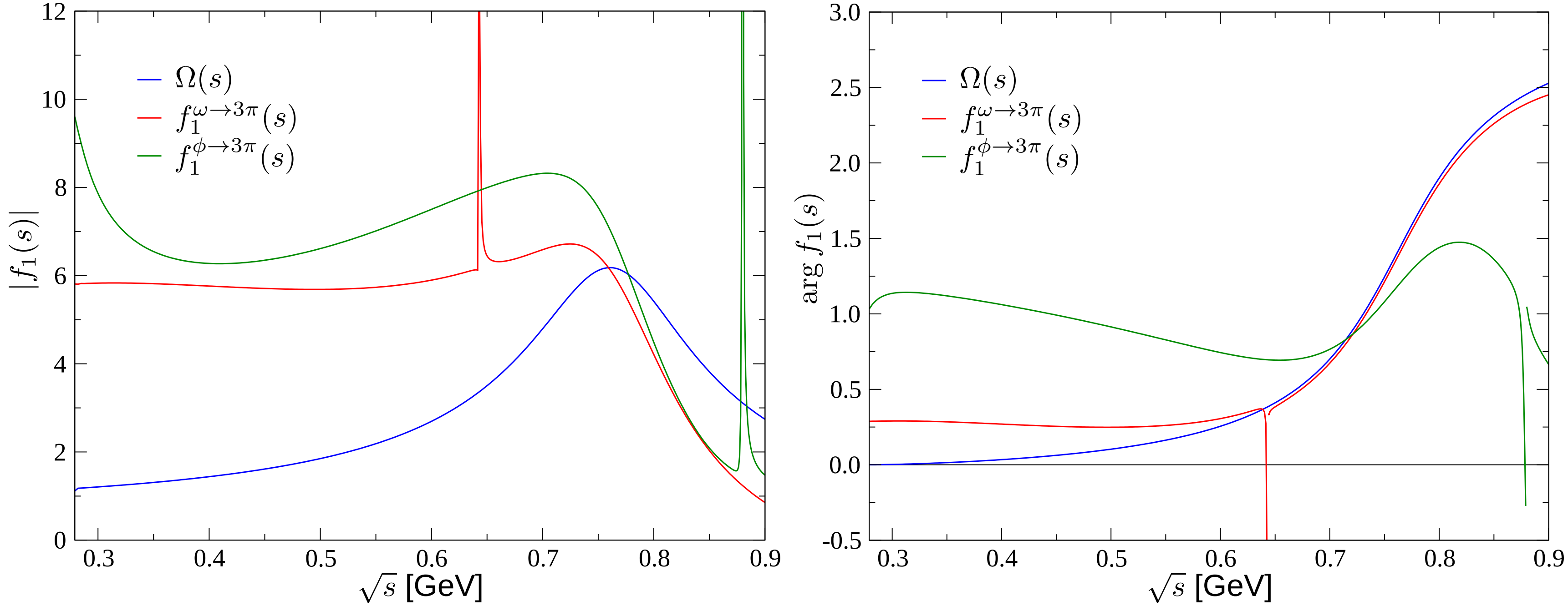


- the vector particle V is unstable $\Rightarrow$ 3-particle cuts become manifest in $\kappa(s)$ $\Rightarrow$ generates complex analytic structure

$V \to 3\pi$ partial-wave amplitude

- partial-wave projection $f_1(s)$:

$$f_1(s) = \frac{3}{4}\int_{-1}^{1} dz\,(1-z^2)\,\mathcal{F}(s,t,u)$$



- phase of the partial-wave amplitude does not vanish at threshold
- divergence at pseudo-threshold expected
  $\Rightarrow$ does not generate non-analytic structure in the TFF

$V \to \pi^0\gamma$ branching ratios

- Estimates for the branching ratios:

$$\mathcal{B}(\omega \to \pi^0\gamma) = (7.48 \ldots 7.75) \times 10^{-2}$$

$$\mathcal{B}^{\exp}(\omega \to \pi^0\gamma) = (8.28 \pm 0.28) \times 10^{-2}$$

$$\mathcal{B}(\phi \to \pi^0\gamma) = (1.28 \ldots 1.37) \times 10^{-3}$$

$$\mathcal{B}^{\exp}(\phi \to \pi^0\gamma) = (1.27 \pm 0.06) \times 10^{-3}$$

- **But:** integrand of $f_{V\pi^0}(0)$ not very well converging
  - ▷ benchmark for approximation of two-pion intermediate states
  - ▷ expected to work better for once-subtracted DR $\Rightarrow$ s-dependence
  - ▷ fix $f_{V\pi^0}(0)$ by using $\Gamma_{V\to\pi^0\gamma}$ as input