Electromagnetic Contributions to Pseudoscalar Masses

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Motivation

- ✦ Disentangling electromagnetic and isospin-violating effects in the pions and kaons is long-standing issue.
- ✦ Crucial for determining light quark masses.
  - Fundamental parameters in Standard Model; important for phenomenology.
  - Size of EM contributions is largest uncertainty in determination of m_u/m_d.

	m_u [GeV]	m_d [GeV]	m_u/m_d
value	1.9	4.6	0.42
statistics	0.0	0.0	0.00
lattice	0.1	0.2	0.01
perturbative	0.1	0.2	--
EM	0.1	0.1	0.04

MILC, arXiv:0903.3598

  - Reduce error by calculating EM effects on the lattice.

Background

- EM error in m_u/m_d dominated by error in $(M_{K^+}^2 - M_{K^0}^2)^\gamma$, where γ indicates the EM contribution.
- Dashen (1960) showed that leading order EM splittings are mass independent:

$$(M_{K^+}^2 - M_{K^0}^2)^\gamma = (M_{\pi^+}^2 - M_{\pi^0}^2)^\gamma$$

- Parameterize higher order effects ("corrections to Dashen's theorem") by

$$(M_{K^+}^2 - M_{K^0}^2)^\gamma = (1+\epsilon)(M_{\pi^+}^2 - M_{\pi^0}^2)^\gamma$$

  - Note: ϵ not exactly same as quantity defined by FLAG (Colangelo, et al., arXiv:1011.4408), which uses experimental pion splittings. But EM splitting ≈ experimental splitting, since isospin violations in pions small. So difference negligible for us at this stage.

Background

- MILC calculations of m_u/m_d after 2004 assumed $\epsilon = 1.2(5)$.
  - Came from estimate by Donoghue of range of continuum phenomenology, based on: Bijnens and Prades, NPB 490 (1997) 239; Donoghue and Perez, PRD 55 (1997) 7075; B. Moussallam, NPB 504 (1997) 381.
- This now seems too large; FLAG (Colangelo, et al., arXiv:1011.4408) quote $\epsilon = 0.7(5)$, based largely on $\eta \to 3\pi$ decay (but also lattice results by several groups).
- Would like to improve on this value with direct lattice calculation of EM effects.
- Fortunately, Bijnens & Danielsson, PRD75 (2007) 014505 showed that EM contributions to $(\text{mass})^2$ differences are calculable through NLO in χPT with *quenched* photons (and full QCD).

Background

- Bijnens and Danielsson result applies to any (mass)^2 difference with same valence masses (but different valence charges).
  - sea quark charges must be same in both cases.
- *So, e.g.*,

$$(M^2_{K^+} - M^2_{K^0})^{\gamma}_{q_{\rm sea}=0} \longrightarrow (M^2_{K^+} - M^2_{K^0})^{\gamma}_{q_{\rm sea}=\rm physical}$$

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  - Difference is not zero, but is calculable at NLO in χPT, in terms of known LO LECs.
    - Unknown NLO LECs cancel.
    - EM quenching error in ϵ only appears at NNLO, should be neglible.

Background

- We would also like to know, *e.g.*, the EM effect on the K^0 itself.
  - this is:

$$(M^2_{K^0})_{(q_{\rm sea}={\rm physical})} - (M^2_{K^{0\prime}})_{(q_{\rm sea}=0)}$$

    - where ′ on a meson indicates that valence charges are set to 0.
  - Unfortunately, a controlled calculation of this with quenched photons not possible, since sea charges different in two terms.
  - We do calculate:

$$(M^2_{K^0} - M^2_{K^{0\prime}})_{q_{\rm sea}=0} \rightarrow (M^2_{K^0} - M^2_{K^{0\prime}})_{q_{\rm sea}={\rm physical}}$$

    - But r.h.s. differs from what we want by uncontrolled (but presumably small) sea quark charge effects: an EM quenching error.

Background

- π_0 has additional issues.
  - Would be costly to simulate true π_0, which has EM disconnected diagrams even in isospin limit.
  - Instead our "π_0" is a (mass)^2 average of **uū** and **dđ** (connected) states.
  - Since all EM contributions to neutral mesons vanish in chiral limit:
    - true $(M^2_{\pi^0})^\gamma$ is small anyway.
    - disconnected contribution is likely to be still smaller.
    - difference $(M^2_{``\pi^0"} - M^2_{\pi^{0\prime}})^\gamma$ is a rough estimate of size of $(M^2_{\pi^0})^\gamma$.
      - but still has quenched EM errors, in addition to the effect of disconnected diagrams.

Chiral Perturbation Theory

✦ Staggered version of NLO SU(3) χPT has been calculated (C.B. & Freeland, arXiv:1011.3994):

$$\Delta M^2_{xy,5} = q^2_{xy}\delta_{EM} - \frac{1}{16\pi^2} e^2 q^2_{xy} M^2_{xy,5} \left[3\ln(M^2_{xy,5}/\Lambda^2_\chi) - 4\right]$$
$$- \frac{2\delta_{EM}}{16\pi^2 f^2}\frac{1}{16}\sum_{\sigma,\xi}\left[q_{x\sigma}q_{xy}M^2_{x\sigma,\xi}\ln(M^2_{x\sigma,\xi}) - q_{y\sigma}q_{xy}M^2_{y\sigma,\xi}\ln(M^2_{y\sigma,\xi})\right]$$
$$+c_1 q^2_{xy}a^2 + c_2 q^2_{xy}(2m_\ell + m_s) + c_3(q^2_x + q^2_y)(m_x + m_y) + c_4 q^2_{xy}(m_x + m_y) + c_5(q^2_x m_x + q^2_y m_y)$$

- x,y are the valence quarks.
- q_x, q_y are quark charges; $q_{xy} \equiv q_x - q_y$ is meson charge.
- δ_{EM} is the LO LEC; ξ is the staggered taste
- σ runs over sea quarks (m_u, m_d, m_s, with $m_u = m_d \equiv m_\ell$)

✦ Errors in $M^2 - (M')^2$ are ~ 0.3% for charged mesons, ~1% for neutrals.

- Need NNLO terms.

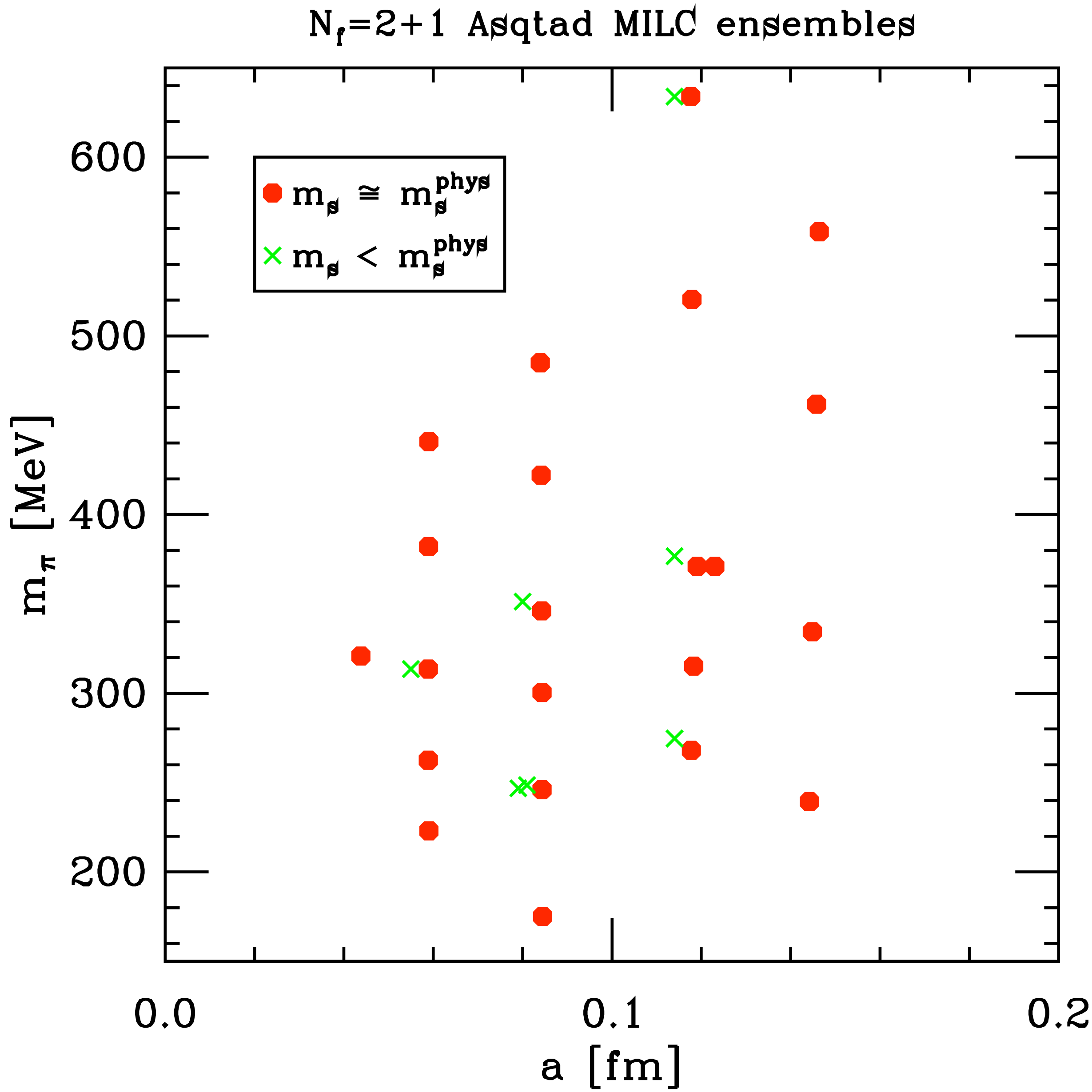
Chiral Perturbation Theory

- For NNLO, staggered χPT has not been calculated
  - Use analytic terms only:
    - Chiral logs small at low mass, where extrapolation is done.
    - Chiral logs well approximated by analytic terms in region near m_s, where they are important.
- Can get analytic terms from spurion analysis.
- But easier just to write down all possible polynomials in mass and charges that satisfy relevant conditions:
  - quadratic in q_x, q_y.
  - symmetric when $m_x \leftrightarrow m_y$, $q_x \leftrightarrow q_y$.
  - obey chiral-symmetric constraints when $q_x = q_y$, because in that case EM terms do not violate symmetry.
    - e.g. $(q_x^2 m_x^2 + q_y^2 m_y^2)$ forbidden, since doesn't go like (m_x+m_y) when $q_x = q_y$, as required by chiral symmetry.

MILC EM Project

- We have been accumulating a library of dynamical QCD plus quenched EM.
  - Improved staggered ("Asqtad") ensembles:
    - 2+1 flavors.
    - $0.12 \text{ fm} \geq a \geq 0.06 \text{ fm}$.
    - ~1000-2000 configs for most ensembles.
    - valence quark charges 1, 2, or 3 × physical charges:
      - $\pm 2/3e$, $\pm 4/3e$, $\pm 2e$ for u-like quarks.
      - $\pm 1/3e$, $\pm 2/3e$, $\pm e$ for d-like quarks.
  - Progress has been reported previously: PoS(LATTICE 2008)127, PoS(Lattice 2010)084, PoS(Lattice 2010)127.

Asqtad Ensembles



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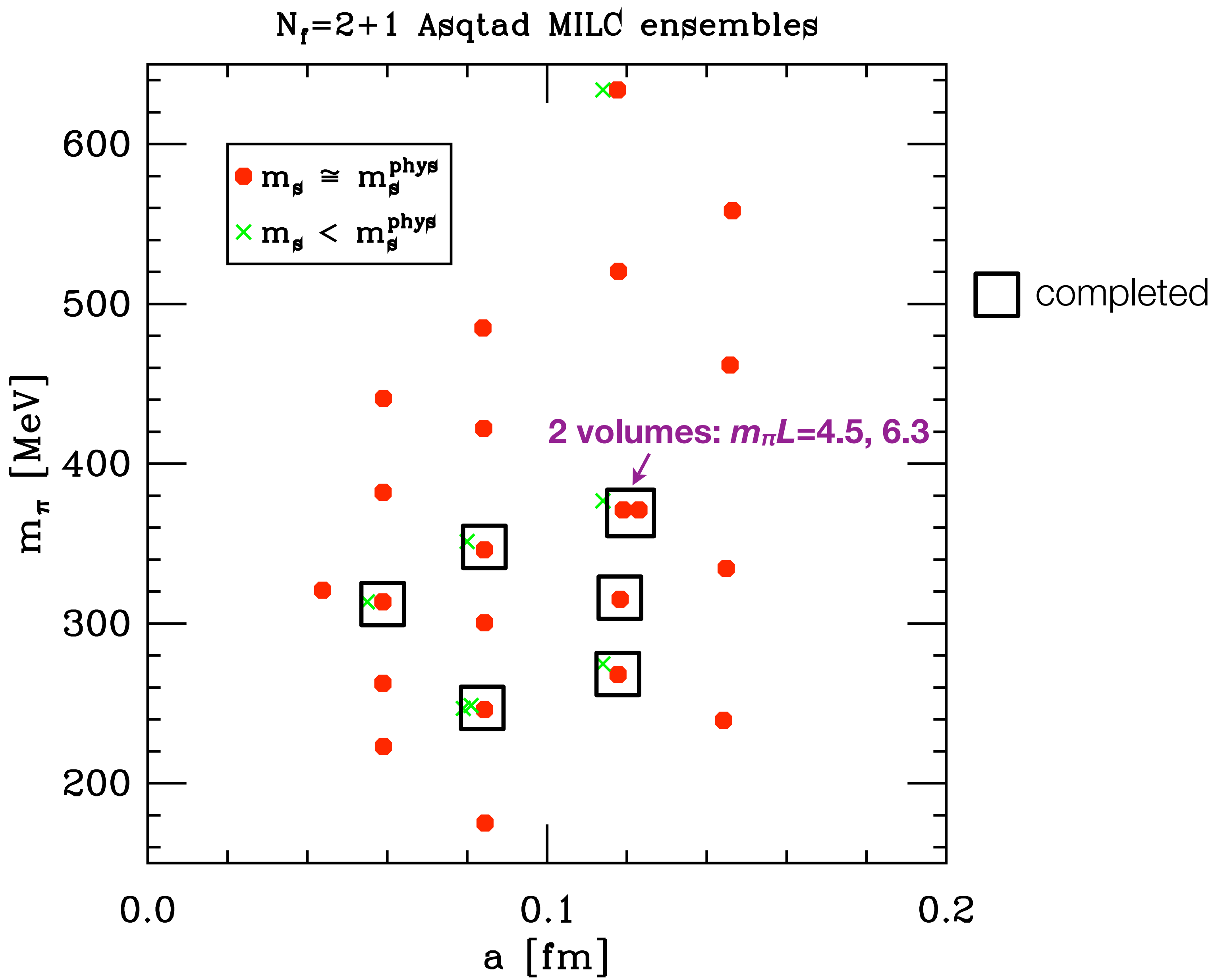

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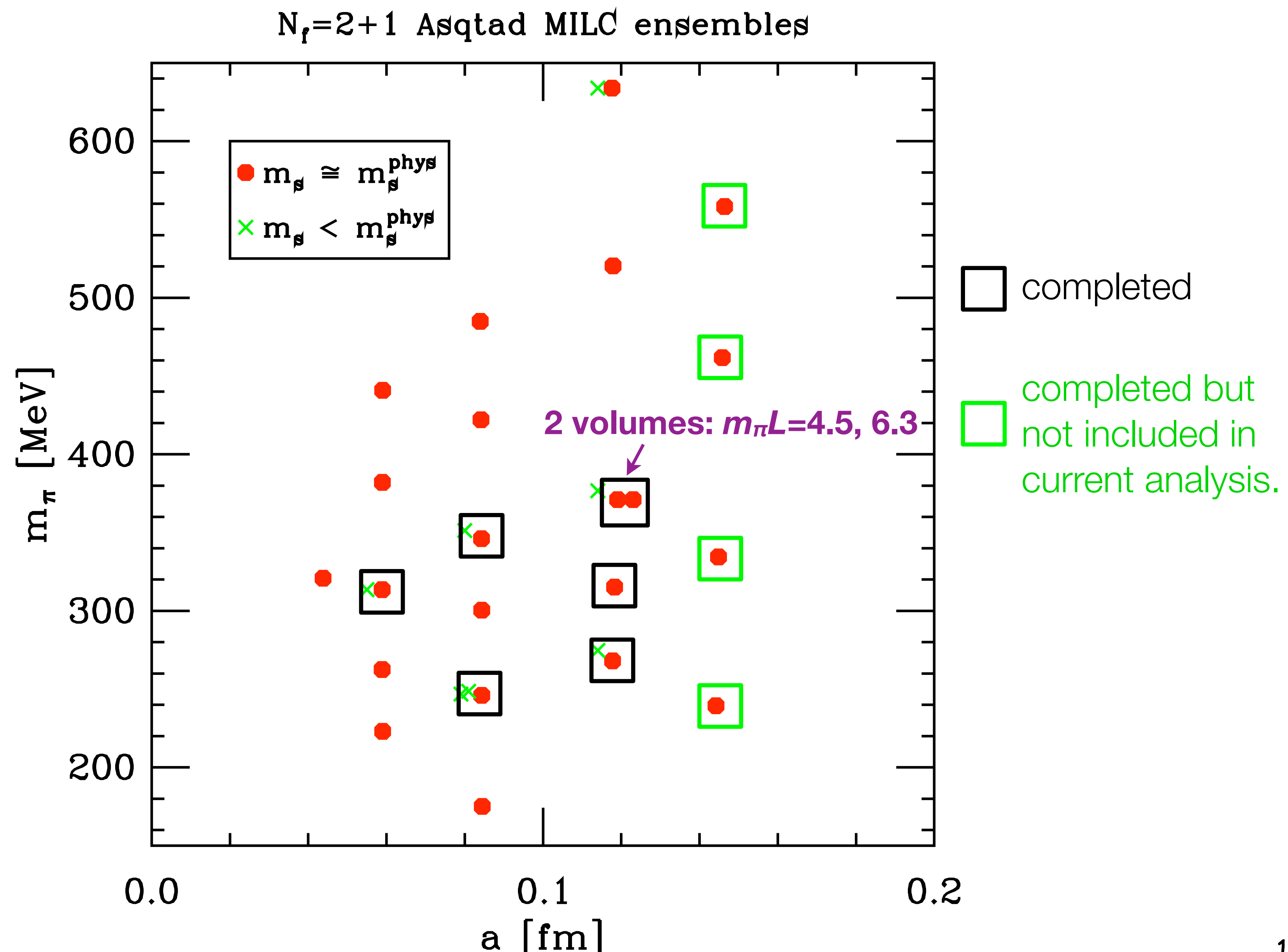

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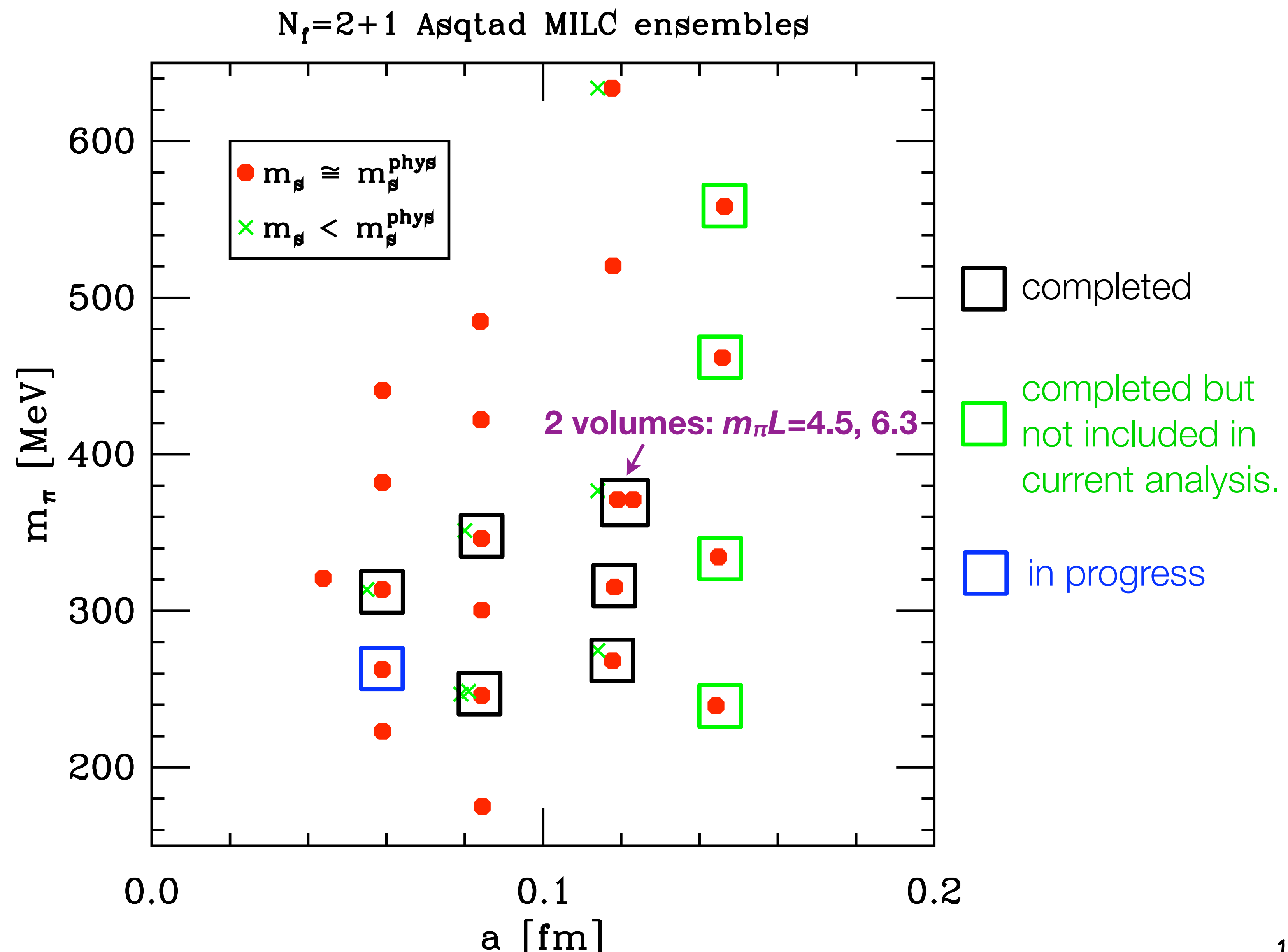
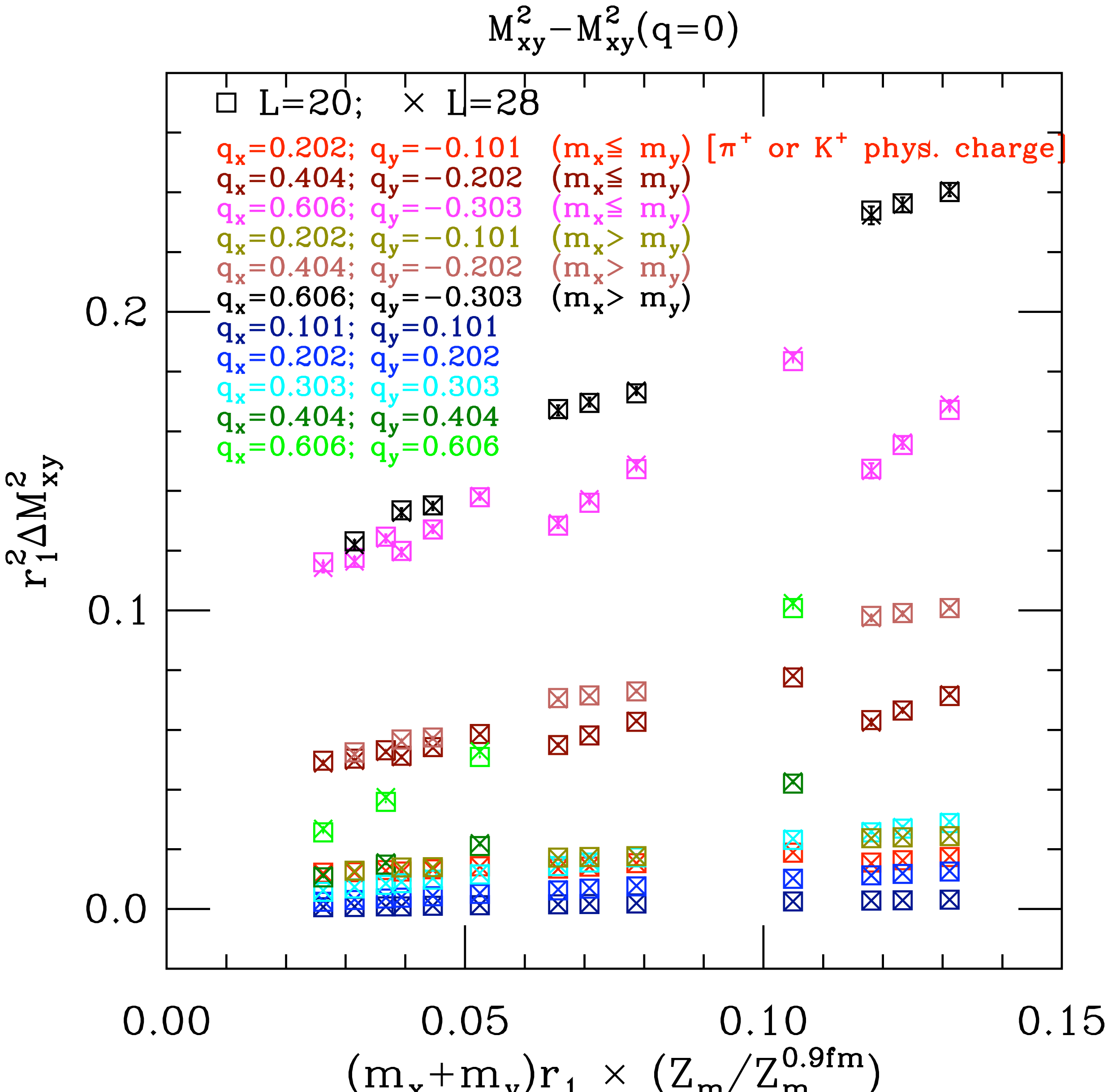
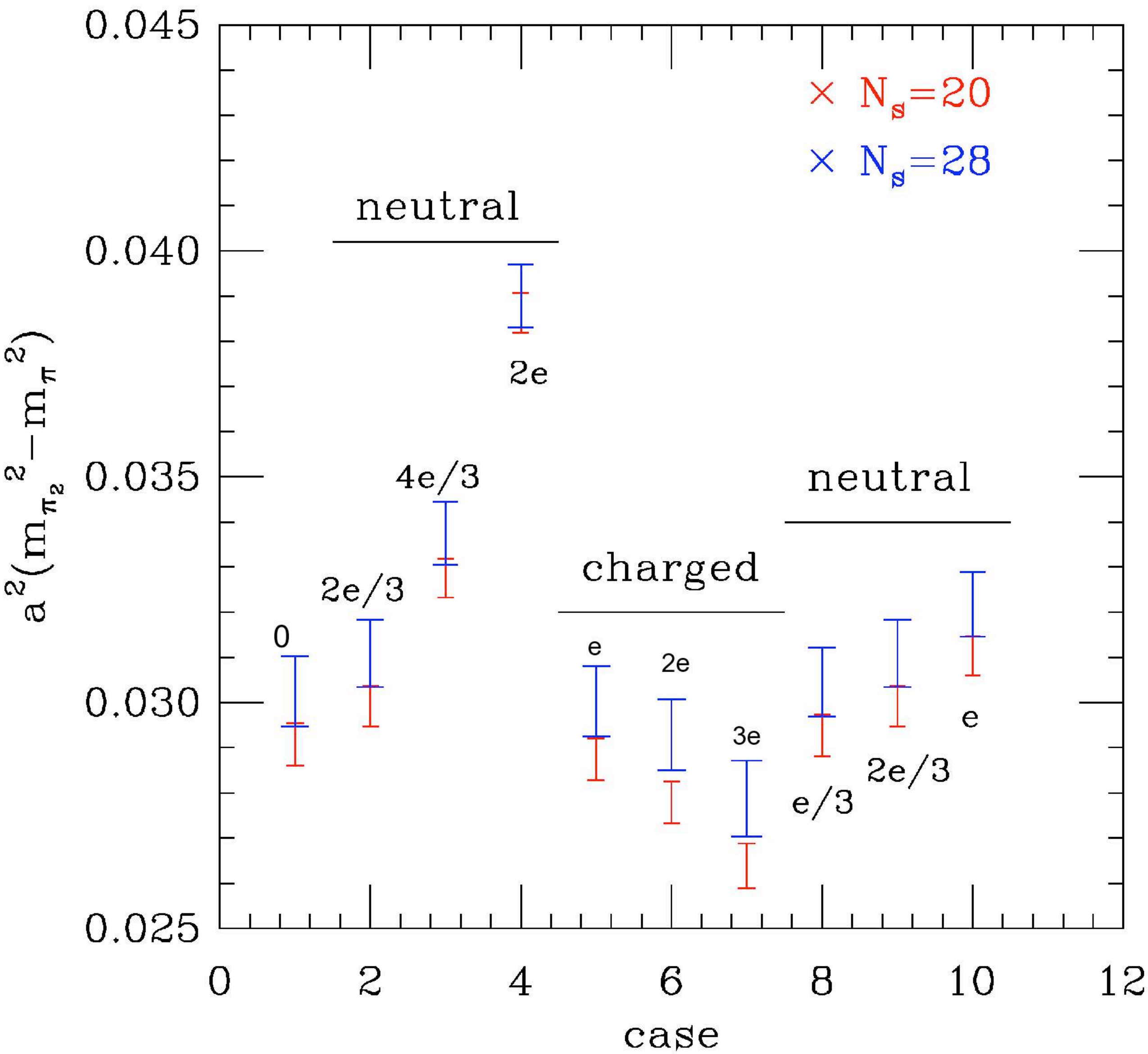


Quick Look at Data



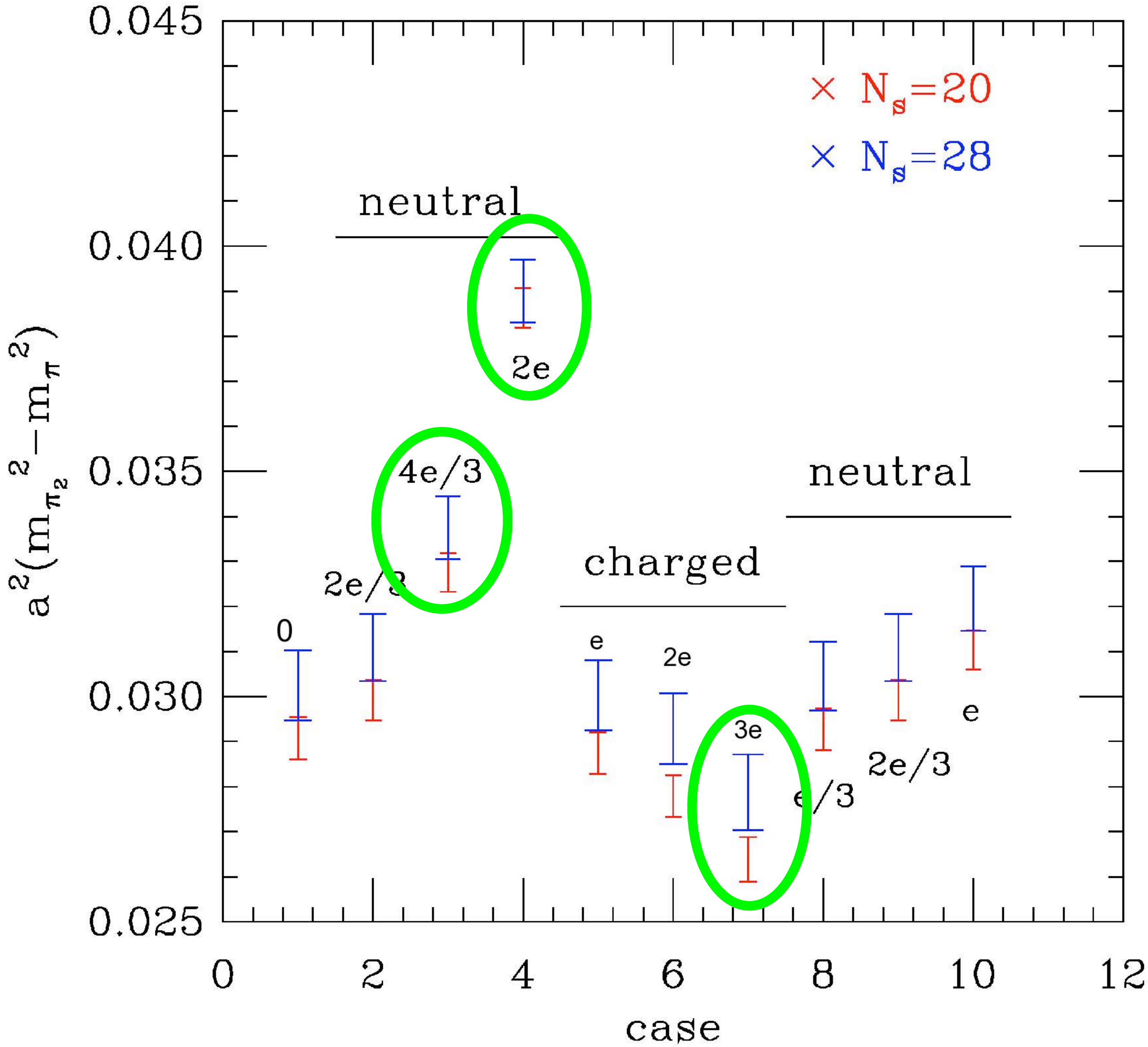
- a = 0.12 fm; $m_\ell = 0.2\ m_s$.
- two volumes: L = 2.3, 3.1 fm.
- not much evidence of finite size effects.
- neutrals are ~smooth function of $(m_x + m_y)$; charged are not.

Taste Splitting



- As charges increase, EM taste-violating effects start to become evident.

Taste Splitting



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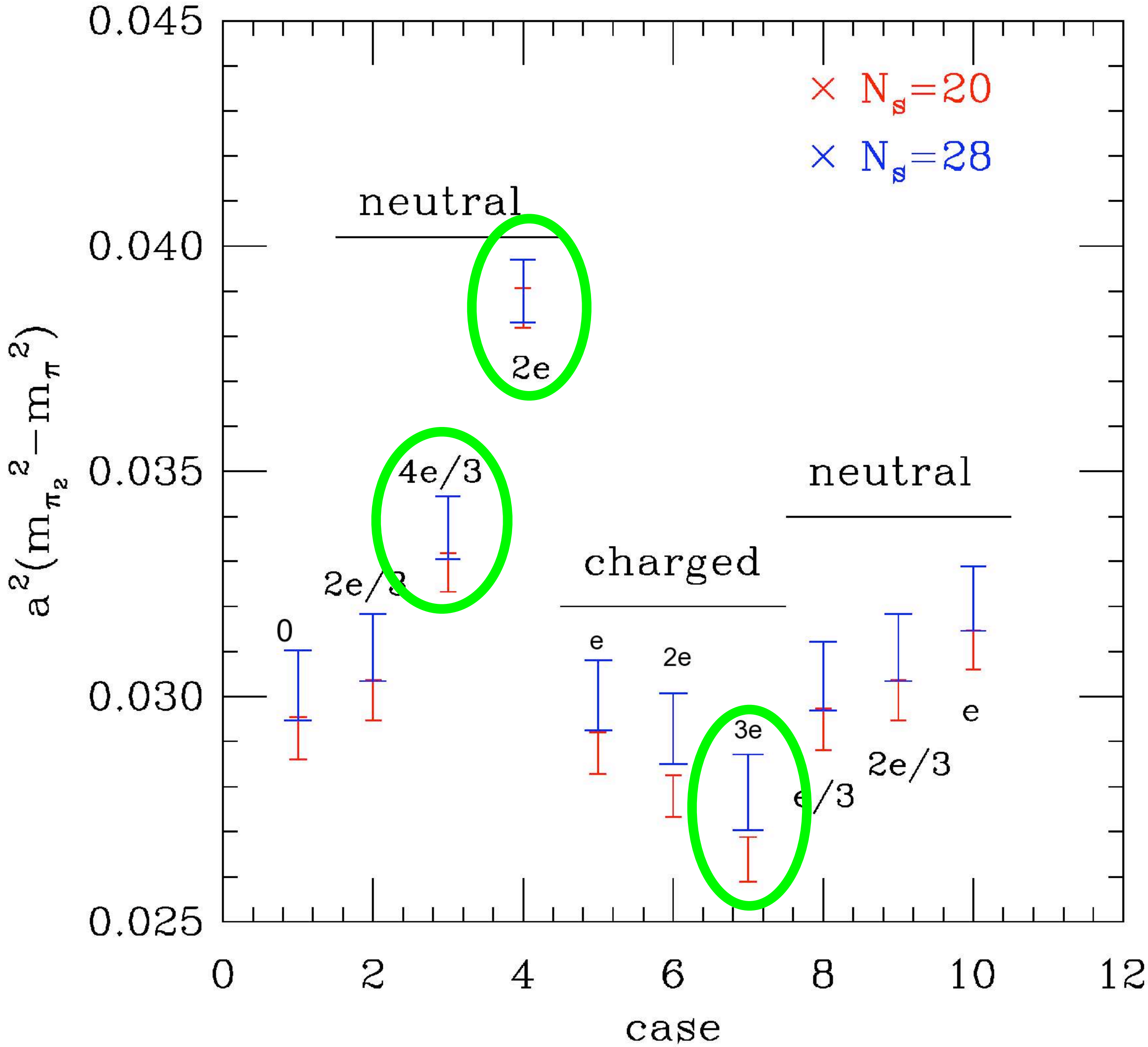

- As charges increase, EM taste-violating effects start to become evident.
- EM taste-violations not included in the χPT .

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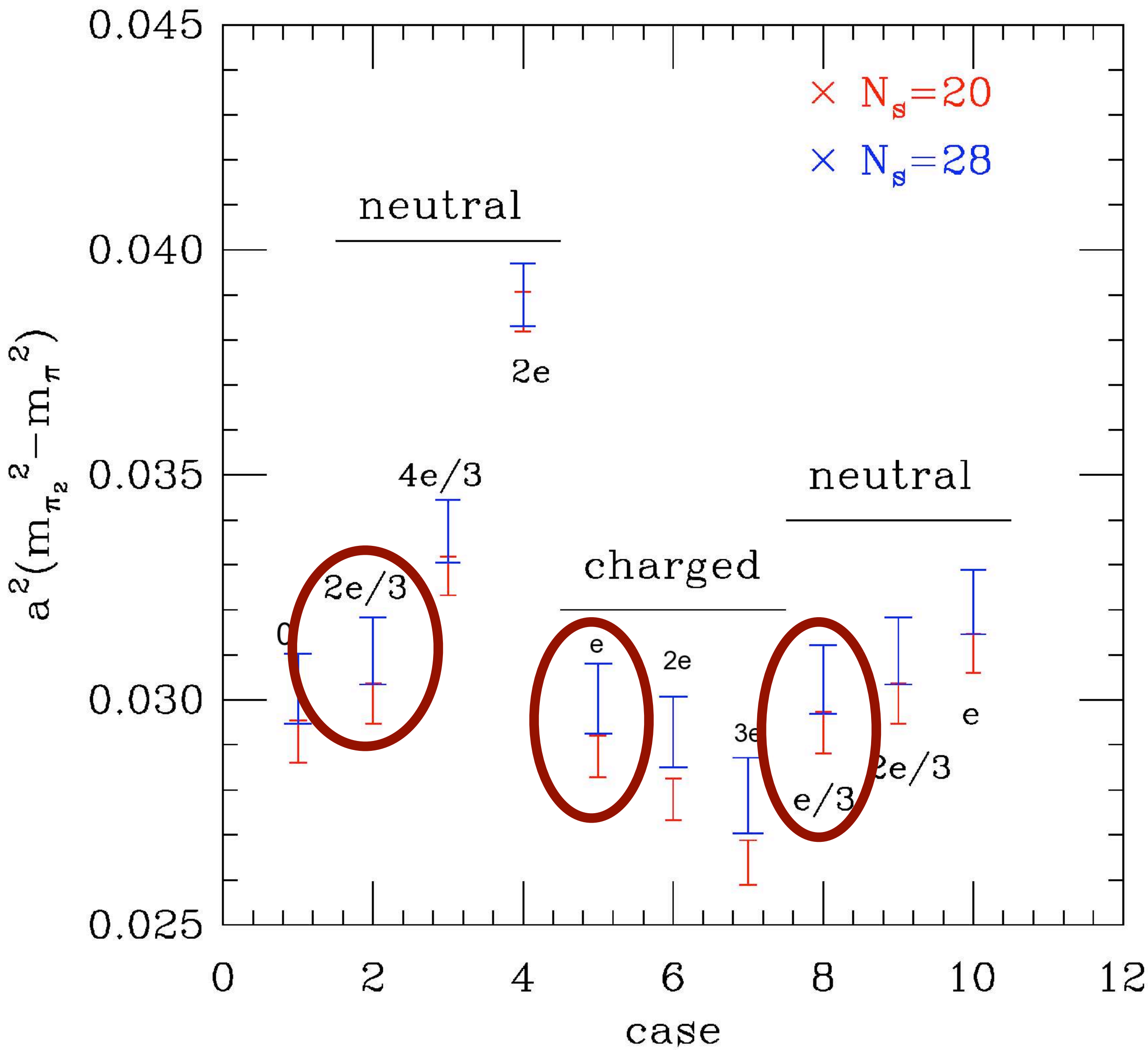
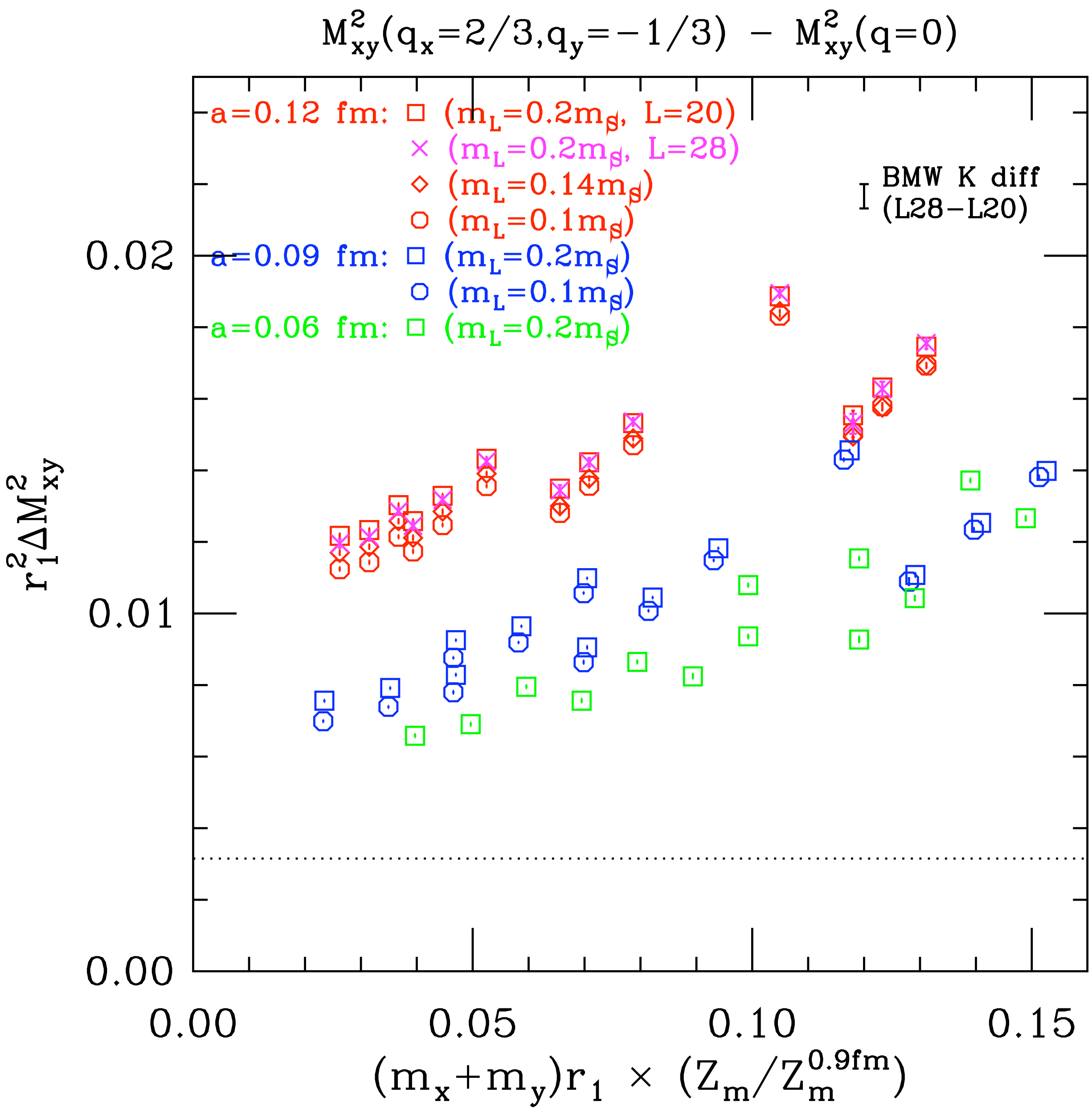


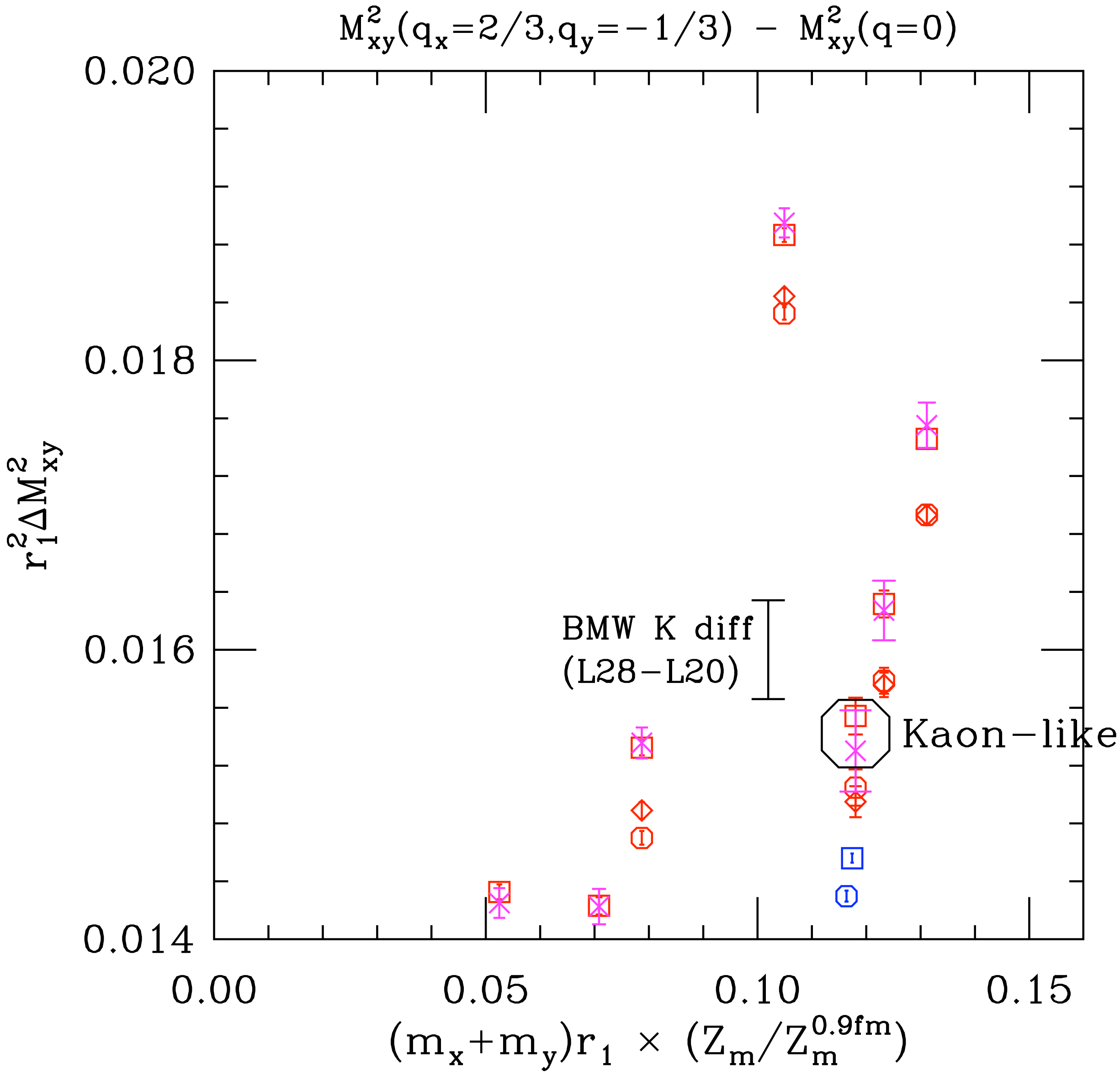
- As charges increase, EM taste-violating effects start to become evident.
- EM taste-violations not included in the χPT .
- Stick with physical charges for now.

Data to Fit



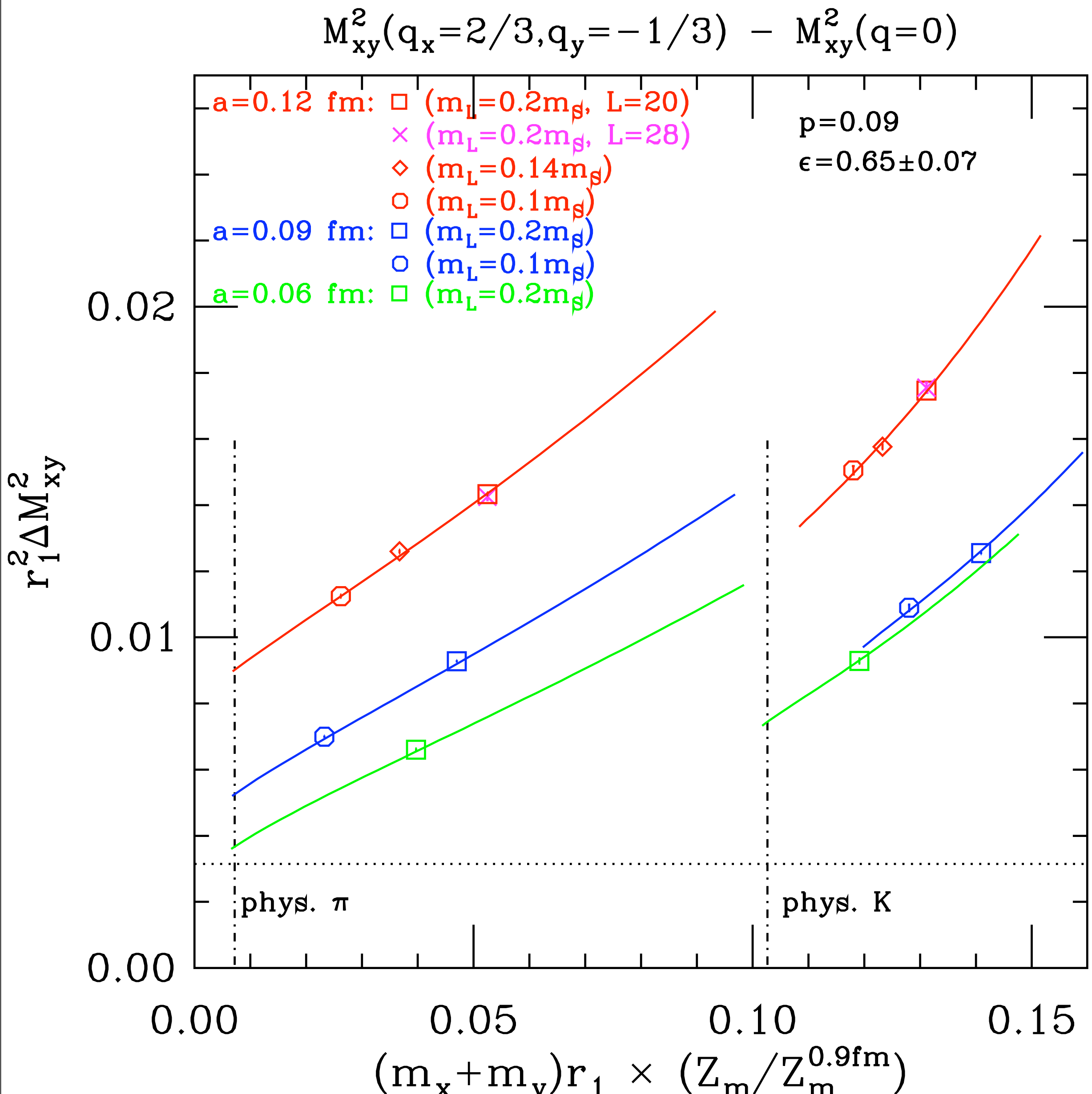
- All ensembles; charge +1 mesons (neutrals not shown).
- Discretization effects are rather large.
- Bar shows expected size of L=20,28 finite size effect, based on what was seen by BMW Collaboration.

Finite Size Effect



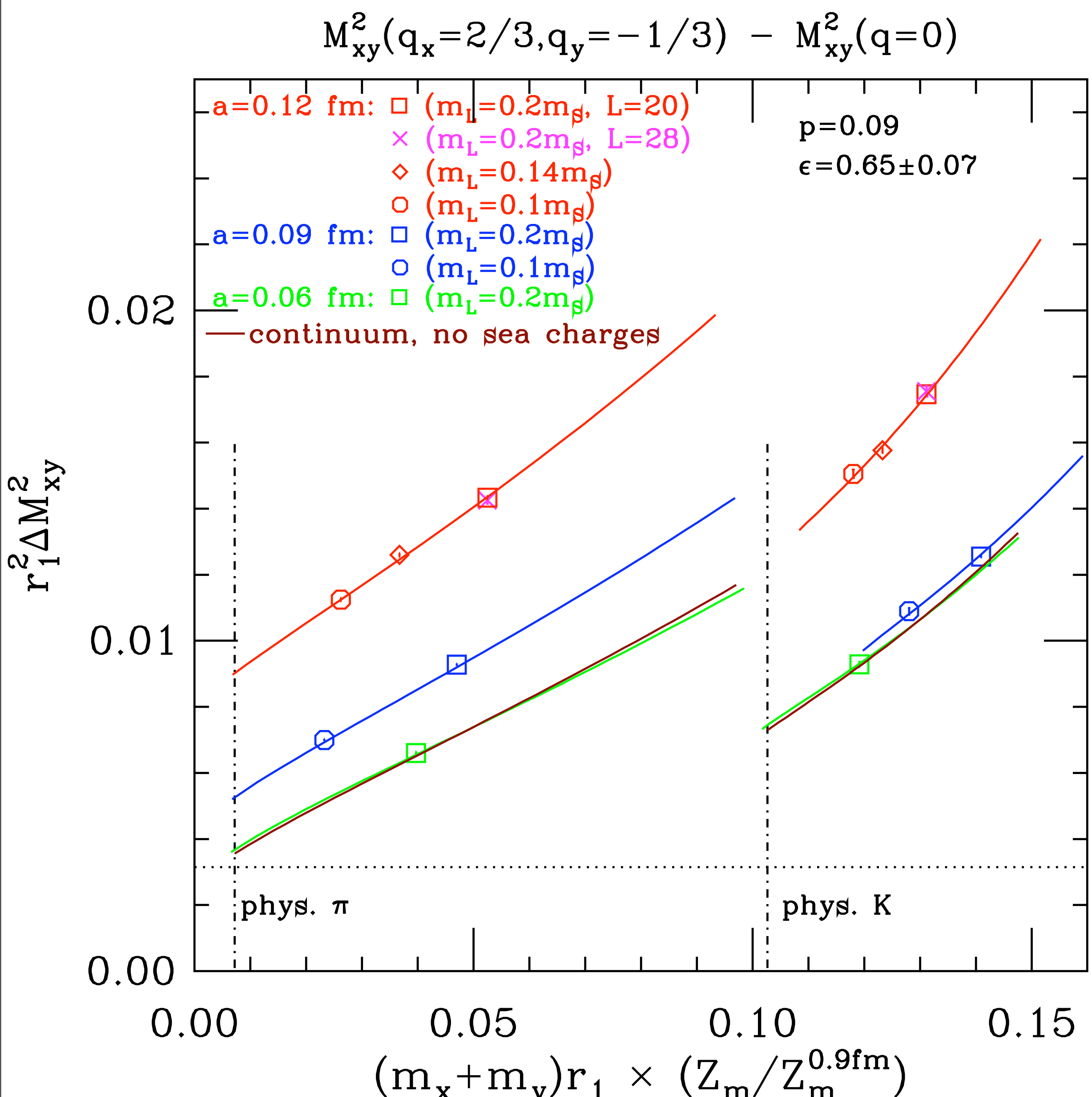
- Blow-up of previous plot.
- Our finite size effect rather small compared to what was seen by BMW Collaboration, but not necessarily inconsistent:
  - 0.35(45) × expected.
- We are increasing statistics on L=28 lattice (×) to improve test.

Chiral Fit and Extrapolation



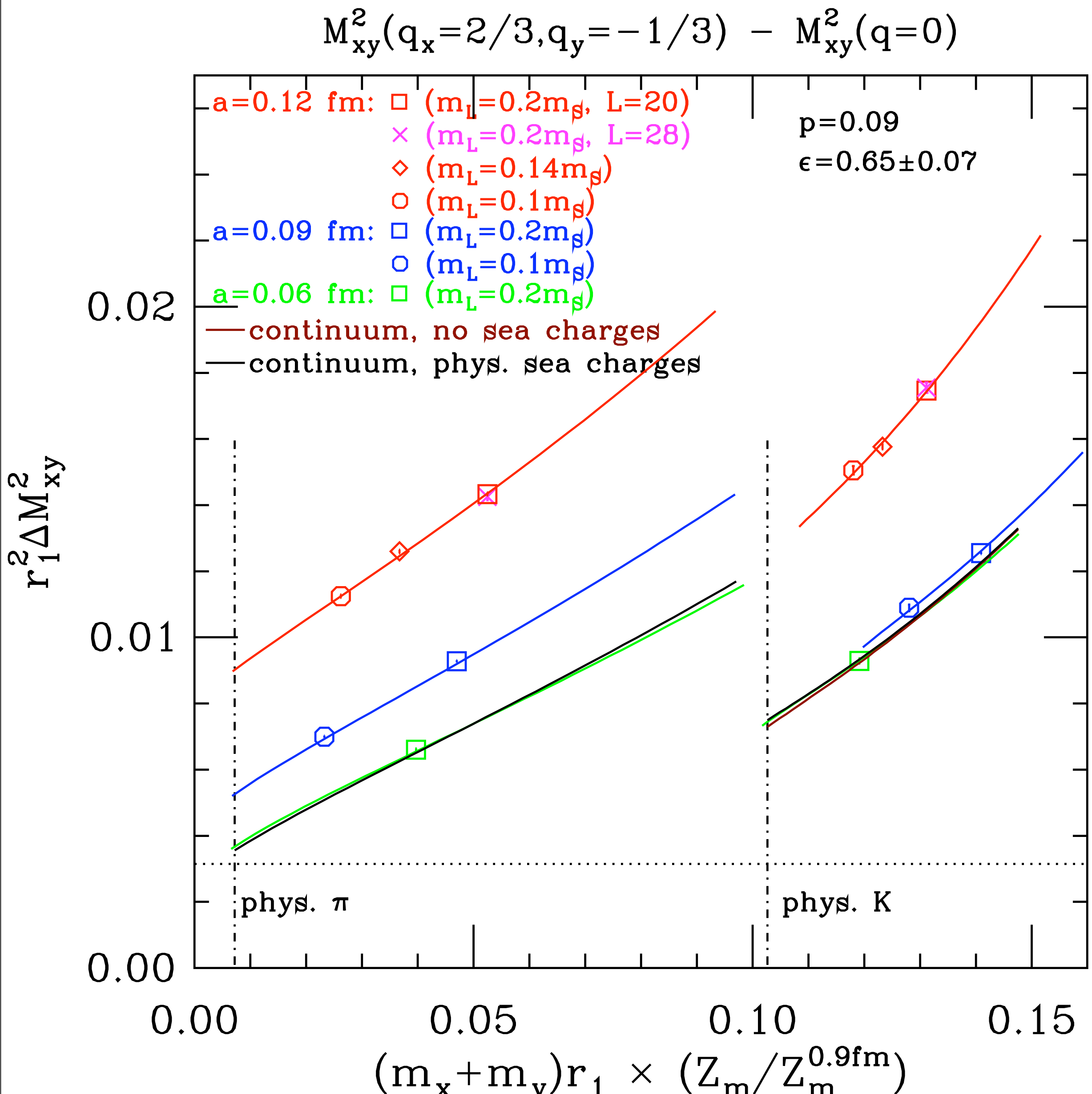
- Only unitary π^+ & K^+ shown, but fit is to all partially quenched points, charged and neutral.
- Different masses & charges for same ensembles are highly correlated, leading to nearly singular covariance matrix.
- This fit is non-covariant (neglects correlations).
- Covariant fits generally have very poor p values; a few of better ones are included in systematic error estimate.

Chiral Fit and Extrapolation



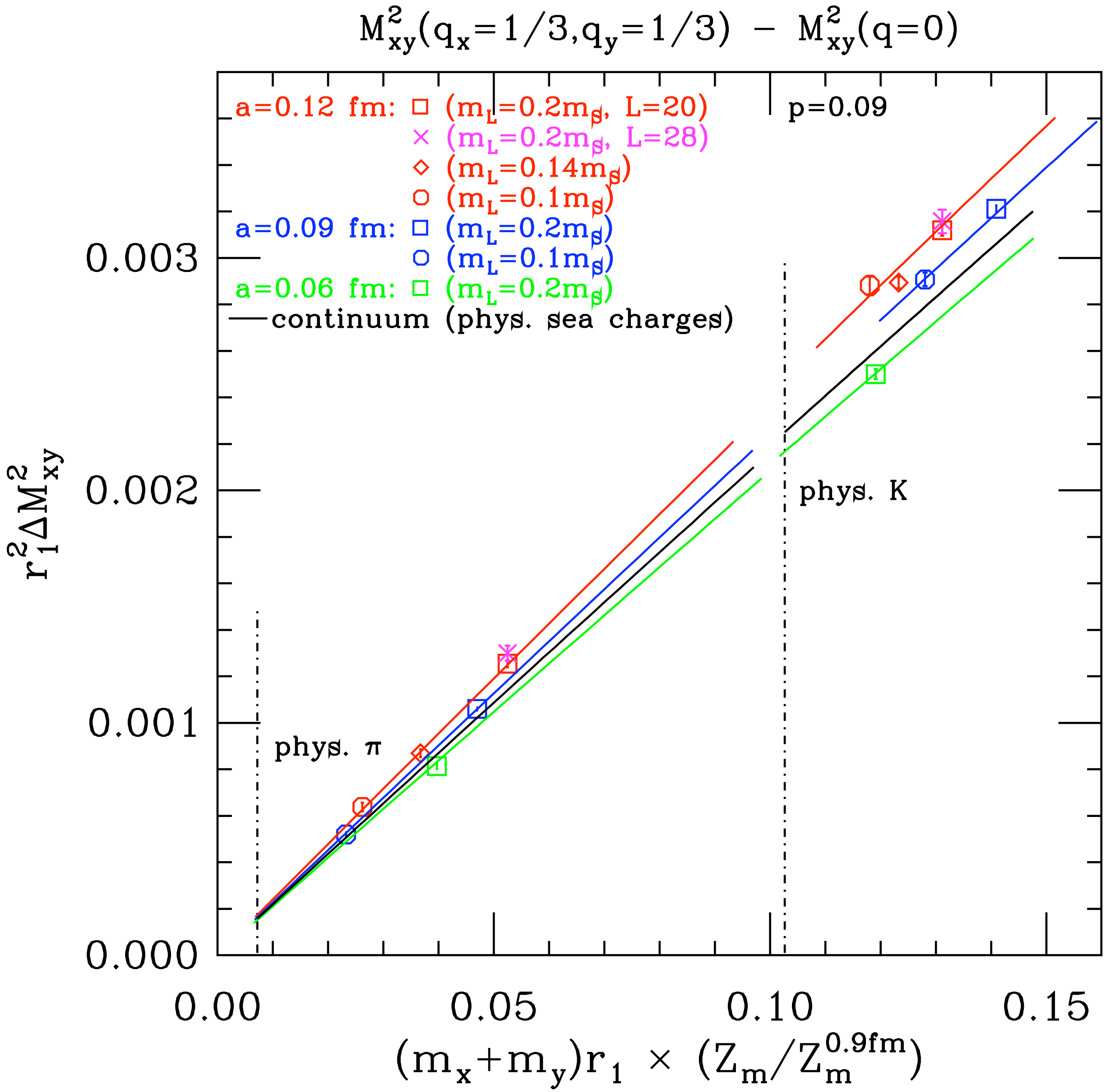
- Extrapolate to continuum, and set valence, sea masses equal.
- Adjust m_s to physical value.
- Keep sea charges = 0.

Chiral Fit and Extrapolation



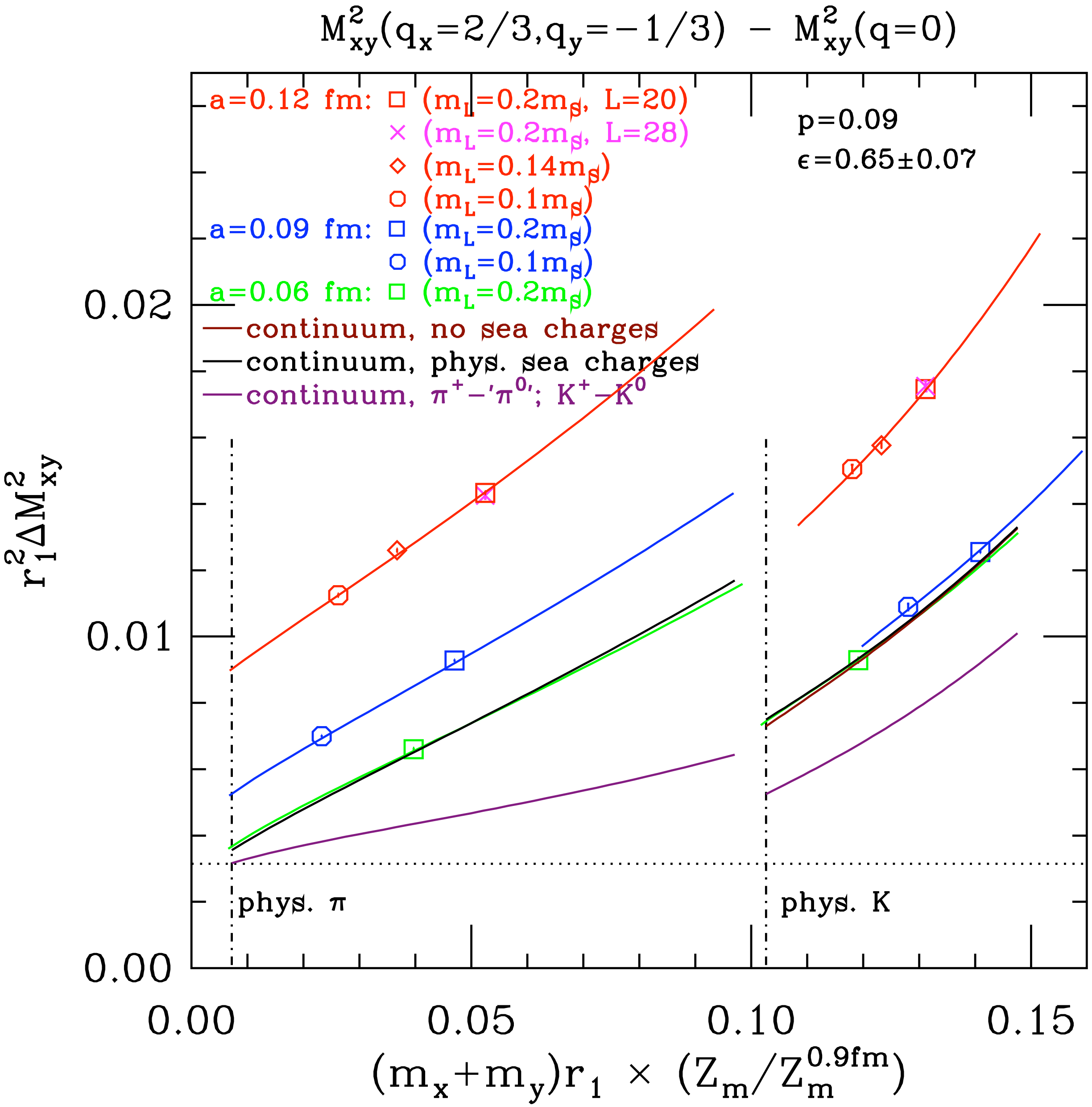
- Set sea quark charges to their physical values, using NLO chiral logs.
- Difference with previous case is very small for kaon; vanishes identically for pion.

Chiral Fit and Extrapolation



- Neutral dđ-like mesons ($q_x = q_y = 1/3$) for same fit.
- Note difference in scale from charged meson plot.
- ~Function of ($m_x + m_y$) only (π and K line up).
- Nearly linear: chiral logs vanish for neutrals.

Chiral Fit and Extrapolation



- Now subtract neutral masses.
- Perfect agreement of π splitting with physical value is an accident:
  - systematic errors are larger than the difference of purple & black lines (i.e., difference between "π^0" and π').
- Can now read off ratio of π and K splittings:

$$\epsilon = 0.65(7)$$

Preliminary Results

$$(M^2_{\pi^+} - M^2_{``\pi^0"})^\gamma = 1270(90)(230)\ \mathrm{MeV}^2$$
$$(M^2_{K^+} - M^2_{K^0})^\gamma = 2100(90)(250)\ \mathrm{MeV}^2$$

$$\epsilon = 0.65(7)(14)$$

$$(M^2_{``\pi^0"})^\gamma = 157.8(1.4)(1.7)\ \mathrm{MeV}^2$$
$$(M^2_{K^0})^\gamma = 901(8)(9)\ \mathrm{MeV}^2$$

} uncontrolled EM quenching error

- Finite volume errors not yet included: seem relatively small at present, but need to be studied more, and quantified.
- Rough estimate of effect of neglecting disconnected EM diagrams in the "π_0" might be half of $(M^2_{``\pi^0"})^\gamma$.
  - Keeping that in mind, and neglecting effects of isospin violation in the π^0, $(M^2_{\pi^+} - M^2_{``\pi^0"})^\gamma$ may be compared with expt. π^+-π^0 splitting: $1261\ \mathrm{MeV}^2$.

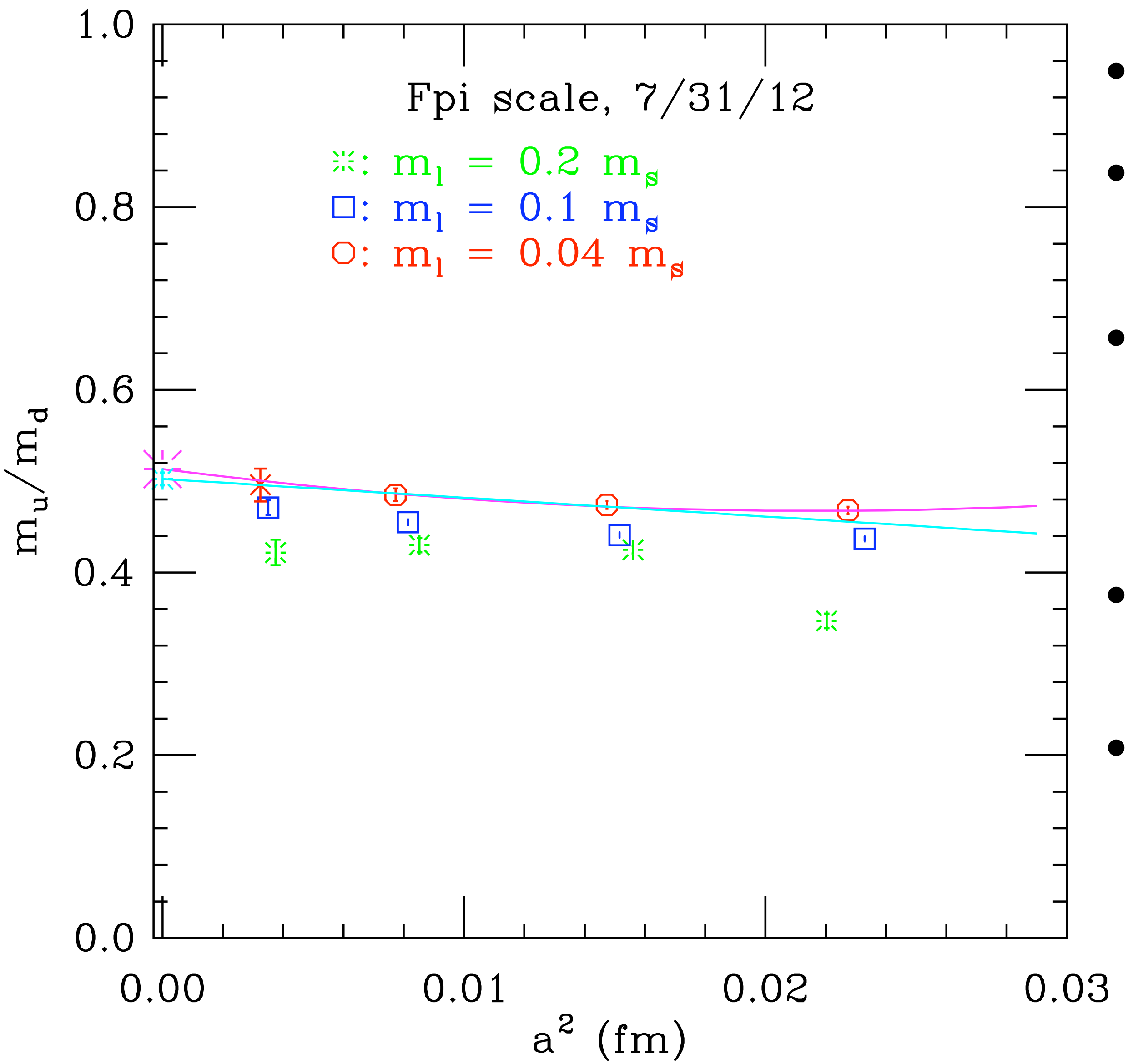
Comparison with Other Work

- ✦ $\varepsilon = 0.60(14)$ [statistics only], Portelli et al. (2010), arXiv:1011.4189.
- ✦ $\varepsilon = 0.628(59)$ [statistics only], Blum et al. (2010), arXiv:1006.1311.
- ✦ $\varepsilon = 0.70(4)(8)(??)$, Portelli et al. (2012), arXiv:1201.2787.
- ✦ $\varepsilon = 0.65(7)(14)(?)$, this work.

?? = discretization errors; ? = finite volume errors

- Good agreement between the groups.
- Errors still need work...

Preliminary Effect on m_u/m_d



- From HISQ lattices.
- Extrapolations omit $m_\ell = 0.2\ m_s$ ensembles.
- Preliminary analysis, not including staggered χPT .
  - Is upward curvature believable?
- Get:
  $m_u/m_d = 0.508(10)(22)$
- EM error reduced by ~factor of 2 (but still the main source of error).

Other Remarks

- Trouble with covariant EM fits is a concern.
  - Only acceptable covariant fits omit a=0.12 fm ensembles; then have large statistical errors.
  - We are running an additional a=0.06 fm ensemble (and others are planned), which should improve this, as well as reducing other systematic errors.
- A lot more physics can be done with our current ensembles, in particular for baryons, and some of that is in progress.
  - EM quenching effect with be present, though.
- We have requested time for a phase-II EM project that would run on the MILC HISQ ensembles, which have significantly smaller discretization errors.
- We are discussing a phase-III project that would generate dynamical EM lattices.

MILC Collaboration

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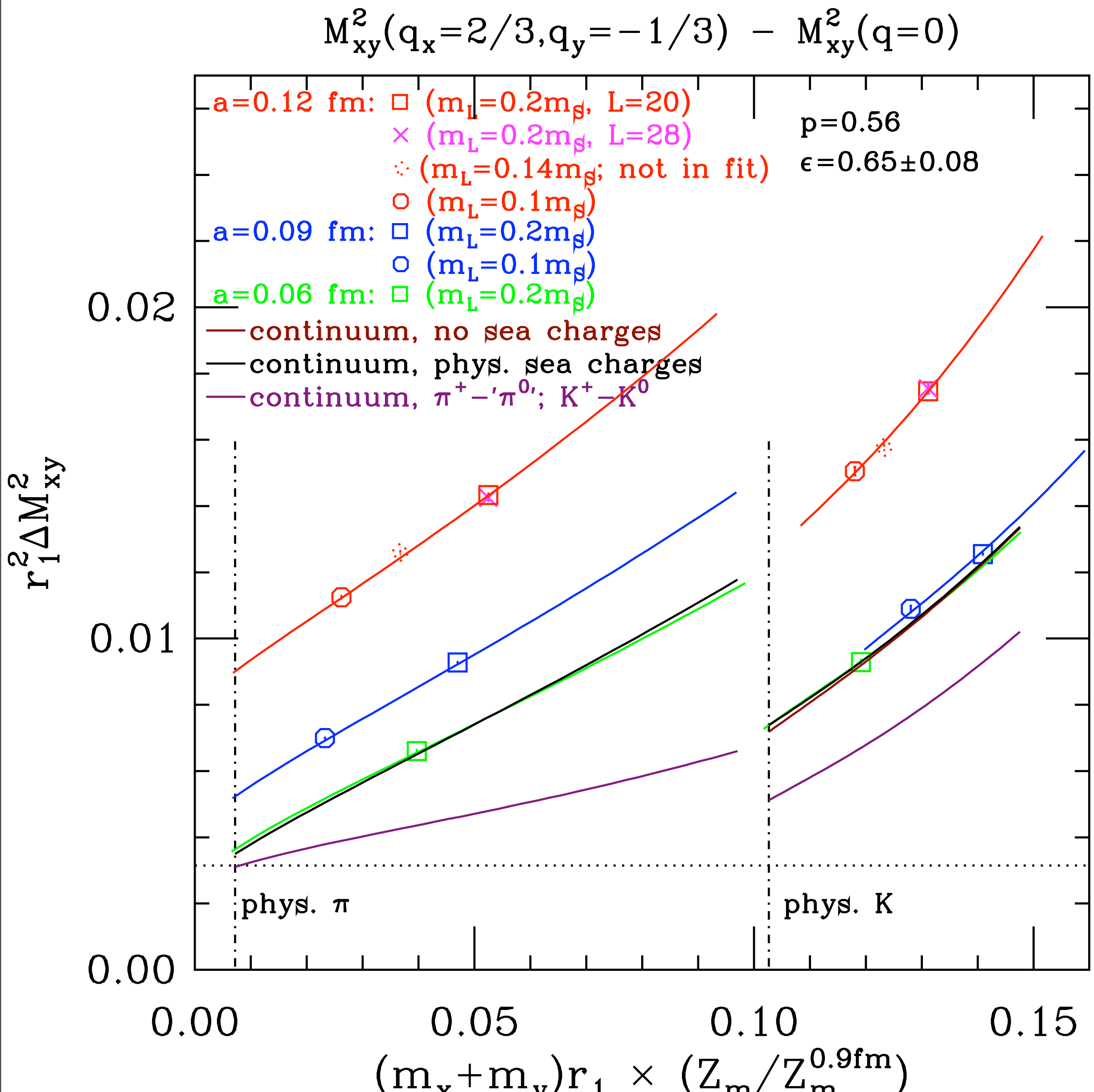
Extra Slides

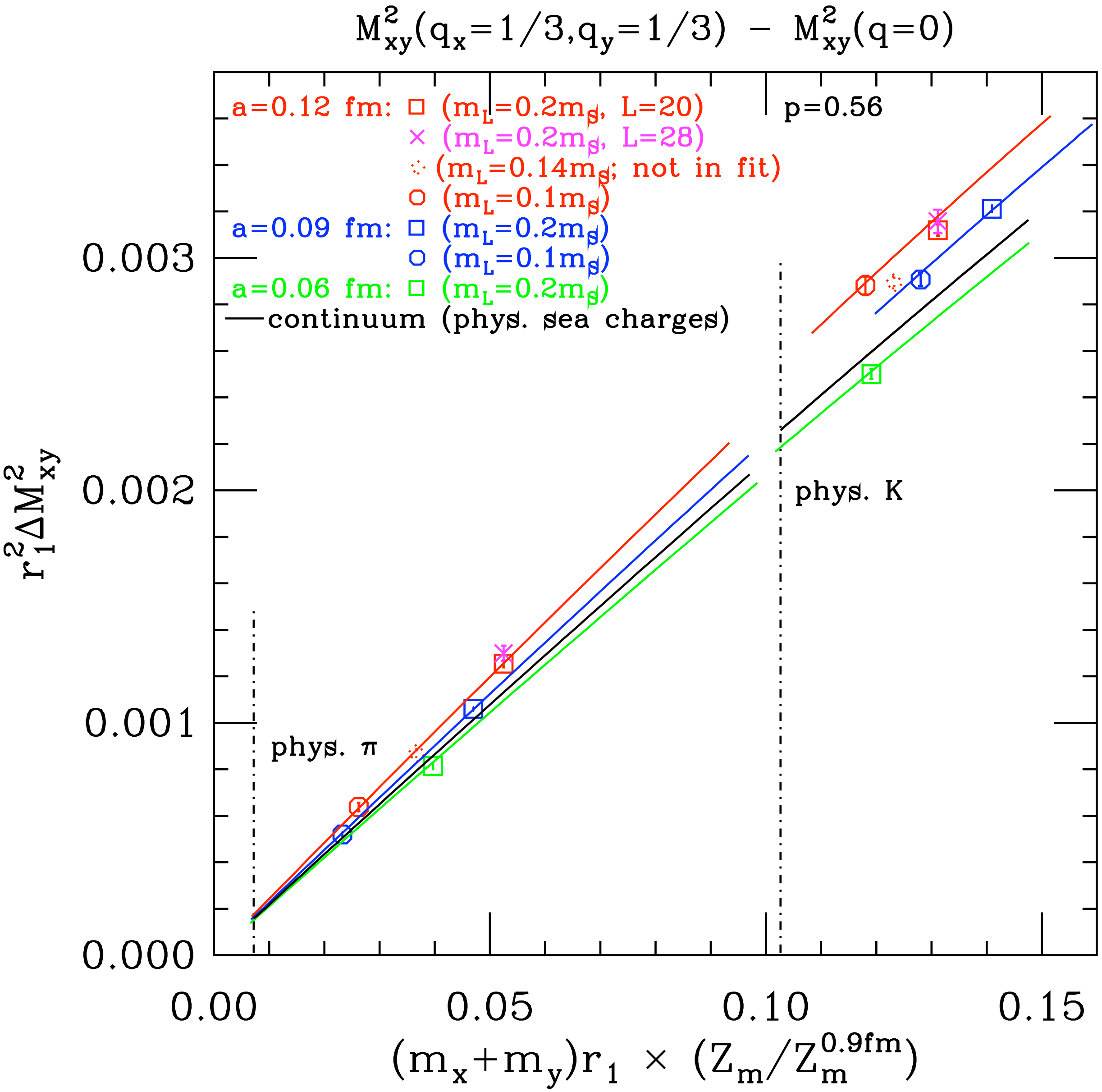
Chiral Fit and Extrapolation



- Alternative fit that drops $a = 0.12$ fm; $m_\ell = 0.2\, m_s$ ensemble:
  - has smallest $m_\pi L$ (= 3.8) of any ensemble; possibly larger finite size effects.
- p value better (0.56 instead of 0.09), but Dashen ratio essentially unchanged:

$$\epsilon = 0.65(8)$$

Chiral Fit and Extrapolation



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