

Determination of CP and CPT violation parameters in the neutral kaon system using the Bell-Steinberger relation and WA data

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The CPT symmetry

CPT symmetry is linked to the basic mathematical tools that we use in particle physics:

QFT + Lorentz invariance + Locality \Rightarrow CPT

These tools have intrinsic limitations (we're not able to include gravity in a consistent way) \Rightarrow we should expect CPT viol. at some level (hard to predict a reference scale/size for CPT viol.)



Phenomenologically driven search

Neutral kaon system
ideal testing ground

$$|m_K - m_{\bar{K}}| < 10^{-18} m_K \sim O(m_K^2/M_{\text{PL}})$$

With this system, the most powerful and simple phenomenological tool for CPT tests is the Bell-Steinberger relation

Time evolution of the neutral kaons

$$i \frac{d}{dt} \begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix} = [\mathbf{M} - i \Gamma/2] \begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix}$$

CPT invariance

$$\mathbf{M}_{11} = \mathbf{M}_{22} \quad \Gamma_{11} = \Gamma_{22}$$

Diagonalization

$$K_{S,L} = N \left([1 + \epsilon_{S,L}] K^0 \pm [1 - \epsilon_{S,L}] \bar{K}^0 \right)$$

$$\epsilon_{S,L} = \epsilon \pm \delta$$

$$\delta = \frac{i(m_K - m_{\bar{K}}) + (\Gamma_K - \Gamma_{\bar{K}})/2}{\Gamma_S - \Gamma_L} \cos \phi_{sw} e^{i\phi_{sw}}$$

↓ $\Delta\Gamma = 0$

$$\phi_{sw} = \tan^{-1} \frac{2(m_L - m_S)}{\Gamma_S - \Gamma_L} = 43.5^\circ$$

$$\left| \frac{m_K - m_{\bar{K}}}{m_K} \right| \cong 3 \times 10^{-14} | \mathbf{Im} \delta |$$

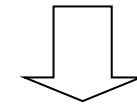
Unitarity and Bell Steinberger relation

Even if CPT is violated, we can assume that **unitarity** is preserved

$$\Gamma_{11} = \sum_f A^*(K^0 \rightarrow f) A(K^0 \rightarrow f)$$

$$\Gamma_{12} = \sum_f A^*(\bar{K}^0 \rightarrow f) A(K^0 \rightarrow f)$$

Expressing the decay amplitudes in the K_S K_L basis and using the definitions of ϵ and δ



$$\left(\frac{\Gamma_S + \Gamma_L}{\Gamma_S - \Gamma_L} + i \tan \phi_{sw} \right) \left(\frac{\text{Re } \epsilon}{1 + |\epsilon|^2} - i \text{Im } \delta \right) = \frac{1}{\Gamma_S - \Gamma_L} \sum_f A^*(K_S \rightarrow f) A(K_L \rightarrow f)$$

Exp. inputs

Two outputs

Exp. Inputs: only $\pi\pi$, 3π and $\pi\nu$ give appreciable contribution, $\geq 10^{-7}$

Im $\delta \neq 0$ could only be due to: violation of CPT, violation of unitarity, new exotic invisible final states

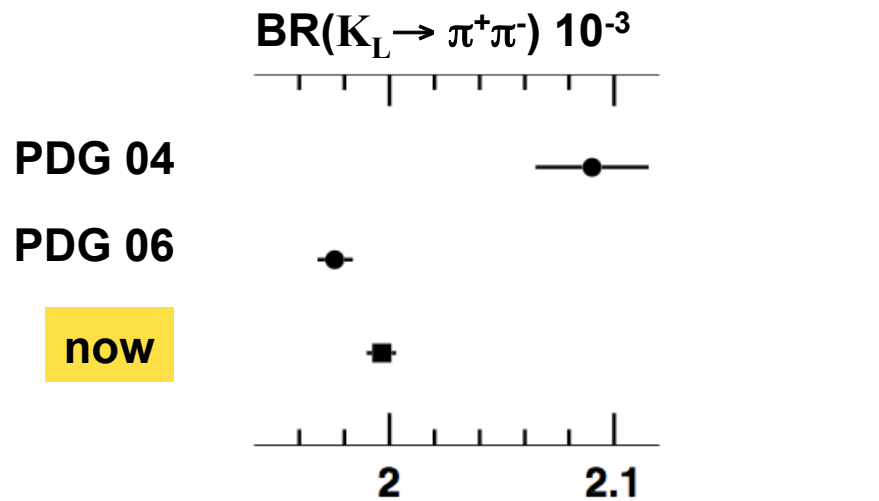
A bit of recent history

- CPLEAR '99 First pioneer work
- PDG (~2000) quote the CPLEAR result
- KLOE , G. D'Ambrosio and G. Isidori JHEP 12 (2006) 011
- First WA result in 2007 (online in PDG07) published in PDG08
- KTeV, Phys.Rev.D83 (2011)
- WA update including new KTeV result PDG12

$\pi^+\pi^-(\gamma), \pi^0\pi^0$ decays (1)

$$\alpha_{\pi\pi} = \frac{1}{\Gamma_S} \langle A^*(K_S \rightarrow \pi\pi) A(K_L \rightarrow \pi\pi) \rangle = \eta_{\pi\pi} \text{BR}(K_S \rightarrow \pi\pi)$$

- $|\eta_{\pi+\pi-}|$ from KTeV, NA48 and KLOE BRs



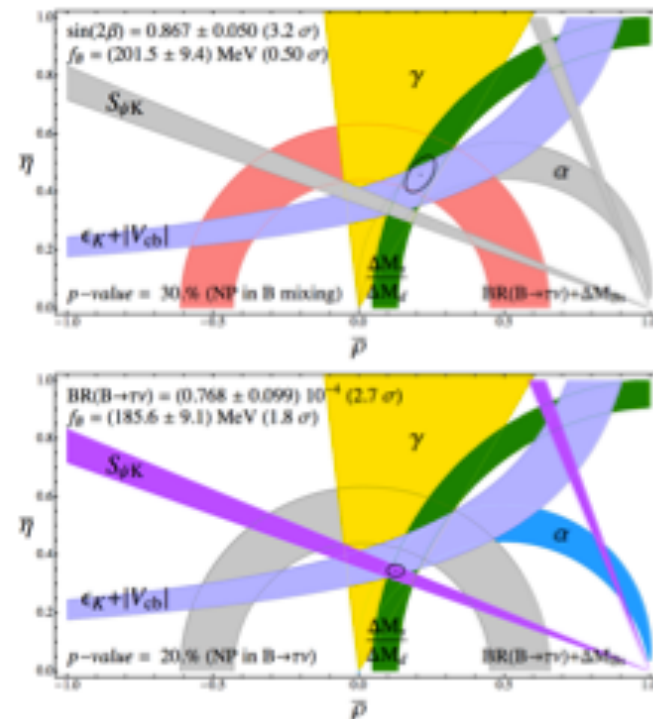
KTeV $\text{BR}(\pi^+\pi^-/K_{e3}) = 4.856(29) \times 10^{-3}$

NA48 $\text{BR}(\pi^+\pi^-/K_{e3}) = 4.826(27) \times 10^{-3}$

KLOE $\text{BR}(\pi^+\pi^-/K_{\mu 3}) = 7.275(68) \times 10^{-3}$

6 DE contribution of **1.52(7)%*** subtracted

- fundamental ingredient for CKM
- Renewed relevance thanks to improvements in B_K determinations
- One of the “tension” in UT analysis:



[E Lunghi, A Soni, arXiv:1104.2117]

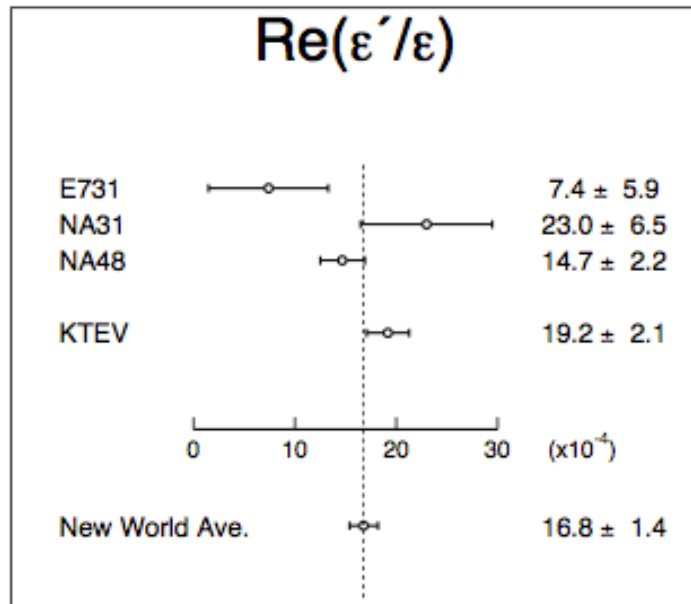
$\pi^+\pi^-(\gamma), \pi^0\pi^0$ decays (2)

$$\alpha_{\pi\pi} = \frac{1}{\Gamma_S} \langle A^*(K_S \rightarrow \pi\pi) A(K_L \rightarrow \pi\pi) \rangle = \eta_{\pi\pi} \text{BR}(K_S \rightarrow \pi\pi)$$

new KTeV CP, CPT measurement [Phys.Rev.D83 (2011)]

$$\phi_{+-} = (43.76 \pm 0.64)^\circ, \quad \phi_{00} = (44.06 \pm 0.68)^\circ$$

“bonus” from direct CP measurement:



- improved average :

**PDG10 fit to $\Delta M, \tau_S, \phi_{+-}, \phi_{00}$
(without assuming CPT)**

$$\phi_{+-} = (43.4 \pm 0.7)^\circ$$

$$\phi_{00} = (43.7 \pm 0.8)^\circ$$

**PDG12 fit to $\Delta M, \tau_S, \phi_{+-}, \phi_{00}$
(without assuming CPT)**

$$\phi_{+-} = (43.4 \pm 0.5)^\circ$$

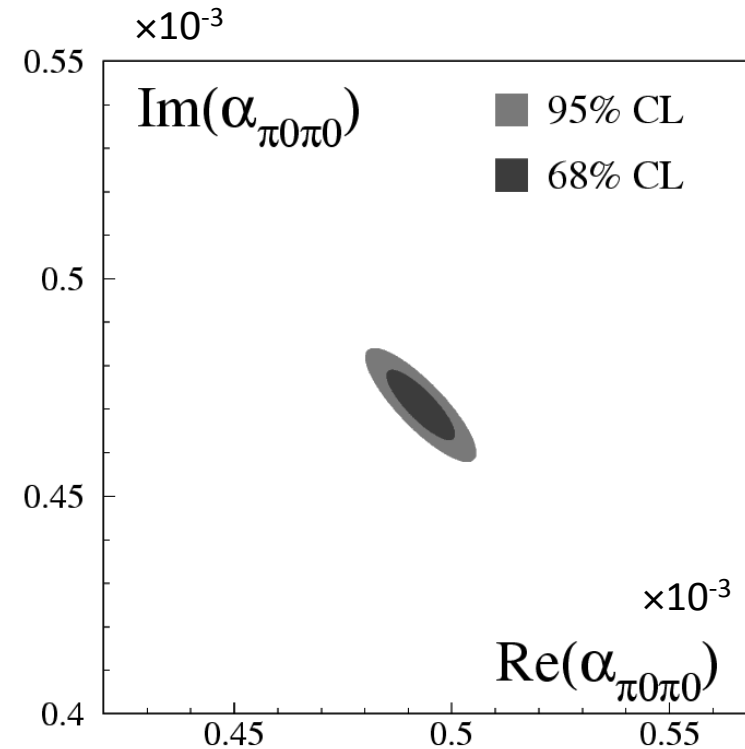
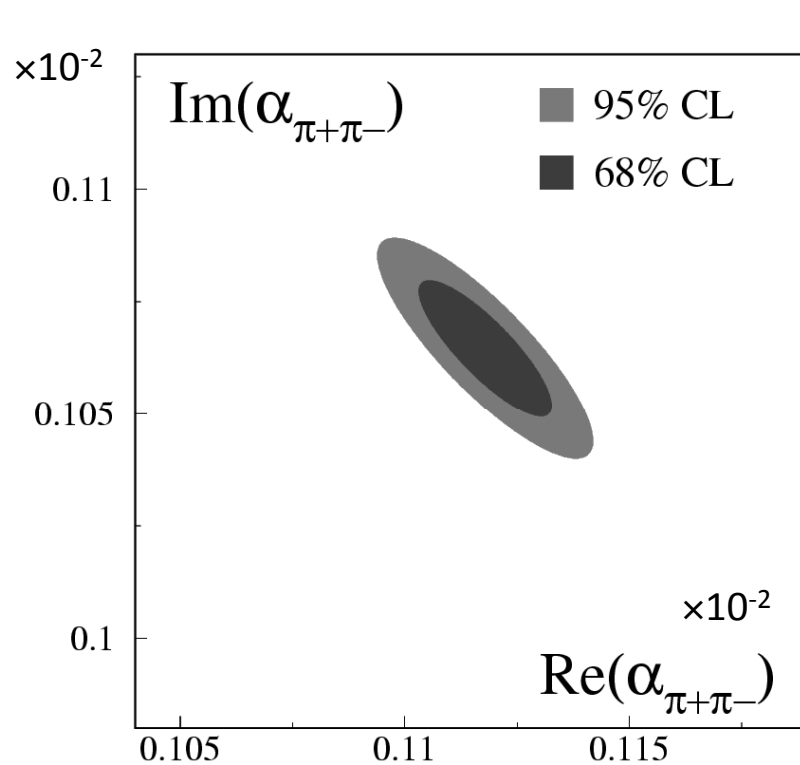
$$\phi_{00} = (43.7 \pm 0.6)^\circ$$

[C.-J. Lin PDG team]

$\pi^+\pi^-(\gamma), \pi^0\pi^0$ decays (3)

$|\eta_{\pi\pi}|$ from KTeV, NA48 and KLOE BRs; $\phi_{\pi\pi}$ from PDG12 fit

$$\alpha_{\pi^+\pi^-} = (1.11 \pm 0.01 + i(1.06 \pm 0.01)) \times 10^{-3} \quad \alpha_{\pi^0\pi^0} = (0.493 \pm 0.005 + i(0.471 \pm 0.005)) \times 10^{-3}$$



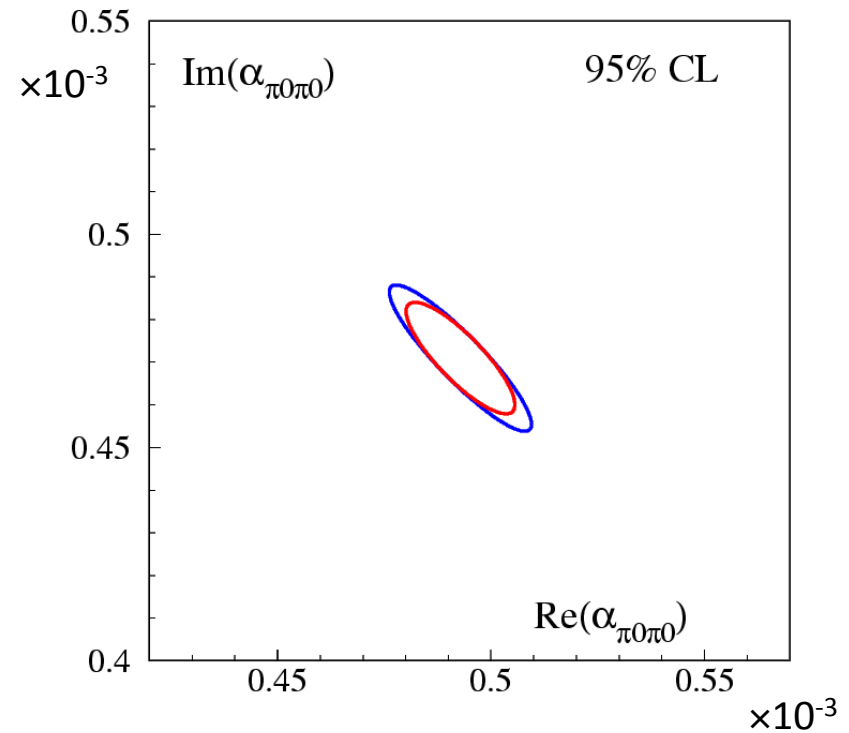
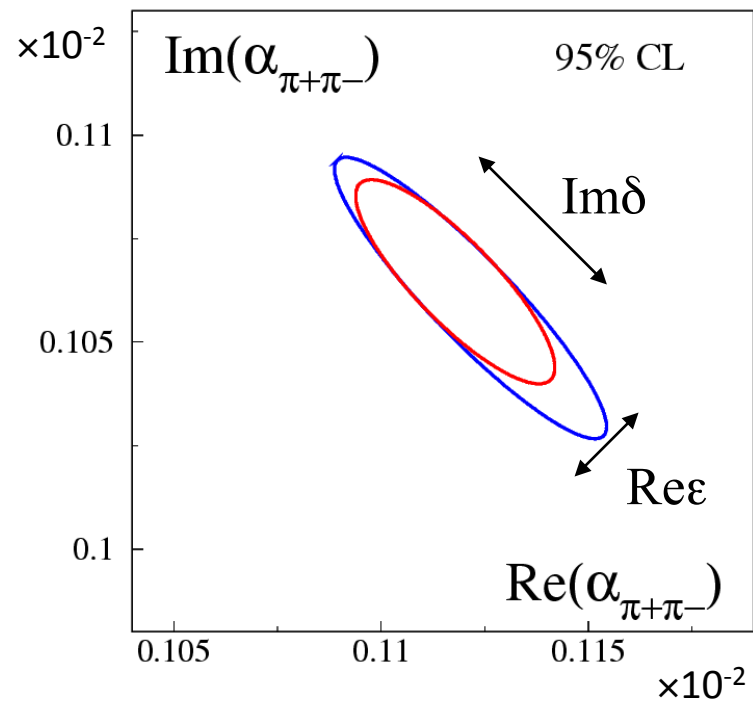
$\pi^+\pi^-(\gamma), \pi^0\pi^0$ decays (3)

PDG10 fit

$$\phi_{+-} = (43.4 \pm 0.7)^\circ \quad \phi_{00} = (43.7 \pm 0.8)^\circ$$

PDG12 fit

$$\phi_{+-} = (43.4 \pm 0.5)^\circ \quad \phi_{00} = (43.7 \pm 0.6)^\circ$$



Semileptonic decays (1)

Decay amplitudes

$$A(K^0 \rightarrow \pi^- l^+ \nu) = A_0 (1 - y)$$

$$A(K^0 \rightarrow \pi^+ l^- \nu) = A_0^* (1 + y^*) (x_+ - x_-)^*$$

$$A(\bar{K}^0 \rightarrow \pi^+ l^- \nu) = A_0^* (1 + y^*)$$

$$A(\bar{K}^0 \rightarrow \pi^- l^+ \nu) = A_0 (1 - y) (x_+ - x_-)$$

~~CPT~~

$\Delta S \neq \Delta Q$

$\Delta S \neq \Delta Q$
and ~~CPT~~

$$A_S - A_L = 4 (\text{Re} \delta + \text{Re } x_-)$$

direct test of CPT

$$A_S + A_L = 4 (\text{Re} \varepsilon - \text{Re } y)$$

key input for BSR

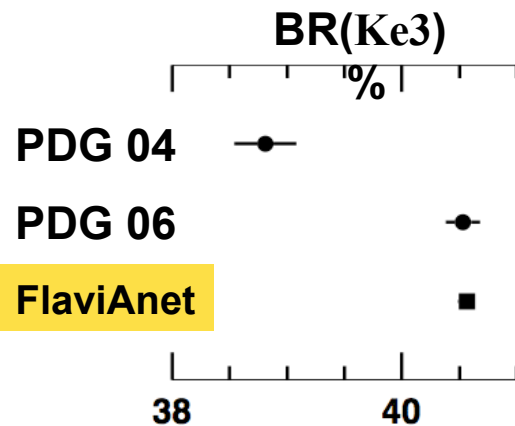
$$\begin{aligned} \frac{1}{\Gamma_S} \langle A_S(\pi l \nu) A_L(\pi l \nu) \rangle &= 2 \frac{\tau_S}{\tau_L} \text{BR}(K_L \rightarrow \pi l \nu) \left[\text{Re} \varepsilon - \text{Re } y - i (\text{Im } x_+ + \text{Im } \delta) \right] \\ &= 2 \frac{\tau_S}{\tau_L} \text{BR}(K_L \rightarrow \pi l \nu) \underbrace{\left[(A_S + A_L)/4 - i (\text{Im } x_+ + \text{Im } \delta) \right]}_{\alpha_{\pi l \nu}} \end{aligned}$$

Semileptonic decays (2)

*

$$\frac{1}{\Gamma_S} \langle A_S(\pi l\nu) A_L(\pi l\nu) \rangle = 2 \frac{\tau_S}{\tau_L} \text{BR}(K_L \rightarrow \pi l\nu) \underbrace{\left(\frac{A_S + A_L}{4} - i (\text{Im}x_+ + \text{Im}\delta) \right)}_{\alpha_{\pi l\nu}}$$

- BR_L from KLOE, KTeV and NA48



- $\tau_S = 0.08958(5)$ ns, constrained by **NA48 '02** and **KTeV '03** values
- $\tau_L = 50.84(23)$ ns, from **KLOE '06**

- $A_L = (3.34 \pm 0.07) \times 10^{-3}$ from **KTeV**

- $A_S = (1.5 \pm 10.0) \times 10^{-3}$ from **KLOE**

- $\text{Im}x_+$ from CPLEAR time-dependent asymmetries:

$$A_\delta = \frac{P(\bar{K}^0 \rightarrow \bar{K}^0(t)) - P(K^0 \rightarrow K^0(t))}{P(\bar{K}^0 \rightarrow \bar{K}^0(t)) + P(K^0 \rightarrow K^0(t))}$$

$$= 4 \text{Re}\delta + f(\text{Im}x_+, \text{Im}\delta, \text{Re}x_-, t)$$

Semileptonic decays (3)

$$A_T^{\text{exp}}(\tau) \approx \frac{\bar{R}_+(\tau) - R_-(\tau)}{\bar{R}_+(\tau) + R_-(\tau)} - 2\text{Re}(y) - 2\text{Re}(x_-) = 4(\text{Re}(\epsilon - y) - \text{Re}(x_-)) + 2 \frac{\text{Re}(x_-) \left(e^{-\frac{1}{2}\Delta\Gamma\tau} + \cos(\Delta m\tau) \right) + \text{Im}(x_+) \sin(\Delta m\tau)}{\cosh\left(\frac{1}{2}\Delta\Gamma\tau\right) - \cos(\Delta m\tau)},$$

$$A_T = \frac{N(\bar{K}^0 \rightarrow e^+\pi^-\nu)(t) - N(K^0 \rightarrow e^-\pi^+\nu)(t)}{N(\bar{K}^0 \rightarrow e^+\pi^-\nu)(t) + N(K^0 \rightarrow e^-\pi^+\nu)(t)}$$

Combined fit to:

A_S , (KLOE) A_L (\sim KTeV), A_T and A_δ (CPLEAR)

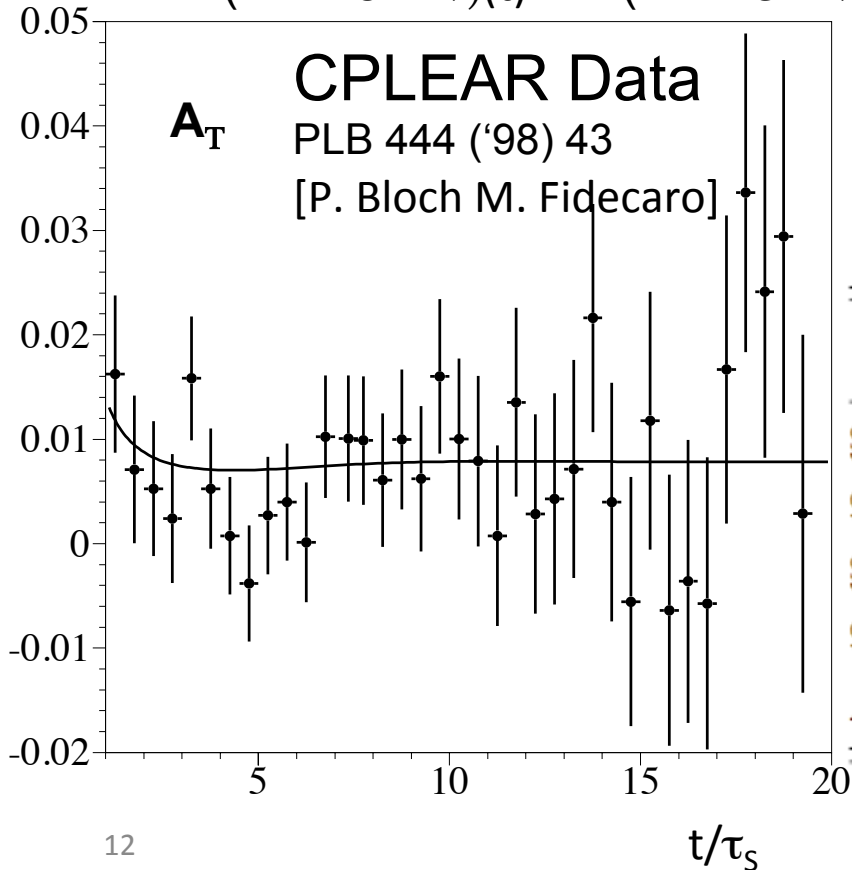


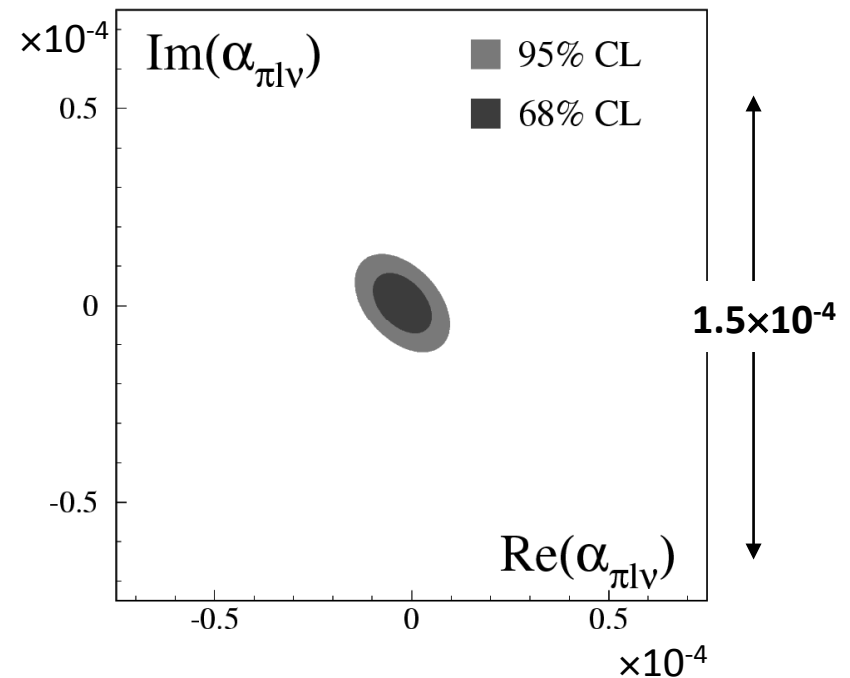
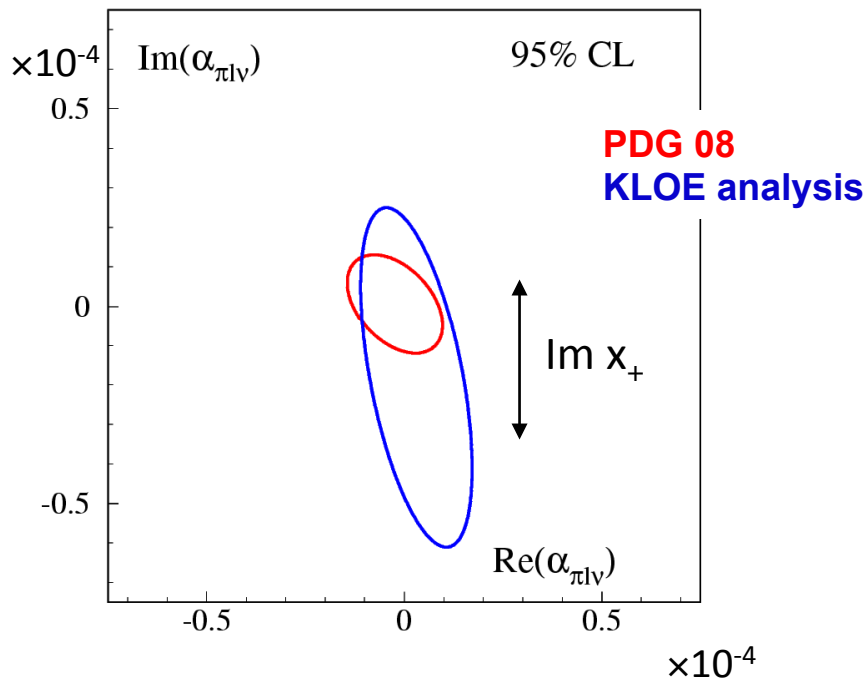
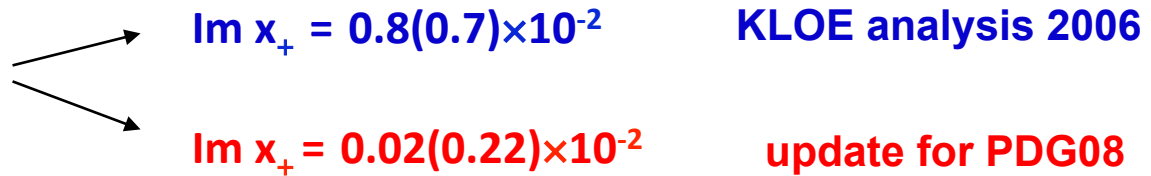
Table 1: Values, errors, and correlation coefficients for $\Re(\delta)$, $\Im(\delta)$, $\Re(x_-)$, $\Im(x_+)$, and $A_S + A_L$ obtained from a combined fit, including KLOE [4] and CPLEAR [13].

	value	Correlations coefficients			
$\Re(\delta)$	$(3.0 \pm 2.3) \times 10^{-4}$	1			
$\Im(\delta)$	$(-0.66 \pm 0.65) \times 10^{-2}$	-0.21	1		
$\Re(x_-)$	$(-0.30 \pm 0.21) \times 10^{-2}$	-0.21	-0.60	1	
$\Im(x_+)$	$(0.02 \pm 0.22) \times 10^{-2}$	-0.38	-0.14	0.47	1
$A_S + A_L$	$(-0.40 \pm 0.83) \times 10^{-2}$	-0.10	-0.63	0.99	0.43

Semileptonic decays (4)

Improved determination of $\text{Im}x_+$ from a combined fit of CPLEAR time-dependent asymmetries and A_L and A_S from KTeV and KLOE

$\text{Im} x_+ = 1.2(2.2) \times 10^{-2}$
original CPLEAR result



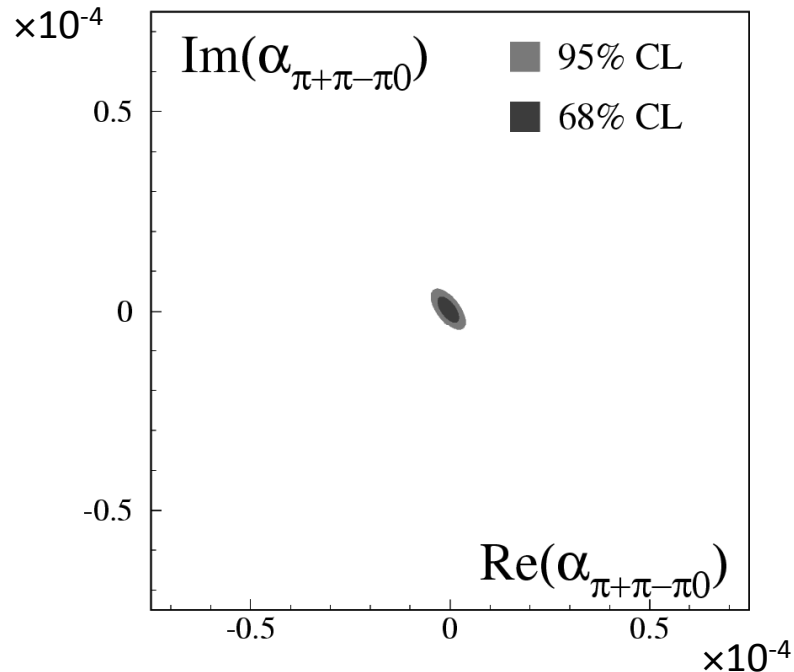
$$\alpha_{\pi l\nu} = (-2.3 \pm 4.9 + i(1.0 \pm 5.1)) \times 10^{-6}$$

3π decays

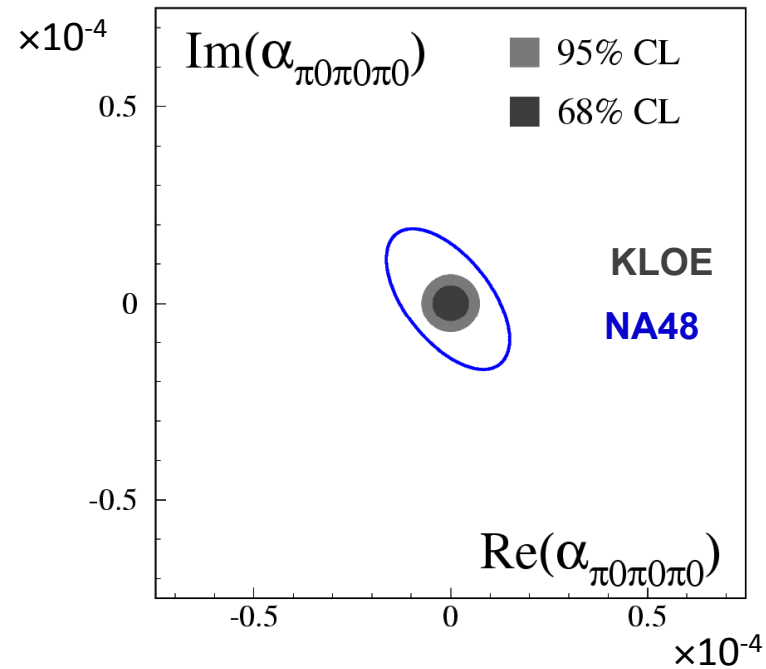
$$\alpha_{3\pi} = \frac{1}{\Gamma_S} \langle A^*(K_S \rightarrow 3\pi) A(K_L \rightarrow 3\pi) \rangle = \frac{\tau_S}{\tau_L} \eta_{3\pi}^* \text{BR}(K_L \rightarrow 3\pi)$$

Re(η_{+-0}), Im(η_{+-0}) from CPLEAR

| η_{000} | from KLOE UL on BR($K_S \rightarrow 3\pi^0$)
 ×2 improvement w.r.t. NA48 η_{000}



$$\alpha_{\pi^+\pi^-\pi^0} = (0 \pm 2 + i(0 \pm 2)) \times 10^{-6}$$

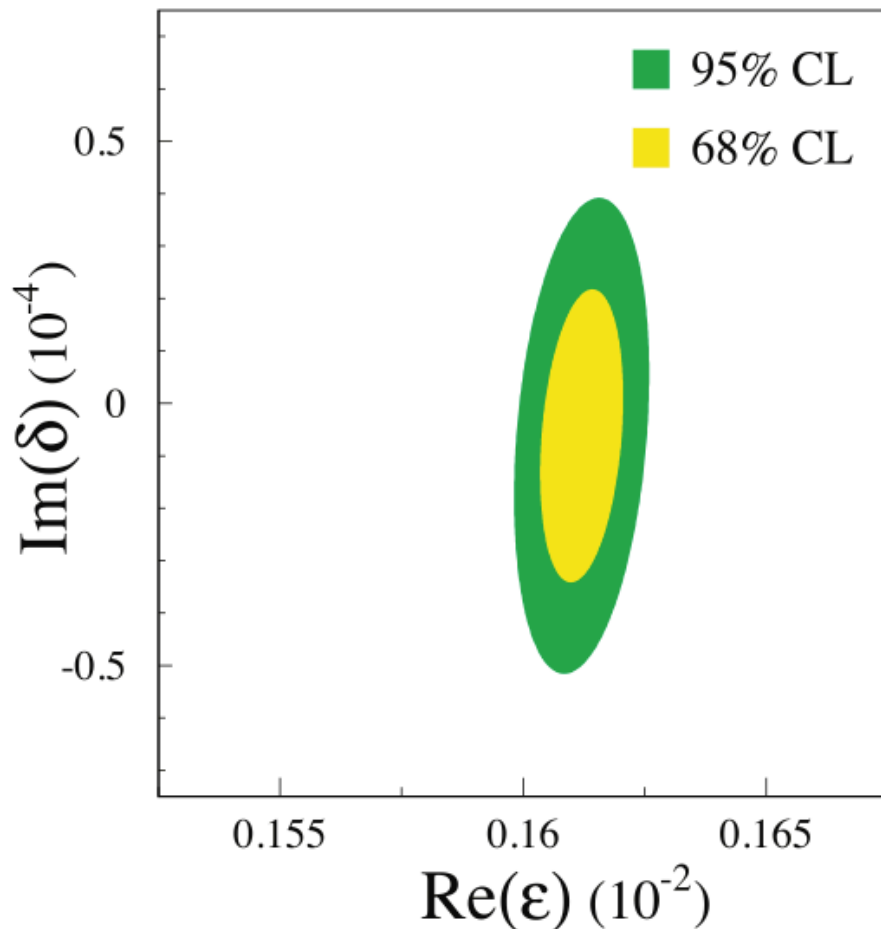


$$|\alpha_{3\pi^0}| < 7 \times 10^{-6} \text{ at 95\% CL}$$

Results for $\text{Re}\varepsilon$ and $\text{Im}\delta$

$$\text{Re } \varepsilon \propto \cos \phi_{\text{SW}} \sum_f \text{Re } \alpha_f + \sin \phi_{\text{SW}} \sum_f \text{Im } \alpha_f$$

$$\text{Im } \delta \propto \cos \phi_{\text{SW}} \sum_f \text{Re } \alpha_f - \sin \phi_{\text{SW}} \sum_f \text{Im } \alpha_f \rightarrow |\alpha_{+-}| \times (\phi_{+-} - \phi_{\text{SW}})$$



$$\text{Re } \varepsilon = (161.1 \pm 0.5) \times 10^{-5}$$

$$\text{Im } \delta = (-0.7 \pm 1.4) \times 10^{-5}$$

$$\sigma(\text{Im } \delta) = 1.3 \oplus 0.6$$

$\pi\pi \quad \pi\nu$

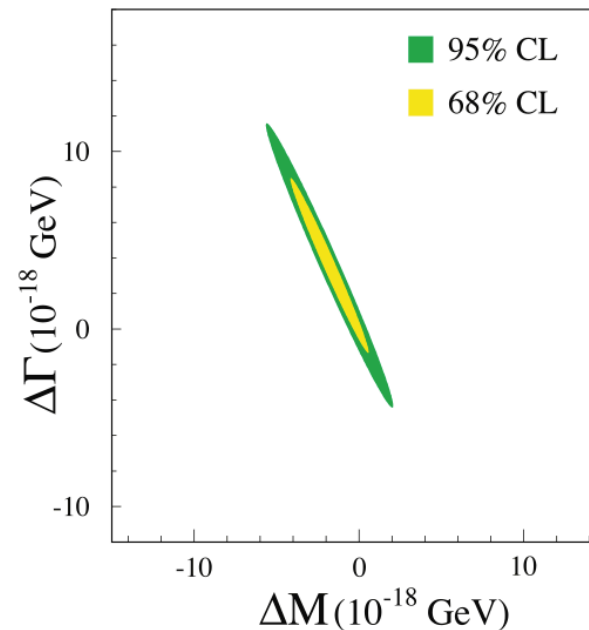
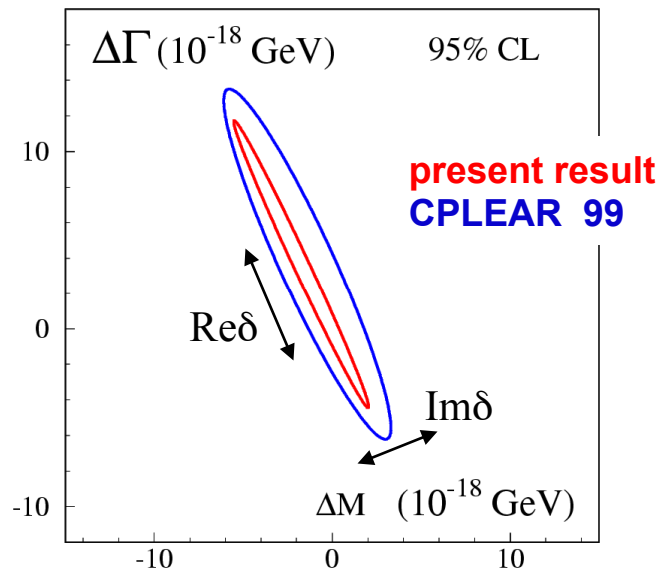
A bit of history: results

	CPLEAR99	KLOE06	PDG08	Present result
Analysis strategy	fit to BSR and $\pi l\nu$ asymm.	Evaluation of α 's		
$\text{Im } \delta (\times 10^5)$	2.4 ± 5.0	0.4 ± 2.1	-0.6 ± 1.9	-0.1 ± 1.4
$\sigma(\text{Im } \delta)$	$4.5 \oplus 2.1$	$1.8 \oplus 1.2$	$1.8 \oplus 0.6$	$1.3 \oplus 0.6$
	3π $\pi\pi$ $\pi l\nu$	$\pi\pi$ $\pi l\nu$	$\pi\pi$ $\pi l\nu$	$\pi\pi$ $\pi l\nu$
Comments	$\pi l\nu$ vs time	new $K_L \rightarrow \pi\pi$, $K_S \rightarrow 3\pi^0$, A_S and A_L		
			better treatment of CPLEAR data	new $\phi_{\pi\pi}$
$\text{Re } \varepsilon (\times 10^5)$	164.9 ± 2.5	159.6 ± 1.3	161.2 ± 0.6	161.2 ± 0.6

CPT test: $m(K)-m(\bar{K})$

$$\delta = \frac{1}{2} \frac{M_{11} - M_{22} - i(\Gamma_{11} - \Gamma_{22})/2}{m_S - m_L - i(\Gamma_S - \Gamma_L)/2} \rightarrow \begin{cases} \Delta M \propto -\sin \phi_{SW} \operatorname{Re} \delta + \cos \phi_{SW} \operatorname{Im} \delta \\ \Delta \Gamma \propto \cos \phi_{SW} \operatorname{Re} \delta + \sin \phi_{SW} \operatorname{Im} \delta \end{cases}$$

$\operatorname{Im} \delta = (-0.1 \pm 1.4) \times 10^{-5}$ from BSR; $\operatorname{Re} \delta = (2.5 \pm 2.3) \times 10^{-4}$ essentially from CPLEAR



Assuming CPT violation only in the mass matrix:

	CPLEAR99	KLOE06	PDG08	Present result
ΔM at $\Delta \Gamma=0$	3.3 ± 7.0	0.5 ± 3.0	-0.9 ± 2.6	-0.1 ± 2.0
$17 (10^{-19} \text{ GeV})$				

Conclusions

- The Bell-Steinberger relation is a very powerful tool to probe some of the basic principles of fundamental interactions (CPT + unitarity)
- Remarkable step forward thanks to new data ($K_L \rightarrow \pi\pi$, $K_S \rightarrow 3\pi^0$, A_S , A_L , ϕ_{+-} and ϕ_{00}) + re-analysis of old CPLEAR data on semileptonic time-asymmetries
- Present uncertainty on $\text{Im}\delta$ is dominated by 2π channels (ϕ_{+-})

$$|m_K - m_{\bar{K}}| < 4 \times 10^{-19} m_K \quad \text{at 95\% CL if } \Delta\Gamma=0$$