A new approach to chiral two-nucleon dynamics

A. M. Gasparyan, E. Epelbaum, M. F. M. Lutz

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Table of contents

Motivation

- Non-perturbative nature of the NN interactions .
  - Potential approaches based on the chiral Lagrangian.

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    M. Albladejo, J.A. Oller 2011 (based on one-pion exchange)
- We suggest an analytic continuation of the scattering amplitude from the subthreshold region to the physical region.
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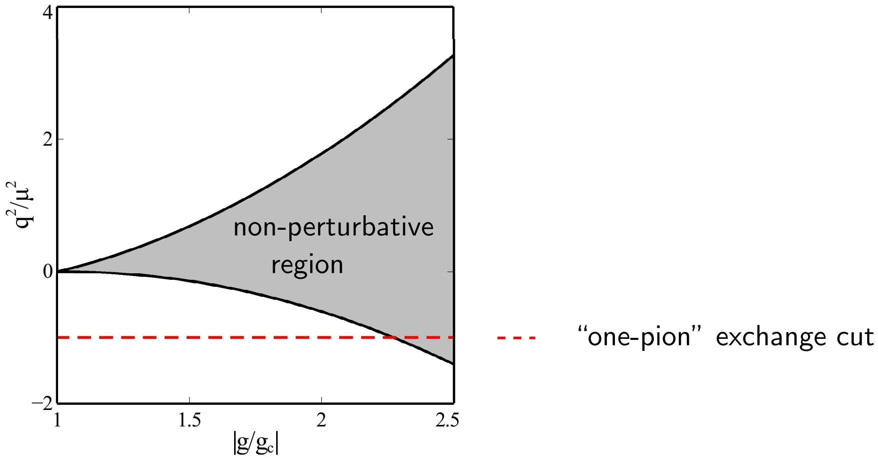
S-wave scattering with a Yukawa potential (Danilkin et al. 2010)



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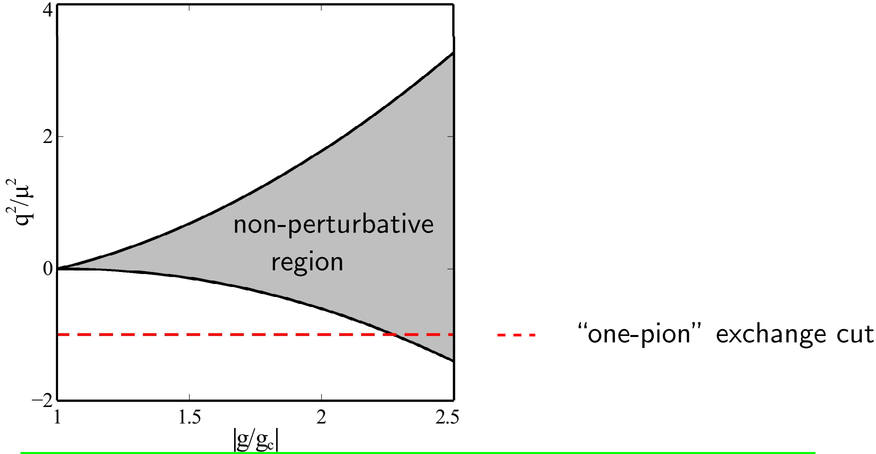

Perturbation theory is applicable below threshold

The scheme

- 1-channel approximation $(NN) \Longrightarrow$ one is limited to energies $T_{lab} \simeq 250\text{MeV}$
- The left-hand cuts closest to the NN threshold are calculated perturbatively in ChPT.
- Analyticity and unitarity are used to extrapolate the amplitude into the physical region.
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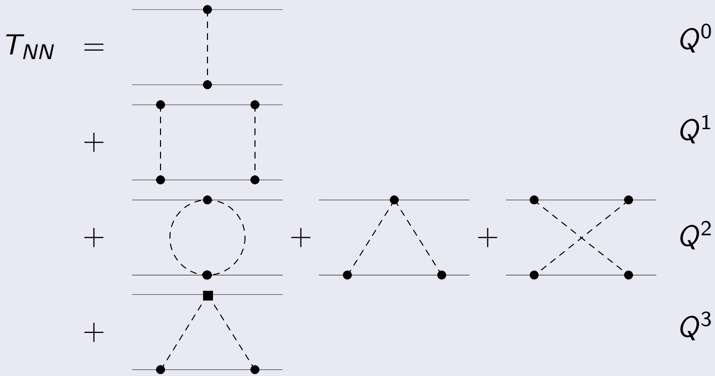
Perturbative expansion of the left-hand cut



Partial Wave Dispersion Relation with one subtraction at $s = \mu_M^2$

Unitarity and Analyticity:

$$T(s) = U(s) + \int_{4m_N^2}^{\infty} \frac{ds'}{\pi} \frac{s-\mu_M^2}{s'-\mu_M^2} \frac{T(s)\,\rho(s')\,T^*(s')}{s'-s-i\epsilon}.$$

$U(s)$ contains only left hand cuts (computed in ChPT)

$\Longrightarrow U(s)$ can be analytically continued beyond threshold region (conformal mapping)

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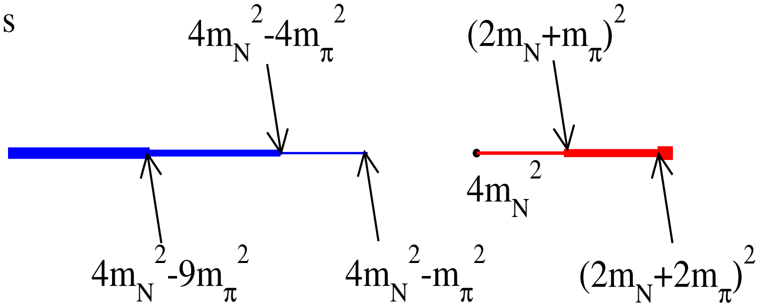
Analytic structure of partial wave amplitudes in the complex s plane



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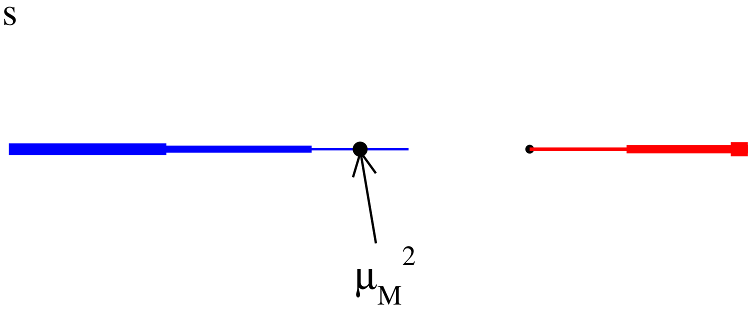
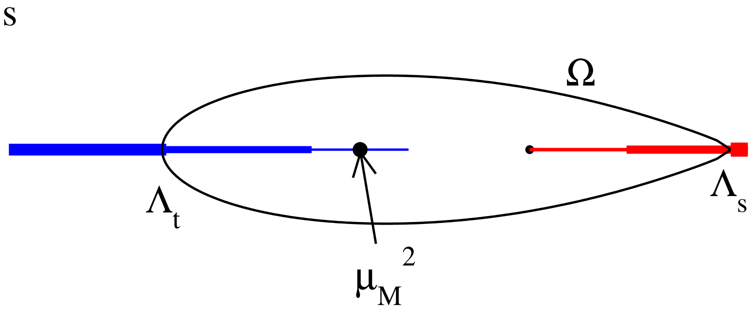


Conformal mapping $\xi(s)$ for generalized potential

$$U(s) = U_{inside}(s) + \sum_{k=0}^{\infty} U_k \left(\xi(s)\right)^k$$



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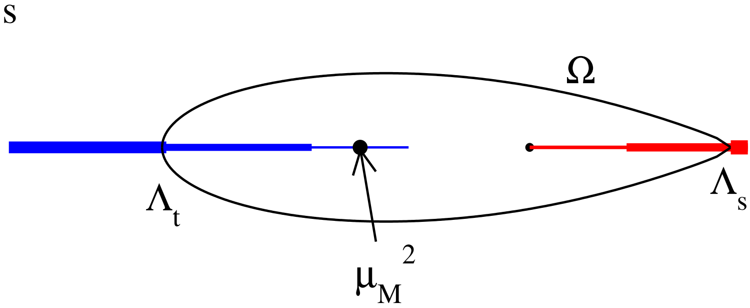
$$U(s) = U_{inside}(s) + \sum_{k=0}^{\infty} U_k \left(\xi(s)\right)^k$$

$$U_{inside}(s) = \int_{\Lambda_t}^{4m_N^2 - m_\pi^2} \frac{\Delta T(s')}{s'-s} \frac{ds'}{\pi}$$



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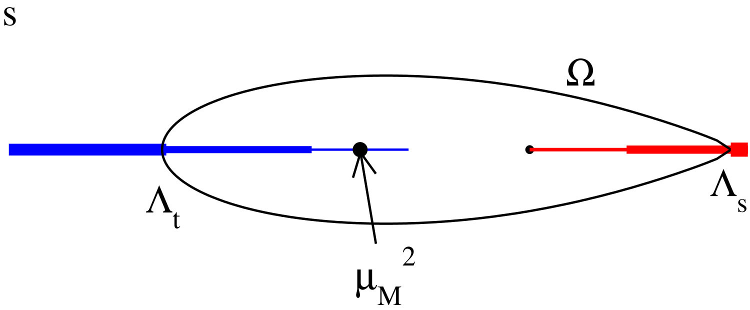
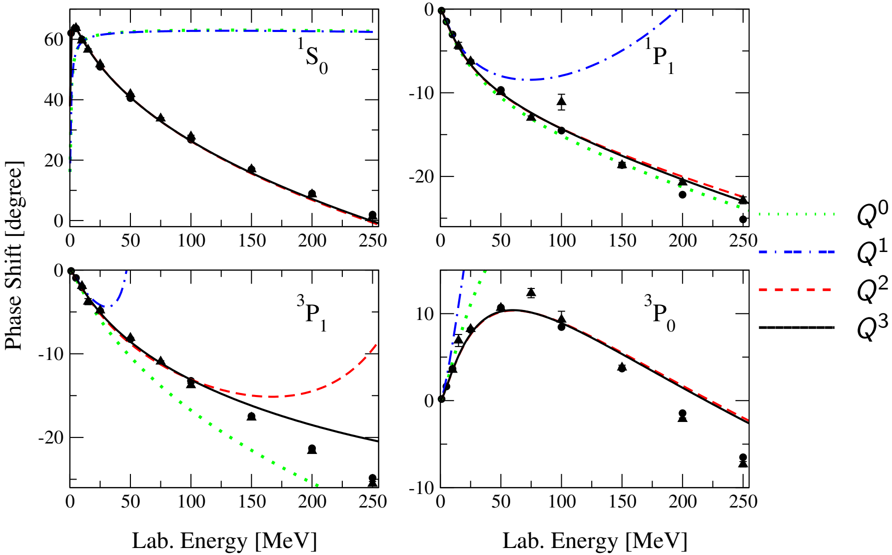
$$U(s) = U_{inside}(s) + \sum_{k=0}^{\infty} U_k \left(\xi(s)\right)^k$$

$$\xi(s){:}\Omega \rightarrow \text{unit circle, } \xi(4m_N^2) = 0$$



Results for *NN* phase shifts without mixing



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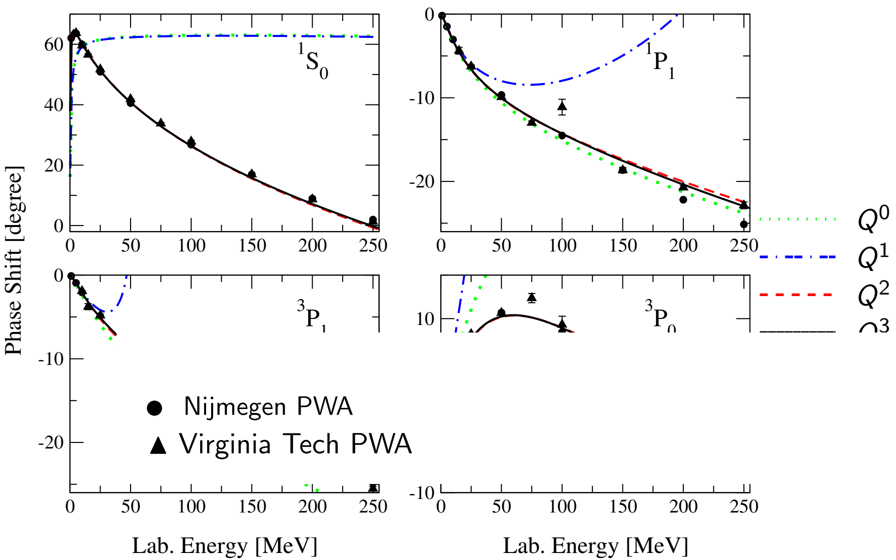

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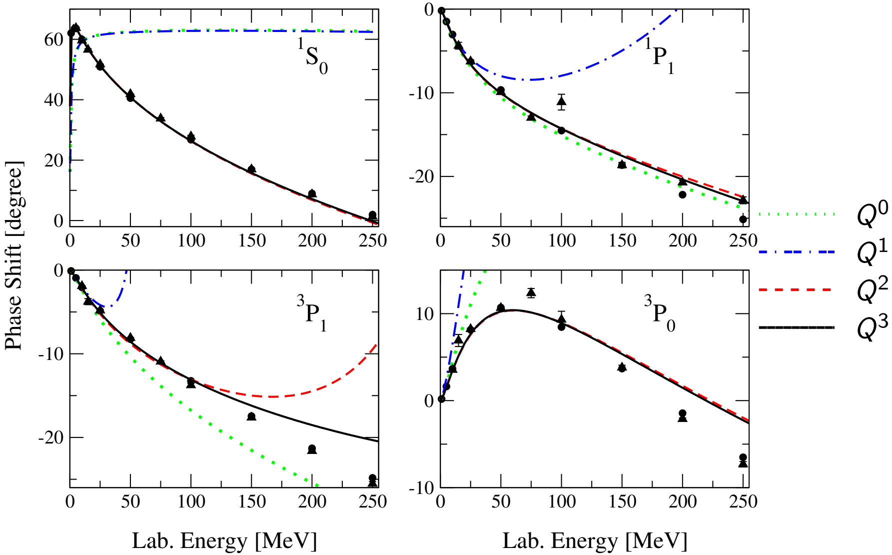

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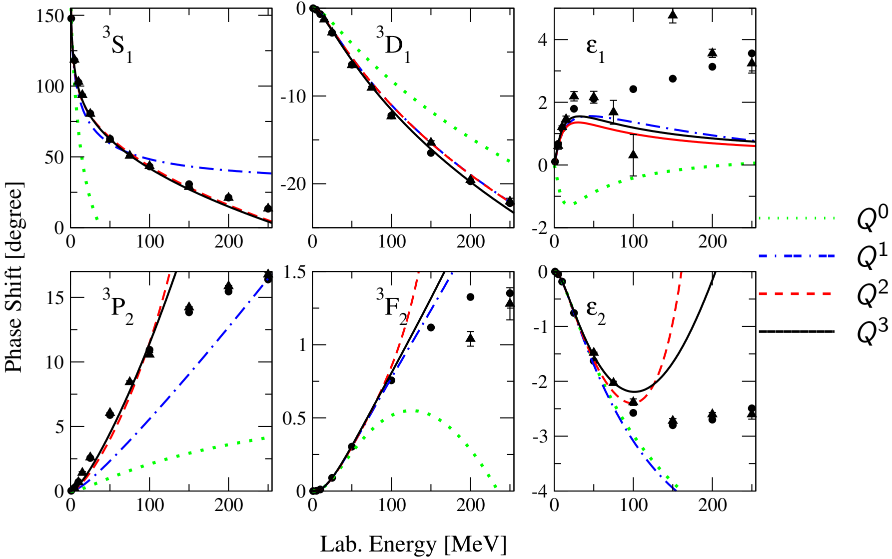

Summary

- Analytic continuation of the subthreshold NN scattering amplitude calculated from the Chiral Lagrangian.
- The results indicate a good convergence pattern when going from the order Q^0 to Q^3. This fact supports the assumption that ChPT expansion converges in the subthreshold region.
- A reasonable description of S and P partial waves is achieved.
- Further developments: higher order effects, including inelastic channels.