Electromagnetic moments and M1 transitions in $A \leq 9$ nuclei including two-body χEFT currents $*$

Saori Pastore @ JLab - Newport News, VA - August 2012

$*$ in collaboration with:
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PRC**78**, 064002 (2008) - PRC**80**, 034004 (2009) - PRC**81**, 034005 (2010) - PRL**105**, 232502, (2010) - PRC**84**, 024001 (2011)

- Standard Nuclear Physics Approach (SNPA)
- Nuclear χEFT approach
- EM current operators up to one loop
- EM observables in $A \leq 9$ systems
- Summary
- Outlook

The Basic Model

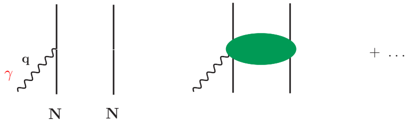
- The nucleus is a system made of A interacting nucleons, its energy is given by

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i<j} \upsilon_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

where υ_{ij} and V_{ijk} are 2- and 3-nucleon interaction operators

- Current and charge operators describe the interaction of nuclei with external fields. They are expanded as a sum of 1−, 2−, ... nucleon operators:

$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \dots \,, \qquad \mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \dots$$



- EM current operator $\mathbf{j}$ satisfies the current conservation relation (CCR) with the nuclear Hamiltonian, hence V, ρ, $\mathbf{j}$ need to be derived consistently

$$\mathbf{q} \cdot \mathbf{j} = [H, \rho]$$

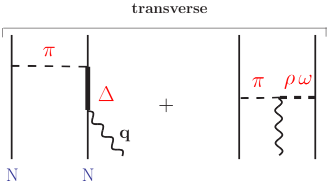
CCR does not constrain transverse (orthogonal to $\mathbf{q}$) currents

Currents from nuclear interactions $*$- Marcucci *et al.* PRC**72**, 014001 (2005)

- Current operator $\mathbf{j}$ constructed so as to satisfy the continuity equation with a realistic Hamiltonian
- Short- and intermediate-behavior of the EM operators inferred from the nuclear two- and three-body potentials

$$\mathbf{j} = \mathbf{j}^{(1)}$$

$$+ \ \mathbf{j}^{(2)}(v) \ +$$



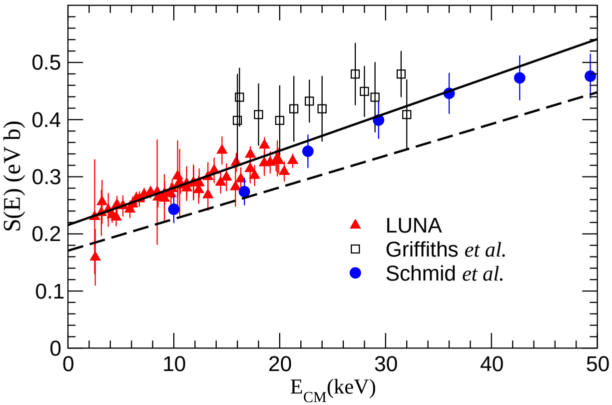
$$+ \ \mathbf{j}^{(3)}(V)$$

$*$ also referred to as Standard Nuclear Physics Approach (SNPA) currents

Currents from nuclear interactions - Marcucci *et al.* PRC**72**, 014001 (2005)

Satisfactory description of a variety of nuclear EM properties [see Marcucci *et al.* (2005) and (2008)]

^{2}H(p,γ)^{3}He capture



- Isoscalar magnetic moments are a few % off (10% in A=7 nuclei)

Nuclear χEFT approach

S. Weinberg, Phys. Lett. **B251**, 288 (1990); Nucl. Phys. **B363**, 3 (1991); Phys. Lett. **B295**, 114 (1992)

- χEFT exploits the χ symmetry exhibited by QCD at low energy to restrict the form of the interactions of π's with other π's, and with N's, Δ's, ...
- The pion couples by powers of its momentum $Q \rightarrow \mathscr{L}_{eff}$ can be systematically expanded in powers of Q/Λ_χ; ($Q \ll \Lambda_\chi \sim 1$ GeV) allowing for a perturbative treatment in terms of Q expansions
- The coefficients of the expansion, Low Energy Constants (LECs) are unknown and need to be fixed by comparison with exp data
- The systematic expansion in Q naturally has the feature

$$\langle \mathscr{O} \rangle_{1-\mathrm{body}} > \langle \mathscr{O} \rangle_{2-\mathrm{body}} > \langle \mathscr{O} \rangle_{3-\mathrm{body}}$$

Previous work

Since Weinberg's papers (1990–92), nuclear χEFT has developed into an intense field of research. A *very* incomplete list:

- *NN* potentials:
  - van Kolck *et al*. (1994–96)
  - Kaiser, Weise *et al*. (1997–98)
  - Epelbaum, Glöckle, Meissner (1998–2005)
  - Entem and Machleidt (2002–03)
- Currents and nuclear electroweak properties:
  - Rho, Park *et al*. (1996–2009), hybrid studies in A=2–4
  - Meissner *et al*. (2001), Kölling *et al*. (2009–2011)
  - Phillips (2003), deuteron static properties and f.f.'s

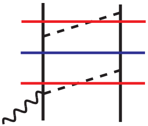
Lots of work in pionless EFT too …

Transition amplitude in time-ordered perturbation theory

$$
\begin{aligned}
T_{\mathrm{fi}} = \langle f \mid T \mid i \rangle &= \langle f \mid H_1 \sum_{n=1}^{\infty} \left(\frac{1}{E_i - H_0 + i\eta} H_1 \right)^{n-1} \mid i \rangle \\
&= \langle f \mid H_1 \mid i \rangle + \sum_{|I\rangle} \langle f \mid H_1 \mid I \rangle \frac{1}{E_i - E_I} \langle I \mid H_1 \mid i \rangle + \ldots
\end{aligned}
$$

- A contribution with N interaction vertices and L loops scales as

$$
\underbrace{e \left(\prod_{i=1}^{N} Q^{\alpha_i - \beta_i/2} \right)}_{H_1\,\text{scaling}} \times \underbrace{Q^{-(N-N_K-1)}\, Q^{-2N_K}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integration}}
$$



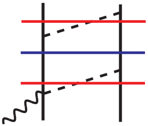
α_i = number of derivatives in H_1 and β_i = number of π's at each vertex

N_K = number of pure nucleonic intermediate states

- Due to the chiral expansion, the transition amplitude T_{fi} can be expanded as

$$
T_{\mathrm{fi}} = T^{\mathrm{LO}} + T^{\mathrm{NLO}} + T^{\mathrm{N2LO}} + \ldots \quad \text{and} \quad T^{\mathrm{NnLO}} \sim (Q/\Lambda_\chi)^n T^{\mathrm{LO}}
$$

Power counting



- N_K energy denominators scale as Q^{-2}

$$\frac{1}{E_i - E_I}|I\rangle = \frac{1}{E_i - E_N}|I\rangle \sim Q^{-2}|I\rangle$$

- $(N - N_K - 1)$ energy denominators scale Q^{-1} in the static limit; they can be further expanded in powers of $(E_i - E_N)/\omega_\pi \sim Q$

$$\frac{1}{E_i - E_I}|I\rangle = \frac{1}{E_i - E_N - \omega_\pi}|I\rangle \sim -\left[\underbrace{\frac{1}{\omega_\pi}}_{Q^{-1}} + \underbrace{\frac{E_i - E_N}{\omega_\pi^2}}_{Q^0} + \underbrace{\frac{(E_i - E_N)^2}{\omega_\pi^3}}_{Q^1} + \dots\right]|I\rangle$$

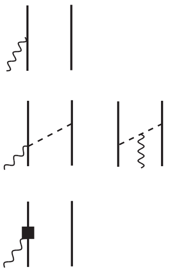
- Terms accounted into the Lippmann-Schwinger Eq. are subtracted from the reducible amplitude
- EM operators depend on the off-the-energy shell prescription adopted for the non-static OPE and TPE potentials
- Ultimately, the EM operators are unitarily equivalent: Description of physical systems is not affected by this ambiguity

EM current up to $n = 1$ (or up to N3LO)

LO : $\mathbf{j}^{(-2)} \sim e\mathbf{Q}^{-2}$

NLO : $\mathbf{j}^{(-1)} \sim e\mathbf{Q}^{-1}$

N^{2}LO : $\mathbf{j}^{(-0)} \sim e\mathbf{Q}^{0}$



- $n = -2, -1, 0$, and 1-(loops only): depend on known LECs namely g_A, F_π, and proton and neutron μ
- $n = 0$: $(Q/m_N)^2$ relativistic correction to $\mathbf{j}^{(-2)}$
- unknown LECs enter the $n = 1$ contact and tree-level currents (the latter originates from a $\gamma\pi N$ vertex of order eQ^2)

- divergencies associated with loop integrals are reabsorbed by renormalization of contact terms
- loops contributions lead to purely isovector operators
- $\mathbf{j}^{(n\leq 1)}$ satisfies the CCR with χEFT two-nucleon potential $\upsilon^{(n\leq 2)}$

N^{3}LO: $\mathbf{j}^{(1)} \sim e\mathbf{Q}$

unknown LEC's →

EM current up to $n = 1$ (or up to N3LO)

- LECs of contact interactions at Q^0 and 'minimal' contact interactions at Q^2 fixed from fits to *np* phases shifts: LECs taken from Q^4 NN potential of D.R. Entem, R.Machleidt—PRC**68**, 041001 (2003)
- LECs from 'non-minimal' interactions fixed by reproducing EM observables: Different parameterizations are possible
- No three-body currents at N3LO

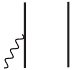
* Note: 2011 EM* operators different from 2009 EM** operators

- Different 'non-minimal' contact LECs
- Different parameterization of EM LECs
- Revised derivation of current of type (i) in 2009 EM (more on this issue on extra slides if interested)

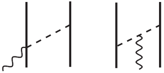
* in preparation, **PRC**80**, 034004 (2009)

EM current up to $n = 1$ (or up to N3LO) - bis

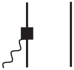
LO : $\mathbf{j}^{(-2)} \sim \mathbf{eQ}^{-2}$



NLO : $\mathbf{j}^{(-1)} \sim \mathbf{eQ}^{-1}$



N^{2}LO : $\mathbf{j}^{(-0)} \sim \mathbf{eQ}^{0}$



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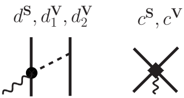
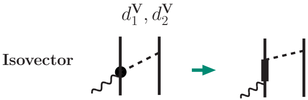
N^{3}LO: $\mathbf{j}^{(1)} \sim \mathbf{eQ}$

unknown LEC's →



Five LECs: d^S, d_1^V, and d_2^V could be determined by pion photo-production data on the nucleon



d_2^V and d_1^V are known assuming Δ-resonance saturation

$\left(d_2^V = 4\mu^* h_A/9m(m_\Delta - m)\right.$ and $\left. d_1^V = 4d_2^V\right)$

Left with 3 LECs: Fixed in the $A = 2-3$ nucleons' sector

- d^S and c^S: from EXPT μ_d and $\mu_S(^3\mathrm{H}/^3\mathrm{He})$
- c^V: from EXPT $\mu_V(^3\mathrm{H}/^3\mathrm{He})$ magnetic moment

Λ	NN/NNN	$10 \times d^S/\Lambda^2$	c^S/Λ^4	d_1^V/Λ^2	c^V/Λ^4
600	AV18/UIX	–2.033	5.238	4.980	–1.025

Different parameterizations have been studied in the $A = 2,3$ systems, and tested into the $A = 6,8$ nuclei (more on this topic on extra slides, if interested)

Minimize expectation value of H

$$E_V = \frac{\langle\Psi_V|H|\Psi_V\rangle}{\langle\Psi_V|\Psi_V\rangle} \geq E_0$$

using trial function

$$|\Psi_V\rangle = \left[\mathscr{S}\prod_{i<j}(1+U_{ij}+\sum_{k\neq i,j}U_{ijk})\right]\left[\prod_{i<j}f_c(r_{ij})\right]|\Phi_A(JMTT_3)\rangle$$

- single-particle $\Phi_A(JMTT_3)$ is fully antisymmetric and translationally invariant
- central pair correlations $f_c(r)$ keep nucleons at favorable pair separation
- pair correlation operators U_{ij} reflect influence of υ_{ij} (AV18)
- triple correlation operator U_{ijk} added when V_{ijk} (IL7) is present

Ψ_V are spin-isospin vectors in $3A$ dimensions with $\sim 2^A \binom{A}{Z}$ components

Lomnitz-Adler, Pandharipande, Smith, NP **A361**, 399 (1981) Wiringa, PRC **43**, 1585 (1991)

Given a decent trial function Ψ_V, we can further improve it by "filtering" out the remaining excited state contamination:

$$\Psi(\tau) = \exp[-(H-E_0)\tau]\Psi_V = \sum_n \exp[-(E_n-E_0)\tau]a_n\psi_n$$

$$\Psi(\tau\to\infty) = a_0\psi_0$$

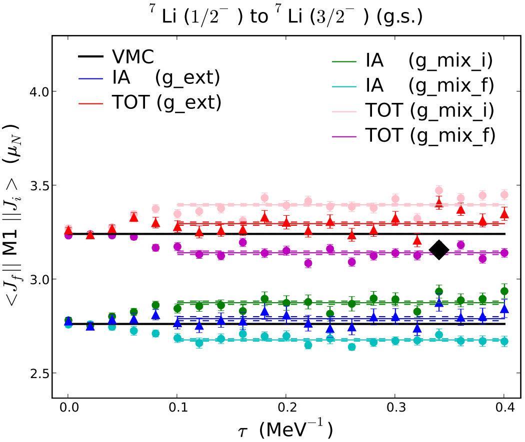
Evaluation of $\Psi(\tau)$ is done stochastically (Monte Carlo method) in small time steps $\Delta\tau$ using a Green's function formulation.
In practice, we evaluate a "mixed" estimates

$$\langle O(\tau)\rangle = \frac{{}_f\langle\Psi(\tau)|O|\Psi(\tau)\rangle_i}{\langle\Psi(\tau)|\Psi(\tau)\rangle} \approx \langle O(\tau)\rangle^i_{\mathrm{Mixed}} + \langle O(\tau)\rangle^f_{\mathrm{Mixed}} - \langle O\rangle_V$$

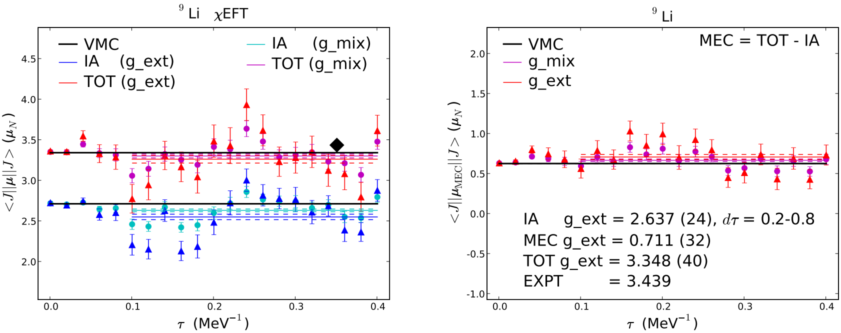
$$\langle O(\tau)\rangle^i_{\mathrm{Mixed}} = \frac{{}_f\langle\Psi_V|O|\Psi(\tau)\rangle_i}{{}_f\langle\Psi_V|\Psi(\tau)\rangle_i} \;;\; \langle O(\tau)\rangle^f_{\mathrm{Mixed}} = \frac{{}_f\langle\Psi(\tau)|O|\Psi_V\rangle_i}{{}_f\langle\Psi(\tau)|\Psi_V\rangle_i}$$

Pudliner, Pandharipande, Carlson, Pieper, & Wiringa, PRC **56**, 1720 (1997)
Wiringa, Pieper, Carlson, & Pandharipande, PRC **62**, 014001 (2000)
Pieper, Wiringa, & Carlson, PRC **70**, 054325 (2004)

Examples of GFMC propagation: M1 Transition in $A = 7$



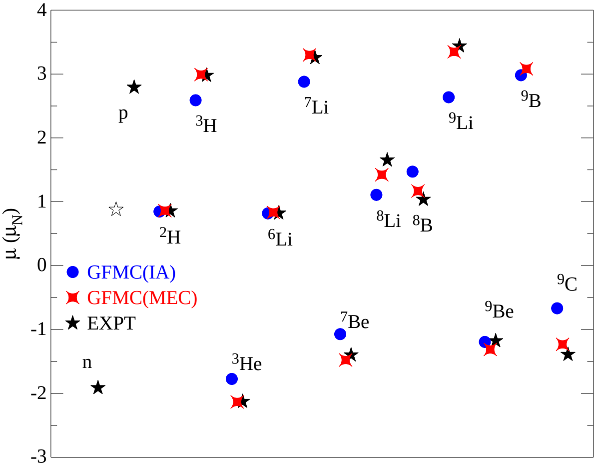
Examples of GFMC propagation: Magnetic moment in $A = 9$



Reduce noise by increasing the statistic for the IA results

GFMC calculation of magnetic moments in $A \leq 9$ nuclei: Summary

Predictions for $A > 3$ nuclei



Preliminary results

$$\mu(\text{IA}) = \mu_N \sum_i [(L_i + g_p S_i)(1 + \tau_{i,z})/2 + g_n S_i (1 - \tau_{i,z})/2]$$

Magnetic moments in $A \leq 9$ nuclei: SNPA and χEFT

	A	s.s.	SNPA	χEFT*	EXP
IS	7	[43]	0.840 (18)	0.911 (11)	0.929
IV		[43]	4.595 (36)	4.779 (22)	4.654
IS	8	[431]	1.178 (24)	1.292 (16)	1.344
IV		[431]	-0.146 (24)	-0.142 (16)	-0.310
IS	9 $[T = 3/2]$	[432]	0.927 (45)	1.058 (29)	1.024
IV		[432]	-1.415 (30)	-1.527 (19)	-1.610
IS	9 $[T = 1/2]$	[441]	0.787 (16)	0.884 (12)	n.a.
IV		[441]	4.207 (36)	4.395 (24)	n.a.

Preliminary results

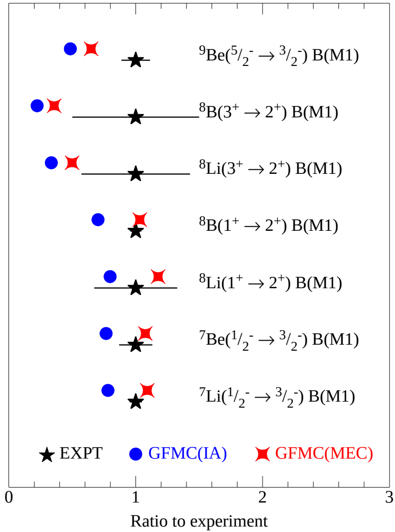
Overall improvement of isoscalar (IS) component of the magnetic moment

$$\mu = \mu_S + \tau_z \mu_V$$

GFMC calculation of M1 transitions in $A \leq 9$ nuclei: Summary

$$\mathrm{M1(IA)} = \mu_N \sum_i [(L_i + g_p S_i)(1+\tau_{i,z})/2 + g_n S_i (1-\tau_{i,z})/2]$$

Prediction for ^{9}Li$(1/2^- \to 3/2^-)$
M1 transition
$\Gamma^{\mathrm{IA}} = 5.93(10)\ (10^{-1}\ \mathrm{eV})$
$\Gamma^{\mathrm{TOT}} = 7.89(25)\ (10^{-1}\ \mathrm{eV})$



Preliminary results

Summary

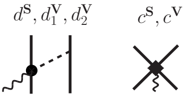
- EM current operators have been derived in χEFT up to $n = 1$
- Predictions from hybrid calculations of magnetic moment and M1 transitions in $A \leq 9$ nuclei are in good agreement with experimental data: Corrections of order $>$ LO are important to bring theory in agreement with experimental data

Outlook: electroweak properties of light nuclei

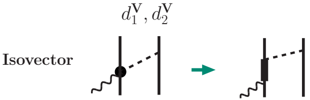
* EM structure of light nuclei
  - Extend hybrid calculations to different combinations of 2N and 3N potentials to study charge radii, charge and magnetic form factors of $A \leq 10$ systems (on going project)

* Weak structure of light nuclei
  - Extend hybrid calculations to weak properties of light nuclei

EXTRA SLIDES



Five LECs: d^S, d_1^V, and d_2^V could be determined by pion photo-production data on the nucleon



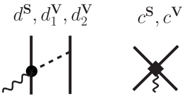
d_2^V and d_1^V are known assuming Δ-resonance saturation ($d_2^V/d_1^V = 1/4$)

Left with 5 LECs: Fixed in the $A = 2 - 3$ nucleons' sector

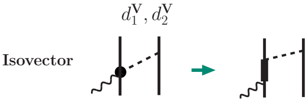
- Isoscalar sector:
  - d^S and c^S from EXPT μ_d and $\mu_S(^3\mathrm{H}/^3\mathrm{He})$

Λ	NN/NNN	$10 \times d^S/\Lambda^2$	c^S/Λ^4
600	AV18/UIX	–2.033	5.238

EM observables at N3LO: fixing LECs p.2/3



Five LECs: d^S, d_1^V, and d_2^V could be determined by pion photo-production data on the nucleon



d_2^V and d_1^V are known assuming Δ-resonance saturation ($d_2^V/d_1^V = 1/4$)

Left with 4 LECs: <u>Fixed in the $A = 2 - 3$ nucleons' sector</u>

- Isovector sector:
  - * I = c^V and d_1^V from EXPT $\mu_V(^3\mathrm{H}/^3\mathrm{He})$ m.m. and EXPT $npd\gamma$ xsec.

    or
  - * II = c^V from EXPT $npd\gamma$ xsec. and d_1^V from Δ-saturation

    or
  - * III = c^V from EXPT $\mu_V(^3\mathrm{H}/^3\mathrm{He})$ m.m. and d_1^V from Δ-saturation*

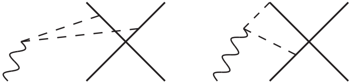
Λ	NN/NNN	Current	d_1^V/Λ^2	c^V/Λ^4
600	AV18/UIX	I	75.0	257.5
		II	4.98	-11.57
		III	4.98	-1.025

* $d_1^V = 4\mu^* h_A/9m(m_\Delta - m)$

Table: m.m.'s of ^{3}H, ^{7}Li, and ^{8}Li, with currents I or III and Λ=600 MeV from VMC

Nucleus	Current	IA	NLO	N2LO	N3LO	LECs(tree)	LECs(ct)	SUM
^{3}H	I	2.590	0.253	-0.033	0.091	1.612	-1.555	2.958
	III					0.102	-0.011	2.992
^{7}Li	I	2.899	0.254	-0.064	0.081	1.718	-1.855	3.033
	III					0.109	-0.011	3.268
^{8}Li	I	1.258	0.223	-0.039	0.088	1.084	-1.541	1.073
	III					0.066	-0.015	1.581

2009 EM current vs 2011 EM currents p. 1/2



$g)$ $i)$

- Non-static corrections entering single-nucleon operators accounted into the derivation of current i)

$$i)\,\mathrm{OLD} = i\frac{e g_A^2}{F_\pi^2}\,\tau_{1,z}\int \frac{\mathbf{q}_1-\mathbf{q}_2}{\omega_1^3\,\omega_2^3}\,\frac{\omega_1^2+\omega_1\,\omega_2+\omega_2^2}{\omega_1+\omega_2}\Big[C_S\,\boldsymbol{\sigma}_1\cdot(\mathbf{q}_1\times\mathbf{q}_2) - C_T\,\boldsymbol{\sigma}_2\cdot(\mathbf{q}_1\times\mathbf{q}_2)\Big] + 1 \rightleftharpoons 2$$

$$i)\,\mathrm{NEW} = 2\,i\,\frac{e g_A^2\,C_T}{F_\pi^2}\,\tau_{1z}\int_{\mathbf{q}_1,\mathbf{q}_2}\frac{\omega_1^2+\omega_1\,\omega_2+\omega_2^2}{\omega_1^3\omega_2^3(\omega_1+\omega_2)}\,(\mathbf{q}_1-\mathbf{q}_2)\,\boldsymbol{\sigma}_2\cdot\mathbf{q}_2\times\mathbf{q}_1 + 1 \rightleftharpoons 2$$

- i) NEW in agreement with Kölling 2009/2011* but for a factor of 2, which has no impact because (i + g) = 0

* PRC**80**, 045502 (2009)/ PRC **84**, 054008 (2011)

- A different derivation in Kölling 2009/2011* leads to an additional term $\sim (\boldsymbol{\sigma}_i \times \mathbf{q}) \times \mathbf{q}$ in the N3LO current at tree level, which however does not contribute to the magnetic moment
- The N3LO contact current of Pastore 2009 is in agreement with that of Kölling 2011 after Fierz-reordering, apart from differences in the term $\propto C_5$:

$$\mathbf{j}_{\rm ct}^{\rm N3LO} = -\frac{i\,C_5}{4}\,(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \times (e_1\,\mathbf{k}_1 + e_2\,\mathbf{k}_2)$$

* PRC**80**, 045502 (2009)/ PRC **84**, 054008 (2011)

Magnetic moment (m.m.) operator

- comparison with Kölling *et al.*:

  i) LO, NLO, N2LO, N3LO TPE, N3LO CT, N3LO TREE m.m.'s agree, but for the N3LO CT term $\propto C_5$
  ii) currents associated with one loop corrections to the OPE are missing in these calculations of m.m.'s; renormalization of OPE currents has been carried out in Kölling 2011*

- comparison with Park *et al.***:

  i) Sachs' m.m. is missing (no problem in two-body systems),
  ii) TPE box contribution at N3LO generates an extra term $\propto (\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_z$

* Ultimately, in actual calculations these differences are presumably mitigated by fitting LECs to experimental data

* loop corrections to OPE: terms $\propto L(k)$ in Eq. (4.28) of Kölling 2011;

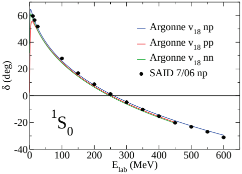
** NP **A596**, 515, (1996)

NUCLEAR HAMILTONIAN

$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

K_i: Non-relativistic kinetic energy, m_n-m_p effects included

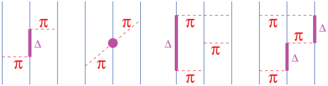
Argonne v_{18}: $v_{ij} = v_{ij}^{\gamma} + v_{ij}^{\pi} + v_{ij}^{I} + v_{ij}^{S} = \sum v_p(r_{ij}) O_{ij}^p$



- 18 spin, tensor, spin-orbit, isospin, etc., operators
- full EM and strong CD and CSB terms included
- predominantly local operator structure
- fits Nijmegen PWA93 data with χ^2/d.o.f.=1.1

Wiringa, Stoks, & Schiavilla, PRC **51**, (1995)

Urbana & Illinois: $V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{3\pi} + V_{ijk}^{R}$



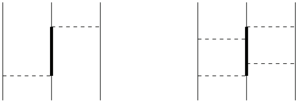
- Urbana has standard 2π P-wave + short-range repulsion for matter saturation
- Illinois adds 2π S-wave + 3π rings to provide extra T=3/2 interaction
- Illinois-7 has four parameters fit to 23 levels in $A \leq 10$ nuclei

Pieper, Pandharipande, Wiringa, & Carlson, PRC **64**, 014001 (2001)
Pieper, AIP CP **1011**, 143 (2008)

courtesy of R.B.Wiringa

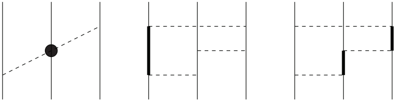
THREE-NUCLEON POTENTIALS

Urbana $V_{ijk} = V_{ijk}^{2\pi P} + V_{ijk}^{R}$



Carlson, Pandharipande, & Wiringa, NP **A401**, 59 (1983)

Illinois $V_{ijk} = V_{ijk}^{2\pi P} + V_{ijk}^{2\pi S} + V_{ijk}^{3\pi \Delta R} + V_{ijk}^{R}$

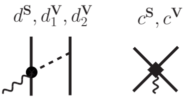


Pieper, Pandharipande, Wiringa, & Carlson, PRC **64**, 014001 (2001)

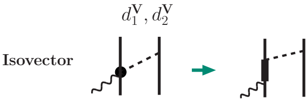
Illinois-7 has 4 strength parameters fit to 23 energy levels in $A \leq 10$ nuclei.
In light nuclei we find (thanks to large cancellation between $\langle K \rangle$ & $\langle v_{ij} \rangle$):

$$\langle V_{ijk} \rangle \sim (0.02 \text{ to } 0.07)\ \langle v_{ij} \rangle \sim (0.15 \text{ to } 0.5)\ \langle H \rangle$$

We expect $\langle \mathcal{V}_{ijkl} \rangle \sim 0.05\ \langle V_{ijk} \rangle \sim (0.01 \text{ to } 0.03)\ \langle H \rangle \sim 1$ MeV in ^{12}C .



Five LECs: d^S, d_1^V, and d_2^V could be determined by pion photo-production data on the nucleon

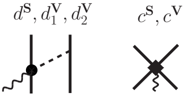


d_2^V and d_1^V are known assuming Δ-resonance saturation ($d_2^V/d_1^V = 1/4$)

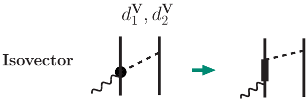
Left with 5 LECs: Fixed in the $A = 2-3$ nucleons' sector

- Isoscalar sector:
  - d^S and c^S from EXPT μ_d and $\mu_S(^3\mathrm{H}/^3\mathrm{He})$

Λ	NN/NNN	$10 \times d^S/\Lambda^2$	c^S/Λ^4
600	AV18/UIX	–2.033	5.238



Five LECs: d^S, d_1^V, and d_2^V could be determined by pion photo-production data on the nucleon



d_2^V and d_1^V are known assuming Δ-resonance saturation ($d_2^V/d_1^V = 1/4$)

Left with 4 LECs: Fixed in the $A = 2-3$ nucleons' sector

- Isovector sector:
  - * I = c^V and d_1^V from EXPT $\mu_V(^3\mathrm{H}/^3\mathrm{He})$ m.m. and EXPT $npd\gamma$ xsec.

    or
  - * II = c^V from EXPT $npd\gamma$ xsec. and d_1^V from Δ-saturation

    or
  - * III = c^V from EXPT $\mu_V(^3\mathrm{H}/^3\mathrm{He})$ m.m. and d_1^V from Δ-saturation*

Λ	NN/NNN	Current	d_1^V/Λ^2	c^V/Λ^4
600	AV18/UIX	I	75.0	257.5
		II	4.98	-11.57
		III	4.98	-1.025

* $d_1^V = 4\mu^* h_A/9m(m_\Delta - m)$

Table: m.m.'s of ^{3}H, ^{7}Li, and ^{8}Li, with currents I or III and Λ=600 MeV from VMC

Nucleus	Current	IA	NLO	N2LO	N3LO	LECs(tree)	LECs(ct)	SUM
^{3}H	I	2.590	0.253	-0.033	0.091	1.612	-1.555	2.958
	III					0.102	-0.011	2.992
^{7}Li	I	2.899	0.254	-0.064	0.081	1.718	-1.855	3.033
	III					0.109	-0.011	3.268
^{8}Li	I	1.258	0.223	-0.039	0.088	1.084	-1.541	1.073
	III					0.066	-0.015	1.581

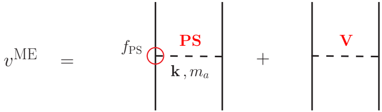
Magnetic moment at N^{3}LO

- Magnetic moment operator due to two-body current density $\mathbf{J}(\mathbf{x})$

$$\boldsymbol{\mu}(\mathbf{R},\mathbf{r}) = \frac{1}{2}\left[\mathbf{R}\times\int \mathrm{d}\mathbf{x}\,\mathbf{J}(\mathbf{x}) + \int \mathrm{d}\mathbf{x}\,(\mathbf{x}-\mathbf{R})\times\mathbf{J}(\mathbf{x})\right]$$

- Sachs' and translationally invariant magnetic moments

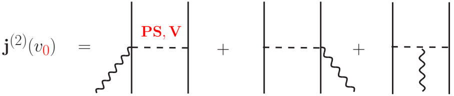
$$\begin{aligned}
\boldsymbol{\mu}_{\text{Sachs}}(\mathbf{R},\mathbf{r}) &= -i\,\frac{\mathbf{R}}{2}\times\int \mathrm{d}\mathbf{x}\,\mathbf{x}\,[\rho(\mathbf{x})\,,\,\upsilon_{12}] \\
\boldsymbol{\mu}_{\text{T}}(\mathbf{r}) &= -\frac{i}{2}\int_{\mathbf{k}} \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}}\,\nabla_q\times\mathbf{j}(\mathbf{q},\mathbf{k})\Big|_{\mathbf{q}=0}
\end{aligned}$$



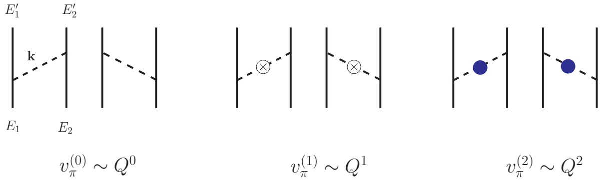
- Exploiting the meson exchange (ME) mechanism, one assumes that the static part υ_0 of υ is due to pseudoscalar (PS) and vector (V) exchanges
- υ^{ME} is expressed in terms of 'effective propagators' υ_{PS}, υ_V, υ_{VS}, fixed such to reproduce υ_0, for example

$$\upsilon_{PS} = [\upsilon^{\sigma\tau}(k) - 2\upsilon^{t\tau}(k)]/3$$

  with $\upsilon^{\sigma\tau}$ and $\upsilon^{t\tau}$ components of υ_0
- The current operator is obtained by taking the non relativistic reduction of the ME Feynman amplitudes and replacing the bare propagators with the 'effective' ones



OPEP beyond the static limit



On-the-energy-shell, non-static OPEP at N2LO (Q^2) can be equivalently written as

$$\upsilon_\pi^{(2)}(\nu=0) = \upsilon_\pi^{(0)}(\mathbf{k})\,\frac{(E_1'-E_1)^2+(E_2'-E_2)^2}{2\,\omega_k^2}$$

$$\upsilon_\pi^{(2)}(\nu=1) = -\upsilon_\pi^{(0)}(\mathbf{k})\,\frac{(E_1'-E_1)\,(E_2'-E_2)}{\omega_k^2}$$

$$\upsilon_\pi^{(0)}(\mathbf{k}) = -\frac{g_A^2}{F_\pi^2}\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,\frac{\boldsymbol{\sigma}_1\cdot\mathbf{k}\,\boldsymbol{\sigma}_2\cdot\mathbf{k}}{\omega_k^2}$$

$\upsilon_\pi^{(2)}(\nu)$ corrections are different off-the-energy-shell $(E_1+E_2\neq E_1'+E_2')$

- TPE contributions are affected by the choice made for the parameter ν

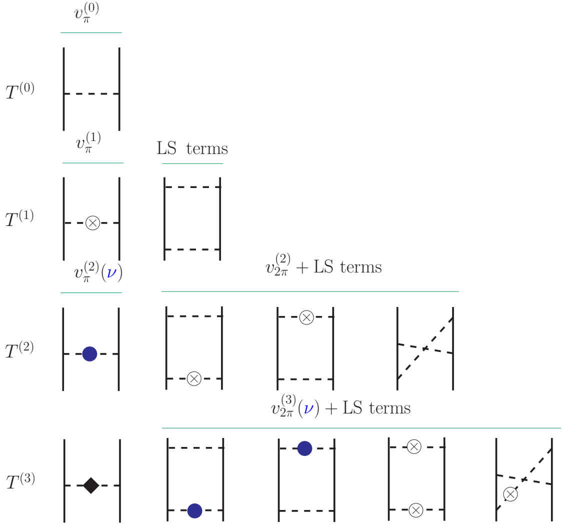
The two-nucleon potential $\upsilon = \upsilon^{(0)} + \upsilon^{(1)} + \upsilon^{(2)} + \dots$ (with $\upsilon^{(n)} \sim Q^n$) is iterated into the Lippmann-Schwinger (LS) equation *i.e.*

$$\upsilon + \upsilon\, G_0\, \upsilon + \upsilon\, G_0\, \upsilon\, G_0\, \upsilon + \dots\,, \qquad\qquad G_0 = 1/(E_i - E_I + i\,\eta)$$

$\upsilon^{(n)}$ is obtained subtracting from the transition amplitude $T_{\rm fi}^{(n)}$ terms already accounted for into the LS equation

$$\begin{aligned}
\upsilon^{(0)} &= T^{(0)}\,, \\
\upsilon^{(1)} &= T^{(1)} - \left[\upsilon^{(0)}\, G_0\, \upsilon^{(0)}\right]\,, \\
\upsilon^{(2)} &= T^{(2)} - \left[\upsilon^{(0)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)}\right] - \left[\upsilon^{(1)}\, G_0\, \upsilon^{(0)} + \upsilon^{(0)}\, G_0\, \upsilon^{(1)}\right]\,, \\
\upsilon^{(3)}(\nu) &= T^{(3)} - \left[\upsilon^{(0)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)}\right] - \left[\upsilon^{(1)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)} + \text{permutations}\right] \\
&\quad \underbrace{- \left[\upsilon^{(1)}\, G_0\, \upsilon^{(1)}\right] - \left[\upsilon^{(2)}(\nu)\, G_0\, \upsilon^{(0)} + \upsilon^{(0)}\, G_0\, \upsilon^{(2)}(\nu)\right]}_{\text{LS terms}}
\end{aligned}$$

From amplitudes to potentials: an example with OPE and TPE only



- To each $\upsilon_\pi^{(2)}(\nu)$ corresponds a $\upsilon_{2\pi}^{(3)}(\nu)$

Unitary equivalence of $\upsilon_{\pi}^{(2)}(\nu)$ and $\upsilon_{2\pi}^{(3)}(\nu)$

- Different off-the-energy-shell parameterizations lead to unitarily equivalent two-nucleon Hamiltonians

$$H(\nu) = t^{(-1)} + \upsilon_{\pi}^{(0)} + \upsilon_{2\pi}^{(2)} + \upsilon_{\pi}^{(2)}(\nu) + \upsilon_{2\pi}^{(3)}(\nu)$$

$t^{(-1)}$ is the kinetic energy, $\upsilon_{\pi}^{(0)}$ and $\upsilon_{2\pi}^{(2)}$ are the static OPEP and TPEP

- The Hamiltonians are related to each other via

$$H(\nu) = \mathrm{e}^{-iU(\nu)} H(\nu=0)\, \mathrm{e}^{+iU(\nu)} \,, \qquad i\,U(\nu) \simeq i\,U^{(0)}(\nu) + i\,U^{(1)}(\nu)$$

from which it follows

$$H(\nu) = H(\nu=0) + \left[t^{(-1)} + \upsilon_{\pi}^{(0)} \,, i\,U^{(0)}(\nu)\right] + \left[t^{(-1)}, i\,U^{(1)}(\nu)\right]$$

- Predictions for physical observables are unaffected by off-the-energy-shell effects

From amplitudes to EM charge and current operators

- In presence of EM interaction the transition amplitude T_γ is expanded as

$$T_\gamma = T_\gamma^{(-3)} + T_\gamma^{(-2)} + T_\gamma^{(-1)} + \ldots, \qquad T_\gamma^{(n)} \sim e\,Q^n$$

and the charge and current operators are related to $T_\gamma^{(n)}$ via

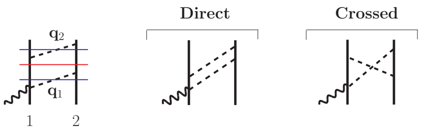
$$\upsilon_\gamma^{(n)} = A^0\,\rho^{(n)} - \mathbf{A}\cdot\mathbf{j}^{(n)} = T_\gamma^{(n)} - \text{ LS terms}$$

that is

$$\begin{aligned}
\upsilon_\gamma^{(-3)} &= T_\gamma^{(-3)}\,, \\
\upsilon_\gamma^{(-2)} &= T_\gamma^{(-2)} - \left[\upsilon_\gamma^{(-3)}\,G_0\,\upsilon^{(0)} + \upsilon^{(0)}\,G_0\,\upsilon_\gamma^{(-3)}\right]\,, \\
\upsilon_\gamma^{(-1)} &= T_\gamma^{(-1)} \underbrace{- \left[\upsilon_\gamma^{(-3)}\,G_0\,\upsilon^{(0)}\,G_0\,\upsilon^{(0)} + \text{permutations}\right] - \left[\upsilon_\gamma^{(-2)}\,G_0\,\upsilon^{(0)} + \upsilon^{(0)}\,G_0\,\upsilon_\gamma^{(-2)}\right]}_{\text{LS terms}}
\end{aligned}$$

$\ldots\ldots$

Technical issue II - Recoil corrections at N^{3}LO

$\mathbf{j}^{\mathrm{N^3LO}} =$ 

- Reducible contributions

$$
\begin{aligned}
\mathbf{j}_{\mathrm{red}} \quad &\sim \quad \int \upsilon^{\pi}(\mathbf{q}_2)\,\frac{1}{E_i - E_I}\,\mathbf{j}^{\mathrm{NLO}}(\mathbf{q}_1) \\
&- \quad \int 2\,\frac{\omega_1+\omega_2}{\omega_1\,\omega_2}\,V_{\pi NN}(2,\mathbf{q}_2)\,V_{\pi NN}(2,\mathbf{q}_1)\,V_{\pi NN}(1,\mathbf{q}_2)\,V_{\gamma\pi NN}(1,\mathbf{q}_1)
\end{aligned}
$$

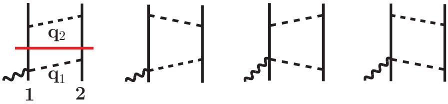
- Irreducible contributions

$$
\begin{aligned}
\mathbf{j}_{\mathrm{irr}} \quad &= \quad \int 2\,\frac{\omega_1+\omega_2}{\omega_1\,\omega_2}\,V_{\pi NN}(2,\mathbf{q}_2)\,V_{\pi NN}(2,\mathbf{q}_1)\,V_{\pi NN}(1,\mathbf{q}_2)\,V_{\gamma\pi NN}(1,\mathbf{q}_1) \\
&- \quad \int 2\,\frac{\omega_1^2+\omega_2^2+\omega_1\,\omega_2}{\omega_1\,\omega_2(\omega_1+\omega_2)}[V_{\pi NN}(2,\mathbf{q}_2),V_{\pi NN}(2,\mathbf{q}_1)]_{-}\,V_{\pi NN}(1,\mathbf{q}_2)V_{\gamma\pi NN}(1,\mathbf{q}_1)
\end{aligned}
$$

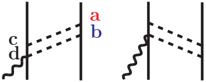
- Observed partial cancellations at N^{3}LO between recoil corrections to reducible diagrams and irreducible contributions

The box diagram: an example at N^{3}LO

Reducible



Irreducible direct

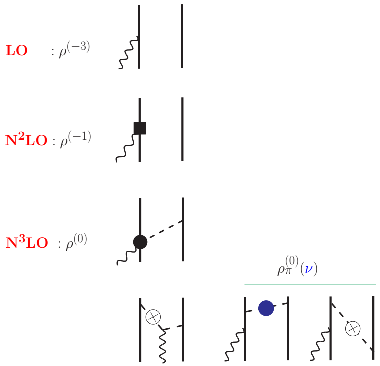


Irreducible crossed

$$
\begin{aligned}
\text{direct} &= f_d(\omega_1,\omega_2)\, V_a\, V_b\, V_c\, V_d \\
\text{crossed} &= f_c(\omega_1,\omega_2)\, V_b\, V_a\, V_c\, V_d
\end{aligned}
\qquad V_b V_a = V_a V_b - [V_a, V_b]_-
$$

$$
\begin{aligned}
\text{irreducible} &= [\,f_d(\omega_1,\omega_2) + f_c(\omega_1,\omega_2)]\, V_a\, V_b\, V_c\, V_d \\
&- f_c(\omega_1,\omega_2)\, [V_a, V_b]_-\, V_c\, V_d
\end{aligned}
$$

EM charge up to $n = 0$ (or up to N3LO)



- $n = -3$

  $\rho^{(-3)}(\mathbf{q}) = e\,(2\pi)^3\delta(\mathbf{p}_1 + \mathbf{q} - \mathbf{p}'_1)\,(1 + \tau_{1,z})/2 + 1 \rightleftharpoons 2$

- $n = -1$:

  $(Q/m_N)^2$ relativistic correction to $\rho^{(-3)}$

- $n = 0$:

  i) 'static' tree-level current (originates from a $\gamma\pi N$ vertex of order $e\,Q$)

  ii) 'non-static' OPE charge operators, $\rho_\pi^{(0)}(\nu)$ depends on $\upsilon_\pi^{(2)}(\nu)$

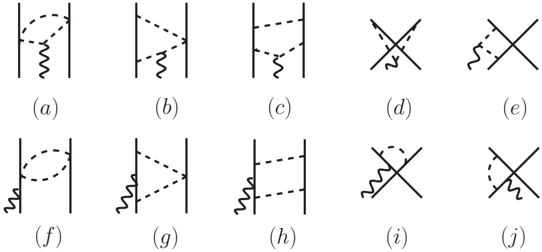
- $\rho_\pi^{(0)}(\nu)$'s are unitarily equivalent

$$\rho_\pi^{(0)}(\nu) \quad = \quad \rho_\pi^{(0)}(\nu = 0) + \left[\rho^{(-3)}\,,\, i\,U^{(0)}(\nu)\right]$$

- No unknown LECs up to this order (g_A, F_π)

EM charge @ $n = 1$ (or N4LO) 1.

N^{4}LO : $\rho^{(1)}$

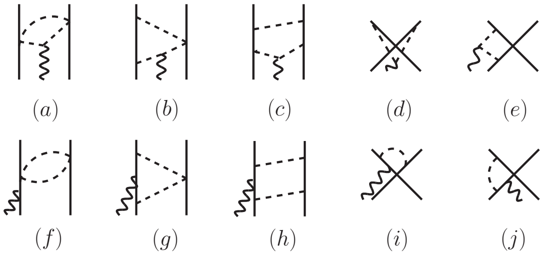


- (a), (f), (d), and (i) vanish
- Divergencies associated with (b) + (g), (c) + (h), and (e) + (j) cancel out—as they must since there are no counter-terms at N4LO
- $\rho_{\rm h}^{(1)}(\nu)$ depends on the parametrization adopted for $\upsilon_\pi^{(2)}(\nu)$ and $\upsilon_{2\pi}^{(3)}(\nu)$
- $\rho_{\rm h}^{(1)}(\nu)$'s are unitarily equivalent

$$\rho_{\rm h}^{(1)}(\nu) \quad = \quad \rho_{\rm h}^{(1)}(\nu = 0) + \left[\rho^{(-3)}, i\, U^{(1)}(\nu)\right]$$

EM charge @ $n = 1$ (or N4LO) 2.

N^{4}LO : $\rho^{(1)}$



- Charge operators (ν-dependent included) up to $n = 1$ satisfy the condition

$$\rho^{(n>-3)}(\mathbf{q} = 0) = 0$$

which follows from charge conservation

$$\rho(\mathbf{q} = 0) = \int \mathrm{d}\mathbf{x}\, \rho(\mathbf{x}) = e\frac{(1+\tau_{1,z})}{2} + 1 \rightleftharpoons 2 = \rho^{(-3)}(\mathbf{q} = 0)$$

- $\rho^{(1)}$ does not depend on unknown LECs and it is purely isovector