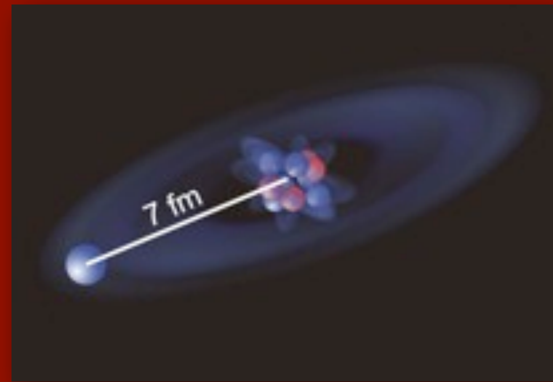
Neutron-rich Helium Isotopes based on Hyper-spherical Harmonics

Sonia Bacca | Theory Group | TRIUMF

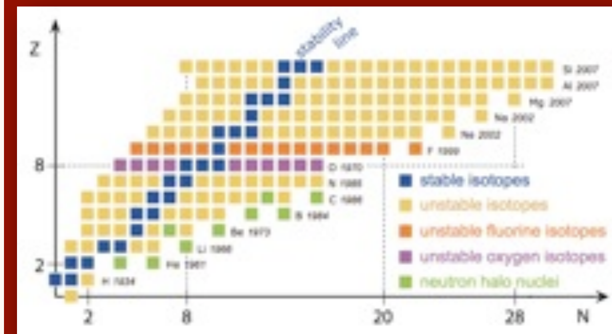
- Motivation
- The Hyper-spherical Harmonics approach
- Results for ^{6}He:
  - Energies and radii
  - Nuclear electric polarizability
- Outlook

The 7th International Workshop on Chiral Dynamics

August 6 - 10, 2012
Jefferson Lab



Nuclear Halo

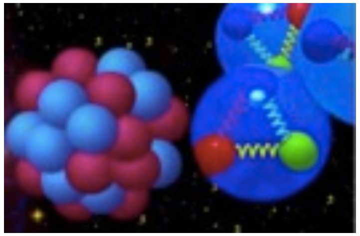


Nuclear Chart

Astrophysics

Neutron-rich Helium Isotopes

- Develop a unified theory for nuclei and connect it to QCD via Chiral Effective Field Theory



V, J^μ

$$V = V_{NN} + V_{3N} + \dots$$
$$J^\mu = J^\mu_N + J^\mu_{NN} + \dots$$

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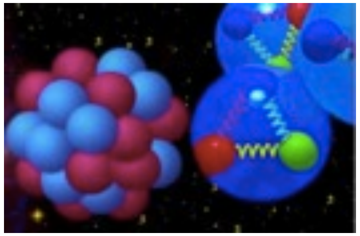

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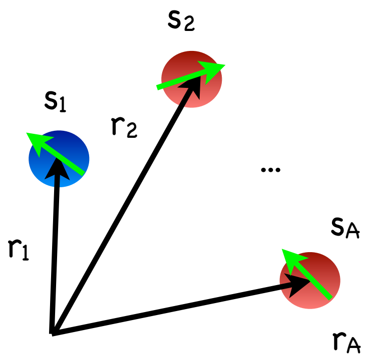
  start from neutrons and protons and solve the non-relativistic quantum mechanical problem of A-interacting nucleons



$$H|\psi_i\rangle = E_i|\psi_i\rangle \qquad H = T + V_{NN} + V_{3N} + \ldots$$

with no approximation or controllable approximations

Calculate low-energy observables form the A-body wave function and compare with experiment to test nuclear forces

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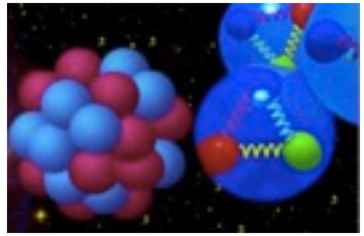

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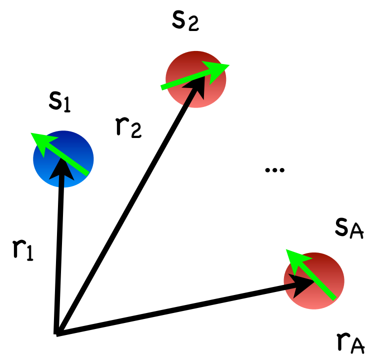

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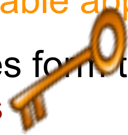
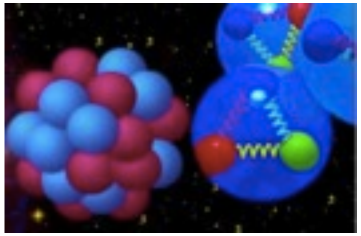


Neutron-rich Helium Isotopes



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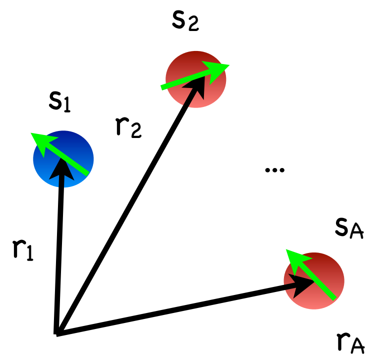

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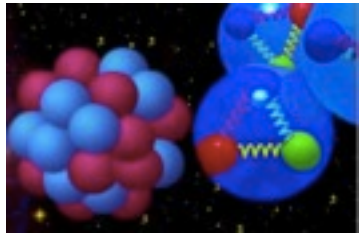

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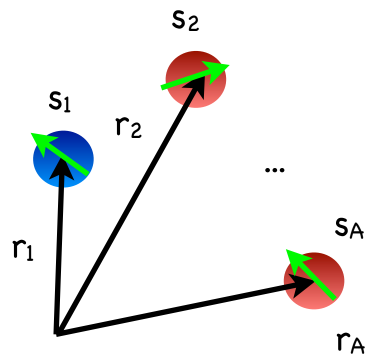

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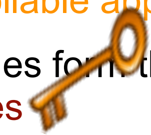

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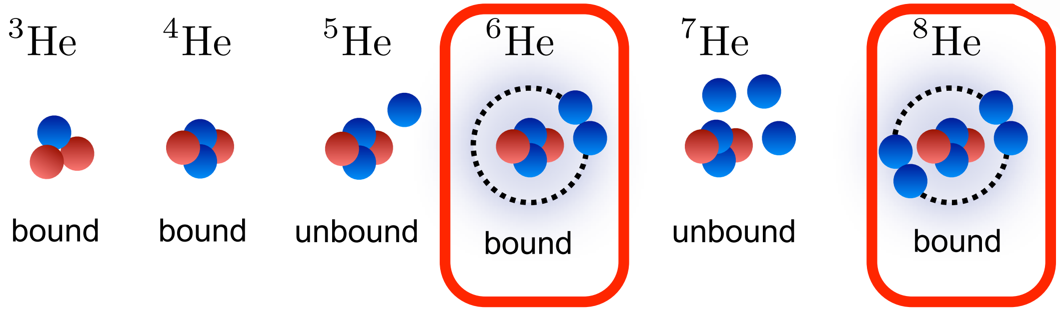
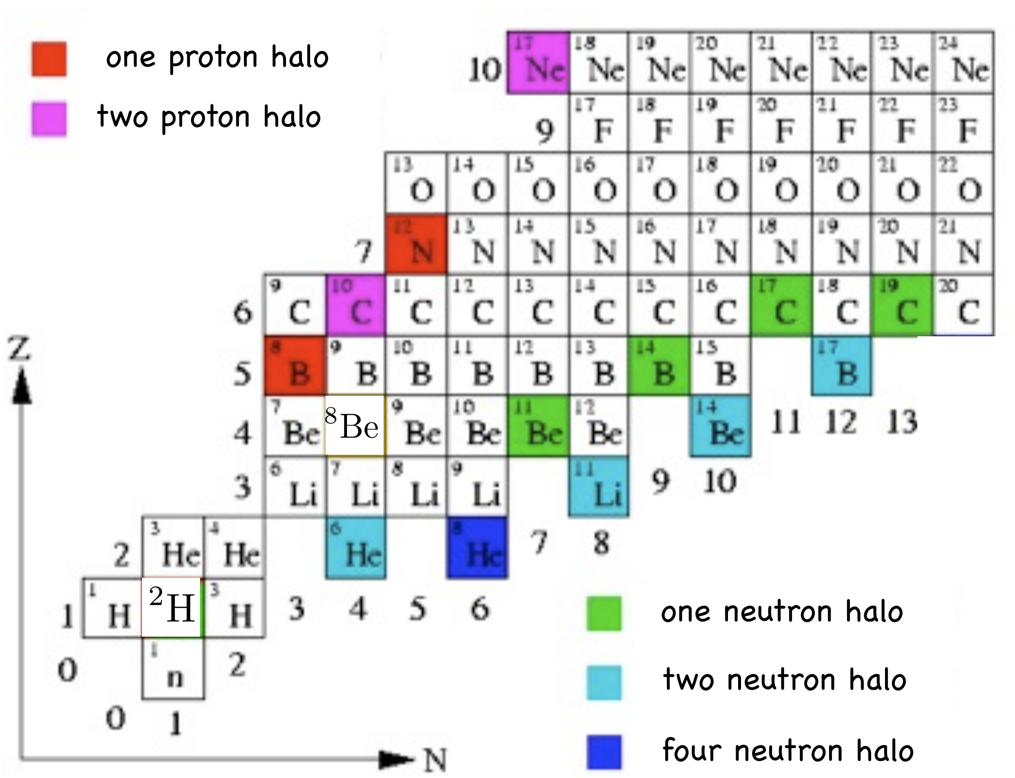
Helium chain

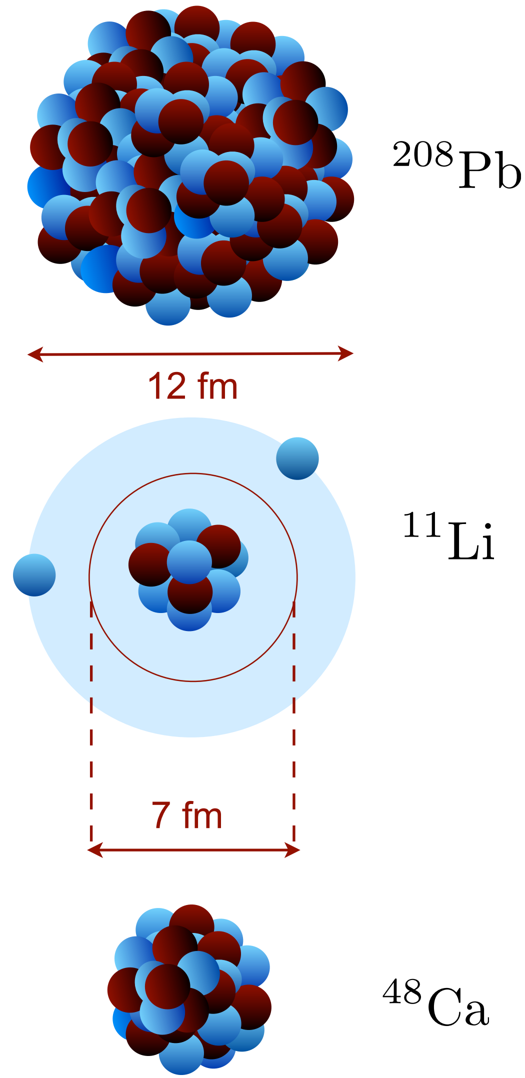


Are heavier bound isotopes?

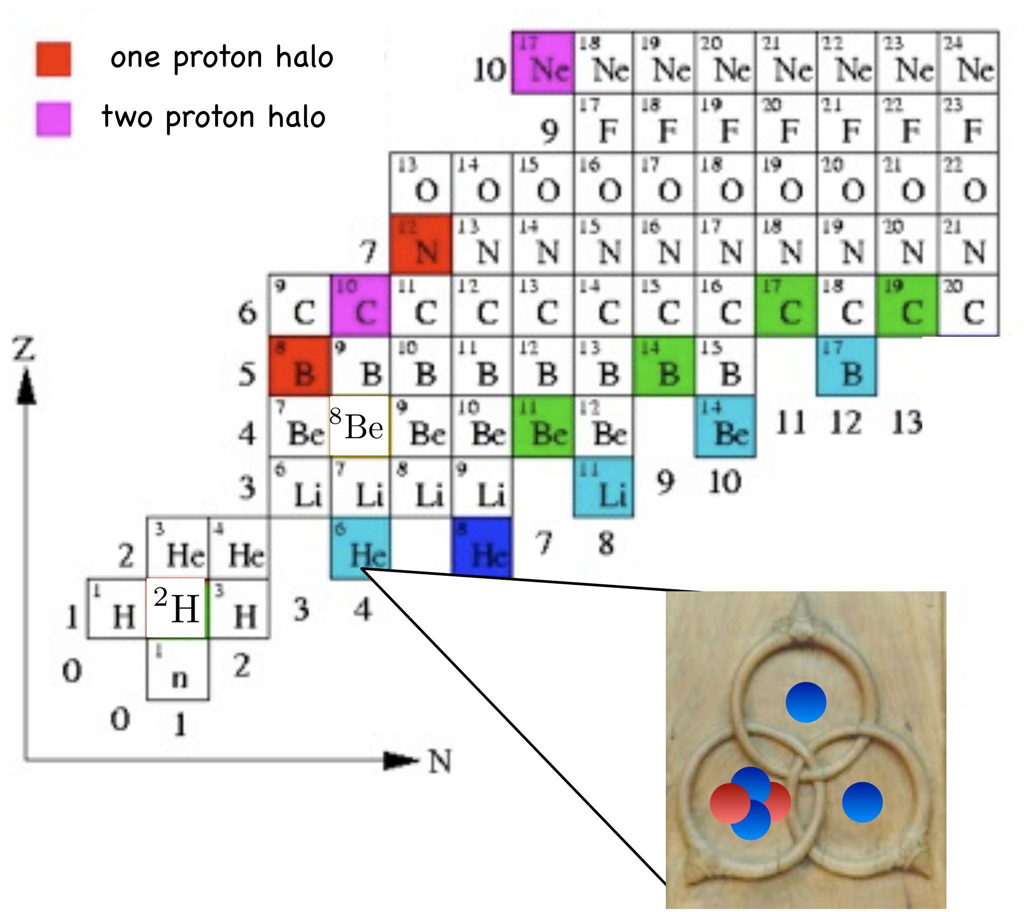
Monday, 6 August, 12

Halo Nuclei

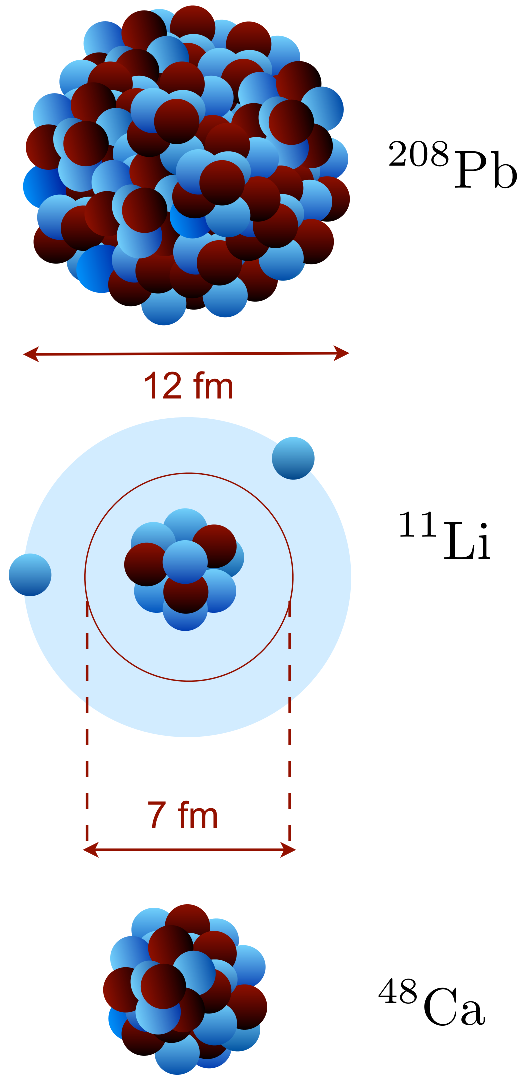

one proton halo
two proton halo
one neutron halo
two neutron halo
four neutron halo
Z
N


^{208}Pb
12 fm
^{11}Li
7 fm
^{48}Ca

Halo Nuclei



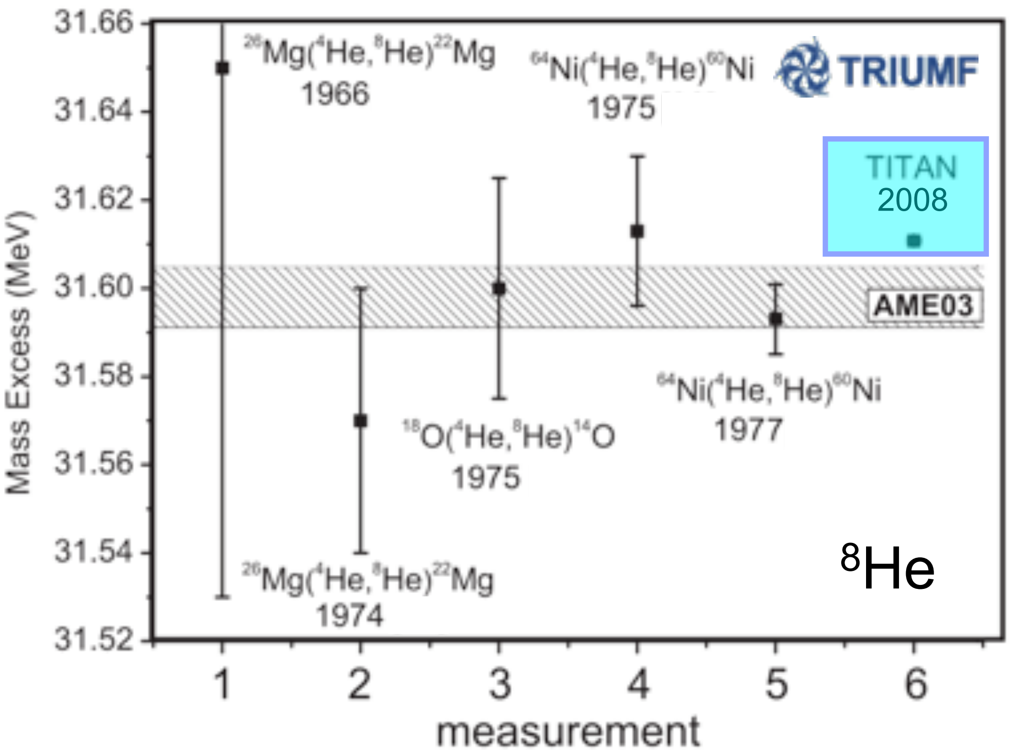
Lightest halo nucleus
and Borromean system



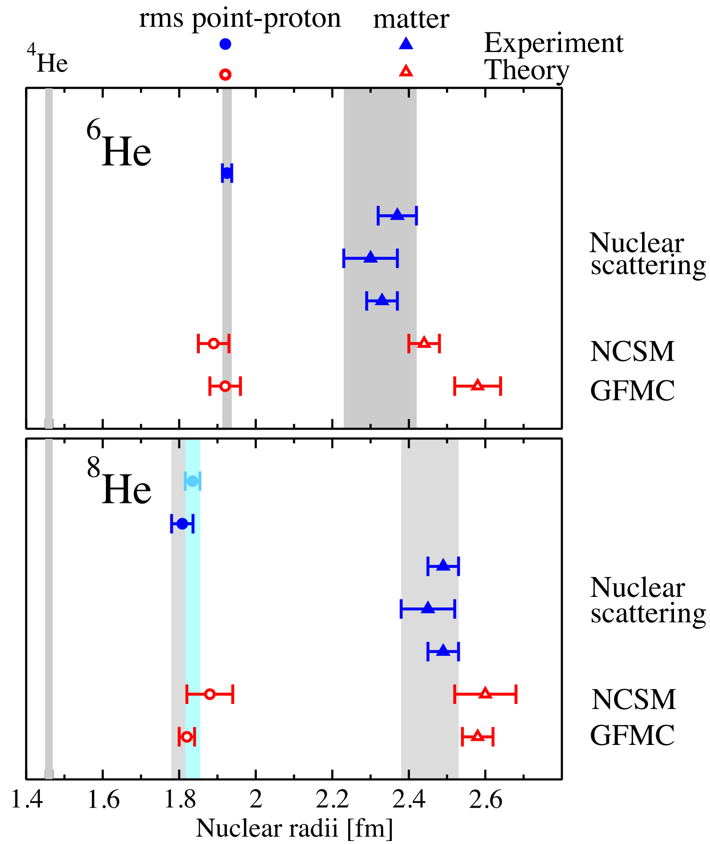
Halo Nuclei - Experiment

New Era of Precision Measurements for masses and radii

- High-precision Penning trap and laser spectroscopy techniques allow accurate measurements of **energies and charge radii** of exotic isotopes ➡ challenge for ab initio calculations



TRIUMF, Ryjkov et al. PRL 101, 012501 (2008)



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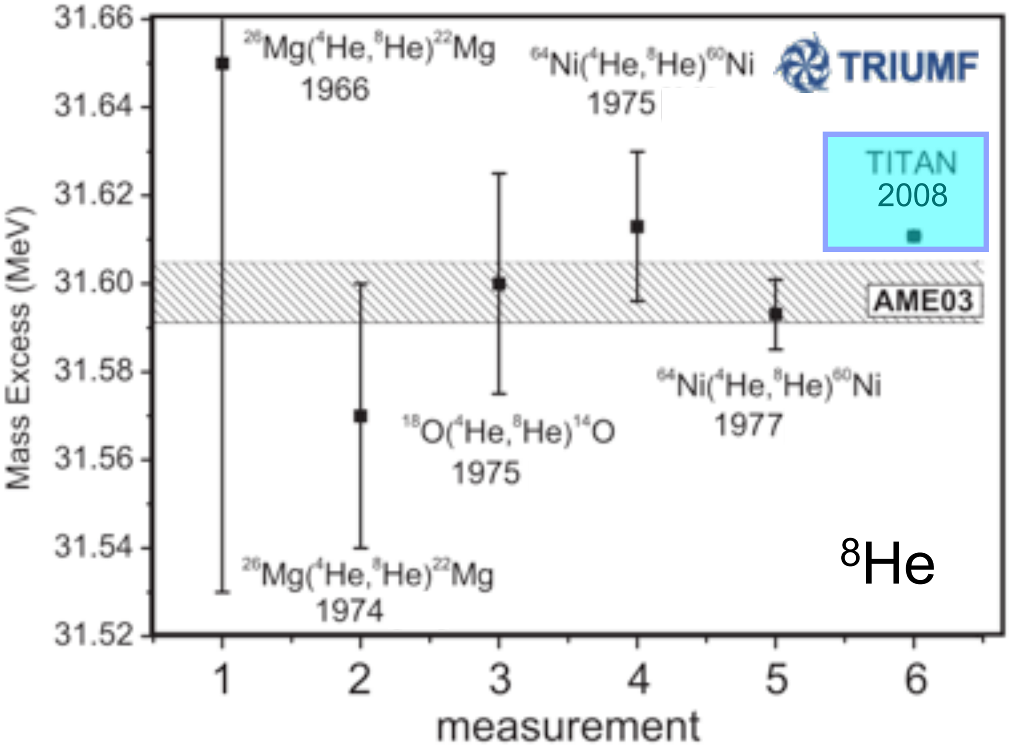

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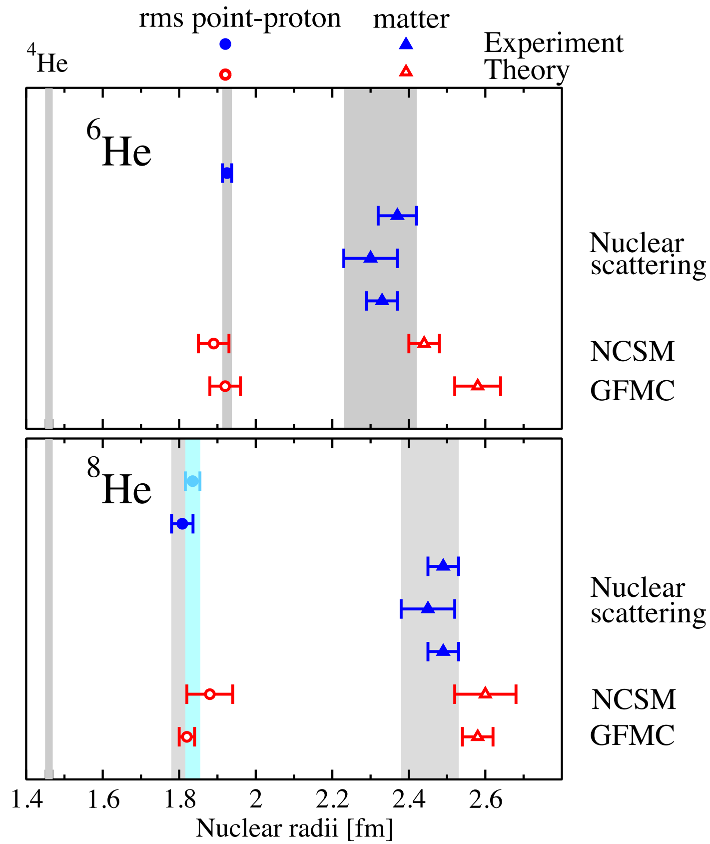

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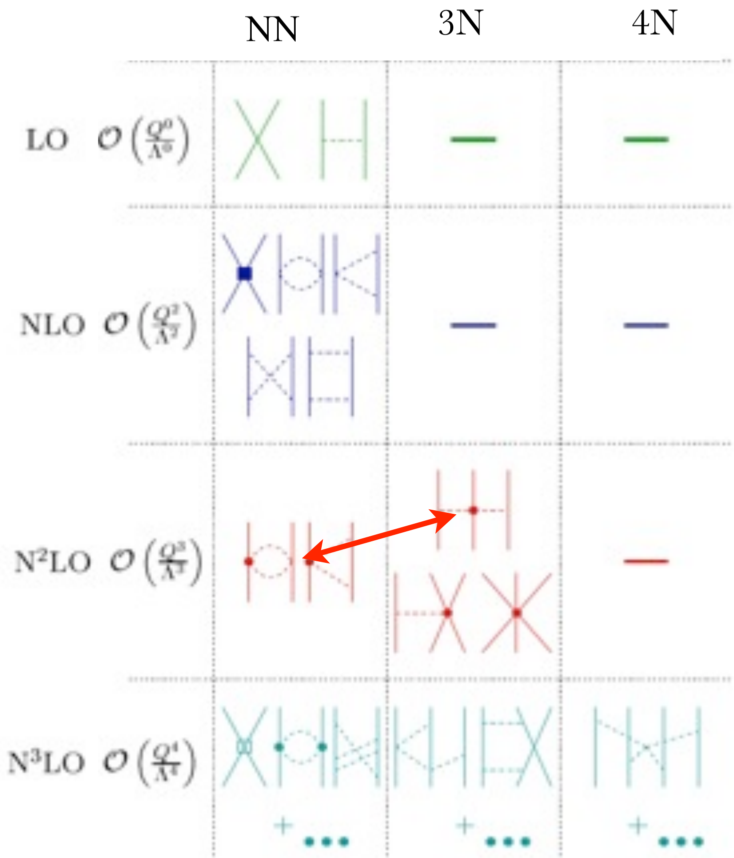
Missing: Predictions for EFT

Nuclear Forces from chiral EFT

$$H(\Lambda) = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + V_{4N}(\Lambda) + \ldots$$



$$V_{NN} > V_{3N} > V_{4N}$$

Weinberg, van Kolck, Kaplan, Savage, Wise,
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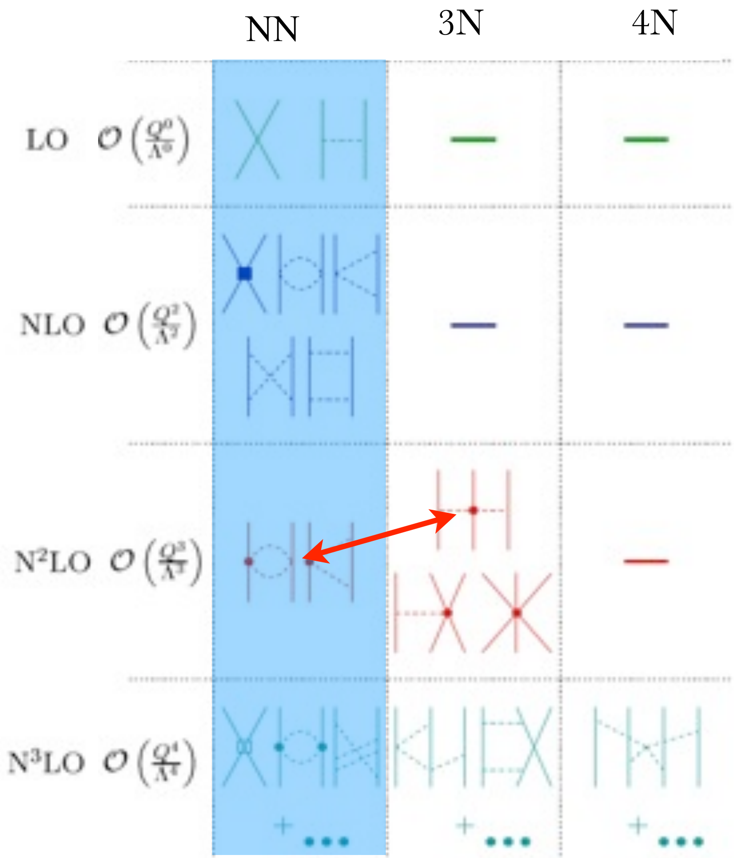

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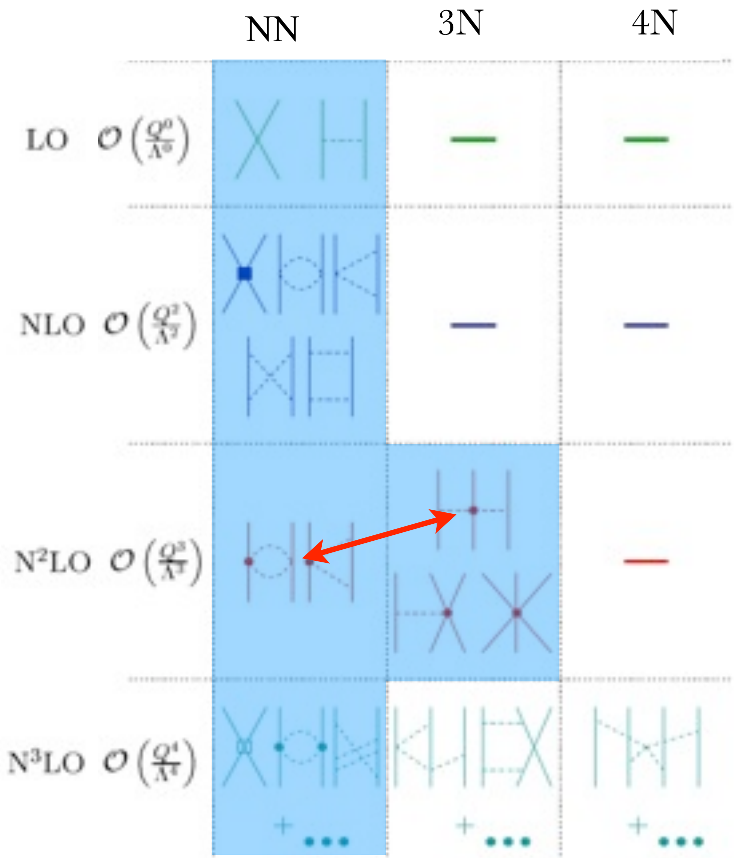

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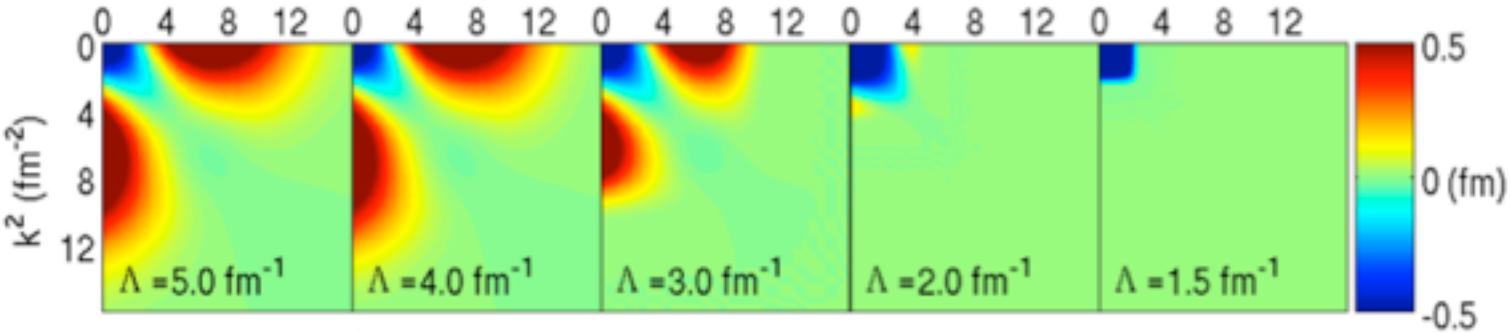
Low-momentum Forces from chiral EFT

Effective field theory potentials and low-momentum evolution

Evolution of 2N forces: phase-shift equivalent

Low-momentum interactions: Bogner, Kuo, Schwenk (2003) need smaller basis

Like acting with a unitary transformation $U^{-1}VU$ still preserve phase-shifts and properties of 2N systems



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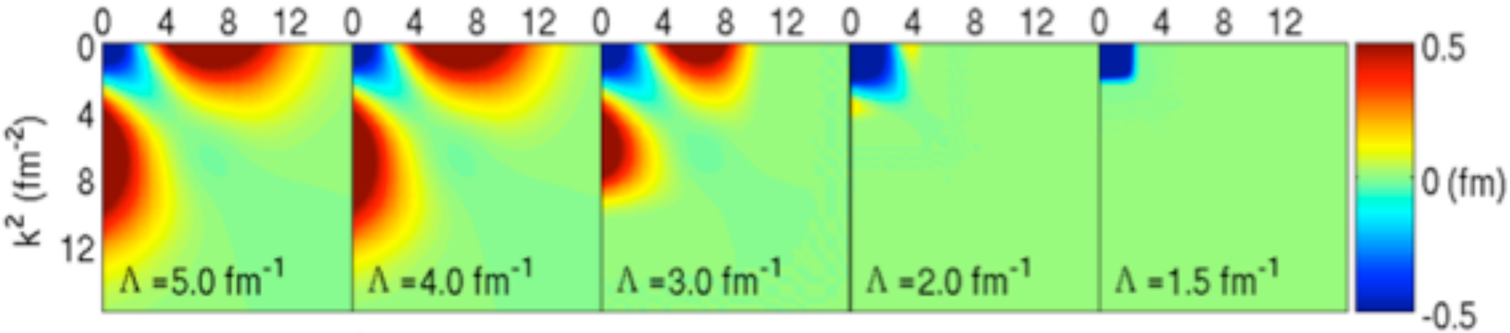

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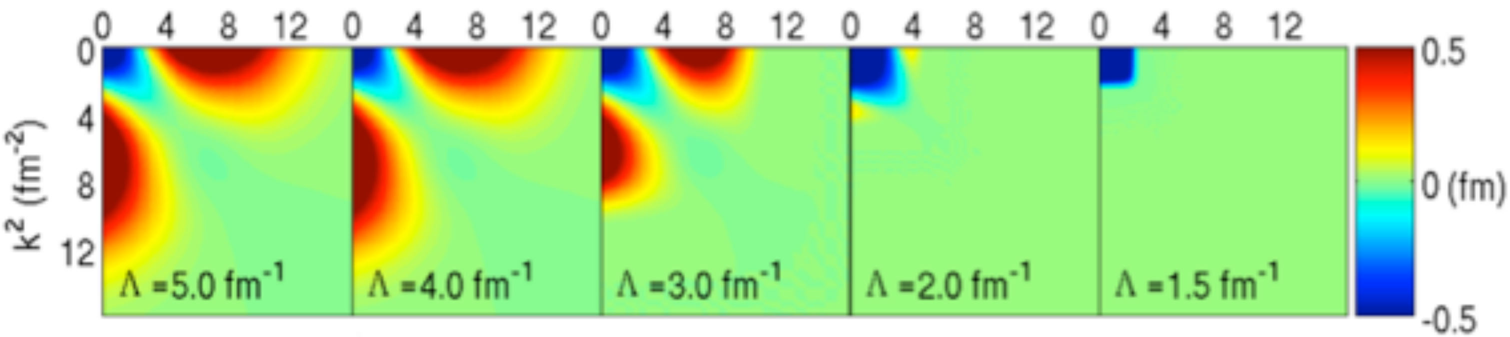

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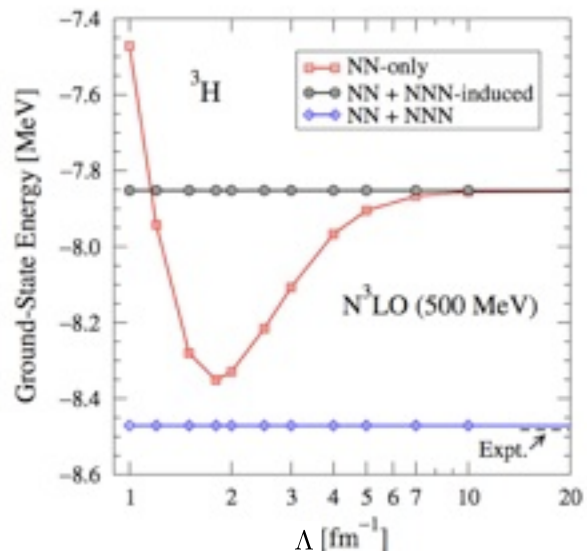
Can evolve consistently 3N forces: Jurgenson, Navratil, Furnstahl, (2009)



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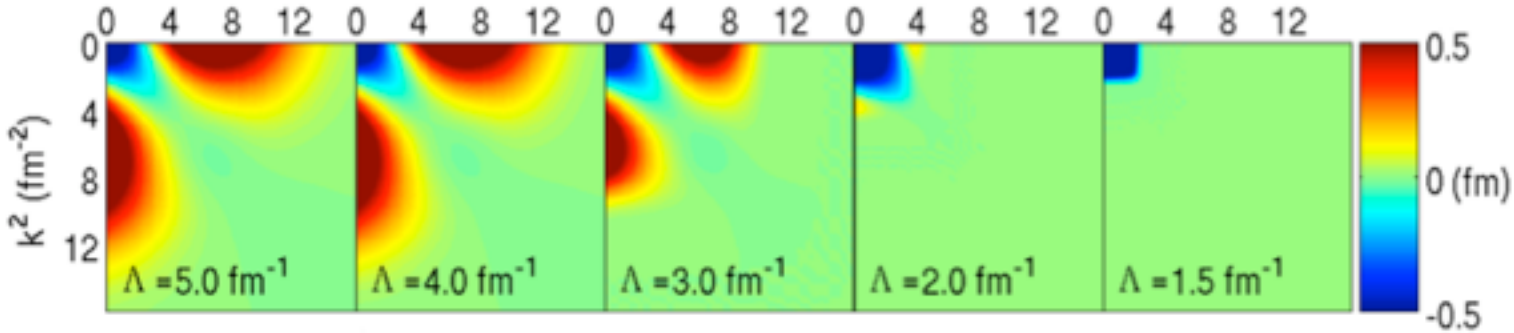

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Variation of the cutoff provides a tool to estimate the effect of the short-range 3N forces

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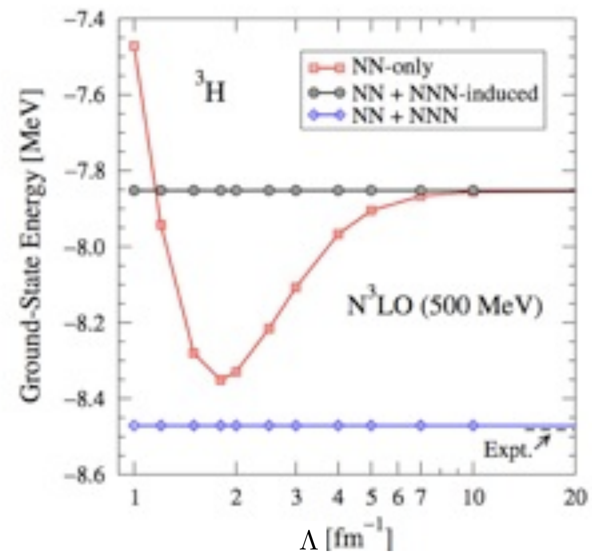

Hyperspherical Harmonics Expansions

A basis set, that can be used to solve the Schroedinger equation by expanding the w.f. on a complete basis states

$$H\left|\psi\right\rangle = E\left|\psi\right\rangle \qquad \left|\psi\right\rangle = \sum_i^{\infty} c_i \left|\psi_i\right\rangle$$

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cannot store an infinite vector

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Eigenvalue problem for an Hermitian matrix

$$\mathbf{H} = \mathbf{H}^\dagger$$

Finding eigenvalues and eigenvectors is equivalent to diagonalize the matrix N^3 operation

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Computationally challenging for growing N, and growing A

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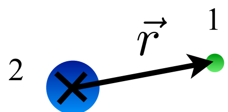
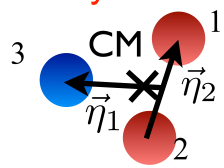
Hydrogen atom



Three-body Nucleus



Hyperspherical Harmonics Expansions

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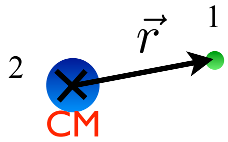

- Solve the problem in the CM frame

$$[T + V(r)]\,\psi(\vec{r}) = E\psi(\vec{r})$$

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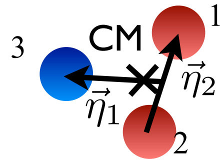

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$$[T + V(\eta_1,\eta_2)]\,\psi(\vec{\eta}_1,\vec{\eta}_2) = E\psi(\vec{\eta}_1,\vec{\eta}_2)$$

Hyperspherical Harmonics Expansions

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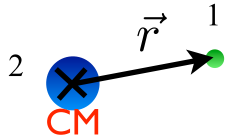

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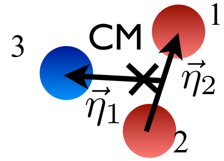
$$[T + V(r)]\,\psi(\vec{r}) = E\psi(\vec{r})$$

- Use spherical coordinates

$$\vec{r} = (r, \underbrace{\theta, \phi}_{\Omega})$$

$$\psi(\vec{r}) \sim Y_{\ell m}(\Omega) u_\ell(r)$$

Three-body Nucleus



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$$\rho = \sqrt{\eta_1^2 + \eta_2^2} \qquad \Omega = (\theta_1, \phi_1, \theta_2, \phi_2, \alpha)$$

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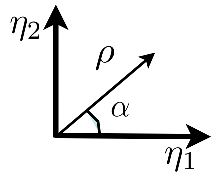

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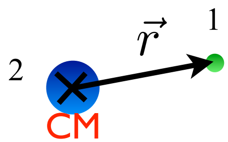

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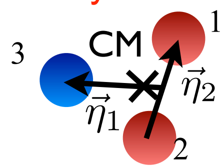
$$T = T_r - \frac{\hat{\ell}^2}{r^2}$$

$$\hat{\ell}^2 \; Y_{\ell m}(\Omega) = \ell(\ell+1) Y_{\ell m}(\Omega)$$

Three-body Nucleus



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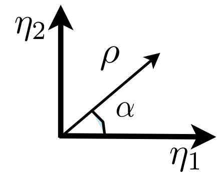
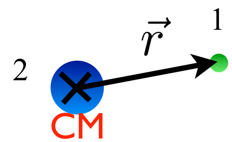


$$T = T_\rho - \frac{\hat{K}^2}{\rho^2}$$

$$\hat{K}^2 \; \mathcal{Y}_{[K]}(\Omega) = K(K+4) \mathcal{Y}_{[K]}(\Omega)$$

Hyperspherical Harmonics Expansions

Hydrogen atom



- Solve the problem in the CM frame

$$[T + V(r)]\,\psi(\vec{r}) = E\psi(\vec{r})$$

- Use spherical coordinates

$$\vec{r} = (r, \underbrace{\theta, \phi}_{\Omega})$$

$$\psi(\vec{r}) \sim Y_{\ell m}(\Omega) u_\ell(r)$$

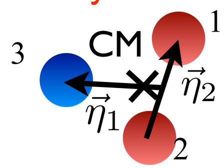
$$T = T_r - \frac{\hat{\ell}^2}{r^2}$$

$$\hat{\ell}^2\; Y_{\ell m}(\Omega) = \ell(\ell+1) Y_{\ell m}(\Omega)$$

- Solve the radial equation

$$\left[T_r - \frac{\ell(\ell+1)}{r^2} + V(r) - E\right] u_\ell(r) = 0$$

Three-body Nucleus



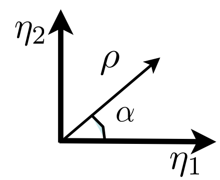
- Solve the problem in the CM frame

$$[T + V(\eta_1, \eta_2)]\,\psi(\vec{\eta}_1, \vec{\eta}_2) = E\psi(\vec{\eta}_1, \vec{\eta}_2)$$

- Use hyperspherical coordinates

$$\rho = \sqrt{\eta_1^2 + \eta_2^2} \qquad \Omega = (\theta_1, \phi_1, \theta_2, \phi_2, \alpha)$$

$$\psi(\vec{\eta}_1, \vec{\eta}_2) \sim \mathcal{Y}_{[K]}(\Omega) R_{[K]}(\rho)$$



$$T = T_\rho - \frac{\hat{K}^2}{\rho^2}$$

$$\hat{K}^2\; \mathcal{Y}_{[K]}(\Omega) = K(K+4)\mathcal{Y}_{[K]}(\Omega)$$

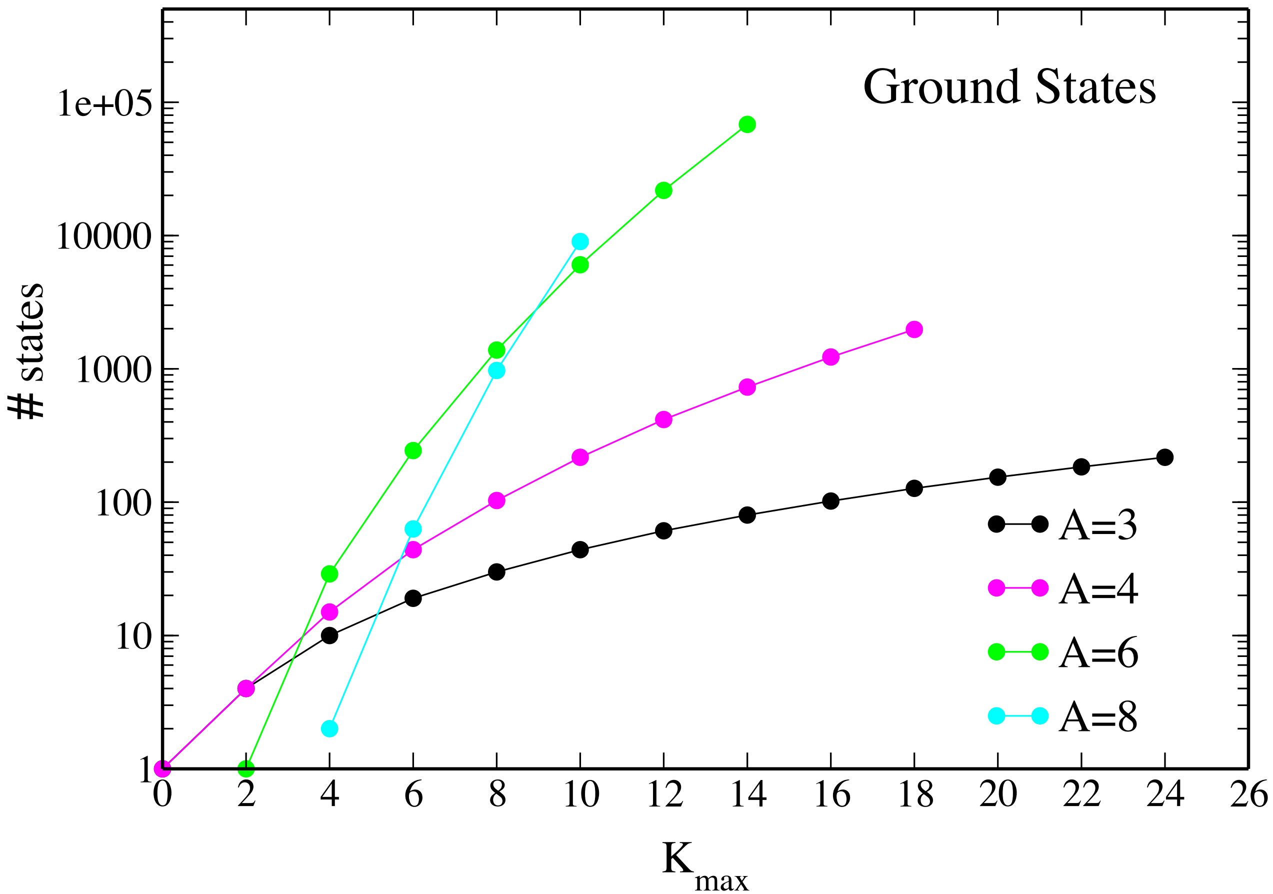
- Solve the hyperradial equation

$$\left[T_\rho - \frac{K(K+4)}{\rho^2} + V(\rho) - E\right] R_K(\rho) = 0$$

Hyperspherical Harmonics Expansions

$$|\psi\rangle = \sum_{[K]}^{K_{max}} \sum_{\nu}^{\nu_{max}} c_{[K]\nu}\, \mathcal{Y}_{[K]}(\Omega)\; e^{-\rho/2b} L_{\nu}(\rho)$$

$K_{max} * \nu_{max}$ = # states



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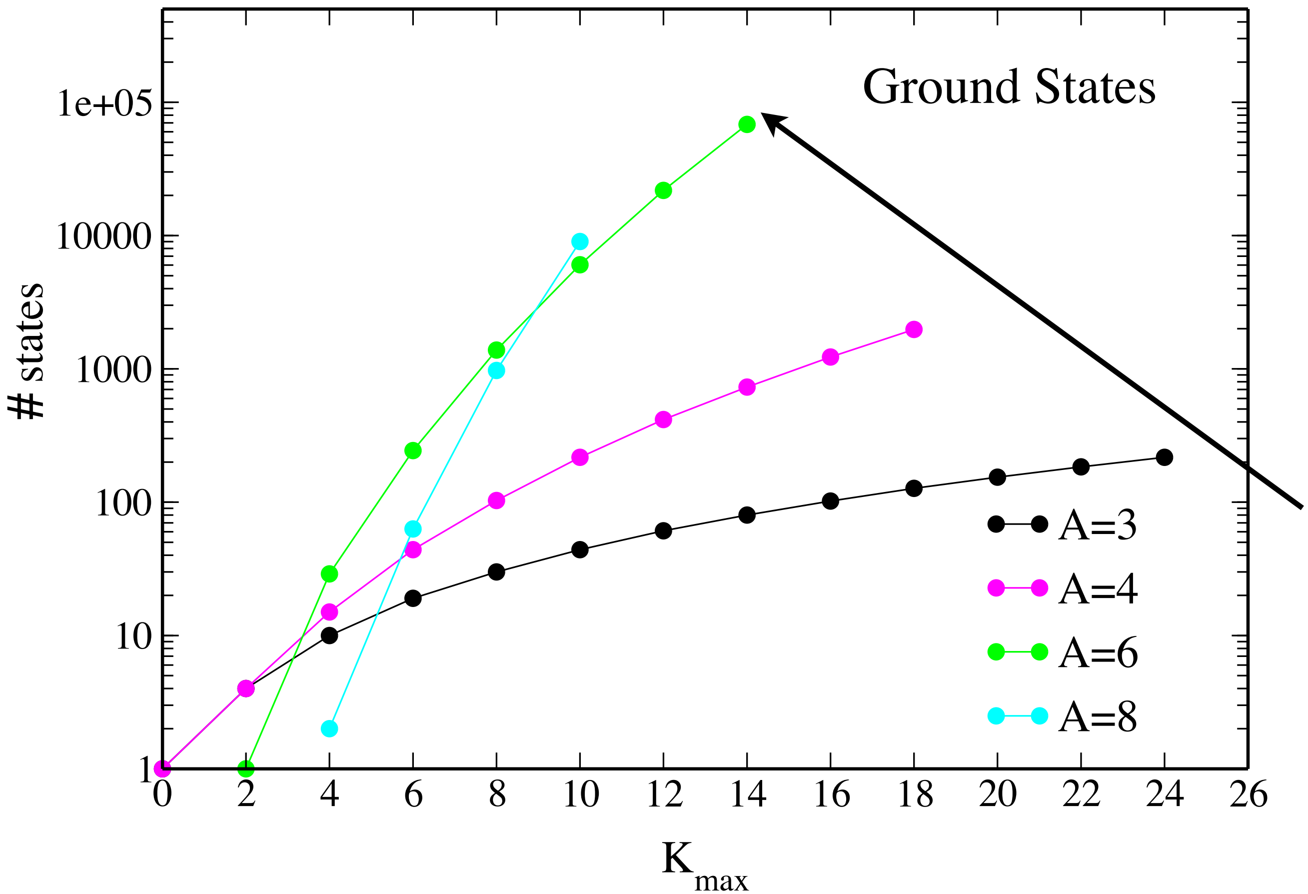
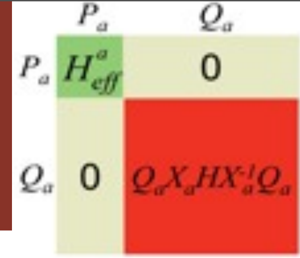


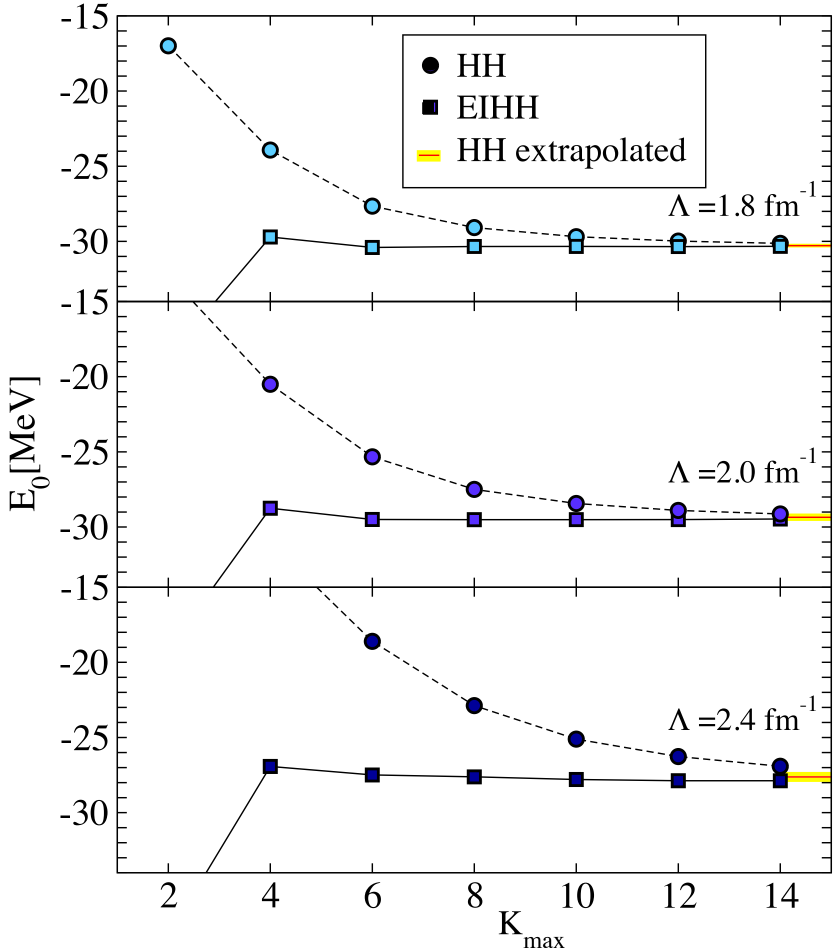
Total number of states:

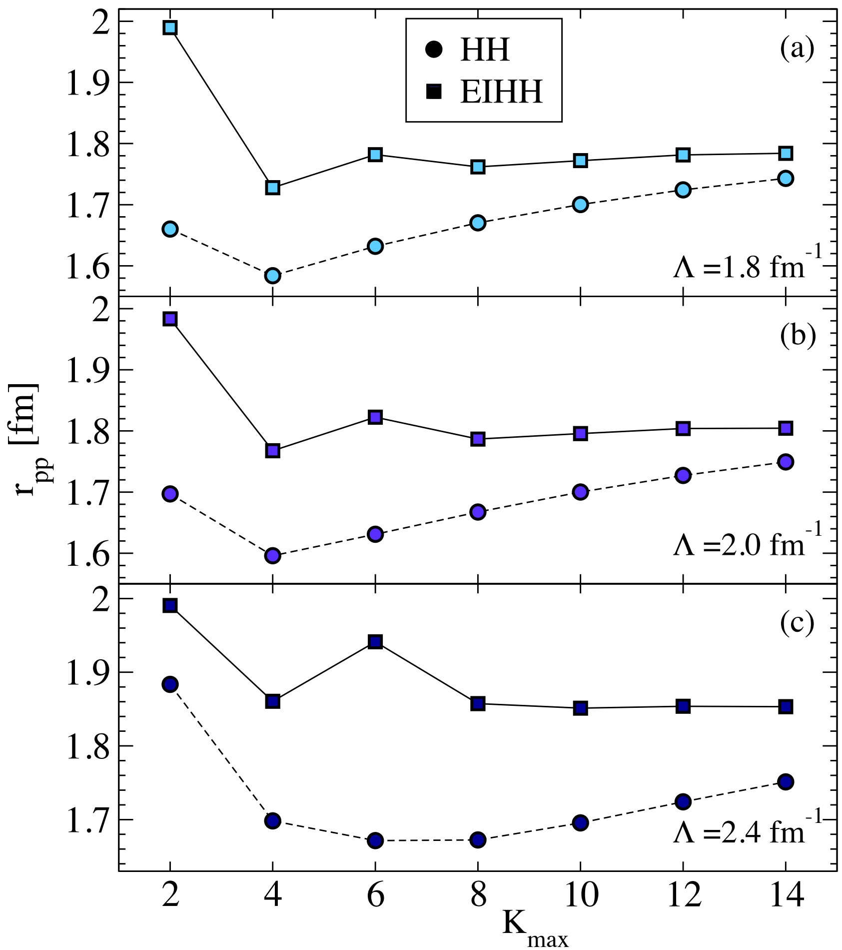
10^6 dense matrix

^{6}He from hyper-spherical harmonics



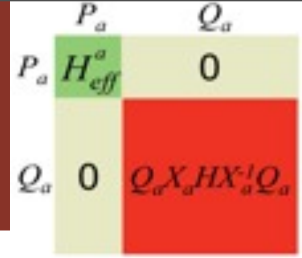
Interaction: $V_{\mathrm{low}\ k}$ from N^{3}LO (500 MeV)



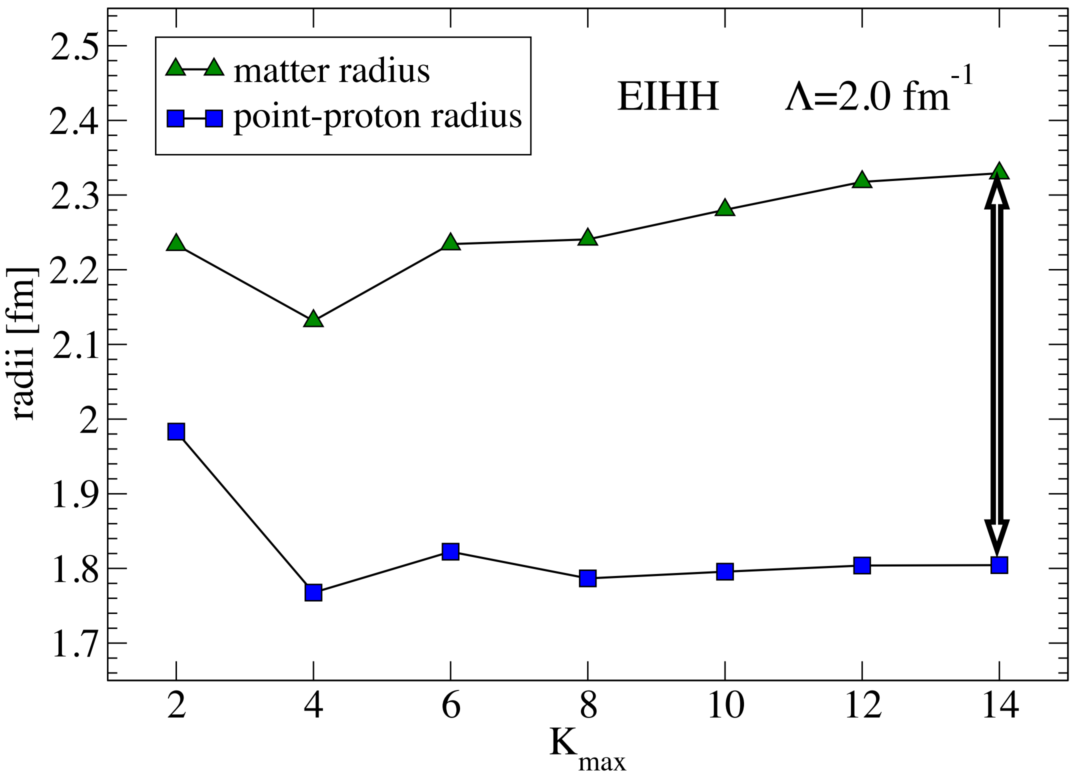


- EIHH agrees with extrapolated HH results from EPJ A 42, 553 (2009)
- EI is key to reach a reliable convergence of radii

^{6}He from hyper-spherical harmonics

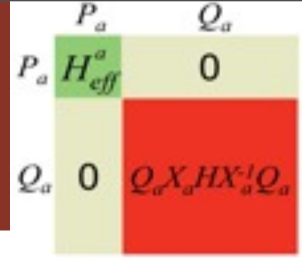


Signatures of the halo



- Point-Proton radii converge better and are smaller than matter radii ⇨ halo structure

^{8}He from hyper-spherical harmonics?



^{8}He from coupled cluster theory

Hilbert space: 15 major shell

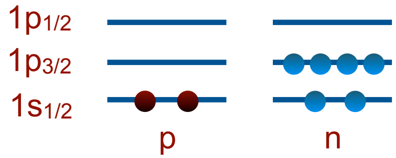
Interaction: $V_{\rm low\ k}$ from N^{3}LO (500 MeV)

Values in MeV

Λ	E[CCSD]	E[Lambda-CCSD(T)]	Δ
1.8	-30.33	-31.21	0.88
2.0	-28.72	-29.84	1.12
2.4	-25.88	-27.54	1.66

S.B et al., EPJ A 42, 553 (2009)

^{8}He closed sub-shell nucleus



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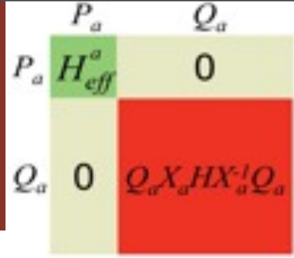

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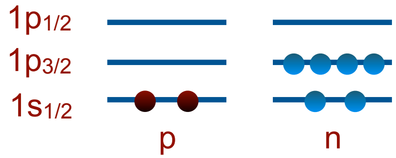

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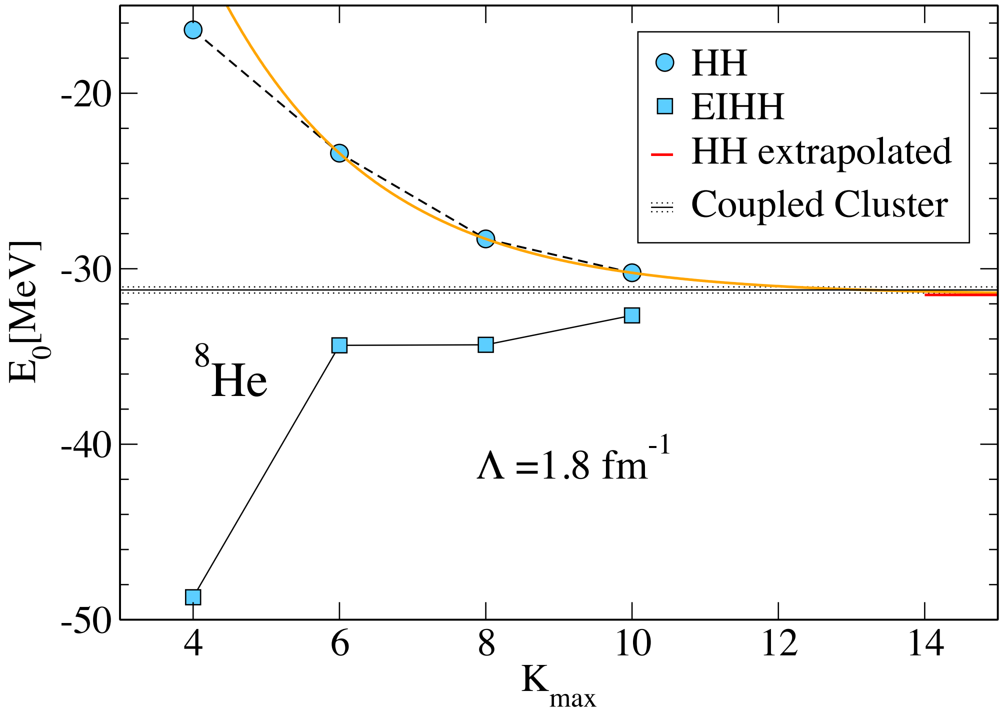

- Difference between HH and EIHH is about 2.4 MeV
- EIHH seems less effective than for ^{6}He
- Extrapolating HH results get

$$E_\infty = -31.49 \mathrm{MeV}$$

TRIUMF

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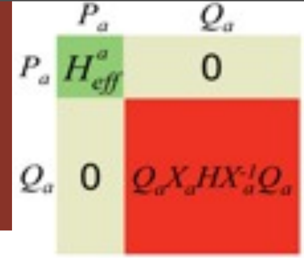

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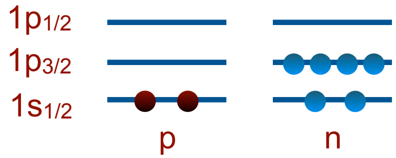

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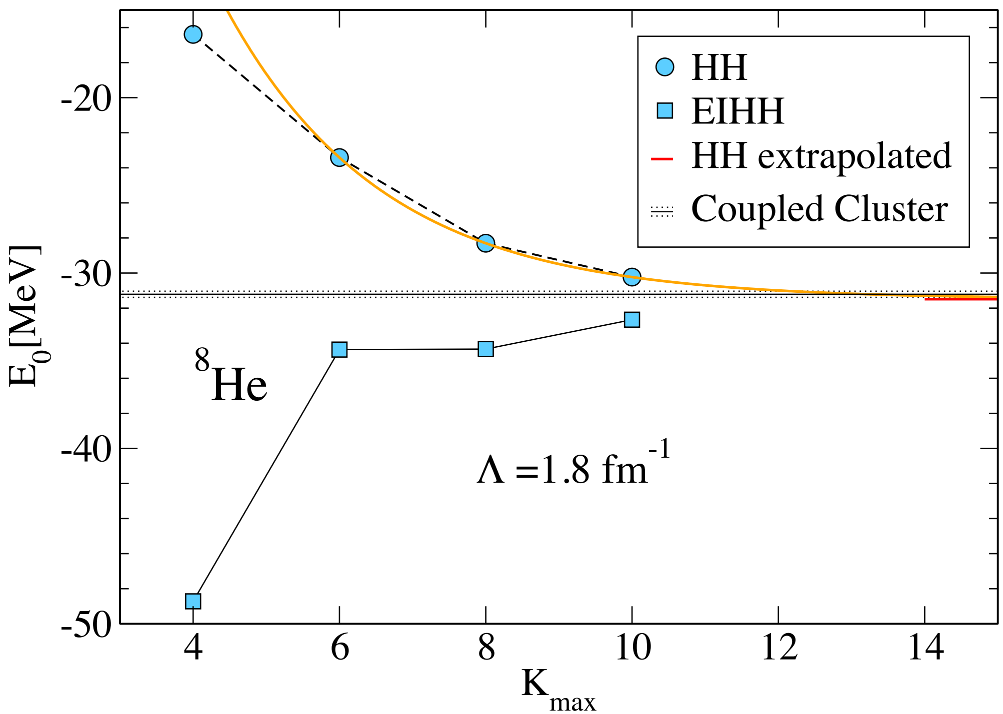

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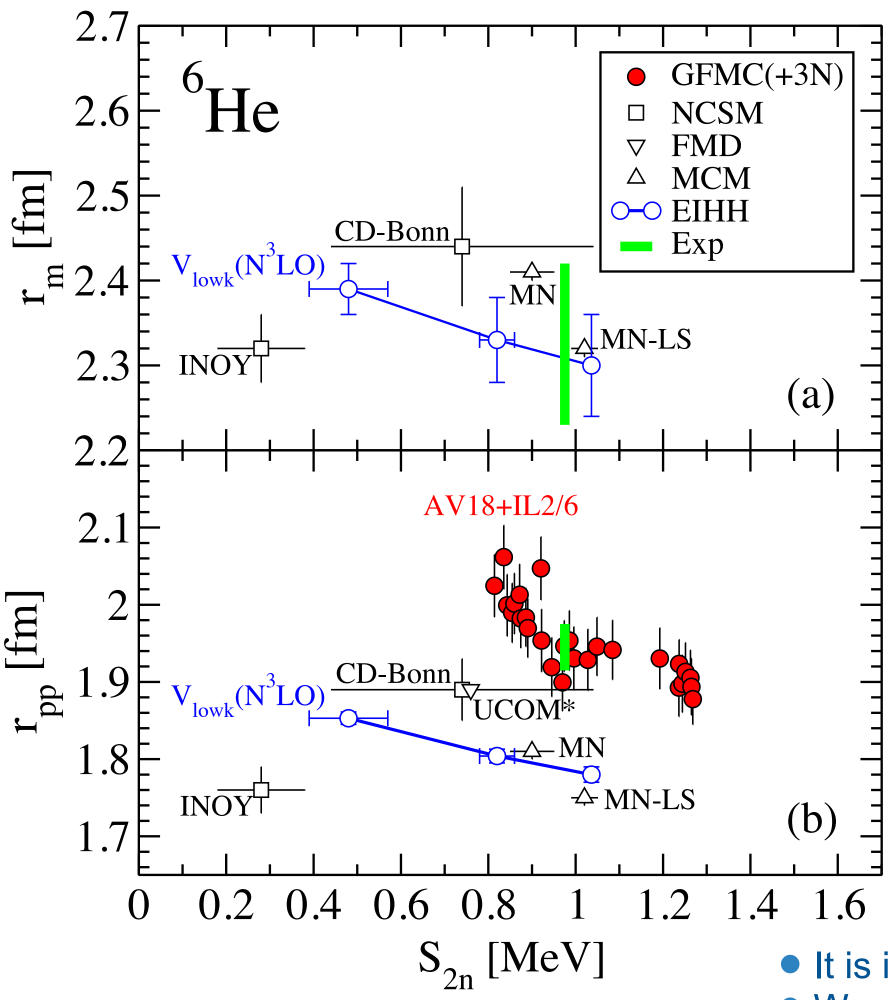
August 7th 2012

TRIUMF

Comparison with experiment



Phys. Rev. Lett. 108, 052504 (2012) & arXiv:1202.0516

(a) Experimental matter radius relatively uncertain

(b) Experimental charge radius well constrained

Relativistic corrections

$$r_{\rm pp}^2 = r_{\rm c}^2 - R_p^2 - \frac{N}{Z} R_n^2 - \frac{3}{4M_p^2} - r_{\rm so}^2$$

0.877(7) fm from electron scattering and H spectroscopy

0.84184(67) from spectroscopy of muonic hydrogen

Calculated ab-initio ~-0.082 fm²

- It is important to compare more than one observable together
- We observe a correlation between radii and separation energy
- Theory needs (improved) 3NFs

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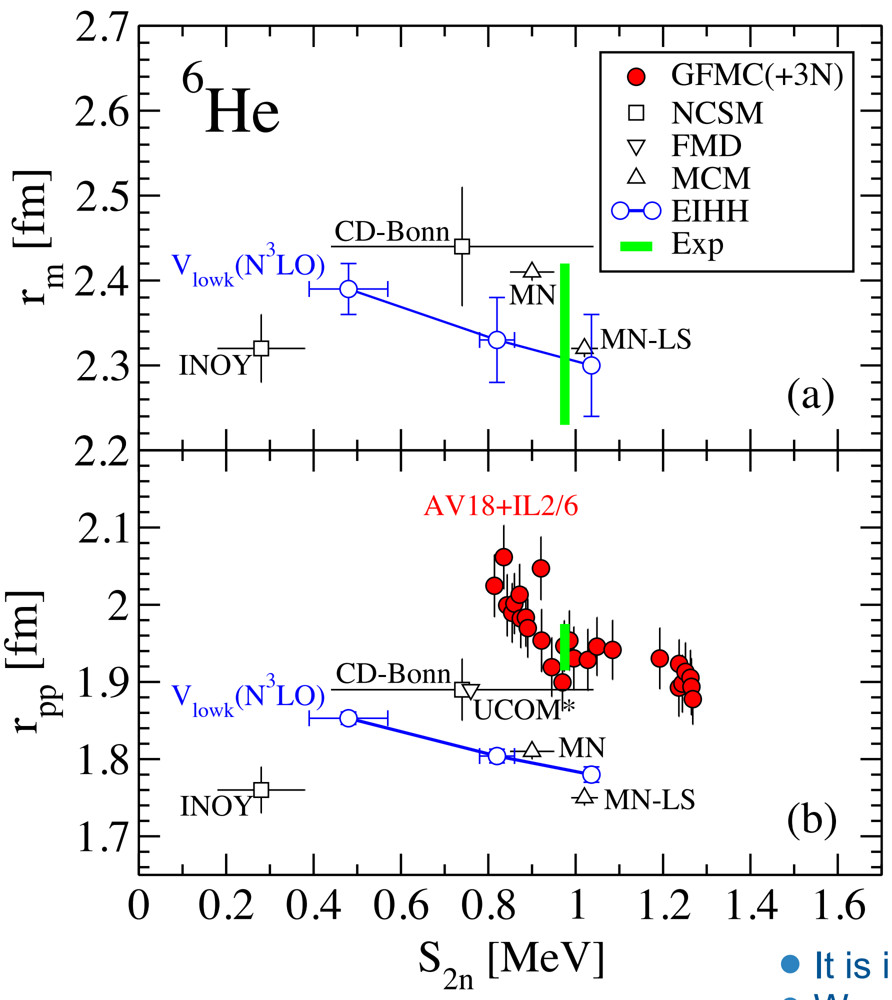

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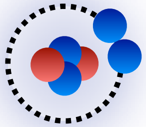
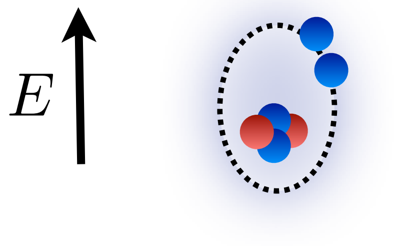
Future:
Include 3NF from EFT

Nuclear Electric Polarizability of ^{6}He



^{6}He

Nuclear Electric Polarizability of ^{6}He

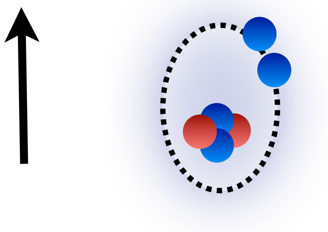


$^6\mathrm{He}^*$

$$D = \alpha_D E$$

Nuclear Electric Polarizability of ^{6}He

E



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$$D = \alpha_D E$$

It is a sum rule of the photo-disintegration cross section

$$\alpha_D = \frac{1}{2\pi^2}\int_{\omega_{th}}^{\infty} d\omega \frac{\sigma_\gamma(\omega)}{\omega^2}$$

Can be calculated using the Lanczos algorithm as

$$\alpha_D \longrightarrow \langle\psi_0|\hat{D}_z\hat{D}_z|\psi_0\rangle \cfrac{1}{E_0 - a_0 - \cfrac{b_1^2}{E_0 - a_1 - \cfrac{b_2^2}{E_0 - a_2 - \cfrac{b_3^2}{\dots}}}}$$

with starting vector $|\phi_0\rangle = \frac{\hat{D}_z|\psi_0\rangle}{\sqrt{\langle\psi_0|\hat{D}_z\hat{D}_z|\psi_0\rangle}}$

Nuclear Electric Polarizability of ^{6}He

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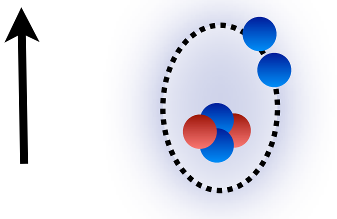

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The Helium Isotopes from NCSM with EFT potentials Stetcu et al., PRC **79**, 064001 (2009)

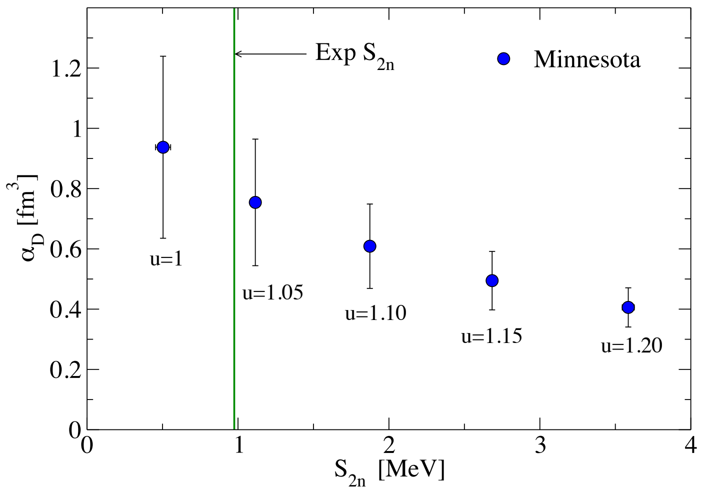
Nucleus	α_E^{calc}(fm^3)	Ref.	α_E^{exp}(fm^3)	Ref.
^{3}He	0.149(5)		0.250(40)	[53]
	0.145	[49]	0.130(13)	[54]
	0.153(15)	[55]		
^{4}He	0.0683(8)(14)		0.072(4)	[31]
	0.0655(4)	[56]	0.076(8)	[55]
	0.076	[49]		
^{6}He	?		1.99(40)	[55]

From HH with AV18+UIX PRC **74**, 061001 (2006) ← 0.0655(4)

Nuclear Electric Polarizability of ^{6}He

Calculations from EIHH with the simple semi-realistic Minnesota potential which gives α_D compatible to the realistic potentials for ^{4}He.

Correlation $\alpha_D - S_{2n}$



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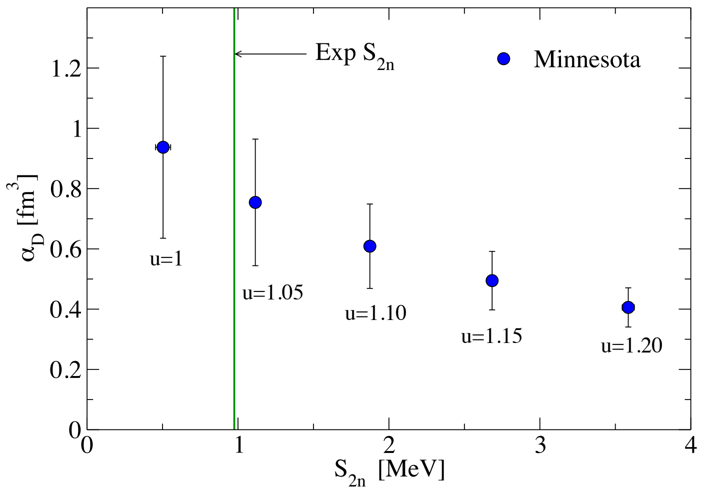

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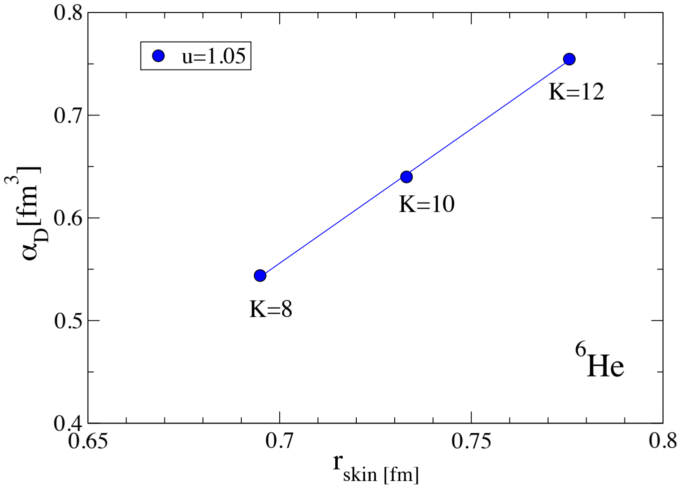

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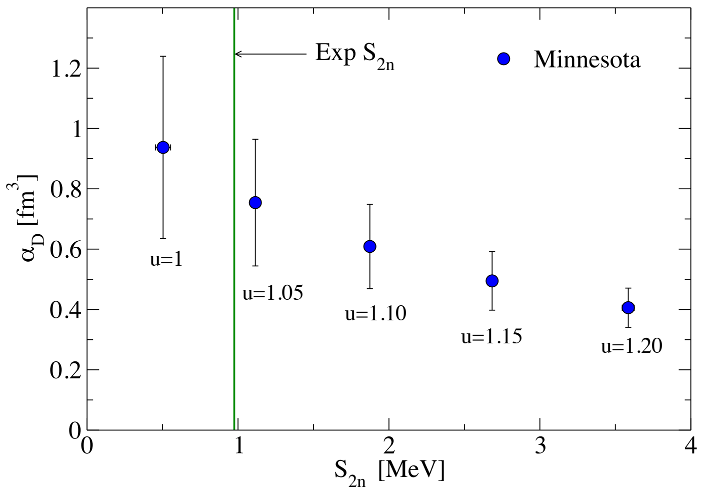

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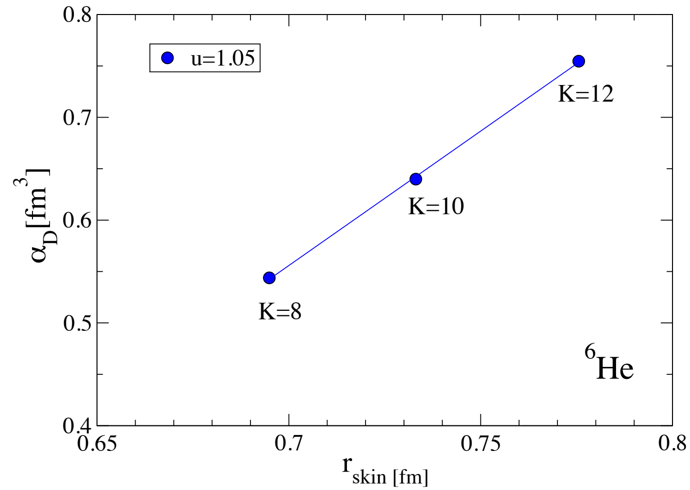

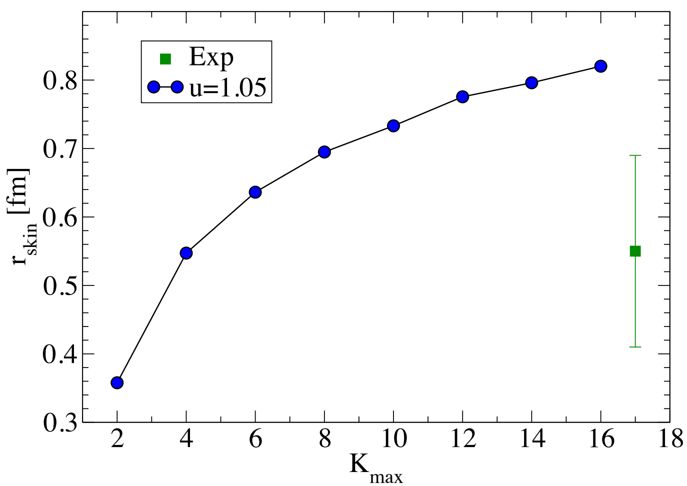

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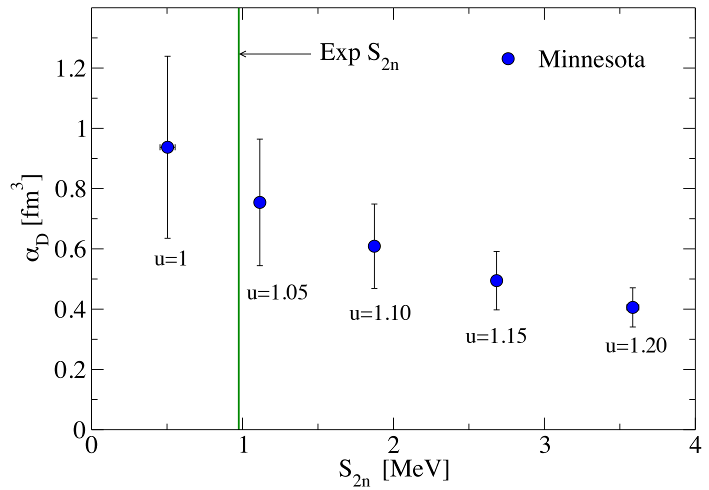

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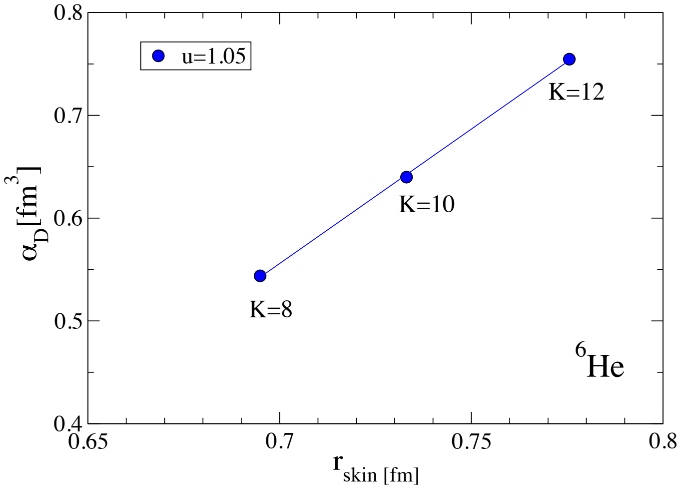

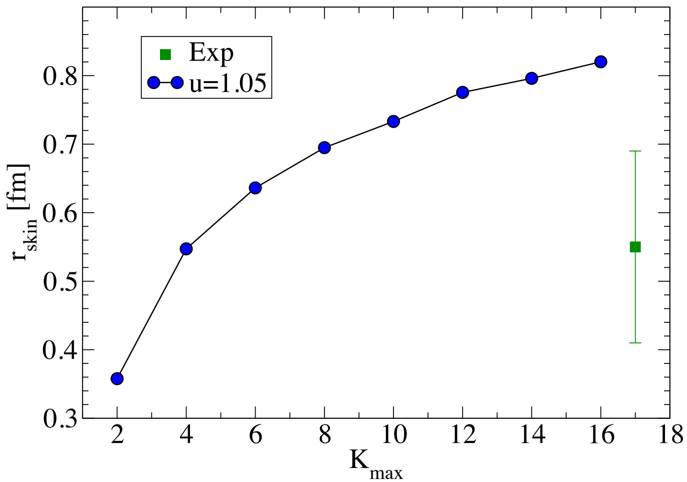

Estimate from calculations $\alpha_D = 0.87(13)\ {\rm fm}^3$

K.Pachucki and A.M.Moro, PRA 75, 032521 (2007) estimate based on Aumann *et al.* data for B(E1) $\alpha_D^{exp} = 1.99(40)\ {\rm fm}^3$

Sonia Bacca

Monday, 6 August, 12

Nuclear Electric Polarizability of ^{6}He

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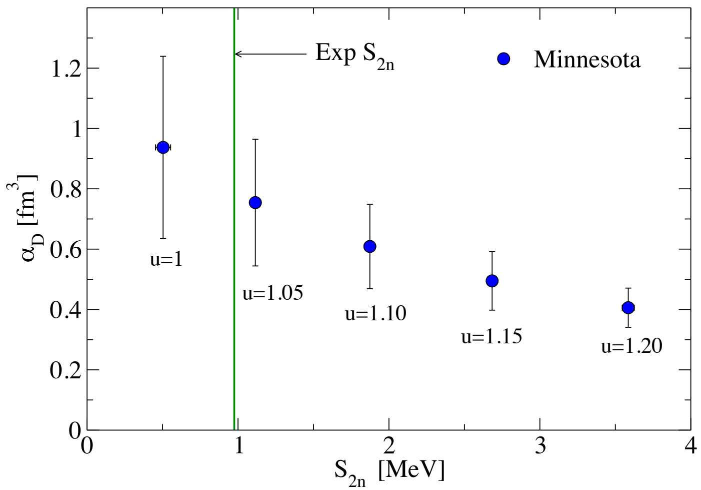

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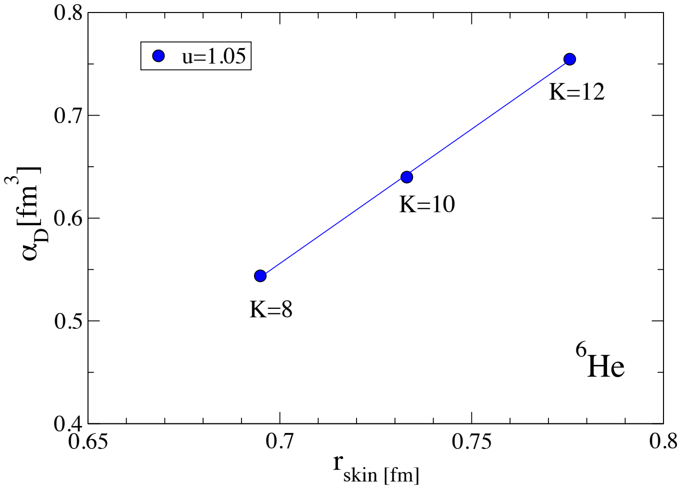

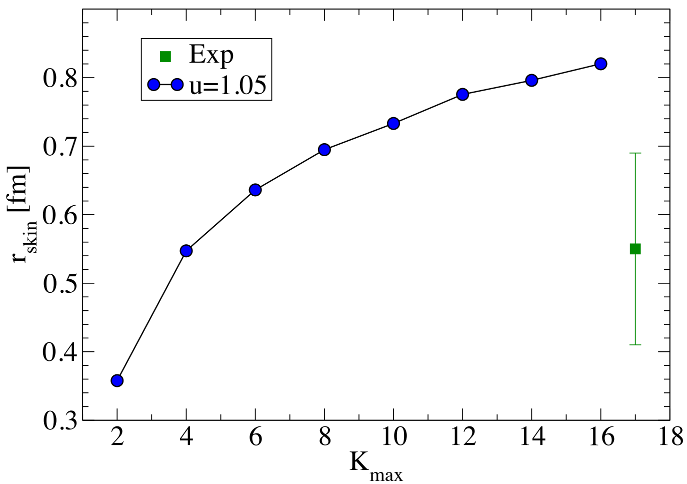

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Potential disagreement between theory and experiment

Future:
Prediction from EFT

Outlook

- Hyper-spherical harmonics provide a powerful tool to perform accurate studies of light nuclei for g.s. (and excited states) properties to test nuclear forces

- Room to study further 3NF effects and to add exchange currents for consistent EFT calculations

Thanks to my collaborators:

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UBC — Raymond Goerke

Winfried Leidemann, Giuseppina Orlandini

TECHNISCHE UNIVERSITAT DARMSTADT — Achim Schwenk

TITAN ISAC-TRIUMF — TRIUMF's Ion Trap for Atomic and Nuclear Science

Thank you!