Symmetries of the similarity renormalization group for nuclear forces

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JLAB, August-9th 2012

Chiral Dynamics 2012

References

- V. S. Timoteo, S. Szpigel and E. Ruiz Arriola arXiv:1108.1162 [nucl-th]
- E. Ruiz Arriola, nucl-th/1009.4161

Introduction

- How much do we need to know light nuclei to predict heavy nuclei ?
- Nucleon size $a \sim 1\mathrm{fm}$
- Nuclear Force $\sim 1/m_\pi = 1.4\mathrm{fm}$
- Nuclear matter (interparticle distance)

$$\rho_{nm} = 0.17\mathrm{fm}^{-3} = \frac{1}{(1.8\mathrm{fm})^3}$$

- Fermi Momentum

$$k_F = 270\mathrm{MeV} \qquad \lambda_F = \pi/k_F = 2.3\mathrm{fm} \gg 1/\sqrt{m_\pi M_N} = 0.5\mathrm{fm}$$

Can we ignore explicit core and explicit pions ?

- Nuclear many body Hamiltonian H

$$H = \sum_i T_i + \sum_{i<j} V_{2,ij} + \sum_{i<j<k} V_{3,ijk} + \sum_{i<j<k<l} V_{4,ijkl} + \dots$$

- NN: $V_{2,ij}$ (deuteron+NN scattering data)
- 3N: Triton+ N-deuteron scattering
- 4N: $\alpha-$particle, dd,tp etc, scattering
- Typical Range of multinucleon forces $e^{-m_\pi d} \sim 0.2$

$$V_{NN} \sim e^{-m_\pi d} \quad V_{NNN} \sim e^{-2m_\pi d} \quad V_{NNNN} \sim e^{-3m_\pi d}$$

- Typical NN wavelengths $\geq 1/\sqrt{m_\pi M_N} \sim 0.5\mathrm{fm}$

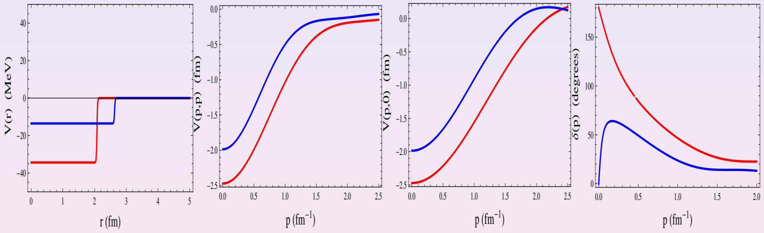
$\rightarrow$ Few wavelengths within a range
(Coarse grained Effective interactions) SYMMETRIES

WIGNER SYMMETRY

Long wavelength limit (Short range interactions)

- $^1S_0$ and $^3S_1$ channels (Square well potential)

$\alpha_{1S0} = -23.74\text{fm} \quad r_{1S0} = 2.75\text{fm} \quad \alpha_{3S1} = 5.4\text{fm} \quad r_{3s1} = 1.75\text{fm}$



$$V^{JS}_{l',l}(p',p) = M_N \int_0^\infty j_{l'}(p'r) j_l(pr) V^{JS}_{l'l}(r) r^2$$

- Wigner symmetry: $V_{1S0}(p,p') \sim V_{3S1}(p,p')$
- Exact if $\int d^3x V_{1S0}(\vec{x}) = \int d^3x V_{3S1}(\vec{x})$ but NOT if $\alpha_{1S0}, \alpha_{3S1} \to \infty$

Long wavelength limit (EFT-approach)

- Effective NN-Lagrangian

$$\mathcal{L} = -\frac{1}{2} C_S (N^\dagger N)^2 - \frac{1}{2} C_T (N^\dagger \vec{\sigma} N)^2$$

- Potential in momentum space

$$V_0(p', p) = C_0 \theta(\Lambda - p)\theta(\Lambda - p')$$

- Lippmann-Schwinger equation

$$\rightarrow C_0(\Lambda) = \frac{4\pi^2 \alpha_0}{M_N} \frac{1}{1 + \Lambda \alpha_0 / \pi} \qquad \alpha_0 = \alpha_{1S0}, \alpha_{3S1}$$

- Wigner symmetry exact if $\alpha_0 = -\infty$ [Mehen,Steward,Wise,1999]
  BUT there is Regularization dependence

SU(4) Wigner symmetry

- Generators

$$
\begin{aligned}
T^a &= \frac{1}{2}\sum_A \tau_A^a\,, \quad \text{ISOSPIN} \\
S^i &= \frac{1}{2}\sum_A \sigma_A^i\,, \quad \text{SPIN} \\
G^{ia} &= \frac{1}{2}\sum_A \sigma_A^i \tau_A^a\,, \quad \text{GAMOW} - \text{TELLER}
\end{aligned}
$$

- Casimir operator (two body)

$$
C_{SU(4)} = T^a T_a + S^i S_i + G^{ia} G_{ia}\,,
$$

- Irreducible representations (λ, μ, ν)

$$
C_{SU(4)} = \mu(\mu + 4) + \nu(\nu + 2) + \lambda^2
$$

- Selection rules in Gamow-Teller weak decays between supermultiplets

$$\langle \lambda\mu\nu | G^{ia} | \lambda'\mu'\nu' \rangle = 0$$

- SU(4) mass formula [Franzini+Radicatti 63]

$$E = c_1 A(A+1) + c_2 \left[\mu(\mu+4) + \nu(\nu+2) + \lambda^2 - \frac{15}{4} A \right]$$

- Anomalously large double binding energy for even-even $N = Z$ nuclei [Van Isaacker,Warnerr,Brenner,1995].
- SU(4) inequalities for nuclei on the lattice [Chen, Lee ,Schaffer 2004]
- SU(4) is crucial for Nuclear Lattice importance sampling [Lee, Epelbaum, 2008]

- One nucleon state

$$\mathbf{4} = (p\uparrow, p\downarrow, n\uparrow, n\downarrow) = (S = 1/2, T = 1/2) \qquad \text{Quartet}$$

- Two nucleon states

$$C^{ST}_{SU(4)} = \frac{1}{2}(\sigma + \tau + \sigma\tau) + \frac{15}{2} \,,$$

$$\tau = \tau_1 \cdot \tau_2 = 2T(T+1) - 3 \,, \sigma = \sigma_1 \cdot \sigma_2 = 2S(S+1) - 3$$

- Sextet and decuplet $(-1)^{S+L+T} = -1$

$$\begin{aligned} \mathbf{6}_A &= (1,0) \oplus (1,0) \quad L = 0, 2, \ldots \quad ({}^1S_0, {}^3S_1), ({}^1D_2, {}^3D_{1,2,3}) \\ \mathbf{10}_S &= (0,0) \oplus (1,1) \quad L = 1, 3, \ldots \quad ({}^1P_1, {}^3P_{0,1,2}) \end{aligned}$$

- At large N_c Wigner symmetry holds ONLY for even-L channels [Kaplan,Savage,Manohar,97]
  This is confirmed by the data !![Calle Cordon,Ruiz Arriola,08]

EFFECTIVE INTERACTIONS

Standard ways to coarse grain

- Nuclear shell model (energy)

$$\hbar\omega = 41 A^{-\frac{1}{3}} \mathrm{MeV} \qquad b = 1 A^{\frac{1}{6}} \mathrm{fm}$$

- $V_{\rm low,k}$ (momentum) [Kuo,Brown,Holt,Bogner,2000]

$$\Lambda_{\rm CM} = 450 \mathrm{MeV} \qquad \pi/\Lambda = 1.5 \mathrm{fm}$$

- Nuclear lattice (space) [D. Lee]

$$a = 2 \mathrm{fm} \qquad k \le \pi/a = 300 \mathrm{MeV}$$

- Coarse grained delta-shells [Navarro,Amaro,Ruiz Arriola,2011]

$$\Delta r \sim 1/\sqrt{M_N m_\pi} \quad n\Delta r \sim 1/m_\pi$$

Similarity Renormalization Group

[Wilson,Glazek,Wegner]

- Unitary transformation with generator η_s $H_s = e^{\eta_s} H e^{-\eta_s}$
- Running Hamiltonian (SRG trajectory)

$$\frac{dH_s}{ds} = [\eta_s, H_s] \ , \qquad (1)$$

- Initial condition $H_{s=0} = H$
- Wegner generator

$$\eta_s = [D(H_s), H_s]$$

- Glazek-Wilson

$$\eta_s = [T, H_s]$$

SRG long distance symmetries

- Operator evolution

$$\frac{dO_s}{ds} = [[T, V_s], O_s] \ . \tag{2}$$

- Symmetry group generator X

$$\frac{dX_s}{ds} = [[T, V_s], X_s] \ . \tag{3}$$

Jacobi's identity $[[A, B], C] + [[C, A], B] + [[B, C], A] = 0$

$$\frac{dX_s}{ds} = -[[X_s, T], V_s] - [[V_s, X_s], T] = 0 \, . \tag{4}$$

Long distance symmetry is a fixed-point of the SRG evolution

Momentum space representation

$$\frac{dV_s(\vec{p}',\vec{p})}{ds} = -(E_p - E_p')^2 V_s(\vec{p}',\vec{p}) + \int \frac{d^3q}{(2\pi)^3} \left(E_p + E_{p'} - 2E_q\right) V_s(\vec{p}',\vec{q}) V_s(\vec{q},\vec{p})\,. \qquad (5)$$

At large momenta (off-diagonal suppression)

$$V_s(\vec{p}',\vec{p}) \approx V_{s=0}(\vec{p}',\vec{p}) e^{-(E_{p'}-E_p)^2 s} + \ldots \qquad (6)$$

Zero momentum states

$$\frac{dV_s(\vec{0},\vec{0})}{ds} = -2\int \frac{d^3q}{(2\pi)^3} E_q \langle \vec{q} | V_s V_s^\dagger | \vec{q} \rangle \le 0\,, \qquad (7)$$

Momentum grid $p_{\rm max} = 5000 {\rm MeV}$ and $N = 250$ points

General Structure of the potential

Mopentum space

$$\begin{aligned} V(\vec{p}',\vec{p}) &= V_C + \vec{\tau}_1\cdot\vec{\tau}_2\, W_C + (V_S + \vec{\tau}_1\cdot\vec{\tau}_2\, W_S)\,\vec{\sigma}_1\cdot\vec{\sigma}_2 \\ &+ (V_{LS} + \vec{\tau}_1\cdot\vec{\tau}_2\, W_{LS})\, i(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{p}'\times\vec{p}) \\ &+ (V_T + \vec{\tau}_1\cdot\vec{\tau}_2\, W_T)\, S_{12}(\vec{p}'-\vec{p}) \\ &+ (V_Q + \vec{\tau}_1\cdot\vec{\tau}_2\, W_Q)\, S_{12}(\vec{p}'\times\vec{p}) \\ &+ (V_P + \vec{\tau}_1\cdot\vec{\tau}_2\, W_P)\, S_{12}(\vec{p}'+\vec{p})\,, \end{aligned}$$

where the tensor operator is defined as

$$S_{12}(\vec{q}) = \left[\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q} - \frac{1}{3}q^2\vec{\sigma}_1\cdot\vec{\sigma}_2\right]\,, \qquad (8)$$

Pauli principle

$$V_{1',2';1,2}(\vec{p}',\vec{p}) = -V_{2',1';1,2}(-\vec{p}',\vec{p})$$

Partial wave decomposition

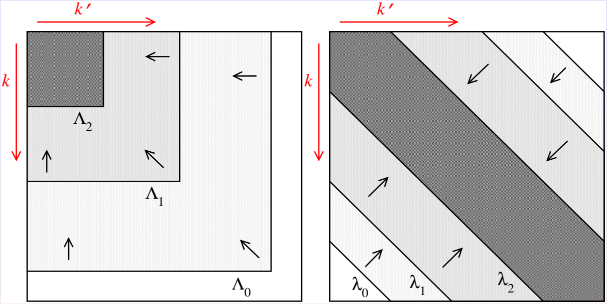
$$\langle \vec{p}' | V_\lambda^S | \vec{p} \rangle = N \sum_{JMLL'} \mathcal{Y}_{LS}^{JM}(\hat{p}') V_{LL'}^{JS}(p', p) \mathcal{Y}_{L'S}^{JM\dagger}(\hat{p}) \quad (9)$$

, Coupled-channel equations,

$$\frac{dV_s(p,p')}{ds} = -(p^2 - p'^2)^2 \; V_s(p,p') + \frac{2}{\pi} \int_0^\infty dq \; q^2 \; (p^2 + p'^2 - 2q^2) \; V_s(p,q) \; V_s(q,p'), \quad (10)$$

where $V_s(p,p')$ is used as a brief notation for the projected NN potential matrix elements,

Integrating out vs Similarity Renormalization Group



- $V_{\text{lowk}} \rightarrow$ Scattering reproduced until the cut-off.

$$\delta_{\text{lowk}}(k, \Lambda) = \delta(k)\theta(\Lambda - k)$$

- V_{SRG} Scattering reproduced at ALL eneries.

$$\delta_{\text{SRG}}(k, \lambda) = \delta(k)$$

Infrared fixed points of SRG

- Fixed points for discretized equations

$$\frac{dV_{ik}}{ds} = -(E_i - E_k)^2 V_{ik} + \sum_k (E_i + E_k - 2E_l) V_{il} V_{lk} = 0 \quad (11)$$

- Diagonal potential in momentum space basis

$$V_{\alpha\beta} = v_\alpha \delta_{\alpha\beta}$$

- $R-$matrix

$$R(p', p; k) = V(p', p) + \frac{2}{\pi} P \int_0^\infty q^2 dq \frac{V(p', q) R(q, p; k)}{k^2 - q^2}$$

- Excluding the mean value

$$R(p, p; p) = -\frac{\tan\delta(p)}{p} = \lim_{\lambda \to 0} V_\lambda(p, p)$$

Sum rules

$$V_{^3P_C} = \frac{1}{9}\left(V_{^3P_0} + 3V_{^3P_1} + 5V_{^3P_2}\right), \tag{12}$$

$$V_{^3P_T} = -\frac{5}{72}\left(2V_{^3P_0} - 3V_{^3P_1} + V_{^3P_2}\right), \tag{13}$$

$$V_{^3P_{LS}} = -\frac{1}{12}\left(2V_{^3P_0} + 3V_{^3P_1} - 5V_{^3P_2}\right), \tag{14}$$

for triplet D-waves

$$V_{^3D_C} = \frac{1}{15}\left(3V_{^3D_1} + 5V_{^3D_2} + 7V_{^3D_3}\right), \tag{15}$$

$$V_{^3D_T} = -\frac{7}{120}\left(3V_{^3D_1} - 5V_{^3D_2} + 2V_{^3D_3}\right), \tag{16}$$

$$V_{^3D_{LS}} = -\frac{1}{60}\left(9V_{^3D_1} + 5V_{^3D_2} - 14V_{^3D_3}\right), \tag{17}$$

for triplet F-waves

$$V_{^3F_C} = \frac{1}{21}\left(5\,V_{^3F_2} + 7\,V_{^3F_3} + 9\,V_{^3F_4}\right), \tag{18}$$

$$V_{^3F_T} = -\frac{5}{112}\left(4\,V_{^3F_2} - 7\,V_{^3F_3} + 3\,V_{^3F_4}\right), \tag{19}$$

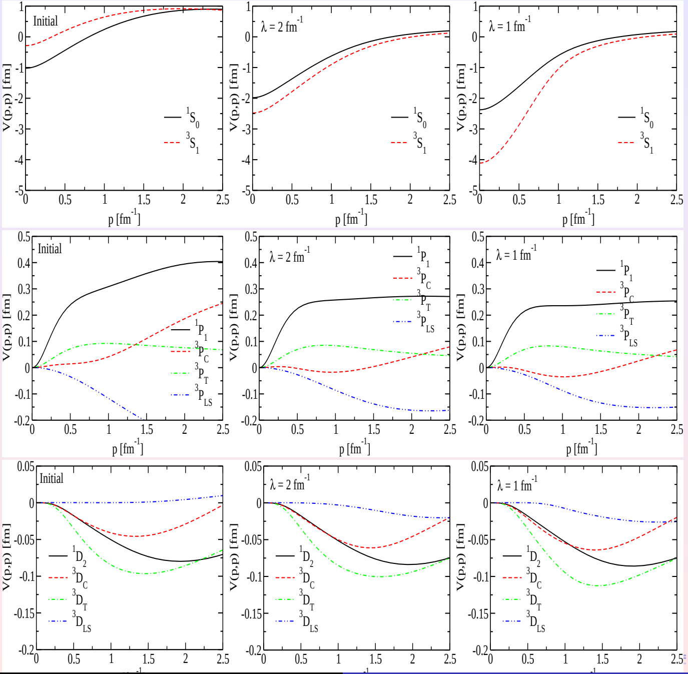
$$V_{^3F_{LS}} = -\frac{1}{168}\left(20\,V_{^3F_2} + 7\,V_{^3F_3} - 27\,V_{^3F_4}\right), \tag{20}$$

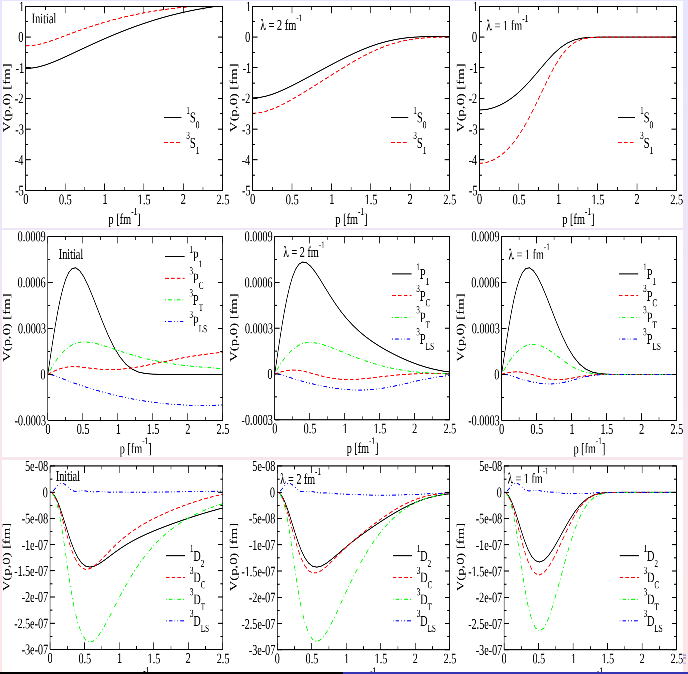
and for triplet G-waves

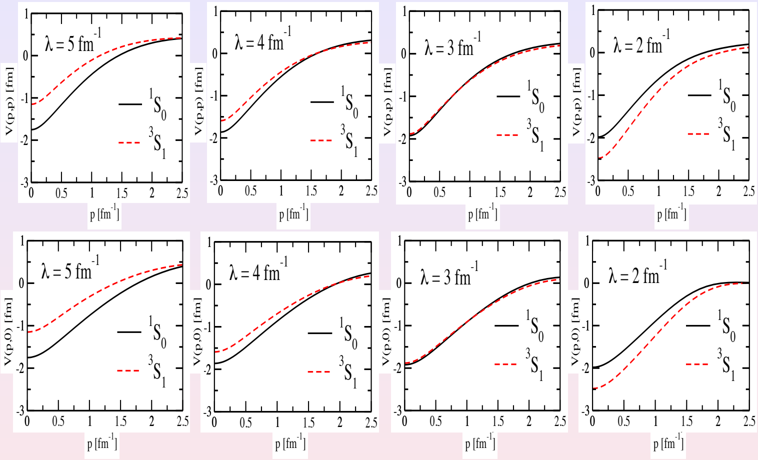
$$V_{^3G_C} = \frac{1}{27}\left(7\,V_{^3G_3} + 9\,V_{^3G_4} + 11\,V_{^3G_5}\right), \tag{21}$$

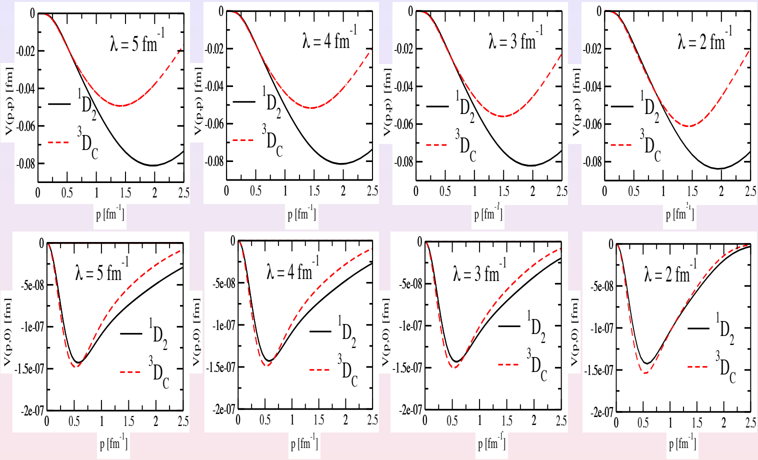
$$V_{^3G_T} = -\frac{77}{2160}\left(5\,V_{^3G_3} - 9\,V_{^3G_4} + 4\,V_{^3G_5}\right), \tag{22}$$

$$V_{^3G_{LS}} = \frac{1}{360}\left(-35\,V_{^3G_3} - 9\,V_{^3G_4} + 44\,V_{^3G_5}\right). \tag{23}$$

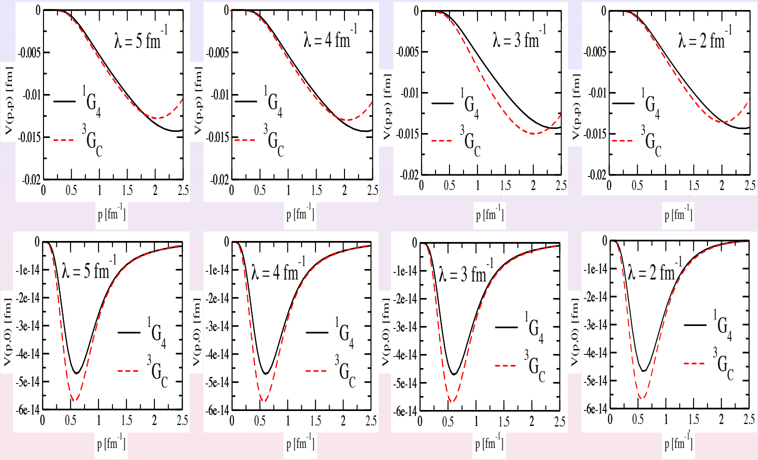

Initial
λ = 2 fm⁻¹
λ = 1 fm⁻¹
V(p,p) [fm]
p [fm⁻¹]
1S0
3S1
1P1
3PC
3PT
3PLS
1D2
3DC
3DT
3DLS


Initial
λ = 2 fm^-1
λ = 1 fm^-1
V(p,0) [fm]
p [fm^-1]
1S0
3S1
1P1
3PC
3PT
3PLS
1D2
3DC
3DT
3DLS


λ = 5 fm^-1
λ = 4 fm^-1
λ = 3 fm^-1
λ = 2 fm^-1
V(p,p) [fm]
V(p,0) [fm]
p [fm^-1]
1S0
3S1



λ = 5 fm-1
λ = 4 fm-1
λ = 3 fm-1
λ = 2 fm-1
V(p,p) [fm]
V(p,0) [fm]
p [fm-1]
1D2
3DC


λ = 5 fm^-1
λ = 4 fm^-1
λ = 3 fm^-1
λ = 2 fm^-1
1G4
3GC
V(p,p) [fm]
V(p,0) [fm]
p [fm^-1]

Wigner and Serber Symmetries

- Wigner symmetry

$$V_{^1S_0} = V_{^3S_C}\,, \quad V_{^1D_2} = V_{^3D_C}\,, \quad V_{^1G_4} = V_{^3G_C}\,. \qquad (24)$$

- Serber symmetry

$$0 = V_{^3P_C} = V_{^3F_C} = V_{^3H_C} = \dots \qquad (25)$$

- The Wigner SRG scale:

$$\lambda_{\rm Wigner} = 3 {\rm fm}^{-1}$$

- Symmetry pattern consistent with large N_c

BINDING ENERGIES

Binding Energies

- Mean field Slater Determinant

$$\psi(\vec{p}_1, \ldots, \vec{p}_A) = \mathcal{A}\left[\phi_{n_1,l_1,s,m_{s1},t,m_{t_1}}(\vec{p}_1)\ldots\phi_{n_A,l_A,s,m_{sA},t,m_{t_A}}(\vec{p}_A)\right] . (26$$

- Single particle states (Harmonic oscillator)

$$P_{nl}(p) = N_{nl} e^{-\frac{1}{2}b^2p^2}(bp)^l L_{n-1}^{l+\frac{1}{2}}\left(b^2p^2\right) \quad (27)$$

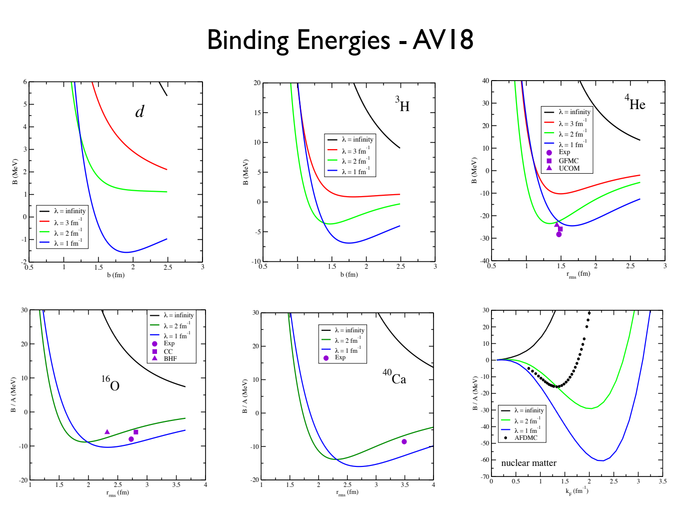
- Two body interaction (Talmi-Moshinsky)

$$\langle V_2\rangle_A = \sum_{nlJS} g_{nlJS}\langle nl|V^{JST}|nl\rangle \, , \quad (28)$$

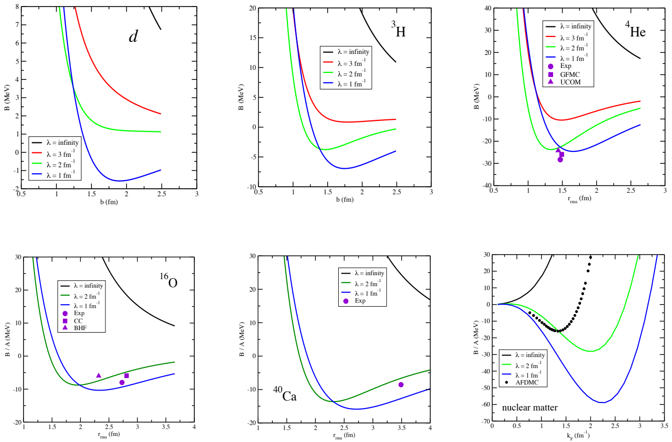
- Nuclei: Shell model (mean field)

$$\mathrm{d} : (1s)^2 \quad \mathrm{t} : (1s)^3 \quad {}^4\mathrm{He} : (1s)^4 \, ,$$
$${}^{16}\mathrm{O} : (1s)^4(1p)^{12} \quad {}^{40}\mathrm{Ca} : (1s)^4(1p)^{12}(2s)^4(1d)^{20}$$

Binding Energies

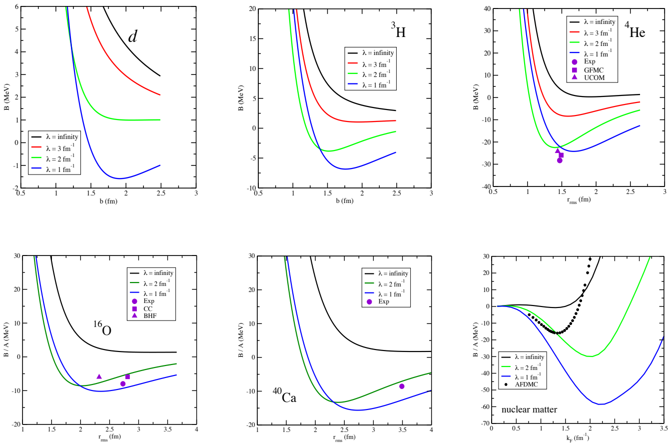


Binding Energies - NijII



Binding Energies

Binding Energies - N3LO



- From a variational point of view: GFM=SRG+Shell model $B_{\rm 4He} = -24{\rm MeV}$;

$$\begin{aligned} \min_\Psi \langle \Psi | H^{\lambda=\infty}_{\rm Av18} | \Psi \rangle &= \min_b \langle (1s)^4 | H^{\lambda \sim {\rm fm}^{-1}}_{\rm Av18} | (1s)^4 \rangle \\ &= \min_b \langle (1s)^4 | H^{\lambda \sim {\rm fm}^{-1}}_{\rm Nijm} | (1s)^4 \rangle \\ &= \min_b \langle (1s)^4 | H^{\lambda \sim {\rm fm}^{-1}}_{\rm N3LO} | (1s)^4 \rangle \end{aligned}$$

- Core + TPE are marginal for ^{4}He. Other nuclei ?
- SRG Wigner scale $\lambda_{\rm Wigner} = 3{\rm fm}^{-1}$ implies large 3-body and 4-body forces
- Are the N-body forces SU(4)-invariant ?

CONCLUSIONS

- Wigner symmetry looks very different when finite range forces are considered.
- In V_{lowk} Wigner symmetry is saturated, i.e. it holds above a cut-off
- In SRG Wigner symmetry happens at one SRG cut-off