Isospin breaking in pion–deuteron scattering and the pion–nucleon scattering lengths

Martin Hoferichter

Helmholtz-Institut für Strahlen- und Kernphysik (Theorie) and Bethe Center for Theoretical Physics, Universität Bonn

with V. Baru, C. Hanhart, B. Kubis, A. Nogga, and D. R. Phillips

Phys. Lett. B 694 (2011) 473, Nucl. Phys. A 872 (2011) 69

Jefferson Lab, Newport News, August 6, 2012

Bonn-Cologne Graduate School of Physics and Astronomy

Outline

1. Hadronic atoms and πN scattering
2. Isospin breaking in threshold $\pi^- d$ scattering
   - Mass effects
   - Virtual photons
3. πN scattering lengths
   - Modified effective range expansion
   - Subtraction of virtual-photon effects

Motivation

- $\pi\pi$ **scattering**: firm prediction from Roy equations + ChPT Colangelo et al. 2000

$$a_0^0 = (0.220 \pm 0.005)\, M_\pi^{-1} \qquad a_0^2 = (-0.0444 \pm 0.0010)\, M_\pi^{-1}$$

$\hookrightarrow$ tested experimentally to high accuracy: K_{e4}, $K \to 3\pi$ NA48/2 2010

- What about πN **scattering**?
  - **Isoscalar** scattering length a^+: chirally suppressed, poor chiral expansion
    $\hookrightarrow$ PWA inconclusive
  - **Isovector** scattering length a^-: needed for a determination of the πN coupling constant via the Goldberger–Miyazawa–Oehme sum rule

$\hookrightarrow$ Data from hadronic atoms prime source of information

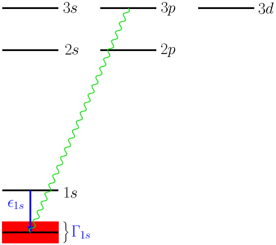
Hadronic atoms

- Pionic hydrogen πH: EM bound state ($e^- \to \pi^-$)

$$E_{QED} = m_{\rm p} + M_\pi - \frac{\mu_H \alpha^2}{2n^2} + \cdots$$

- Strong interaction modifies the spectrum
  - **Energy shift**

$$\epsilon_{1s} = E_{1s} - E_{1s}^{\rm QED} = -2\alpha^3 \mu_H^2 \underbrace{\left(a^+ + a^- + \Delta a_{\pi^- p}\right)}_{a_{\pi^- p}} (1 + \cdots)$$



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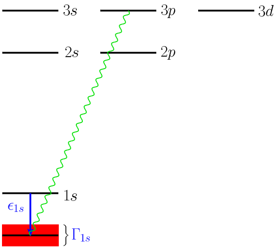


- Similar for πD, but width due to $\pi^- d \to nn, \gamma nn$
- Recent experimental results Gotta et al. 2005, 2010

$$\epsilon_{1s} = (-7.120 \pm 0.012)\,\text{eV} \qquad \Gamma_{1s} = (0.823 \pm 0.019)\,\text{eV}$$

$$\epsilon_{1s}^D = (2.356 \pm 0.031)\,\text{eV}$$

Interpretation of hadronic-atom data

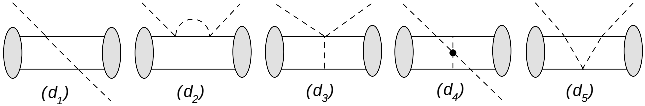
- Basic relations Deser, Goldberg, Baumann, Thirring 1954

$$\epsilon_{1s} = -2\alpha^3\mu_H^2\left(a^+ + a^- + \Delta a_{\pi^- p}\right)(1+\cdots)$$

$$\Gamma_{1s} = 4\alpha^3\mu_H^2 p_1\left(1+\frac{1}{P}\right)\left(-\sqrt{2}a^- + \Delta a_{\pi^- p\to\pi^0 n}\right)^2(1+\cdots) \qquad P = \frac{\sigma(\pi^- p\to\pi^0 n)}{\sigma(\pi^- p\to\gamma n)} = 1.546\pm 0.009$$ Spuller et al. 1977

$$\epsilon_{1s}^D = -2\alpha^3\mu_D^2 \,\mathrm{Re}\, a_{\pi^- d}(1+\cdots), \qquad \mathrm{Re}\, a_{\pi^- d} = \frac{2\mu_D}{\mu_H}\left(a^+ + \Delta a^+\right) + a^{(3)}_{\pi^- d}$$

- Three equations for $a^\pm$
- Further theory input needed
  - Higher orders in NREFT Gasser, Lyubovitskij, Rusetsky 2008
  - Isospin-violating corrections $\Rightarrow$ ChPT MH, Kubis, Meißner 2009
  - Three-body corrections in πD $\Rightarrow$ ChEFT Weinberg 1992,...



$(-24.9\pm 0.9)\cdot 10^{-3}M_\pi^{-1}$ $\quad (-0.2\pm 0.3)\cdot 10^{-3}M_\pi^{-1}$ $\quad (2.6\pm 0.5)\cdot 10^{-3}M_\pi^{-1}$ Baru et al. 2011

Isospin violation in $\pi^- d$ scattering

- Nomenclature
  - **Charge symmetry**: invariance under rotation by π in isospin space
  - **Charge independence** = **Isospin symmetry**: invariance under an arbitrary rotation in isospin space $\hookrightarrow$ assumed in the definition of $a^{\pm}$
- Example: $\Delta_\pi = M_\pi^2 - M_{\pi^0}^2$ contributes to IV, but not to CSB

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- Example: $\Delta_\pi = M_\pi^2 - M_{\pi^0}^2$ contributes to IV, but not to CSB
- Need $a^{(3)}_{\pi^- d}$ with an accuracy of better than 10 % $\Rightarrow$ IV in 3B sector?
- Virtual photons: **multi-scale problem**
  - **ChPT regime**: $p \sim M_\pi$
  - **Atomic-atom regime**: $p \sim \alpha M_\pi \Rightarrow$ NREFT calculation
  - **Deuteron wave function**: $p \sim \sqrt{m_\mathrm{p}\epsilon}$
  - **Three-body dynamics**: $p \sim \sqrt{M_\pi \epsilon}$
- Virtual photons could be enhanced by $\sqrt{M_\pi/\epsilon}$

Stage setting

Numbers

$$\mathrm{Re}\, a_{\pi^- d} \sim -25 \cdot 10^{-3} M_\pi^{-1} \qquad a^- \sim 80 \cdot 10^{-3} M_\pi^{-1} \qquad a^+ \sim 0$$

- ChPT expansion parameter $p \sim \chi = M_\pi / m_\mathrm{p}$
- Estimate $(N^\dagger N)^2 \pi\pi$ contact term
  - Power counting $\mathcal{O}(p^2)$
  - Wave-function dependence of integrals
- **Uncertainty estimate** for strong 3B diagrams: $1 \cdot 10^{-3} M_\pi^{-1} \sim 5\%$

$\hookrightarrow$ can ignore effects (significantly) below that threshold

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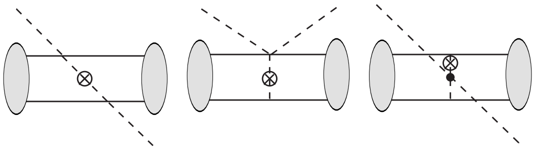
- We count $e \sim \mathcal{O}(p)$ and $m_\mathrm{d} - m_\mathrm{u} = \mathcal{O}(e^2)$
- IV effects from **mass differences** and **virtual photons**

Mass effects

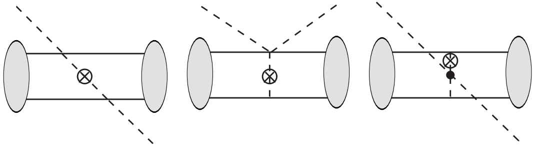


- **Mass-difference insertions** in LO diagrams from Δ_π and $\Delta_N = m_n - m_p$
- Numerically only relevant for double scattering

$$\rho = 2M_\pi \Delta_N - \Delta_\pi$$

$\hookrightarrow$ 2% correction

Mass effects



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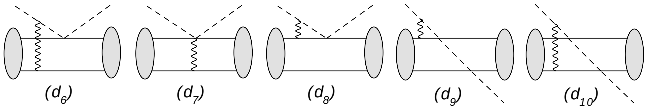
- Mass differences

$$\Delta_\pi = 2Ze^2F_\pi^2 \qquad \Delta_N = -4Bc_5(m_d - m_u) + f_2 e^2 F_\pi^2$$

$\hookrightarrow$ quark-mass effects are suppressed

- Another 1% correction from IV at vertices, starting at NLO

Virtual photons



- (d_6) and (d_8)–(d_{10}) "**would-be infrared singular**"

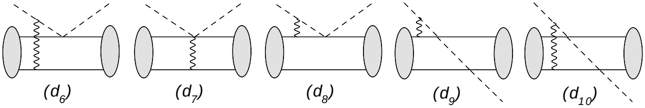
$$\left\langle \frac{1}{\mathbf{q}^2(\mathbf{q}^2+\delta)} \right\rangle \qquad \delta = 2\sqrt{M_\pi^2+\mathbf{q}^2}\left(\varepsilon + \frac{\mathbf{p}^2+(\mathbf{p}-\mathbf{q})^2}{2m_p}\right)$$

$\hookrightarrow$ potentially enhanced by $\sqrt{M_\pi/\varepsilon}$

- ChPT counting applicable in infrared enhanced diagrams?
- Separate those contributions already included in the Deser formula

$\hookrightarrow$ **deuteron pole**

Virtual photons: isovector case



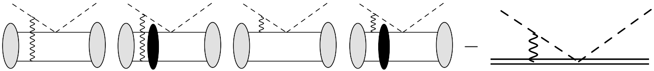
- **Pauli principle**: $(-1)^{L+T+S} = -1 \Rightarrow$ no S-wave NN interaction
- LO result using asymptotic wave functions

$$a_{T=1}^{(d_6)} \sim -a^- \int d^3p\, d^3q \frac{\Psi^\dagger(\mathbf{p}-\mathbf{q})\Psi(\mathbf{p})}{\mathbf{q}^2\big(\mathbf{q}^2+2M_\pi(\varepsilon+\mathbf{p}^2/m_\mathrm{p})\big)} = -a^- \frac{8\pi}{3\sqrt{2}}\frac{1}{\sqrt{M_\pi\varepsilon}} \qquad \Psi(\mathbf{p}) \sim \frac{\sqrt[4]{m_\mathrm{p}\varepsilon}/\pi}{\mathbf{p}^2+m_\mathrm{p}\varepsilon}$$

$$a_{T=1}^{(d_8)} \sim a^- \int d^3p\, d^3q \frac{\Psi^\dagger(\mathbf{p})\Psi(\mathbf{p})}{\mathbf{q}^2\big(\mathbf{q}^2+2M_\pi(\varepsilon+\mathbf{p}^2/m_\mathrm{p})\big)} = a^- \frac{8\pi}{3\sqrt{2}}\frac{1}{\sqrt{M_\pi\varepsilon}}$$

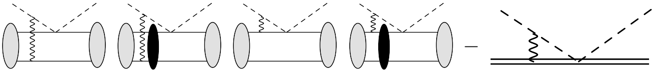
$\hookrightarrow$ enhanced contributions from $p \sim \sqrt{M_\pi\varepsilon}$ cancel

Virtual photons: isoscalar case

$a_{T=0}^{(d_6)+(d_8)} =$ 

- *S*-wave *NN* interaction now allowed
- Need to subtract the deuteron pole ⇒ **Deser formula**

Virtual photons: isoscalar case

$a^{(d_6)+(d_8)}_{T=0} =$ 

$$\sim a^+ \int \frac{d^3k}{(2\pi)^3}\frac{1}{\mathbf{k}^2}\left\{\frac{|F(\mathbf{k})|^2-1}{\mathbf{k}^2/2M_\pi - i\eta}\right.$$

$$\left. + \int \frac{d^3p}{(2\pi)^6}\frac{1}{\varepsilon+\mathbf{p}^2/m_\mathrm{p}+\mathbf{k}^2/2M_\pi-i\eta} G^S_\mathbf{p}(\mathbf{k})\frac{1}{2}\big(G^S_\mathbf{p}(\mathbf{k})+G^S_\mathbf{p}(-\mathbf{k})\big)\right\}$$

$$F(\mathbf{k}) = \int d^3q \Psi^\dagger(\mathbf{q})\Psi(\mathbf{q}-\mathbf{k}/2) \quad \Rightarrow \quad |F(\mathbf{k})|^2-1 = \mathscr{O}(\mathbf{k}^2)$$

$$G^S_\mathbf{p}(\mathbf{k}) = \int d^3q \Psi^\dagger(\mathbf{q})\Psi^S_\mathbf{p}(\mathbf{q}-\mathbf{k}/2) = \mathscr{O}(\mathbf{k})$$

- S-wave NN interaction now allowed
- Need to subtract the deuteron pole $\Rightarrow$ **Deser formula**
- Use **orthogonality of deuteron and continuum wave functions**

$\hookrightarrow$ No infrared enhanced contributions from $p \sim \sqrt{M_\pi \varepsilon}$ altogether!

Virtual photons: summary

- Still infrared enhanced contributions from $p \sim \sqrt{m_p \epsilon}$ possible
- Explicit calculation yields

$$a_{T=0}^{(d_6)+(d_8)} = -0.034 a^+$$

$\hookrightarrow$ indeed dominated by $p \sim \sqrt{m_p \epsilon}$, but too small to be relevant

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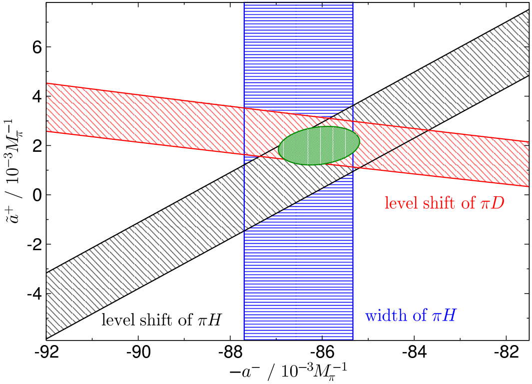
- Remaining effect from **momenta of order M_π**

$$a^\mathrm{EM} = (0.95 \pm 0.01) \cdot 10^{-3} M_\pi^{-1}$$

$\hookrightarrow$ **residual isovector contributions** (and (d_7))

- Consistent with ChPT estimate
- Same conclusions for the higher-order diagrams (d_9) and (d_{10})

Combined analysis of πH and πD



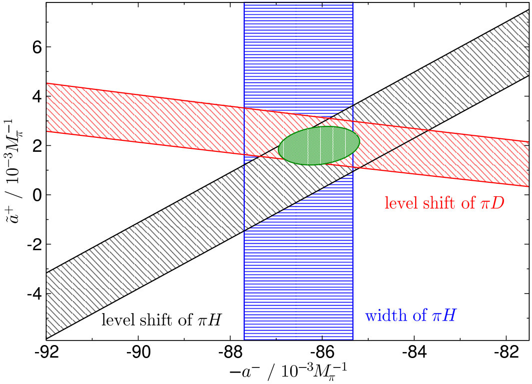
$$\tilde{a}^+ = a^+ + \frac{1}{4\pi(1+M_\pi/m_p)}\left\{\frac{4(M_\pi^2-M_{\pi^0}^2)}{F_\pi^2}c_1 - 2e^2 f_1\right\}$$

πN scattering lengths

$$a^+ = (7.6 \pm 3.1)\cdot 10^{-3}M_\pi^{-1} \qquad \tilde{a}^+ = (1.9 \pm 0.8)\cdot 10^{-3}M_\pi^{-1}$$

$$a^- = (86.1 \pm 0.9)\cdot 10^{-3}M_\pi^{-1}$$

Combined analysis of πH and πD



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isospin limit	channel	scattering length	channel	scattering length
$a^+ + a^-$	$\pi^- p \to \pi^- p$	86.1 ± 1.8	$\pi^+ n \to \pi^+ n$	85.2 ± 1.8
$a^+ - a^-$	$\pi^+ p \to \pi^+ p$	-88.1 ± 1.8	$\pi^- n \to \pi^- n$	-89.0 ± 1.8
$-\sqrt{2}a^-$	$\pi^- p \to \pi^0 n$	-121.4 ± 1.6	$\pi^+ n \to \pi^0 p$	-119.5 ± 1.6
a^+	$\pi^0 p \to \pi^0 p$	2.1 ± 3.1	$\pi^0 n \to \pi^0 n$	5.5 ± 3.1

Removing Coulomb effects: the *pp* scattering length

- Consider first a more familiar example: ***pp* scattering**
  1. Split total phase shift into **pure Coulomb** σ^C + **remainder** δ_{pp}^C

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$$k\left(\cot\delta^C_{pp} - i\right) = -\frac{4\pi}{m}\frac{e^{2i\sigma^C}}{T_{pp}(k)} \qquad\qquad k = |\mathbf{k}|$$

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3. **Modified effective range expansion** Bethe 1949

$$k\left[C_\eta^2\left(\cot\delta_{pp}^C - i\right) + 2\eta H(\eta)\right] = -\frac{1}{a_{pp}^C} + \frac{1}{2}r_0 k^2 + \cdots$$

$$C_\eta^2 = \frac{2\pi\eta}{e^{2\pi\eta}-1} \qquad \eta = \frac{\alpha m}{2k} \qquad H(\eta) = \psi(i\eta) + \frac{1}{2i\eta} - \log(i\eta) \qquad \psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

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4. Removal of **residual Coulomb** effects **scale-dependent**

$$\frac{1}{a_{pp}} = \frac{1}{a^C_{pp}} + \alpha m\left[\log\frac{1}{\alpha M r_0} - 0.33\right]$$ Jackson, Blatt 1950

$$\frac{1}{a_{pp}} = \frac{1}{a^C_{pp}} + \alpha m\left[\log\frac{\mu\sqrt{\pi}}{\alpha M} + 1 - \frac{3}{2}\gamma_E\right]$$ Kong, Ravndal 1999

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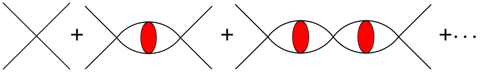

- Difference due to **Coulomb-dressed bubble sum**

$$\frac{1}{a_{pp}} = \frac{1}{a_{pp}^C} + \alpha m \left[\log \frac{1}{\alpha M r_0} - 0.33 \right]$$

Jackson, Blatt 1950

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Kong, Ravndal 1999

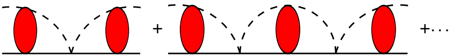
- a_{pp} supposed to correspond to strong part of the potential, but **Coulomb-nuclear interference** depends on short-distance part of the nuclear force
- Numbers for **singlet channel**
  - $a_{pp}^C = (-7.8063 \pm 0.0026)\,\text{fm}$ — Bergervoet et al. 1988
  - $a_{np} = (-23.749 \pm 0.008)\,\text{fm}$ — Koester, Nistler 1975
  - $a_{nn} = (-18.8 \pm 0.5)\,\text{fm}$ — González et al. 2006
  - $a_{pp} = (-17.3 \pm 0.4)\,\text{fm}$ — Miller et al. 1990

$\hookrightarrow$ $a_{pp}^C - a_{pp}$ **huge effect**!

Back to πN scattering

- **Deser formula**: shift and $a_{\pi^- p}$ in NREFT

$$\epsilon_{1s} = -2\alpha^3\mu_H^2 a_{\pi^- p}\left(1 + 2\alpha(1-\log\alpha)\mu_H a_{\pi^- p} + \cdots\right)$$



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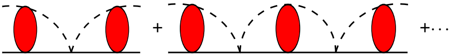

- **ChPT convention** for scattering length Lyubovitskij, Rusetsky 2000

$$e^{-2i\sigma^C} T_{\pi^- p} = \frac{\pi \alpha \mu_H a_{\pi^- p}}{k} - 2\alpha\mu_H \left(a_{\pi^- p}\right)^2 \log \frac{k}{\mu_H} + a_{\pi^- p} + \mathcal{O}(k, \alpha^2)$$

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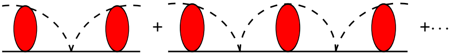

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- **Compare with mERE**: expand first in α, then in k

$$e^{-2i\sigma^C} T_{\pi^- p} = \frac{\pi\alpha\mu_H a^C_{\pi^- p}}{k} - 2\alpha\mu_H \big(a^C_{\pi^- p}\big)^2 \left(\gamma_E + \log\frac{k}{\alpha\mu_H}\right) + a^C_{\pi^- p} + \mathcal{O}(k,\alpha^2)$$

$\hookrightarrow$ same $\log\alpha$ as in Deser formula!

Back to πN scattering

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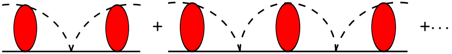

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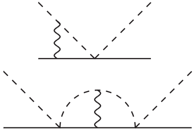
ChPT vs. mERE scattering length

$$\underbrace{a_{\pi^- p}}_{86.1\pm 1.8} = a^C_{\pi^- p} + \underbrace{2\alpha\mu_H \big(a^C_{\pi^- p}\big)^2 (\log\alpha - \gamma_E)}_{-0.5} + \mathcal{O}(\alpha^2)$$

Subtraction of virtual-photon effects

- Application in **dispersion relations** $\hookrightarrow$ analytic properties
- Effects calculable in ChPT, e.g.
  - Coulomb pole $\sim 1/k$ at NLO, $\mathcal{O}(p^3)$
  - $\log k$ first at two loops, $\mathcal{O}(p^5)$



- Application in **dispersion relations** $\hookrightarrow$ analytic properties
- Effects calculable in ChPT, e.g.
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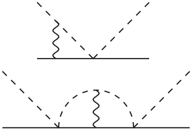

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  - **Finite terms** $\Rightarrow$ fine
  - **UV divergent photon loops** $\Rightarrow$ need to separate mass-difference and virtual-photon contributions to LECs $\Rightarrow$ scale dependence

Subtraction of virtual-photon effects

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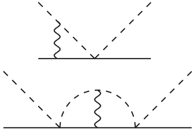

- Subtract virtual-photon contributions
  - **Finite terms** $\Rightarrow$ fine
  - **UV divergent photon loops** $\Rightarrow$ need to separate mass-difference and virtual-photon contributions to LECs $\Rightarrow$ scale dependence
- How large are these effects?
  - Full: $a_{\pi^- p} - a_{\pi^+ p} = (173.4 \pm 1.6) \cdot 10^{-3} M_\pi^{-1}$
  - Virtual photons: $a^\gamma_{\pi^- p} - a^\gamma_{\pi^+ p} = (2.1 \pm 1.8) \cdot 10^{-3} M_\pi^{-1}$
  - Virtual-photon subtracted: $a^{\not\gamma}_{\pi^- p} - a^{\not\gamma}_{\pi^+ p} = (171.3 \pm 2.4) \cdot 10^{-3} M_\pi^{-1}$

$\hookrightarrow$ much smaller than in a_{pp}

Applications

- **Goldberger–Miyazawa–Oehme sum rule**

$$\frac{g_c^2}{4\pi} = \left(\left(\frac{m_\text{p}+m_\text{n}}{M_\pi}\right)^2 - 1\right)\left\{\left(1+\frac{M_\pi}{m_\text{p}}\right)\frac{M_\pi}{4}\left(a^\gamma_{\pi^- p} - a^\gamma_{\pi^+ p}\right) - \frac{M_\pi^2}{2}J^-\right\}$$

$$J^- = \frac{1}{4\pi^2}\int_0^\infty dk\, \frac{\sigma^\text{tot}_{\pi^- p}(k) - \sigma^\text{tot}_{\pi^+ p}(k)}{\sqrt{M_\pi^2 + k^2}}$$

πN coupling constant

$$g_c^2/4\pi = 13.69 \pm 0.12 \pm 0.15 = 13.69 \pm 0.20$$

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$$g_c^2/4\pi = 13.69 \pm 0.12 \pm 0.15 = 13.69 \pm 0.20$$

- Input for a dispersive analysis of πN (based on $\pi^\pm p$) $\hookrightarrow$ see C. Ditsche, We 16:40

πN scattering lengths in isospin basis

$$a_\gamma^{1/2} \doteq \frac{1}{2}\left(3a^\gamma_{\pi^-p} - a^\gamma_{\pi^+p}\right) = (170.5 \pm 2.0)\cdot 10^{-3} M_\pi^{-1}$$

$$a_\gamma^{3/2} \doteq a^\gamma_{\pi^+p} = (-86.5 \pm 1.8)\cdot 10^{-3} M_\pi^{-1}$$

Conclusions

- **Combined analysis** of πH and πD
  - Isospin-breaking corrections
  - Virtual photons: ChPT counting proves adequate in the end
- **Comparison to *pp***: definition of πN scattering lengths
  - Essentially **Coulomb-modified scattering lengths** (up to two-loop ChPT effects)
  - Residual virtual-photon effects much smaller than in *pp*
  - **Removal of virtual photons** generates only **small scale dependence**
  - Application in dispersion relations

Key results

$$a^+ = (7.6 \pm 3.1) \cdot 10^{-3} M_\pi^{-1} \qquad a^- = (86.1 \pm 0.9) \cdot 10^{-3} M_\pi^{-1}$$

$$g_c^2/4\pi = 13.7 \pm 0.2$$

$$a_\gamma^{1/2} = (170.5 \pm 2.0) \cdot 10^{-3} M_\pi^{-1} \qquad a_\gamma^{3/2} = (-86.5 \pm 1.8) \cdot 10^{-3} M_\pi^{-1}$$

Photon diagrams with intermediate-state *NN* interactions

- **Separable *NN* interaction** (integral dominated by low-momentum modes)
- Potential

$$V(\mathbf{p},\mathbf{p}') = \lambda g(\mathbf{p}) g(\mathbf{p}') \qquad g(\mathbf{p}) = \frac{1}{\mathbf{p}^2+\beta^2}$$

- λ tuned to reproduce the binding momentum $\gamma = \sqrt{m_\text{p}\epsilon}$
- $\beta = 1.4\,\text{fm}^{-1}$ parameterizes effective range of *pn* scattering
- Hulthén wave functions

$$\Psi(\mathbf{p}) = N\frac{g(\mathbf{p})}{\mathbf{p}^2+\gamma^2} \qquad N = \frac{1}{\pi}\sqrt{\gamma\beta(\gamma+\beta)^3}$$

Separation of virtual-photon effects

- Finite terms

$$a^{\gamma}_{\pi^- p} + a^{\gamma}_{\pi^+ p} = -\frac{1}{4\pi(1+M_\pi/m_\mathrm{p})}\frac{e^2 g_\mathrm{A}^2 M_\pi}{16\pi F_\pi^2} = -0.5\cdot 10^{-3} M_\pi^{-1}$$

- Terms requiring renormalization

$$a^{\gamma}_{\pi^- p} - a^{\gamma}_{\pi^+ p} = -\frac{M_\pi}{2\pi(1+M_\pi/m_\mathrm{p})}\left\{\frac{e^2 g_\mathrm{A}^2}{16\pi^2 F_\pi^2}\left(1+4\log 2+3\log\frac{M_\pi^2}{\mu^2}\right)\right.$$
$$\left.-2e^2\left(\tilde{g}_6^\mathrm{r}+\tilde{g}_8^\mathrm{r}-\frac{5}{9F_\pi^2}\tilde{k}_1^\mathrm{r}\right)\right\} = (2.1\pm 1.8)\cdot 10^{-3} M_\pi^{-1}$$

- Retain pion mass difference $\sim e^2 Z$, but subtract virtual photons $\sim e^2$
  $\hookrightarrow$ use known β-functions of the LECs

$$\tilde{k}_i^\mathrm{r} = \frac{\sigma_i|_{Z=0}}{\sigma_i} k_i^\mathrm{r} \qquad \tilde{g}_i^\mathrm{r} = \frac{\eta_i|_{Z=0}}{\eta_i} g_i^\mathrm{r}$$